Advanced receivers for submillimeter and far infrared astronomy
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Chapter 4

Fundamentals of SIS and HEB mixers*

4.1 SIS mixers: Introduction

A Superconductor-Insulator-Superconductor (SIS) junction is a “sandwich” of two superconductors separated by a very thin insulator layer. The quantum mechanical nature of SIS tunnel junctions lies in the way charge carriers with quantized energy levels (Cooper pairs and quasiparticles) tunnel through the barrier. Heterodyne mixers based on Cooper pair tunneling are known as Josephson mixers, whereas mixers based on quasi (single) particle tunneling are referred to as SIS mixers. It is the latter, and its application to advanced receivers designs, that we concern ourselves with in this thesis.

The quantum generalization of super-Schottky diodes was laid out by Tucker and Millea in 1978 [1], with a follow up paper in 1979 by Tucker [2] on the theory of quantum limited detection in superconducting tunnel junctions. The key features of this theory are the prediction of quantum limited mixer noise and unlike classical mixers, the possibility of RF-IF conversion gain. For this phenomenon to occur the tunnel junction has to exhibit an extremely nonlinear I/V curve. In the case of a SIS junction, this is achieved by the very sharp onset of quasiparticle tunneling beyond a dc threshold voltage equal to the energy gap of the superconductor (Fig. 4.1).

In the first part of this Chapter, we cover some of the fundamental underpinnings needed in the understanding and design of SIS mixers. Some of the discussion is seen to follow the excellent review by Tucker and Feldman in 1985 [3]. The second part of

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### 4.1.1 Photon-assisted quasiparticle tunneling

The physical phenomena that provides the basis of SIS mixer theory is photon-assisted tunneling by quasiparticles across a thin, typically 10 – 20 Å, Al$_2$O$_3$ or as of late AlN barrier (Chap. 7). This effect was first discovered by Dayem and Martin in 1962 [4], and theoretically explained by Tien and Gordon soon afterwards in 1963 [5]. It is made possible by a significant overlap of the wave functions on either side of the tunnel barrier. And because insulators have typically a ∼ 0.5 eV barrier to electrons, tunneling is the only significant source of current at the small bias voltages (< 10 mV) and temperatures (< 5 K) at which niobium junctions operate.

In a superconductor at the lowest-energy quantum state (T=0 K), electrons are bound into Cooper pairs. This occurs in certain metals due to phonon-mediated attraction between two electrons with equal momenta but opposite spin, and at a temperature below which it is energetically favorable for Cooper pairs to “condense” into the ground, or lowest-energy state [6]. In an SIS junction at 0 K, the superconductors on either side of the barrier are thus seen to be in the ground state. From the BCS theory (Bardeen, Cooper, Schriefer (1957)) [7] we understand that each electron in a Cooper pair has a binding energy $\Delta$. The minimum energy for excitation of a Cooper pair above the ground state is therefore $2\Delta$. This breaking energy is known as the superconducting energy gap and is the energy required to produce two single-particle excitations from a Cooper pair. Following BCS as given in Tinkham [6] the energy of the quasiparticle excitation is

$$E = \sqrt{\varepsilon^2 + \Delta^2}$$

where $\varepsilon$ is the normal state quasiparticle energy measured relative to the Fermi level. The density of states $N_S(E)$ for quasiparticles in a superconductor then becomes

$$N_S(E) = N_N(\varepsilon) \frac{d\varepsilon}{dE}.$$  \hspace{1cm} (4.2)

And because we are only interested in energies $\varepsilon$ a few meV from the Fermi energy, we are allowed to take $N_N(\varepsilon)=N_N(0)$. Thus $N_N(0)$ represents the density of state in the normal metal above the superconducting transition. Using Eq. 4.1 we find

$$N_S(E) = N_N(0)\frac{E}{\sqrt{E^2 - \Delta^2}}, \hspace{1cm} E > \Delta$$

and

$$N_S(E) = 0, \hspace{1cm} E < \Delta.$$  \hspace{1cm} (4.4)

From this discussion it is evident that no quasiparticle states are allowed for $E < \Delta$, e.g. in the gap. The divergence in the density of states near the gap is illustrated graphically in the semi-conductor energy level diagram of Fig. 4.1 for two scenarios; without and with photon assisted tunneling. It is interesting to observe that it is the
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Figure 4.1: Semiconductor picture for a SIS tunnel junction. Left) Density of states vs. energy for quasiparticle tunneling at the same energy level. \( \mu_R \) and \( \mu_L \) are the electron Fermi levels on both sides of the barrier. Applying a voltage potential \( V_0 \) shifts the Fermi level of one side with respect to the other. Right) photon assisted tunneling.

Referring to Fig. 4.1a, the onset of quasiparticle tunneling takes place at a dc voltage \( eV_0 = 2\Delta \). At this potential a single electron is able to tunnel through the barrier leaving an un-paired quasiparticle behind. In Fig. 4.1b we depict the process we are most concerned with, photon-assisted tunneling. In this scenario the photon energy + dc bias voltage should be greater than \( 2\Delta \) (\( h\nu + eV_0 > 2\Delta \)) for electrons to be able to tunnel through the barrier. Referring again to the energy diagram, the superconducting energy gap voltage may now be defined as

\[
V_{\text{gap}} = \frac{2\Delta}{e}.
\]  

(4.5)

In Fig. 4.2a we show an ideal I/V curve at 0 K. The sharp rise in current at the gap is, as was seen, due to the divergent density of states just above and below the gap. From Tinkham [6] the direct current through the tunnel junction may be calculated as

\[
I(V_0) = C \int_{-\infty}^{\infty} \frac{dN_S(E) \, dN_S(E + eV_0)}{dE} \left[ f(E + eV_0) - f(E) \right] dE,
\]  

(4.6)

where \( N_S \) is given by Eqs. 4.3 & 4.4 and \( f(E) \) the Fermi-Dirac distribution

\[
f(E) = \frac{1}{e^{E/(k_B T)} + 1}.
\]  

(4.7)

In Eq. 4.7, \( k_B \) is Boltzmann’s constant and \( T \) the temperature of the superconducting material.

In practice a SIS junction is not ideal, and certainly not operated at 0 K. For this reason we show in Fig. 4.2b a measured unpumped I/V curve of a high current density
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Figure 4.2: (Left) Ideal high-current density AlN-barrier SIS I/V curve at 0 K (Eq. 4.6). The gap voltage is 2.77 mV, R_n 10.6 Ω, and I_c ∼ 200 µA. (Right) The actual measured I/V curve. All parameters are the same, except for a 70 µV gap smear and R_{eq} / R_n ratio (leakage) of 13.7. Superimposed is a photon assisted tunnel current corresponding to 345 GHz submillimeter irradiation. The critical current I_c is given by Eq. 4.9.

AlN-barrier SIS junction. Superimposed on the I/V curve is the critical current (Sec. 4.1.2). The important junction defining parameters are: gap voltage, subgap leakage current, gap smearing, and the critical current. The gap voltage is a function of the superconductor (2∆). The critical current is a constant determined by the superconducting energy gap and normal state resistance of the junction (Sec. 4.1.2), and the gap smearing and leakage current a quality factor related to the manufacturing process and operating temperature. At 0 K (no thermal excitation of quasiparticles), any residual subgap leakage is the result of single electron and/or Multiple-Andreev Reflections (MAR) through pinholes in the barrier (Sec. 7.2.4). The sharpness of the gap is found to relate to the mixer conversion gain via Z_{01} in Eq. 4.39, itself a function of the dc I/V curve and its Kramers-Kronig transform (Sec. 4.1.4).

Superimposed on the unpumped I/V curve in Fig. 4.2b is a 345 GHz quantized photon assisted tunnel current. This tunnel current is made possible when V_0 + nhν/e ≥ V_{gap}, with n an integer corresponding to the number of photons absorbed by a quasiparticle. The absorption probability (Eq. 4.7) decreases rapidly as the photon number increases. For n=1 the photon step at V_0 = V_{gap} - hν/e arises from electrons absorbing a single photon and being able to tunnel across the barrier. The photon step directly above the gap at V_0 > V_{gap} arises from electrons absorbing a photon and making a transition to a higher energy level where the density of states is lower. However, when V_0 > V_{gap} + hν/e emission as well as absorption of a photon with energy hν is possible. This is evident in the current rise at V_0 = V_{gap} + hν/e. Thus a SIS junction biased at V_0 will respond to radiation in the frequency range e(V_{gap}-V_0) < hν < e(V_{gap}+V_0).
4.1.2 The Josephson effect

In addition to quasiparticle induced tunnel current, a barrier potential between two superconductors (SIS) also supports Cooper pair tunneling. This phenomena was first predicted by Josephson in 1962, and is known as the “dc Josephson effect” \[8\]. For his prediction Josephson shared in 1973 the Nobel Prize in Physics with Esaki and Giaever, co-discoverers of the tunneling phenomena in solids. The Cooper pair induced tunnel current was found to depend on the phase difference \( \varphi \) of the superconducting wave functions across the barriers

\[
I = I_c \sin(\varphi). \tag{4.8}
\]

\( I_c \) is the critical current which, referring to Fig. 4.2b, is the maximum possible zero-voltage current that can be passed. Since at zero-voltage the phase can take any value, the current will vary between \(-I_c\) and \(+I_c\).

From microscopic theory it is understood that \( I_c \) is a constant that depends on the barrier and junction area according to

\[
I_c = \frac{\pi \Delta}{2eR_n} \tanh \left( \frac{\Delta}{2k_BT} \right) \approx \frac{\pi V_{gap}}{4 R_n}. \tag{4.9}
\]

For bulk niobium, with a \( 2\Delta \) energy gap voltage of 2.8 mV, the critical current amounts to \( \sim 1.9 \) mV/R\(_n\). Rewriting Eq. 4.9 in terms of a of current density yields

\[
J_c = \frac{\pi V_{gap}}{4 (R_n A)}. \tag{4.10}
\]

If a constant voltage is applied across the barrier, we understand from the Josephson relations that the phase difference varies in time according to

\[
\frac{d\varphi}{dt} = \frac{2eV_0}{h}. \tag{4.11}
\]

Integrating Eq. 4.11 with respect to time yields \( \varphi = \varphi_0 + 2eV_0 t/h \). This implies an oscillating current at a frequency \( \nu = 2eV_0/h \). The latter equation is referred to as the “ac-Josephson effect”, and corresponds to a frequency of 484 GHz/mV. Though useful for an accurate determination of voltage standards, Cooper pair tunneling is undesirable in SIS junctions.

In a LO pumped mixer Josephson oscillations are excited at discrete voltages \( S_n \), known as Shapiro steps \[9\]. \( S_n(\text{mV}) = nh\nu_0/2e = n\nu_0/484 \) GHz, where \( n \) is an integer. Biasing on a Shapiro step results in large excess noise in the mixer with subsequent unstable receiver behavior. Fortunately magnetic fields may be used to suppress Cooper pair tunneling in a SIS quasiparticle mixer. This is based on the fact that \( I_c \) is a function of the magnetic field flux \( \Phi \) threaded through the barrier region. For rectangular junctions the critical current is modulated in a Fraunhofer like diffraction pattern

\[
I_c = I_c(0) \left| \frac{\sin(x)}{x} \right|, \quad x = \frac{\pi \Phi}{\Phi_0}. \tag{4.12}
\]
\( \Phi_0 \) the magnetic flux quantum \( h/2e \). To fully suppress the Josephson effect one flux quantum should be threaded through the barrier region. The magnetic field necessary is approximately \( B = \Phi_0/t(d+2\lambda) \), where \( t \) is the length of the junction size perpendicular to the direction of \( B \)-field, \( d \) the barrier thickness, and \( \lambda \) the magnetic penetration depth. For a typical \( 1 \ \mu m^2 \) junction with \( \lambda=100 \) nm and a \( 1 \) nm barrier thickness the magnetic field required is \( \sim 100 \) Gauss. It may be seen [6] that for circular junctions the modulation of the critical current follows an Airy pattern

\[
I_c = 2I_c(0) \left| \frac{J_1(x)}{x} \right|, \quad x \sim \frac{\Phi}{\Phi_0},
\]

where \( J_1 \) is the Bessel function of the first kind. We refer to Sec. 4.1.1 for an actual measurement of the critical current suppression in a SIS junction as a function of magnetic field.

### 4.1.3 Network representation

Tucker [2] and Feldman [3] laid out the quantum theory of heterodyne mixing in SIS mixers. On a very global level, this theory consists of three components: The interaction of the local oscillator signal on the SIS tunnel junction (LO pumped I/V curve), the mixing or down conversion process of RF signals to an intermediate frequency (IF), and the addition of noise in this conversion process.

In the context of the discussion, we introduce a network representation describing the coupling of a current source (LO) to a load (SIS junction). The voltage diagram is shown in the left panel of Fig. 4.3.

For heterodyne mixing the current, or admittance, representation shown in the right panel of Fig. 4.3 is the more useful one. In this case the LO signal is shunted by an internal embedding admittance \( Y_{emb} \) in parallel with the complex junction admittance \( Y_{RF} \). The embedding admittance represents the mixer mount, and includes

![Figure 4.3: a) Voltage representation of a LO signal source with internal embedding impedance \( Z_{emb} \) driving the complex SIS load \( Z_{RF} \). b) Current representation of the same lossless circuit. The admittance network is a more useful model to describe the interaction of the LO current with the embedding and junction admittances. The parasitic junction capacitance is included in the embedding admittance. The currents are a function of dc bias voltage \( V_0 \) and applied LO voltage \( V_{LO} \).]
the SIS junction parasitic capacitance (Table 4.1). As shall be seen, $Y_{RF}$ is a function of bias and LO voltage, and can be obtained from the dissipative and reactive current components through the junction, Eqs. 4.24 & 4.25, once the LO pump level ($\alpha \equiv eV_{LO}/\hbar \omega$) has been established. $Y_{emb}$ on the other hand can be obtained from electro-magnetic (em) simulation analyses [10, 11]. This has not always been the case. Before the advent of em-simulation tools, obtaining an accurate estimation for the embedding admittance was very difficult. Scaled mixer mount models were frequently employed for example.

From network theory we know that the maximum power transfer occurs when $Y_{emb} = Y_{RF}^*$ and thus $G_{emb} = G_{RF}$. This is a resonance condition and much effort is typically expanded to maximize the frequency range (RF bandwidth) over which this condition can be approximated (Sec. 4.1.8). The actual power dissipated in the junction in this scenario is

$$P_{av} = \frac{|I_{LO}^2|}{8G_{emb}}. \quad (4.14)$$

### 4.1.4 Current in an SIS junction

Due to the quantum nature of the quasiparticle tunnel current, nonlinear equations described by a Taylor series or Fourier analysis are no longer possible. To this extent the averaged quantized nonlinear quasiparticle tunnel current, based on the transfer Hamiltonian formulation of tunneling put forward by Cohen et al. [12], and work by Ambegaokar and Baratoff [13] to describe Cooper pair tunneling, was formulated by Werthamer [14] in 1966 as

$$\langle I(t) \rangle = Im \int_{-\infty}^{\infty} d\omega' d\omega'' W(\omega')W^*(\omega'')e^{-i(\omega'-\omega'')t} j(V_0 + \hbar \omega'/e). \quad (4.15)$$

$V_0$ is the SIS bias voltage, and $W(\omega)$ the phase factor. The time dependent voltage across the junction is then obtained from the Fourier transform of $W(\omega)$

$$\int_{-\infty}^{\infty} d\omega' W(\omega')e^{-i\omega' t} = \exp\left[-\frac{ie}{\hbar} \int dt' [V(t') - V_0]\right]. \quad (4.16)$$

In Eq. 4.15 $j(V)$ is a complex function, known as the current response function, and is given by

$$j(V) = iI_{dc}^{\text{Im}}(V_0) + I_{KK}(V_0). \quad (4.17)$$

For the special case of a time-independent potential $W(\omega) = \delta(\omega)$, the left side of Eq. 4.15 reduces to the dc (no LO irradiation) $I/V$ curve of the tunnel junction,

$$I_{dc}^{\text{Re}}(V_0) = Im[j(V_0)]. \quad (4.18)$$

Note that $Im[j(V_0)]$ is the dissipative part of the response function and can be measured directly (Fig. 4.4). The real part of the response function $Re[j(V_0)]$ characterizes the reactive portion of the tunnel current, and it can be related to the dissipative part through a Kramers-Kronig transform.
\[ I_{KK}(V_0) = \text{Re}[j(V_0)] = P \int_{-\infty}^{\infty} \frac{dV'}{\pi} \frac{P^0_\text{dc}(V') - V'/R_n}{V' - V_0}. \quad (4.19) \]

\( P^0_\text{dc} \) denotes the (measured) unpumped dc I/V curve, \( V' \) the measured I/V curve voltage, \( R_n \) the junction normal state resistance, and \( P \) an indication that the Cauchy principal value is taken. In practice the \( I_{KK} \) is evaluated numerically from the unpumped dc I/V curve. We find therefore that the ac response for quasiparticle tunneling is completely characterized by the measured I/V curve through Eqs. 4.15 – 4.19. The reactance obtained from \( I_{KK}(V) \) is non-classical, and is in addition to the ordinary geometric junction capacitance.

To derive the current expressions of a SIS junction under irradiation of a monochromatic local oscillator signal we make the assumption that the large geometric capacitance effectively shunts all harmonics. Strictly speaking this is not necessarily the case for modern SIS junctions at frequencies \( \hbar \omega \leq eV_{\text{gap}}/2 \), and likely results in an over estimation of the available mixer gain [3]. Ignoring harmonic effects, the time dependent voltage across the tunnel barrier is of the form

\[ V(t) = V_0 + V_{\text{LO}} \cos(\omega t). \quad (4.20) \]

Solving Eq. 4.15 by means of Eq. 4.16 the local oscillator current in the junction can be found

\[ I_{LO}(t) = \text{Im} \sum_{n,m = -\infty}^{\infty} J_n(\alpha)J_{n+m}(\alpha)e^{+im\omega t}i(V_0 + n\hbar\omega/e), \quad (4.21) \]

where \( J_n \) is the \( n \)-th Bessel function of the first kind, and \( \alpha \) the LO pumping parameter \( eV_{\text{LO}}/\hbar \nu \). The average current induced by the local oscillator thus contains components at all harmonic frequencies with amplitudes

\[ 2a_m = \sum_{n = -\infty}^{\infty} J_n(\alpha)[J_{n+m}(\alpha) + J_{n-m}(\alpha)]P^0_\text{dc}(V_0 + n\hbar\omega/e), \]

\[ 2b_m = \sum_{n = -\infty}^{\infty} J_n(\alpha)[J_{n+m}(\alpha) - J_{n-m}(\alpha)]I_{KK}(V_0 + n\hbar\omega/e). \quad (4.22) \]

However since the harmonics from the LO are, to a first order, shunted by the geometric junction capacitance we are justified to limit the summation of \( m \) to +1, 0, -1. The LO pumped dc I/V characteristic is provided by \( m=0 \) as

\[ I_{\text{dc}}(V_0, V_{\text{LO}}) = a_0 = \sum_{n = -\infty}^{\infty} J_n^2(\alpha)P^0_\text{dc}(V_0 + n\hbar\omega/e). \quad (4.23) \]

The dissipative, in phase current \( I_{\text{LO}}' \) through the junction at the LO frequency \( \omega \) is given by \( 2a_1 \).
\[ I'_{\text{LO}}(V_0, V_{\text{LO}}) = 2a_1 = \sum_{n=-\infty}^{\infty} J_n(\alpha) [J_{n-1}(\alpha) + J_{n+1}(\alpha)] I^0_{\text{dc}}(V_0 + n\hbar \omega/e) , \quad (4.24) \]

and the reactive current component through the junction \( I''_{\text{LO}} \) is obtained from \( 2b_1 \) as

\[ I''_{\text{LO}}(V_0, V_{\text{LO}}) = 2b_1 = \sum_{n=-\infty}^{\infty} J_n(\alpha) [J_{n-1}(\alpha) - J_{n+1}(\alpha)] I_{KK}(V_0 + n\hbar \omega/e) , \quad (4.25) \]

In this way the complex LO induced current through the junction may be defined

\[ I_{\omega \text{LO}}(V_0, V_{\text{LO}}) = I'_{\text{LO}} + iI''_{\text{LO}} . \quad (4.26) \]

When we now refer back to Fig. 4.3b we see that the total LO current for a lossless RF circuit equals \( I_{\text{LO}}(V_0, V_{\text{LO}}) = Y_{\text{emb}} V_{\text{LO}} + I_{\omega \text{LO}}(V_0, V_{\text{LO}}) \). As we have seen, the embedding environment is determined by the waveguide transition, tuning circuit, and the junction’s geometric capacitance. The LO induced SIS current and RF embedding impedance can be calculated in a way described by Skalare [15]. This method is known as the “RF voltage matching method” and matches the RF junction voltage \( V_{\text{LO}}^k = \alpha_k \hbar \omega/e \) at each point \( k \) of the pumped IV curve \( (V^k_0) \). With this set of points (all belonging to the first photon step), the parameters \( |I_{\text{LO}}| \) and \( Y_{\text{emb}} \) which minimize the function

\[ \sum_k |I_{\text{LO}}|^2 - |Y_{\text{emb}} V_{\text{LO}}^k + I_{\omega \text{LO}}(V^k_0, V_{\text{LO}}^k)|^2 | \quad (4.27) \]

may be found.

These days modern em-simulation software [10, 11], as well as custom circuit tools such as, for example, Supermix or Pcircuit [16, 17], provide a means of establishing the RF embedding environment of the SIS tunnel junction. It is this method that we employ in the design of the tunerless, large fractional RF bandwidth, SIS mixers in this thesis. As a final note, \( V_{\text{LO}} \) is strictly speaking a function of \( V_0 \), due to the fact that the RF input impedance of the tunnel junction depends on bias voltage. However a constant \( \alpha \) is often a fairly good first order assumption as seen in Fig. 4.6b [18].

In Fig. 4.4 we show the nonlinear behavior and quantum response of a high current density AlN-barrier SIS junction. The bias voltage is normalized to \( V_{\text{gap}} \), and the impedance to the normal state resistance \( R_n \). Fig. 4.4a depicts the unpumped dc I/V curve and its Kramers-Kronig transform. The single SIS junction current represents half of the measured AlN barrier twin-junction current presented in Chap. 7. This junction is baselined for the balanced- and correlation mixers discussed in Chap. 8. The junction has a quality factor \( R_{sg}/R_n=14 \), gap voltage of 2.77 mV, gap smear of ± 70 µV, and normal state resistance of 10.6 Ω. Fig. 4.4b shows the unpumped I/V curve overlaid with two photon-assisted quasiparticle tunneling steps correspondent to an LO pump level \( (\alpha) \) of 0.4 and 0.8. In Fig. 4.4c we show the junction’s quantum conductance, which may be derived from \( I'_{\text{LO}}/V_{\text{LO}} \) (Eq. 4.24). The junction’s
quantum susceptibility is shown in Fig. 4.4d and is derived from $I''_{LO}/V_{LO}$ (Eq. 4.25). The RF susceptibility changes from capacitive to inductive, and back to capacitive as a function of bias voltage and location of the photon step.

Again referring to Fig. 4.3b, a capacitive RF embedding admittance can, under appropriate conditions, tune out the inductive susceptibility of the tunnel junction close to $V_{gap}$. This causes a strong absorption of the incident LO signal, a slight increase in the LO pump level $\alpha(V_0)$ of the junction, and a positive sloped LO pumped I/V curve. For an inductive embedding susceptibility, the opposite occurs. Close to the gap there is a large mismatch, and $\alpha(V_0)$ decreases. The drop in a LO signal on the junction results in a (undesirable) negative sloped LO pumped I/V curve. For further details we refer to G. de Lange Ph.D. dissertation [18].
### 4.1.5 Small-signal 3-port mixer model

The linear network representation of a heterodyne mixer is shown in Fig. 4.5. With the LO pumping parameter $\alpha$ known (previous Section) the small-signal mixing products can now be determined. Under the simplifying assumption that the large geometric capacitance of the SIS junction shunts all harmonics, the local oscillator frequency $\omega$ is seen (Chap. 3) to mix with the IF output frequency $\omega_0$ as

$$\omega_m = m\omega + \omega_0, \quad m = -1, 0, 1.$$  \hspace{1cm} (4.28)

In this notation $\omega_{-1} = \omega_l$ represents the image sideband, and $\omega_1 = \omega_S$ the signal sideband. In general each of the sidebands is terminated by an admittance $Y_m$.

The purpose of the mixer is to convert the incoming signal to an output frequency $\omega_0$ at the IF output port $Y_0$, itself terminated into the (frequency dependent) complex load admittance $Y_L$ of the IF matching network and low noise amplifier. To take into account the interaction between the ports, the small-signal voltage and current components can be linearly related by an admittance matrix $Y_{mm'}$

$$i_m = \sum_{m'} Y_{mm'} v_{m'}.$$  \hspace{1cm} (4.29)

The values of the admittance matrix elements $Y_{mm'}$ are determined by the nonlinearity of the dc I/V curve (Eqs. 4.18 & 4.19) and the LO pumping parameter $\alpha$. Expanding Eq. 4.29 in the individual elements ($m = -1, 0, 1$) and including the load admittance at the signal ($Y_S$), IF output ($Y_L$), and image ($Y_I$) ports gives

$$\begin{bmatrix} i_1 \\ i_0 \\ i_{-1} \end{bmatrix} = \begin{bmatrix} Y_{11} + Y_S & Y_{10} & Y_{1-1} \\ Y_{01} & Y_{00} + Y_L & Y_{0-1} \\ Y_{-11} & Y_{-10} & Y_{-1-1} + Y_I \end{bmatrix} \begin{bmatrix} v_1 \\ v_0 \\ v_{-1} \end{bmatrix},$$  \hspace{1cm} (4.30)

The admittance matrix elements are obtained by expanding the total current through the junction barrier and identifying those terms that are linear in the small-signal

![Figure 4.5: Schematic diagram of the heterodyne mixer with LO pump signal $\omega$. In the matrix formalism the signal sideband frequency $\omega_1 = \omega_S$, the image sideband $\omega_{-1} = \omega_l$, and the IF $\omega_0 = \omega_{IF}$. All ports are terminated with an admittance $Y_m$.](image-url)
voltage limit. The result [3] is of the form \( Y_{m'n'} = G_{m'n'} + iB_{m'n'} \) with

\[
G_{m'n'}(V_0, V_{LO}) = \frac{e}{2\hbar \omega_{m'}} \sum_{n,n'=\infty}^{\infty} J_n(\alpha) J_n'(\alpha) \delta_{m-m',n'-n} \\
\times \left\{ I_{dc}^0(\alpha) \left( V_0 + n'\hbar \omega/e + \hbar \omega_{m'}/e \right) - I_{dc}^0(V_0 + n'\hbar \omega/e) \right\} \\
+ \left\{ I_{dc}^0(\alpha) \left( V_0 + n\hbar \omega/e - \hbar \omega_{m'}/e \right) - I_{dc}^0(V_0 + n\hbar \omega/e - \hbar \omega_{m'}/e) \right\},
\]

(4.31)

and

\[
B_{m'n'}(V_0, V_{LO}) = \frac{e}{2\hbar \omega_{m'}} \sum_{n,n'=\infty}^{\infty} J_n(\alpha) J_n'(\alpha) \delta_{m-m',n'-n} \\
\times \left\{ I_{KK}^0(\alpha) \left( V_0 + n'\hbar \omega/e + \hbar \omega_{m'}/e \right) - I_{KK}^0(V_0 + n'\hbar \omega/e) \right\} \\
- \left\{ I_{KK}^0(\alpha) \left( V_0 + n\hbar \omega/e - \hbar \omega_{m'}/e \right) - I_{KK}^0(V_0 + n\hbar \omega/e - \hbar \omega_{m'}/e) \right\}.
\]

(4.32)

Thus we find that the small-signal admittance matrix elements separate into conductance elements \( G_{m'n'} \) and reactive elements \( B_{m'n'} \), which are fully determined by the dc I/V curve, its Kramers-Kronig transform, and LO pump level \( \alpha \).

To calculate the mixer conversion gain we need to know the output power delivered to an arbitrary IF load \( Y_L \) at the IF output port of the mixer. This includes the parasitic junction and RF matching network capacitance, shunted by the input admittance of the IF matching network and/or first low noise amplifier. This output power may be written in the form \( G_L |v_0|^2/2 \). To obtain \( v_0 \) the admittance matrix has to be inverted into an impedance matrix with elements \( Z_{m'n'} \)

\[
v_m = \sum_{m'} Z_{m'n'} I_{m'}.
\]

(4.33)

Normalizing the resulting \( Z_{m'n'} \) to \( Z_{00} \), the internal IF output port impedance of the mixer, the output voltage at the IF port may be then obtained

\[
v_0 = Z_{00} \sum_m \lambda_{0m} I_m.
\]

(4.34)

\( \lambda_{0m} = Z_{0m}/Z_{00} \) and does not depend on the output load termination \( Y_0 = Y_L \).

For a double sideband mixer, the dissipated output power from the signal and image sidebands equals

\[
P_{out} = \frac{1}{2} G_L |v_0|^2 = \frac{1}{2} G_L \left[ |Z_{01}|^2 |I_S|^2 + |Z_{0-1}|^2 |I_I|^2 \right].
\]

(4.35)

\( |Z_{01}|, |Z_{0-1}| \) describe the coupling from the signal and image sidebands to the mixer IF output. The available signal power in both sidebands is then given by

\[
P_m = \frac{|I_S|^2}{8G_S} + \frac{|I_I|^2}{8G_I},
\]

(4.36)
and thus the DSB mixer conversion gain \( \frac{P_{\text{out}}}{P_{\text{in}}} \) may be derived as
\[
G_{\text{mix}}^{\text{DSB}}(V_0, G_S, G_I, G_L, V_{\text{LO}}) = 4G_L [G_S |Z_{01}|^2 + G_I |Z_{0-1}|^2].
\] (4.37)
In case of an ideal DSB mixer with equal sideband ratio and signal/image termination, \( Z_{01} = Z_{0-1} \) and Eq. 4.37 reduces to
\[
G_{\text{mix}}^{\text{DSB}}(V_0, G_S, G_L, V_{\text{LO}}) = 2G_L G_S |Z_{01}|^2.
\] (4.38)
For an ideal SSB mixer, the image sideband is terminated into 0 K, and Eq. 4.37 simplifies to
\[
G_{\text{mix}}^{\text{SSB}}(V_0, G_S, G_L, V_{\text{LO}}) = G_L = 4G_L G_S |Z_{01}|^2.
\] (4.39)

### 4.1.6 Noise properties

The noise properties of a mixer can also be described using the multiport network of Fig. 4.5. Fundamentally there are two dominant noise sources in an SIS mixer, noise due to thermally excited quasiparticles and shot noise due to the large local oscillator pump current. Thermal noise from the port terminations may be neglected.

Following Tucker and Feldman [3], the noise power coupled into an IF load is
\[
P_{\text{noise}} = G_L \sum_{m,m'} Z_{0m} Z_{0m'} H_{mm'},
\] (4.40)
\( H_{mm'} \) is known as the current noise correlation matrix and describes the generation and mutual correlation between the two dominant noise sources at different frequencies. Tucker [2] derived the correlation matrix and found the mixer noise temperature in the form
\[
k_B T_{\text{mix}}^{\text{LO}} = \frac{1}{4G_S |\lambda_{01}|^2} \sum_{m,m'} \lambda_{0m} \lambda_{0m'}^* H_{mm'},
\] (4.41)
with
\[
H_{mm'}(V_0, V_{\text{LO}}) = \frac{e}{2 \hbar \omega_{m'}} \sum_{n,n'=\pm \infty} J_n(\alpha) J_{n'}(\alpha) \delta_{m-m',n-n'}
\]
\[
\times \left\{ \coth[(eV_0 + n' \hbar \omega + \hbar \omega_{m'})/2k_B T] f_{dc}(V_0 + n' \hbar \omega/e + \hbar \omega_{m'}/e) \right. \\
+ \left. \coth[(eV_0 + n \hbar \omega - \hbar \omega_{m'})/2k_B T] f_{dc}(V_0 + n \hbar \omega/e - \hbar \omega_{m'}/e) \right\}. \quad (4.42)
\]
Thus the contribution to the total mixer noise \( T_{\text{mix}} \) is due to a combination of shot noise generated from the dc bias \( (V_0) \) and shot noise from the LO induced tunnel current. Not included in the Tucker noise analysis are multiple Andreev reflections (MAR) through pinholes in the (non-ideal) barrier [19]. This may be accounted for as described in Sec. 7.2.4. Another noise source not included is quantum noise due to vacuum zero-point fluctuations of the electromagnetic field. This half-a-photon noise,
\( \hbar \omega / 2k_B \), is due to the Heisenberg uncertainty principle and may either be included as part of the mixer noise, or as part of the mixer input signal \([20, 21]\). In practice the half-a-photon noise is attributed to the blackbody input load in accordance with Callen & Welton \([22]\) and is in agreement with the Tucker theory \([2]\). Including the half-a-photon noise as part of the incoming radiation also works out favorably in terms of the mixer and receiver noise temperature. For a SSB mixer the minimum noise temperature, referred to the output of the mixer, is \( \hbar \omega / k_B \) with half-a-photon added by the SSB mixer. For a DSB receiver, it can be shown that the minimum noise temperature referred to the input of the mixer is 0 K \([2]\), and that the minimum noise temperature referred to the output of the mixer equals \( \hbar \omega / 2k_B \) K \([20]\).

### 4.1.7 SIS Mixer properties

#### 4.1.7.1 RF reflection coefficient

An important property in the design of SIS mixers is the RF input reflection coefficient. For each sideband, this quantity may be calculated once the embedding impedance \( Y_{\text{emb}} \) and intrinsic input admittance \( Y_{\text{RF}} \) of the tunnel junction is known. To obtain the junction admittance, the complex large signal LO current through the SIS junction of Eq. 4.26 is divided by \( V_{\text{LO}}(V_0) \), itself obtained from a fit to the LO pumped I/V curve

\[
Y_{\text{RF}}(V_0, V_{\text{LO}}) = \frac{I_{\text{RF}}^0}{V_{\text{LO}}(V_0)}.
\] (4.43)

As we have seen \( V_{\text{LO}}(V_0) \) is, strictly speaking, a (small) function of bias voltage for a capacitively tuned SIS junction. However, to a good first order approximation \( V_{\text{LO}} \) can be assumed constant on the first photon step below the gap. In addition, judging from Eqs. 4.24 & 4.25 the RF junction admittance at the LO, signal, and image sideband of a DSB mixer will be similar. It is also important to note that \( Y_{\text{RF}} \) is independent of the sideband and IF load termination. When the LO voltage is smaller than the photon step, e.g. \( \alpha = eV_{\text{LO}}/\hbar \omega < 1 \), \( Y_{\text{RF}} \) can be approximated by

\[
Y_{\text{RF}}(V_0, V_{\text{LO}}) = \frac{e}{2\hbar \omega} J_0(\alpha)^2 \left[ I_0^0(V_0 + \hbar \omega/e) - I_0^0(V_0 - \hbar \omega/e) \right] + \frac{i e}{2\hbar \omega} J_0(\alpha)^2 \left[ I_{KK}(V_0 + \hbar \omega/e) - 2I_{KK}(V_0) + I_{KK}(V_0 - \hbar \omega/e) \right].
\] (4.44)

It can be seen that Eq. 4.44 reduces to the classical conductance \( dI_0/dV_0 \) with the susceptance term \( \to 0 \) when \( \hbar \omega/e \) is smaller than the dc nonlinearity of the gap. Assuming that the RF embedding impedance is known \([10, 11, 17]\), the RF reflection coefficient may now be found

\[
S_{11\text{RF}}(V_0, V_{\text{LO}}) = \frac{Y_{\text{emb}}^* - Y_{\text{RF}}}{Y_{\text{emb}}^* + Y_{\text{RF}}}.
\] (4.45)

To minimize reflections between the mixer and the telescope (the secondary mirror) \( S_{11\text{RF}} \) should preferably be kept \( > -6 \text{ dB} \).
4.1.7.2 IF output admittance and conversion gain

In 1981 Shen [23] presented a paper describing the quasiparticle conversion gain in SIS mixers. He formulated the intrinsic mixer output admittance $Y_{IF}$ as

$$Y_{IF}(V_0) = \frac{1}{Z_{00}}$$

(4.46)

where $Z'$ denotes the augmented Z matrix ($Y^{-1}_{mm}$) with IF load admittance $Y_L = Y_0 = 0$ ($Z_L = \infty$). For a double sideband mixer, with $Y_1 = Y_{L}^* = Y_S = G_S + iB_S$, the output admittance is real due to symmetry in the Y matrix. In this case $Y_{IF}$ can also be obtained by subtracting the load conductance from the total mixer conductance at the IF output frequency:

$$Y_{IF}(V_0) = G_{IF} = Y_{00} = \frac{1}{Z_{00}} - G_L.$$  

(4.47)

From the above expressions, it is evident that $Y_{IF}$ does not depend on the IF load conductance $G_L$. The quantity $G_{IF}$ can be derived from the subgap slope of the LO pumped I/V curve (Fig. 4.4b). Knowing $Y_{IF}$ and $Y_L$ the IF reflection coefficient may be calculated

$$S_{11_{IF}}(V_0) = \frac{Y_L^* - Y_{IF}}{Y_L^* + Y_{IF}}.$$  

(4.48)

Following Shen, the mixer conversion gain of Eq. 4.39 can be separated into a coupling factor $\eta_{IF}$ and intrinsic mixer gain factor $L_{01}^{-1}$ according to

$$G_{mix}^{SSB}(V_0, G_S, G_L, V_{LO}) = L^{-1} = \eta_{IF}L_{01}^{-1}$$

$$= \frac{4G_{IF}G_L}{|Y_{IF} + Y_L|^2} \left( \frac{G_S}{G_{IF}} |\lambda_{01}|^2 \right).$$  

(4.49)

From the above analysis, it is clear that a matched condition is obtained at the IF output port of the mixer, when $G_L = |G_{IF}|$ and $B_L = -B_{IF}$. Following this thread, when the mixer output conductance is positive we find that $\eta_{IF} = 1$ and that the SSB mixer gain equals the intrinsic mixer gain, e.g. $L^{-1} = L_{01}^{-1}$. When however the mixer output conductance is negative $\eta_{IF} = 0$ and infinite mixer gain is possible. Feldman [24] has shown that under the condition of a negative IF match, the reflection coefficient at the signal, image, and every harmonic sideband will tend to infinity. Needless to say, this is a condition to be avoided.

The derived analysis is valid under the condition that the susceptance of the geometric SIS junction capacitance is sufficiently small to be ignored. This is certainly the case for intermediate frequencies around 1 GHz. However when the mixer IF is increased to 8 GHz, or above, the parasitic susceptance can no longer be ignored, and causes the mixer gain to roll off. In Sec. 4.1.10 we present a case study and offer a solution with a more integrated IF circuit approach.
Figure 4.6: a) Calculated input and output return loss as a function of SIS bias voltage (345 GHz). The from simulation obtained RF embedding admittance is $160.2 + j28.4$ mS. The IF load admittance is $28.1 + j41.9$ mS. b) $V_{LO}(V_0)$ obtained from a fit against the measured LO pumped I/V curve. Note that $\alpha_{Fit}$ may be approximated as a constant (0.65). $\alpha_{J2}$ represents the LO pumping on the second junction (text). c) Heterodyne response. $T_{hot}^* = 267.3$ K, and $T_{cold}^* = 73.2$ K. In deriving the input noise temperatures (Fig. 5.3) we find a best fit to measurement with a front-end optics loss $t_{RF} = 0.9156$, and physical temperature $T_a = 24.6$ K. d) Derived DSB mixer gain. e) Simulated and measured Y-factor. f) DSB receiver and mixer noise temperature. In the mixer noise we have included $1 \hbar \omega / k_B$ K of photon noise.
4.1.7.3 Calculated receiver performance

With the embedding admittance, LO pump level, and IF load admittance established, Tucker’s three port approximation can then be used to calculate the mixer noise temperature and DSB mixer conversion gain $G_{mix}^{DSB}$. Once these quantities are obtained, the IF output power $P_{IF}$ may be obtained via

$$P_{IF}^{DSB}(V_0, V_{LO}) = k_B \left( [T^*_{load} + T_{mix}] \cdot G_{mix}^{DSB} + T_{IF} \right) BG_{IF}.$$  \hfill (4.50)

$T^*_{load}$ represents the effective noise temperature at the input of the mixer and depends on the optical loss in front of the mixer (Sec. 5.1.2). $B$ is the IF bandwidth and $T_{IF}$ the equivalent IF noise temperature referred to the input of the low noise IF amplifier. To experimentally derive the IF noise temperature we use the shot noise from the SIS junction, when biased above the gap, as a calibrated noise source. For details on this procedure, we refer to Chap. 7. Once the hot and cold load IF response is known, the receiver noise temperature may then be calculated via the well known Y-factor method ($Y = P_{IF}^{hot}/P_{IF}^{cold}$), with

$$T_{rec}^{DSB}(V_0, V_{LO}) = \frac{T_{hot} - YT_{cold}}{Y - 1}.$$  \hfill (4.51)

In Fig. 4.6a we depict the calculated input and output return loss of an AlN based SIS mixer design at 345 GHz (Chap. 7). In the calculations we have fit the measured LO pumped I/V curve with the unpumped I/V curve of Fig. 4.4. This provides $\alpha(V_0)$ according to Eq. 4.23, and is indeed reasonably constant on the first photon step below the gap (Fig. 4.6b). Since there are two junctions (Sec. 7.2.2), distanced 54.9 µm via a 10.31 Ω transmission line (63.58° phase at 345 GHz), the LO voltage across the second junction can also be calculated ($\alpha_{J2}$ in Fig. 4.6b). In Fig. 4.6c we depict the simulated heterodyne response, as well as the fitted $\alpha(V_0)$ LO pumped I/V curve. The shape of the hot and cold (heterodyne) response is slightly different than that obtained from measurement (Fig 7.10). This is attributed to the effect of higher order harmonics, which have been ignored in the simplified 3-port Tucker model. Fig. 4.6d shows the calculated DSB mixer gain. Finally, in Figs. 4.6e, f we plot the calculated DSB receiver and mixer noise temperature and Y-factor against measured values at 345 GHz. Despite the simplification of ignoring higher order harmonics, the 3-port Tucker model fits the measured data quite well. To calculate the receiver noise temperature we use the Callen & Welton [22] formulism with half a photon of noise, due to zero-point vacuum fluctuations, included as part of the input blackbody radiation load.

4.1.8 RF matching networks below the energy gap

As is clear from Fig. 4.4d, for bias voltages on the first quasiparticle step, the junction’s quantum susceptance ($B_{RF}$) is approximately zero with the junction susceptance dominated by the parasitic barrier capacitance, $C_j$. The general expression for the RF conductance of a LO-pumped SIS junction with arbitrary upper and lower sideband response and harmonics is well described by Eq. 4.31.
Under the assumption of being in the low LO power limit where \( \alpha \equiv eV_{lo}/\hbar \omega < 1 \), having equal sideband response functions, and that the LO harmonics are shorted by the large parasitic junction capacitance, we are allowed to use the approximate, but simplified, solution of Eq. 4.44. Here \( G_{RF} = \text{Re}[Y_{RF}] \) and is given by the slope of the line joining points on the pumped IV curve one photon step above and below the dc bias point [25]. \( R_{RF} \left( G_{RF}^{-1} \right) \) ranges from \( R_{n}/3 \) in the lower part of the sub-millimeter band to \( R_{n} \) for frequencies \( \geq 500 \text{ GHz} \). The admittance of the junction can thus be described as

\[
Y_{RF} = (Y_{RF} + i\omega C_j)
\]

with the value of \( C_j \), the geometric junction capacitance, determined by the thickness and material properties of the barrier (Table 4.1).

The RF conductance varies slowly with frequency and for our design purposes can be assumed constant.

The coupling efficiency between the antenna and junction has the form

\[
\eta(\omega) = \frac{4G_{RF}G_p}{|Y_{RF} + Y_p|^2}.
\]

\( Y_p \) is the waveguide probe admittance which, by design, has an approximately constant admittance locus (Chap. 7). To achieve optimum coupling it is required that \( Y_p = Y_{RF} \). The simplest way to achieve this condition is to tune out the junction susceptance with an open or closed transmission line in parallel to the junction (commonly referred to as open or closed ended stub).

The admittance of a transmission line with a characteristic admittance \( Y_0 = Z_0^{-1} \) is given by

\[
Y_s = Y_0 \left( \frac{Y_l + iY_0\tan(\beta l)}{Y_0 + iY_l\tan(\beta l)} \right).
\]

\( \beta \) is the propagation constant = \( \frac{2\pi}{\lambda} \), \( Y_l \) the load admittance at the end of the transmission line, and \( \beta l \) the electrical length. For \( Y_l = 0 \) (open circuit) the input susceptance becomes

<table>
<thead>
<tr>
<th>Material</th>
<th>( J_c=7 \text{ kA/cm}^2 )</th>
<th>( J_c=10 \text{ kA/cm}^2 )</th>
<th>( J_c=25 \text{ kA/cm}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AlO(_x)</td>
<td>60</td>
<td>82</td>
<td>–</td>
</tr>
<tr>
<td>AlN †</td>
<td>65</td>
<td>75</td>
<td>90</td>
</tr>
<tr>
<td>MgO</td>
<td>100</td>
<td>140</td>
<td>–</td>
</tr>
</tbody>
</table>

† Recent measurement at TU Delft indicate that the specific capacitance of AlN may be as low as 60 fF/\( \mu \text{m}^2 \) at \( J_c = 25 \text{ kA/cm}^2 \) [26].
The shunt susceptance of the open ended stub cancels the junction susceptance for \( Y_0 = \omega C_j \) and \( l = 3/8 \lambda \), giving 100% coupling efficiency provided that the probe admittance is real and equals the RF conductance of the junction \( G_{RF} \).

Similar results can be obtained with a shorted 1/8 \( \lambda \) transmission line or an ideal inductor. The latter requires a series capacitance to allow the junction to be dc biased and as such is difficult to realize at submillimeter wavelengths. For a 1/8 \( \lambda \) shorted stub we obtain

\[
B_s = -iY_0 \cot(\beta l) \tag{4.56}
\]

The shorted stub provides a larger frequency range over which a match to the junction can be achieved. This can be observed by comparing the derivatives of Eq. 4.55 and Eq. 4.56.

It can be shown that the open-circuited stub has a theoretical bandwidth of \( \approx 0.35 \kappa^{-1} \) while the short-circuited stub has a bandwidth of \( \approx 0.93 \kappa^{-1} \), where \( \kappa \equiv \omega R_{RF}C_j \sim \omega R_n C_j \). From a fabrication point of view both tuning networks suffer from being critically tuned, and in practice it has proven difficult to design a tuning stub that resonates at the design frequency. This is largely due to uncertainties in the manufacturing process and properties of niobium at 4 K.

These problems can to some extent be overcome by the use of a “twin-junction” matching network [27] or “end-loaded” tuning circuit [28, 29]. The “twin-junction” matching network works by putting a short length of transmission line (\( \sim 63^\circ \) at 345 GHz for the AlN-barrier designs presented in Chaps. 7 & 8) between two SIS junctions. In this manner the complex impedance of one junction, after transformation by the transmission line, is equal to the complex conjugate of the other.

![Diagram](image.png)

Figure 4.7: Layout of the “end-loaded” stub RF matching network. The junction is located inside a 5 \( \times \) 5 \( \mu m^2 \) pad. At 345 GHz the width and electrical length of the “end-loaded” stub are 2 \( \mu m \) and 39.8\(^\circ\) respectively.
In the remainder of this Section, we concentrate on the “end-loaded” stub RF matching network of Figs. 4.7 & 4.8, where a small section transmission line is placed in series with the junction. This results in the transformation of the complex junction admittance, $Y_{RF}$, to a small but real impedance.

Consider a transmission line with a characteristic admittance $Y_0 = \omega C_j$ and a load $Y_l = Y_{RF}$. Using Eq. 4.54 we calculate the electrical length of the “end-loaded” stub ($\beta l_s$) for which the imaginary part of $Y_s$ equals zero.

$$
\tan(\beta l_s) = \frac{Z_0^2}{2} \left( \sqrt{G_{RF}^4 + 4Y_0^4 - G_{RF}^2} \right),
$$

(4.57)

where $Z_0$ is the characteristic impedance ($Y_0^{-1}$) of the transmission line. We can simplify Eq. 4.57 for $G_{RF} < Y_0$,

$$
\tan(\beta l_s) \approx 1 - \frac{1}{2\kappa^2}, \quad \kappa = \omega R_{RF} C_j > 1.2.
$$

(4.58)

Combining Eq. 4.58 and Eq. 4.54 we now solve for the real part of the transformed junction admittance.

$$
G'_{RF} \approx Y_0 \left( \frac{\kappa(2\kappa^2 - 1)}{2\kappa^4 - 2\kappa^3\sqrt{\kappa^2 - 1} - 1} \right), \quad l = l_s.
$$

(4.59)

For $\omega R_{RF} C_j \gg 1$ Eq. 4.59 simplifies to

$$
R'_{RF} \approx \frac{R_{RF}}{2\kappa},
$$

(4.60)

where $R'_{RF}$ is the transformed resistance of a junction connected to an “end-loaded” stub of electrical length $\beta l_s$ (Fig. 4.8). Then expressing Eq. 4.59 in terms of the electrical length of the series transmission line we find

$$
R'_{RF} \approx R_{RF} \cos(2\beta l_s).
$$

(4.61)

Thus, as the frequency increases we find from Eq. 4.58 that the stub length $l_s \rightarrow \pi/4$, which reduces $R'_{RF}$ to a very small value (Fig. 4.9, Eq. 4.61). This is a serious drawback of the “end-loaded” stub. For frequencies above 500 GHz it gets more and
more difficult to transform $R'_{RF}$ to match the probe impedance over a reasonable bandwidth.

To transform $R'_{RF}$ to $Z^*_p$, we used a two section equal-ripple Chebyshev transformer [30]. Given the Bode-Fano criterion [31, 32], which dictates that $\omega RC \leq \pi \ln(\rho_m^{-1})$, this type of transformer provides the maximum bandwidth while allowing a tolerable pass band ripple. If we define $\rho_m$ as the maximum allowed voltage reflection

![Figure 4.9: The real part of the transformed junction impedance $R'_{RF}$ and electrical length of the “end-loaded” stub (Eq. 4.57), as a function of frequency. The junction’s RF conductance is obtained from Tucker’s theory. The calculations assume a 0.5 $\mu$m$^2$ junction area, and a 85 fF/$\mu$m$^2$ specific capacitance.](image1)

![Figure 4.10: Characteristic impedances of an equal-ripple two-section Chebyshev quarter-wave transformer needed to transform $R'_{RF}$ (Fig. 4.9) to a 40 $\Omega$ probe impedance. Above 500 GHz, $Z_2 < 2 \Omega$ which makes implementation difficult.](image2)
coefficient of the passband ripple (Fig. 4.11), we can derive the impedances for the two quarter-wave sections that give the widest possible fractional bandwidth

\[ Z_1^2 = Z_p \sqrt{\frac{R_{RF}'}{R_{RF}}} \sqrt{\frac{1 - \rho_m}{1 + \rho_m}} \]  
\[ Z_2^2 = R_{RF}^2 \sqrt{\frac{Z_p}{R_{RF}}} \sqrt{\frac{1 + \rho_m}{1 - \rho_m}}. \]

(4.62) \hspace{1cm} (4.63)

\( Z_1 \) is the high impedance section that connects to the waveguide probe. \( Z_2 \) is the low impedance section and attaches to the “end-loaded” stub (Fig. 4.7). Using Eq. 4.60 we can express \( Z_1 \) and \( Z_2 \) in terms of \( \kappa \) and the RF junction resistance

\[ Z_1^2 \approx \frac{Z_p}{\kappa} \sqrt{\frac{R_{RF} Z_p}{2}} \sqrt{\frac{1 - \rho_m}{1 + \rho_m}}, \quad \kappa \gg 1 \]  
\[ Z_2^2 \approx \frac{R_{RF}}{4\kappa^3} \sqrt{2R_{RF} Z_p} \sqrt{\frac{1 + \rho_m}{1 - \rho_m}}, \quad \kappa \gg 1. \]

(4.64) \hspace{1cm} (4.65)

Eqs. 4.64 & 4.65 show that \( Z_1 \propto \kappa^{-0.5} \) and \( Z_2 \propto \kappa^{-1.5} \). For frequencies in the upper half of the submillimeter band \( Z_2 \) becomes very small. It is in practice difficult to realize a very low impedance superconducting microstrip transmission line because its aspect ratio, (length/width), becomes rather small. Connecting the high impedance “end-loaded” stub to a low impedance transmission line results in a large discontinuity, which adds an additional parasitic series inductance, thereby increasing the effective electrical length of the “end-loaded” stub, shifting the resonance down in frequency. This is an especially serious effect for frequencies above 500 GHz where the center frequency of the RF matching network is critically dependent on the “end-loaded”

![Graph](image-url)

Figure 4.11: Input reflection coefficient and coupling efficiency of an “end-loaded” stub connected to a 50 Ω, 0.49 µm², Nb/AlOₓ/Nb tunnel junction. The designed in-band RF reflection coefficient is -9 dB, which was a trade off between maximum RF bandwidth and optimum impedance match.
4.1. SIS MIXERS: INTRODUCTION

stub’s electrical length (Fig. 4.9). Fig. 4.10 depicts the calculated impedances of a two section quarter-wave transformer in the submillimeter band for a worst case in-band reflection coefficient, \( \rho_m \), of -9 dB.

To avoid the issue of very small \( R'_{RF} \) values above \( \sim 500 \) GHz we have introduced a “butterfly” RF matching network which constitutes a hybrid between a “end-loaded” and radial stub matching network. In Fig. 4.12 we show a photograph of the actual device as used in the 600 – 720 GHz receiver at the Caltech Submillimeter Observatory, Mauna Kea, HI [33]. Instead of transforming the junction impedance to a real impedance, we make the transformed impedance slightly inductive, which when presented with a shunt capacitance (parallel radial stubs) transforms the impedance to \( \approx 3.5 \) Ω. A single section quarter wave transformer is then used to transform to the desired antenna or probe impedance. The advantages of this kind of matching scheme are several: First, the “end-loaded” stub length can be extended beyond \( \pi/4 \), which eases the constraints on the photolithography. Secondly, there is no large discontinuity. The latter greatly simplifies RF circuit models [11].

The microstrip transmission line properties are calculated based on expressions by Hammerstad & Jensen [34]. To take into account the finite complex conductivity of the superconductor (Eqs. 4.99 – 4.100), the transmission line properties have been modified following Kautz & Whitaker [35, 36]. For a more detailed discussion we refer to Zmuidzinas and Bin et al. [16, 37].

4.1.9 Sensitivity to operating temperature

Radiometer stability can be compromised when the mixer or IF amplifier (LNA) are subjected to temperature fluctuations. A slow change in the physical temperature of the mixer, or amplifier, results in a change in receiver gain. As a consequence, temperature fluctuations manifest themselves as low frequency drift noise at the output of the receiver [38, 39].

Because most, if not all, sensitive receivers require some kind of active cooling
system, it is of interest to quantify the maximum allowed temperature drift given a certain Allan variance system stability time (Appendix A). Let’s assume an IF output signal of the form

\[ s(t) = s_0(1 + g_t t). \]  

(4.66)

Here \( g_t \) is defined as the normalized drift in system gain, and \( s_0 \) the nominal total power at \( t=0 \). Defining \( m_t \) as the slope of the IF output drift with respect to time, it can be seen that \( g_t = m_t/s_0 \). If we take two contiguous measurements, one on source and one as reference, in a time period \( 2T \) and define

\[ z(T) = [s(T) - r(T)]/r(T), \]  

(4.67)

we obtain the variance of the relative drift

\[ \sigma^2_{\text{drift}} = \langle (z(T))^2 \rangle - \langle z(T) \rangle^2. \]  

(4.68)

Since the mean of \( z(T) = 0 \), Eq. 4.68 simplifies to

\[ \sigma^2_{\text{drift}} = \langle [(s(T) - r(T))/r(T)]^2 \rangle = (g_t \cdot T)^2. \]  

(4.69)

This corresponds to a \( \beta = +2 \) slope for the drift contribution in an Allan plot (Figs. 7.14 & 10.5). At the same time we have for the radiometric noise (Eq. 10.1, Appendix A)

\[ \sigma^2_{\text{rf}} = \langle [(s(T) - r(T))/r(T)]^2 \rangle = 2 \cdot \Delta \nu \cdot T, \]  

(4.70)

where \( \Delta \nu \) is the bandwidth and \( T \) is the integration time of the data sample. Using the constant \( c \) to represent \( 1/f \) electronic noise, we obtain the general Allan variance form

\[ \sigma^2_A(T) = (g_t \cdot T)^2 + \frac{2}{\Delta \nu \cdot T} + c. \]  

(4.71)

Differentiating with respect to \( T \) provides the Allan stability time minima

\[ T_A = \left( g_t^2 \Delta \nu \right)^{-\frac{1}{2}}. \]  

(4.72)

Eq. 4.72 can now be re-written so that given a desired Allan stability time we obtain an estimate for the maximum allowed rate of change in system gain

\[ g_t = \left( T_A^3 \cdot \Delta \nu \right)^{-\frac{1}{2}}. \]  

(4.73)

For example, if a radiometer requires a broadband total power continuum detection of 4 GHz and has a required Allan stability time of 1 s, we find a maximum allowed drift in system gain of \( 1.4 \cdot 10^{-3} \%/s \) (5.7 \%/hour).

Having calculated the allowed gain drift, we can now get an idea of the maximum temperature fluctuation a niobium based SIS mixer and low noise cryogenic GaAs HEMT amplifier may be subjected to. Re-writing Eq. 4.66 to include a temperature dependence gives
4.1. SIS MIXERS: INTRODUCTION

Figure 4.13: SIS mixer and LNA gain sensitivity as a function of temperature. Note the large difference in sensitivity to temperature between the LNA and LO pumped SIS mixer. The pumped and unpumped SIS current has been included for reference. In the experiment the SIS junction was constant voltage biased at 2.2 mV, the peak of the total power response at 4.2 K. The computed normalized slope \( g_T = (1/s_o \cdot dP/dT) \) is shown in the bottom half of the figure and used in Eq. 4.75. Above 9.2 K the niobium film ceases to be a superconductor and the LO pumped junction current drops sharply.

\[
s(t) = s_o[1 + g_T(T - T_o)].
\]  

(4.74)

\( g_T \) in the above equation is defined as the (to the IF signal output) normalized temperature dependent drift of the system. If we now let \( m_T \) be the measured drift slope \( (dP/dT) \), a property of the SIS mixer and amplifier, then \( g_T = m_T/s_o \).

To obtain \( m_T \) for a typical GaAs LNA and LO pumped SIS mixer [40], we performed the following experiments: First, the LNA was gradually heated from 4.2 K to 10 K in a time span of 1 hour (LNA input load at 4.2 K), while continuously recording the IF total power and amplifier physical temperature. In the second experiment we pumped the SIS mixer with LO power and over the course of an hour varied its temperature from 2.16 K to 9.6 K. The temperatures referred to in the text were measured at the outside of the mixer and amplifier block. During the LO pumped mixer experiment the LNA remained unheated at 2.16 K. In Fig. 4.13a we show the measured IF output power as a function of temperature. The normalized temperature dependent drift \( (g_T) \) is shown in Fig. 4.13b. It is clear that the SIS mixer has a negative temperature dependence, while the low noise amplifier has a positive and constant temperature dependence.

As the physical temperature of the mixer in Fig. 4.13 is changed from 2.16 K to 6 K we see the mixer output noise drop in an exponential manner. The dotted line in Fig. 4.13a is a best fit Fermi function. As the temperature of the mixer block
is increased from 2.16 K to 6 K only a minimal change in the junction’s unpumped current (shot noise) is observed. The change in total power is to a very large extent caused by the temperature dependent conversion gain of the mixer. Above 7.2 K we observe a jump in total power, which is attributed to the by now large leakage current (shot noise) in the junction. At 9.2 K the niobium film ceases to be a superconductor and the LO pumped junction current drops sharply. Note, that the SIS mixer conversion gain sensitivity to temperature peaks around 4.9 K. This is unfortunately close to the 4.2 K bath temperature SIS mixers usually operate at (1 Bar atmospheric pressure). Moving to a high altitude site (600 mBar) improves the mixer conversion gain by $\sim 7\%$ [41] and reduces the mixer’s sensitivity to temperature fluctuations. Reducing the helium bath temperature to 1.5 K reduces the temperature sensitivity to a minimum. It is interesting to note that the mixers in the HIFI instrument on Herschel [42] operate at a physical temperature of 2.2 K.

Combining Eqs. 4.66 & 4.74 we obtain the maximum allowed temperature change

$$\delta T = t \cdot \frac{g_t}{g_T}.$$  

$g_t$ is derived from Eq. 4.73 with $g_T$ obtained from the bottom panel of Fig. 4.13(b).

In the example of the 4 GHz IF bandwidth total power continuum detection (worst case scenario) we estimate a maximum allowed gain drift of $14$ · $10^{-4}$ %/s, given a 1 s Allan time. At 4.2 K this gain drift equates to an allowed temperature drift of 470 $\mu$K/s for a GaAs LNA, and 66 $\mu$K/s for a LO pumped SIS mixer! In high resolution spectroscopy mode with a channel bandwidth of 100 KHz, a 1 s Allan stability time, and 4.2 km altitude helium bath temperature of 3.6 K, we find a maximum allowed temperature fluctuation of 106 mK/s for the low noise amplifier and 48 mK/s for the SIS mixer (13.8 mK/s at 4.2 K bath temperature).

4.1.10 IF bandwidth

As was seen in Chap. 1, there is a strong interest to increase the IF bandwidth of SIS receivers in order to facilitate extragalactic observations and spectral line survey efficiency. However, increasing the IF bandwidth generally also means increasing the IF operating frequency of the mixer, because it is very difficult to achieve a good IF impedance match to a low noise amplifier (LNA) for more than an octave bandwidth. Complicating the matter is that available cryogenic isolators, positioned between the mixer and low noise amplifier, nearly always have less than one octave of bandwidth. An alternative to increasing the IF bandwidth beyond an octave is to integrate a low noise amplifier directly into the mixer block, thereby minimizing the distance between the junction and LNA [43]. This is the approach taken with SuperCam in Chap. 9.

The IF output impedance $Z'_{00}$ (Eq. 4.46) of a LO pumped SIS junction is typically 8–10 times the normal state resistance [44, 45]. Following the discussion in Sec. 4.1.7.2, the intrinsic mixer IF output admittance $Y'_{IF}$ is shunted by the parasitic capacitance of the junction and that of the RF matching network. Table 4.1 provides the specific capacitance for a variety of SIS tunnel barriers at a given current density. The RF matching network required to match the LO pumped SIS junction admittance (Eq. 4.44) to the waveguide embedding impedance (Sec. 4.1.8, Chap. 7) is commonly
implemented in microstrip mode, and fabricated with either SiO or SiO$_2$ as the dielectric between the wire layer and ground-plane. For example, the JPL process uses 450- and 200 nm thick SiO ($\varepsilon_r=5.6$) dielectric layers whereas UVa uses SiO$_x$ with an ($\varepsilon_r=4.3$). The relatively high dielectric constant of SiO$_x$ guarantees that any type of RF matching network implemented in microstrip introduces significant parasitic capacitance, thereby limiting the obtainable IF bandwidth of the mixer [29, 46].

Now consider the $\omega R_n C$ product of a SIS junction. For a niobium SIS junction with AlO$_x$ barrier and current density ($J_c$) of 7–10 kA/cm$^2$, the $\omega R_n C$ product is $\sim 1$ at 100 GHz. In contrast, an AlN barrier junction with $J_c=25$ kA/cm$^2$ has an $\omega R_n C$ product of 1 at $\sim 180$ GHz (Chap. 7).

It can be seen that the combined junction and RF matching network geometric capacitance is nearly the same for all microstrip tuned SIS devices. This is especially so if one considers that the fabrication process for SIS junctions is universal. We are justified therefore to do a case study of the popular and widely used “end-loaded” stub RF matching network, without losing too much generality [28, 47, 48, 49]. As an example we investigate a SIS junction/RF matching network with a combined parasitic capacitance of 260 fF at the IF Port of the mixer. Superconducting mixer calculations using Supermix [17, 50] have been performed in order to better understand the IF output conductance, optimum IF load impedance and IF frequency limitations of the SIS mixer.

In Fig. 4.14 we plot the junction IF output admittance as a function of bias voltage. For reference sake we show the pumped and unpumped IV curves as well.
Figure 4.15: Mixer gain and RF reflection, $S_{11}(RF)$, as a function of IF frequency for different IF terminations load. Presenting a conjugate load results in mixer gain and a large RF reflection ($+3 \text{ dB}$). Presenting a real load such as $2 R_n$ or $5 R_n$ is adequate at the lower IF frequencies, but results in a large loss in mixer performance at IF frequencies above 8 GHz. The frequency independent mixer gain (horizontal lines) occurs when the IF termination includes a negative shunt capacitance. Unfortunately such a design is physically unrealizable. A good alternative is shown in Fig. 4.16.

The IF frequency is 6 GHz, and 3 harmonics were used in the harmonic balance part of the program. The junction IF output impedance consists of a real part ($10 R_n$) shunted by a capacitive component of 307 fF. This includes the quantum susceptance, and is a more accurate value than the estimated 260 fF mentioned earlier. To see how the junction’s IF admittance varies as a function of IF frequency we ran the mixer simulation from 0.5 to 12 GHz. The result is shown in Fig. 4.15. The IF junction admittance is comprised of a parallel RC network, with $R = 10 R_n$ and $C = 307 \text{ fF}$.

Intuitively one may think that it is best to conjugate match the mixer IF output admittance. However, aside from mixer gain, this also provides a $+3 \text{ dB}$ reflection gain [$S_{11}(RF)$] at the RF input port of the mixer (Eq. 4.30, top curve Fig. 4.15)! Reflection gain is liable to causes significant standing waves (VSWR) between the mixer and the telescope, with baselines distortions likely at the output of the back-end spectrometer. A more suitable value for $S_{11}(RF)$ is $\geq -5 \text{ dB}$, which results in a slightly reduced mixer gain but much more stable instrument (Sec. 7.2.2).

To achieve a broad IF bandwidth, an IF matching network with integrated tuning circuit is needed before the IF signal is taken off chip (via an inductive bondwire). A natural design is the Chebyshev impedance transformer, implemented for example in co-planar wave (CPW) with a capacitance integrated on chip. This is the approach we have taken with the high current density AlN barrier SIS junctions design for the Caltech submillimeter Observatory (Chaps. 7 & 8). This concept has also been very successfully applied to the integrated receiver of Koshelets et al. [51].
In Fig. 4.16 we show an actual 380 – 520 GHz twin-SIS junction mixer chip with integrated IF. The theoretical IF response is flat to ~13 GHz when terminated into a 20 Ohm load. Above 13 GHz the IF frequency response rolls off as a second order pole due to the integrated capacitance. This helps minimize saturation in the SIS junctions [52] (Chap. 7). It should be noted that this circuit is critically

![Figure 4.16: IF response of the new high current density AlN-barrier SIS junctions with on-chip IF matching. For a more detailed description refer to Chaps. 7 & 8.](image)

**Figure 4.16:** IF response of the new high current density AlN-barrier SIS junctions with on-chip IF matching. For a more detailed description refer to Chaps. 7 & 8.

Figure 4.17: Mixer conversion gain as a function of LO power (α) for various IF load terminations. IF frequency is 6 GHz, LO frequency 345 GHz, and the $R_n A$ product 18 Ω-cm$^2$.

![Figure 4.17: Mixer conversion gain as a function of LO power (α) for various IF load terminations. IF frequency is 6 GHz, LO frequency 345 GHz, and the $R_n A$ product 18 Ω-cm$^2$.](image)
tuned, and that deviations from perfection will cause the IF passband to have a more characteristic “double-tuned” response. And finally, in Fig. 4.17 we present the calculated mixer conversion gain as a function of LO power at the junction. Optimal LO pumping is found with $\alpha \equiv e V_w / \hbar \omega = 0.75$. For an $R_n = 37 \, \Omega$ this corresponds to a LO power level at the junction of $\sim 30\,\text{nW}$. For comparison, the LO power required to optimally pump a high current density AlN barrier junction with $R_n A = 7.6$, and similar device area, $\sim 75\,\text{nW}$. The twin SIS junctions of Chap. 7 requires approximately $150\,\text{nW}$.

### 4.1.11 SIS mixer stability as a function of magnetic field

Now that we have an understanding of the theoretical relationship between magnetic field and the ac Josephson effect (Sec. 4.1.2), and the SIS tunnel junction as a phase coherent photon detector, it becomes constructive to investigate the quasiparticle mixer heterodyne system stability (Sec. 2.3.5 and Appendix A) as a function of magnetic field.

For this measurement we used the recently installed “Technology development receiver” (Trex), at the Caltech submillimeter Observatory (CSO) in Hawaii. The mixer employs a twin AlN barrier high current density SIS junction as described in Chap. 7. By design the SIS junctions are rectangular in shape (0.7 x 1.0 $\mu$m on a side), e-beam defined, and oriented 45° to the magnetic field in the plane of the junction (Fig. 7.4). The projected junction geometry thus approximates that of a diamond rather than a rectangle or circle. Judging from the measured shape of the critical current vs. magnetic field in Fig. 4.18 this does indeed appear to be the case [53].

![Figure 4.18: Measured critical current ($I_c$) vs. magnetic field coil current for Trex, the technology development receiver of Chap. 7. Stars indicate at which magnetic field setting the system stability data was taken. The mixer is usually operated around 30 mA to avoid mixer conversion gain (text). However the best total power and spectroscopic stability is obtained at the first Josephson null (7.8 mA), which corresponds to one flux quantum ($\hbar/2e = 2.07 \times 10^{-15}$ Wb) of magnetic field through the barrier.](image-url)
find therefore that junction geometry affects the shape, and how broad a minimum in the critical current suppression can be achieved. In Chap. 9 we discuss SIS junctions fabricated by the University of Virginia (Lichtenberger et al.) for SuperCam, a 64 pixel heterodyne focal plane array. The tunnel junctions are circular, and the critical current vs. magnetic field suppression is expected to follow an Airy pattern.

In the experiment we limit ourselves to a fixed 345 GHz LO frequency, a bias voltage of 2.25 mV, and local oscillator pump level of 85 µA. These bias settings are optimal for this particular junction design (Sec. 7.3.2). From the above discussion, we understand that the 3rd Shapiro step is at 2.138 mV, 0.11 mV below the chosen bias voltage. In this regard the experiment should be considered as “nominal”, and not as a worst case scenario.

The system stability was measured at a variety of carefully chosen magnetic field settings. As seen from Fig. 4.19 the first null corresponds to 7.8 mA. However, as was noted in Chap. 7, appreciably more current is needed (30 mA) to slightly suppress the superconducting energy gap and thereby avoid mixer gain and unwanted interference from ac-Josephson oscillations mixing with the third harmonic of the LO signal (Sec. 7.3.2). The latter can possibly effect the calibration accuracy of the instrument. Slightly complicating the situation is that the parallel twin-SIS junctions form a SQUID, or superconducting quantum interference device. This explains the sawtooth like modulation on top of the measured data, its periodicity set by the self inductance of the twin-junction loop. To derive the continuum and spectroscopic mixer stability (Appendix A) two fixed tuned 35 MHz bandpass filters with center frequency of 5 and 7 GHz were used, similar to the setup in Fig. 10.1. In this way two IF channels are formed in the middle of the 4 – 8 GHz receiver IF passband. This
allows the spectroscopic Allan variance to be established (Sec. 10.2.1). The interest of the experiment is to establish the location, and to which precision the Josephson effect (Cooper pair tunneling through the barrier) needs to be suppressed, as this has significant bearing on automated tuning algorithms and baseline quality/distortion.

Throughout the experiment the receiver beam terminated into a 290 K hot black body. The measured Y-factor was 2.6, which corresponds to a receiver noise temperature of 50 K. As for the LO source, we used a fundamental Gunn oscillator (Carlstrom et al.) and a VDI \[54\] ×4 multiplier. The Gunn was left free running after a substantial warm-up period of \(\sim 12\) hours. Any (slow) drift will only manifest itself in the total power Allan variance at long time scales and is therefore of no concern in this experiment. In the spectroscopic Allan variance the drift component is common to both IF channels, and is removed in the analysis.

Judging from Fig. 4.19 the total power Allan variance exhibits large excess noise (Josephson oscillations) for very small magnetic field settings. A clear optimum is visible at the first Josephson null (7.8 mA, Fig. 4.18). Operating the mixer near the 4\(^{th}\) Josephson null (30 mA) actually degrades the stability performance. A factor two deviation in continuum stability from radiometric performance occurs at \(\sim 8\) s. Given the 35 MHz 8-pole filter passband, and assuming a drift slope \(\beta = 2\) (Appendix A), this equates to a total power stability of \(\sim 38\) s in a (typical) 1.5 MHz spectrometer noise fluctuation bandwidth. The spectroscopic Allan variance deviates a factor two from radiometric noise at \(\sim 16\) s. This is again at the first Josephson null and corresponds to approximately 80 s in a 1.5 MHz noise bandwidth. Note that the large excess noise at zero magnetic field is to a large extent removed by the zero’s order baseline subtraction of the spectroscopic Allan variance calculation. Quite unexpected, 1 s double beam switch (DBS) spectroscopic observations are actually possible (spectroscopic Allan time \(\sim 2\) s) with no applied magnetic field! Of course the line intensity calibration will be terrible (Sec. 2.3.6). As with the continuum stability results, operation near the 4\(^{th}\) Josephson null (30 mA) is non-optimal. In the experiment the two IF channels are 2 GHz apart (5 and 7 GHz), and it may be expected that spectroscopic subtraction improves with closer channel spacing (Josephson noise more correlated).

This information is particular relevant for extragalactic observations, where due to the generally wide emission linewidths and weak line intensities, binning of spectrometer channels to 100 MHz is not uncommon. In this case operation on, or near, the 1\(^{st}\) Josephson null is important. For narrow line observations the data suggests that the magnetic field current setting is reasonably relaxed, with \(\pm 40\%\) from the optimal setting an acceptable limit. As a final note, the experiment was performed on an IR labs LHe cryostat. System stability performance with closed cycle refrigerators is expected to be somewhat reduced due to microphonic modulation of the LO - mixer standing wave (Chap. 10) \[55\].
4.2 Electrodynamics and the non-local anomalous limit

4.2.1 Transmission line model

To propagate the RF and LO signals to the described SIS tunnel junction (mixing element), superconducting transmission lines are typically employed. Below the superconducting energy gap ($2\Delta$) superconductors exhibit no loss to direct currents (dc), and very low loss to alternating currents (ac). Above the energy gap the situation changes as RF and LO photons have enough energy to break electron pairs in the superconducting film. This phenomena causes large absorption loss (attenuation) in the superconducting films in front of the mixing element, with significantly degraded mixer sensitivity as a result. In this Section we take a closer look at the anomalous skin effect in superconductors, and the associated complex conductivity and surface impedance (loss) above and below the energy gap. We start our discussion with the general transmission line model as shown in Fig. 4.20.

$R$ represents the resistance due to the finite conductivity of the conductor, and $G$ the shunt conductance due to dielectric loss in the material between conductors. $L$ and $C$ are the transmission line series inductance and shunt capacitance per unit length. From network theory we have what is known as the “telegraph” equations:

$$\frac{dV_s}{dz} = -(R + i\omega L)I_s, \quad \frac{dI_s}{dz} = -(G + i\omega C)V_s. \quad (4.76)$$

Combining the above with Maxwell’s curl equations for a uniform plane wave yields the frequency dependent complex constant of a lossy medium

$$\gamma = \alpha + i\beta = \sqrt{(R + i\omega L)(G + i\omega C)}. \quad (4.77)$$

The propagating electric field in the z-direction thus has the form

$$E_{xs} = E_{x0} e^{-\alpha z} e^{-i\beta z}. \quad (4.78)$$

$\alpha$ is known as the attenuation constant (Nepers/m), and $\beta$ as the phase constant (Rad/m) with $V(z) = V(0)e^{-\gamma z}$, and $I(z) = I(0)e^{-\gamma z}$. Since the power transmitted is equivalent to $Re[V(z)I(z)^*]$ we find that

![Figure 4.20: Lumped element equivalent circuit of a transmission line.](image-url)
\[ P(z) = P(0)e^{-2\alpha z}. \]  

Thus the power attenuation is \(2\alpha\). The characteristic impedance of the lossy transmission line can now be defined as

\[
Z_0 = \frac{V(z)}{I(z)} = \frac{R + i\omega L}{\gamma} = \sqrt{\frac{R + i\omega L}{G + i\omega C}},
\]

which for a lossless transmission line reduces to the familiar \(Z_0 = \sqrt{L/C}\) expression. In the above expression the phase constant \(\beta = 2\pi/\lambda\) and the phase velocity \(v = \omega/\beta\). For a lossless transmission line \(\alpha = 0\) and \(\gamma = i\beta\). It may be readily observed that \(v = 1/\sqrt{LC}\). Dispersion on the transmission line occurs when \(\beta\) is a non-linear function of frequency (the phase velocity is different for different frequencies). Related to this discussion is group delay, defined as \(v_g = d\omega/d\beta\). For narrow band signals (for example HDTV in the submillimeter wavelength regime) \(v_g\) should ideally be zero.

Finally, the quality factor of a lossy transmission line equals

\[ Q_L = \frac{\omega LC}{RC + GL} = \frac{\beta}{2\alpha}. \]

If we can ignore the substrate conductance \(G\) then we see that Eq. 4.81 reduces to the classical formulism \(Q = \omega L/R\). We will use this knowledge in Chap. 5 where we discuss the effect of transmission line loss above the superconducting energy gap.

### 4.2.2 Plane waves in lossy dielectrics

All dielectric materials have some conductivity \(G\). In the context of our discussion on loss, it is constructive to have a brief overview of dielectric loss. Again from Maxwell’s equations the complex propagation constant for a lossy slab of material can be obtained

\[
\gamma = i\omega\sqrt{\mu\varepsilon}\sqrt{1 - i\frac{\sigma}{\omega\varepsilon}},
\]

with the intrinsic complex impedance [56] derived as

\[
\eta = \sqrt{\frac{i\omega\mu}{\sigma + i\omega\varepsilon}}.
\]

\(\mu\) is the permeability of the lossy material (\(\mu = \mu_r\mu_0\)). For non-magnetic materials \(\mu_r = 1\) and \(\mu\) equals the permeability of free space (\(\mu_0 = 4\pi10^{-7}\) H/m). \(\varepsilon\) is the permittivity, obtained from the material’s dielectric constant \((\varepsilon_r)\) times the permittivity of free space \((\varepsilon_0 = 8.85410^{-12}\) F/m). For free space Eq. 4.83 reduces to \(\eta = \sqrt{\mu_0/\varepsilon_0} = 377\) \(\Omega\).

If there is loss in the material \(\varepsilon\) will be complex \((\varepsilon' - i\varepsilon'')\). The real part is the material dielectric constant, whereas the imaginary part accounts for loss due to damping of vibrating dipole moments [57]. For free space, being lossless and real,
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ε′ = ε₀. The loss tangent of the material is defined as the ratio of the conduction current to displacement current density and becomes

\[ \tan \delta = \frac{\sigma}{\omega \varepsilon} = \frac{\omega \varepsilon'' + \sigma}{\omega \varepsilon'}, \]

(4.84)

with \( \varepsilon = \varepsilon_r \varepsilon_0 (1 - i \tan \delta) \). From general circuit design principles the quality factor (Q) of a tuned, or resonating, circuit is defined as

\[ Q = \frac{\text{Stored Energy}}{\text{Loss per Radian}} = \frac{\omega}{\Delta \omega} = \frac{1}{\tan \delta} \]

(4.85)

Thus a low loss dielectric material exhibits a high Q. This is important not only in the context of Kinetic Inductance Detectors (KID’s) [58], but also the RF tuning circuits we concern ourselves with (Sec. 4.1.8). The \( \omega R_n C \) (Q) of AlOₓ barrier SIS junctions in the submillimeter ranges from 2–8, and even less for AlN barrier SIS devices. This is much lower than the SiOₓ dielectric loss (tanδ at 100 GHz \( \approx 10^{-3} \) [59]), and we are justified in ignoring dielectric material loss, e.g. \( G \approx 0 \) in Fig. 4.20.

4.2.3 Plane waves in good conductors

For a perfect conductor the current density at any point within the conductor is related by

\[ J = \sigma E. \]

(4.86)

In case of a good conductor the loss tangent factor \( \sigma/\omega \varepsilon \gg 1 \) and Eq. 4.82 reduces to

\[ \gamma = i \omega \sqrt{\mu \varepsilon} \sqrt{-i \frac{\sigma}{\omega \varepsilon}} = (1 + i) \sqrt{\frac{\mu \omega \sigma}{2}}. \]

(4.87)

The conduction current at any point within the conductor becomes

\[ J_x = \sigma E_x e^{-\delta_c z} \cos(\omega t - \delta_c z), \quad \delta_c = \sqrt{\frac{2}{\mu \omega \sigma}}. \]

(4.88)

\( \delta_c = \sqrt{2/\mu \omega \sigma} \) is known as the classical skin depth, and represents the distance at which the current density decreases by \( e^{-1} \), or 63.2%. For example, room temperature aluminum has a conductivity of \( \sigma_{300K} = 3.82 \cdot 10^7 \) S/m, so that at 500 GHz \( \delta_{300K} = 115 \) nm. At cryogenic temperatures \( \sigma_{4K} \) in thin-films is known to improve by a factor 5–10 so that \( \delta_{4K}^\text{Al} \) at 500 GHz \( \sim 40-55 \) nm. Given that the electron mean free path \( l_e \) of aluminum is \( \approx 16 \) nm, we find that \( \delta_c \gg l_e \). This condition is known as the “local limit”, e.g. the limit where local-electrodynamics such as presented here apply. The power loss (heat) due to the skin depth in a good, but non-perfect, conductor may be obtained from Eq. 4.79.

The intrinsic impedance of the conductor is \( \eta = i \omega \mu / \gamma \) and simplifies in a good conductor to
\[
\eta(\omega) = (1 + i) \sqrt{\frac{\omega \mu}{2\sigma}} = (1 + i) \frac{1}{\sigma \delta_c}.
\] (4.89)

\(\eta\) is known as the complex surface impedance \((Z_s)\) in a bulk metal. The real part of the surface impedance equals \(R_s(\omega) = \text{Re}(\eta) = \rho / \delta_c \Omega/\square\), and can be related to the average dissipated power (conductor loss) and free space impedance via

\[
P = \frac{2 |E_0|^2 R_s}{\eta_0^2}, \quad \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377 \Omega.
\] (4.90)

The surface resistance contributes to the series impedance of the transmission line and the surface reactance to \(L\), as shown in Fig. 4.20. For the microstrip tuning networks we concern ourselves with (Fig. 4.7), the attenuation due to the conductor loss may be approximated by

\[
\alpha \approx \frac{R_s}{Z_0 W} (N p/m),
\] (4.91)

where \(Z_0\) equals the characteristic impedance, and \(W\) the width of the microstrip transmission line [60].

### 4.2.4 Surface impedance of normal metals

The general (non-bulk) equation for the surface impedance takes the form

\[
Z_s(\omega) = R_s + iX_s = \frac{E_x(0, \omega)}{\int_0^d dz J_x(z, \omega)},
\] (4.92)

where \(d\) is the thickness of the thin-film conductor. As was seen, the real part of the surface impedance contributes to the loss in the (microstrip) transmission line, and the imaginary component to the transmission line inductance. In Eq. 4.92 \(E_x(0, \omega)\) is \(\parallel\) at the surface, with \(J_x(z, \omega)\) the current density into the conductor at a depth \(z\). In the local limit, the electric field penetration depth is long in comparison to the electron mean free path, e.g. \(\delta_c \gg l_e\) and the surface impedance for an arbitrary film thickness \(d\) may be solved from Maxwell’s equations as

\[
Z_s(\omega) = \sqrt{\frac{i \omega \mu_0}{\sigma \coth\left(d \sqrt{i \omega \mu_0 \sigma}\right)}}.
\] (4.93)

At cryogenic temperatures and microwave frequencies (or higher) when the metal mean free path \(\geq \delta_c\), a non-local form must be assumed for \(J\) and \(E\). Pippard (1954) [61] proposed a replacement of the vector local current density \(J\) with a volume current density in a region around \(r\), such that [6, 62]

\[
J(\mathbf{r}) = \frac{3 \sigma}{4 \pi l_e} \int \frac{\mathbf{R} \cdot \mathbf{E}(\mathbf{r} + \mathbf{R}) e^{-R/l_e} dV'}{R^4}.
\] (4.94)

\(dV'\) is an infinitesimally small volume at location \(\mathbf{r} + \mathbf{R}\). In the limit of infinite conductor thickness \((d \to \infty)\) Reuter-Sondheimer [63] obtained a simple expression for the surface impedance in the extreme anomalous limit \((l_e \gg \delta_c)\)
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\[ Z_s(\omega) = \frac{1}{3^{1/3}2\pi^{1/3}}(1 + \sqrt{3}i)\mu_0^{2/3}c^{2/3}(\frac{\ell_c}{\sigma})^{1/3}, \tag{4.95} \]

with the anomalous skin penetration depth as

\[ \delta_a = \left( \frac{2\sqrt{3}\ell_c}{\pi} \right)^{1/2} \cdot \delta_e^{2/3}. \tag{4.96} \]

Judging from Eq. 4.95 we see that the surface impedance in the extreme anomalous limit only depends on the material properties \( \ell_c, \sigma \) which are nearly temperature independent. It is also interesting to observe that in all cases \( \delta_a > \delta_c \). For normal metals \( \sigma \) is real if we ignore possible time relaxation effects at very high frequencies.

As an example, for a typical thin-film wire layer (Fig. 4.7) with thickness \( d = 200 \) nm, aluminum metal with resistivity \( \rho_{4K} = 1/\sigma \sim 0.5 \) \( \mu \Omega \)-cm, the mean free path is \( \sim 250 \) nm [64]. At 500 GHz \( \delta_c = 39 \) nm, and we find that \( \ell_c > \delta_c \) and \( \ell_c \sim d \). This places Al at 4 K in the anomalous, rather than extreme anomalous limit. A method to calculate \( Z_s \) in the anomalous limit is given by Kautz [65] and solved numerically by Bin and Zmuidzinas et al. [16]. Bin found good agreement between the normal and anomalous skin depth, with a 40 \% underestimation of \( Z_s \) when calculated in the extreme anomalous limit. In Fig. 4.22 we plot the complex surface impedance of Al in the local limit (Eq. 4.93), and compare it to that of Nb and NbTIN as calculated from the Mattis-Bardeen theory in the extreme-anomalous or local limit.

4.2.5 Surface impedance of superconductors

From the BCS theory we understand that pair formation between electrons is energetically advantageous. An attractive force between two electrons, with equal but opposite momenta and spin, occurs when the exchange of phonons (quantum of thermal energy) is taken into consideration. At temperatures below \( T_c \) two electrons are bound in what is known as Cooper pairs and condense until an equilibrium is reached. At 0 K all Cooper pairs occupy the same minimum state, known as the BCS ground state where each electron has a binding energy \( \Delta(0) = \frac{1}{1.764k_B T_c} \).

The average separation distance between two electrons forming a Cooper pair is called the coherence length \( \xi_0 \). The coherence length is the smallest size wave packet superconducting charge carriers can form. From an uncertainty-principle argument, \( \xi_0 \approx \hbar v_F/k_B T_c \), where \( v_F \) equals the Fermi velocity. In practice \( \xi_0 \) ranges from 0.1 – 1 \( \mu \)m, and is analogous to the mean free electron path length in the non-local electrodynamics of normal metals. Due to the large coherence length, the Cooper pairs are found to be highly overlapping.

A second, so far undiscussed phenomena is the so called “Meissner effect” (1933) [66]; the fact that magnetic flux is completely excluded from the interior of a superconductor. Similar to the exponential decay of an electric field in a normal metal, the magnetic flux density inside a superconductor was proposed by London [67] to decay as \( B(z) = B_0 e^{-z/\lambda} \). In practice the measured magnetic penetration depth \( \lambda \) is larger than \( \lambda_L \) and can be linked via the Pippard coherence length \( \xi^{-1} = \xi_0^{-1} + \ell_c^{-1} \).
In materials with impurities and alloys \( \lambda = \lambda_L (\xi / \xi_0)^{1/2} \), and has a temperature dependence

\[
\lambda(T) \approx \frac{\lambda(0)}{\sqrt{1 - \left(\frac{T}{T_c}\right)^4}}. \tag{4.97}
\]

This is the situation we concern ourselves with, and it is known as the “dirty limit” with \( l_e \ll \xi \). This condition is similar to that of high \( T_c \) superconductors which exhibit very short coherence lengths and operate in the local regime \( (\xi \ll \lambda) \). It is interesting to note that near \( T_c \) all superconductors become local.

Empirically, the resistivity, critical temperature, and magnetic penetration depth obey the BCS relation \([68, 69]\)

\[
\lambda = \left[ \frac{\rho (\mu \Omega \text{-cm})}{T_c (K)} \right]^{1/2} \times 100 \text{ nm}. \tag{4.98}
\]

As a practical example, thin-film niobium is found to have a normal state resistivity of \( 5 \mu \Omega \text{-cm} \), \( \xi_0 = 37 \text{ nm} \), \( l_e = 11 \text{ nm} \), and a \( T_c = 9.2 \text{ K} \). The latter values correspond to a magnetic penetration depth \( \lambda_{nb}(0) = 75 \text{ nm} \), and compares well with the measured bulk \((d = \infty)\) penetration depth of \( 85 \text{ nm.} \) With \( l_e \ll \xi \) niobium is in the “dirty regime”. NbTiN films have a measured resistivity \([70]\) in the range 70–100 \( \mu \Omega \text{-cm} \), depending on the quality of the film deposition, and with a \( T_c \sim 15.2 \text{ K} \) exhibit a penetration depth around 200–250 nm. This is very similar to NbN, another “dirty limit” superconductor. For all of these superconductors the surface impedance needs

![Normalized Complex Conductivity vs Frequency](image)

Figure 4.21: Calculated complex conductivity for niobium \((T_c = 9.2 \text{ K, } 2\Delta/e = 2.8 \text{ mV})\) and NbTiN \((T_c = 15.3 \text{ K, } 2\Delta/e = 4.4 \text{ mV})\). For niobium the normal state resistivity \( \rho_n = 5 \mu \Omega \text{-cm} \), and for good quality NbTiN films \( \rho_n \sim 80 \mu \Omega \text{-cm}. \)
to be calculated with the skin effect in the anomalous limit, as laid out by Mattis and Bardeen in 1958 [71]. Pöpel [72] carried out the numerical double integration to obtain the surface impedance for thin films and found that if the thickness is $\geq 3\lambda$, the bulk limit solution is a good approximation of the exact solution. For thin niobium films the “bulk approximation” appears appropriate, however for NbTiN films as discussed in Chap. 5 this is not quite the case.

Nonetheless we can simplify our task by representing the superconductor with a complex conductivity $\sigma_s = \sigma_1 - i\sigma_2$ as obtained from the Mattis-Bardeen theory in the extreme-anomalous limit, where a local approximation can be made, and substitute $\sigma_s$ in the expression for the surface impedance in the local limit of Eq. 4.93. Once the normal state conductivity $\sigma_n$, measured just above $T_c$, and the energy gap are known, $\sigma_1$ and $\sigma_2$ can then be obtained from:

$$\frac{\sigma_1}{\sigma_n} = \frac{2}{\hbar \omega} \int_{\Delta}^{\infty} dE [f(E) - f(E + \hbar \omega)] \frac{E^2 + \Delta^2 + \hbar \omega E}{\sqrt{E^2 - \Delta^2} \sqrt{(E + \hbar \omega)^2 - \Delta^2}$$

$$+ \frac{1}{\hbar \omega} \int_{\Delta - \Delta \omega \Delta \omega}^{\hbar \omega - \Delta \omega \Delta \omega} dE [1 - 2f(h \omega - E)] \frac{\hbar \omega E - E^2 - \Delta^2}{\sqrt{E^2 - \Delta^2} \sqrt{(h \omega - E)^2 - \Delta^2}}$$.

(4.99)

$$\frac{\sigma_2}{\sigma_n} = \frac{1}{\hbar \omega} \int_{\Delta - \hbar \omega \Delta \omega}^{\Delta} dE [1 - 2f(E + \hbar \omega)] \frac{E^2 + \Delta^2 + \hbar \omega E}{\sqrt{E^2 - \Delta^2} \sqrt{(E + \hbar \omega)^2 - \Delta^2}}$$.

(4.100)

$f(E)$ is the Fermi-Dirac distribution as given in Eq. 4.7. In Fig. 4.21 we show the (calculated) normalized complex conductivity. The first term of Eq. 4.99 describes the scattering of thermally excited normal electrons. The second term describes the creation of photon-excited quasiparticles, and is zero for $\hbar \omega < 2\Delta$. Eq. 4.100 describes the “kinetic” inductance caused by Cooper pairs. The lower limit of the integral becomes $-\Delta$ for $\hbar \omega > 2\Delta$.

Figure 4.22: Calculated real (left) and imaginary part (right) of the surface impedance for niobium, NbTiN, and Aluminum at 4 K in the local limit (Eq. 4.93). See text for details.
In Fig. 4.22 we show the real and imaginary components of the complex surface impedance for Nb and NbTiN. Overlaid is the surface impedance of thin-film aluminum (200 nm) calculated in the local limit. Bin et al. showed good agreement between the local, or "dirty" limit, calculation and the rigorous non-local Mattis-Bardeen numerical integration for thick films. The approximation of the extreme anomalous limit (Eq. 4.95) showed an under estimation of $R_s$ by as much at 40%. Above $\sim 800$ GHz the real part of the Al surface impedance (loss) is less that that of Nb. It should be noted that below the energy gap the surface impedance of a superconductor is purely inductive ($R_s \sim 0$, Fig. 4.22 left panel) and given by $X_s = \omega \mu_0 \lambda$. For Nb this corresponds to a surface inductance $L_s = \mu_0 \lambda \sim 0.11 \mu\text{H}/\square$, and for NbTiN 0.3 $\mu\text{H}/\square$ [69]. In Chap. 5 we have a closer look at the first all-NbTiN film SIS mixer (Nb junctions), designed to operate in the 800 – 920 GHz atmospheric window. It also served as a demonstration receiver for HIFI [42] mixer bands 3–5 on the Herschel far-IR space observatory. As of this writing the all-NbTiN mixer of Chap. 5 remains the most sensitive heterodyne receiver in this frequency range.

4.3 HEB mixers: Introduction

Hot electron bolometer mixers (HEBs) consist of a strip of thin film superconducting material. As the name implies, HEB mixers work on the principle of the bolometric effect, which utilizes the strongly non-linear temperature dependent resistance of the superconductor as a bases for the heterodyne mixing process (Sec. 2.3). In the terahertz regime, well above the energy gap of the thin-film superconducting material, incident photons are able to break Cooper pairs (Sec. 4.1.1) and a uniform heating across the superconducting bridge may be assumed. To dissipate the incident power and have a reasonable response time, a bolometer needs to be coupled to a thermal reservoir. It is the mechanics of the heat capacity, and thermal conductance that governs the HEB response function.

For the HEB to function as a heterodyne mixer, both LO ($V_{LO}$) and RF ($V_s$) signals need to be present. The power dissipated in the bridge with dc resistance $R_0$ is therefore

$$P_B(t) = \frac{(V_{LO} \cos(\omega_{LO} t) + V_s \cos(\omega_s t))^2}{2R_0}. \quad (4.101)$$

The resulting IF beat frequency is than $V_{IF}(t) = \frac{1}{2} V_{LO} V_s \cos(|\omega_{LO} - \omega_s|) t$. To function as a terahertz heterodyne mixer, the time response of the HEB mixer is required to be on the order of tenth of pico-seconds (IF bandwidth should at least be several GHz and preferably more). This has been the subject of much research, with Chap. 6 devoted to this subject in its entirety.

In Fig. 4.23a we show a scanning electron microscope (SEM) close up of the HEB bridge. Typical dimensions are 1 $\mu$m by 0.1 $\mu$m and up to 4 $\mu$m by 0.4 $\mu$m. To couple the RF and LO signals to the HEB, quasi-optical antenna structures such as shown in Fig. 4.24 are typically employed [74, 75]. This is opposed to submillimeter receivers which are predominantly waveguide based (Chaps. 7-9). To facilitate low loss coupling of terahertz radiation to the HEB contact pads, the antenna structures are ordinarily
made of high quality gold. A cross-section of the contact pads is shown in (Fig. 4.23b). The contact pad consists of 60 nm Au with a 5 nm Ti adhesion layer underneath. In the conventional fabrication process of the HEB mixer, the deposition of the contact pads is done without any cleaning of the NbN film. Thus it is to be expected that a contact resistance exists between the NbN film and the contact pad. The presence of such a contact resistance has been reported in literature [76, 77, 78] and explains why the device resistance of HEB mixers is always higher than expected, as based on the bolometer size and NbN sheet resistance. It goes without saying that the performance of the HEB mixer depends strongly on the (thermal) boundary conditions. A contact resistance between the NbN film and the contact forms a barrier for the hot electrons diffusing out of the bolometer. It also changes the effective length of the bolometer, because the contact resistance determines the length over which RF current flows through the NbN film. For a more detailed discussion on this subject we refer to Baselmans & Hajenius et al. [79, 80].

In Fig. 4.24b we show the HEB connected to a twin-slow antenna [74, 81]. A very similar device is currently in operation in the HIFI instrument of the Herschel space
observatory [42]. The use of a twin-slot open structure antenna over, for example, a very broad bandwidth non-resonant spiral antenna offers the advantage that the “radiated” beam has a fixed polarization and a high level of “Gaussiinity” [82]. We refer to Chap. 5 for further details on the twin-slot antenna in the context of SIS mixers.

As was seen earlier, the physical properties of the NbN film are essential to the HEBs operating principle. Well below the critical temperature of the superconductor the electron-phonon coupling is weak, and to a first order the electron gas may be treated as separate from the phonon system. The electron-electron interaction is however strong, and incident radiation will tend to thermalize the electrons before being dissipated to the surrounding phonon bath. As a result the thermally excited “hot” electrons have an equilibrium temperature that is higher than that of the lattice (phonons). Since heated electrons have a small heat capacity $c_e$, the corresponding thermal response time can be short [83], typically $\leq 3$ ps for NbN films [84]. However, cooling of the “hot” electrons is another matter. This involves the transfer of energy to phonons (quantum of thermal energy), which by design either escape predominantly into the substrate (film thickness $d < 6$ nm), as would be the case of “phonon cooled” hot electron bolometers, or diffuse out of the Au contact pads (length of the superconductor strip $< $ electron diffusion length) in the case of “diffusion cooled”
HEBs. The electron-phonon interaction time is determined by the film thickness as 
\[ \tau_{eph} = 500T^{-1.6}\text{ ps K} \] [83, 87].

As an example, for a NbN HEB with a 10 K electron temperature \( \tau_{eph} \sim 12 \text{ ps} \) (Table 6.1). The heat transfer rate of phonons in the film to phonons in the substrate is known as the phonon escape time \( \tau_{esc} \) and may be approximated by \( 10.5d \text{ ps/nm} \) [88, 89]. For a typical NbN film thickness of 6 nm, this corresponds to an escape time of approximately 60 ps (Table 6.1). The described time constants can be used in the Perrin-Vanneste linearized two-temperature model [90] of Chap. 6 to estimate the IF bandwidth of the hot electron bolometer. It is found that the shorter the phonon escape time (thinner films), and the higher the electron temperature \( (T_c) \), the larger the IF bandwidth becomes [91, 92, 93].

To better appreciate the complex physical interactions that govern the HEB mixer, we depict in the left panel of Fig. 4.25 a diagram with the various processes. The dashed curve in the middle of the triangle indicates the condition for optimal mixer gain. In the right panel of Fig. 4.25 we show four I/V curves under various LO pump conditions. Without application of LO power (heating) there is up to a critical current \( (I_c) \), in this case \( \sim 50 \mu A \), no substantial resistance in the NbN film. As the current approaches the critical current of the superconducting film, free vortices pairs created by the increase in temperatures and current enhanced two-dimensional (2D) phase slip events, or flux flow, give rise to a resistance and thus a voltage drop across the type III superconductor. Understanding this mechanism, Barends et al. [94] showed that this consideration leads to a correct description of the dc I(V) curve.
Figure 4.26: Calculated electron temperature profile along the length of the bridge at $P_{LO}=90$ nW at various dc bias voltages. The metal contacts on both sides of the bridge are assumed to be at a phonon temperature of 4.2 K. The calculation is performed using the heat-balance approach described in Sec. 6.2

With no LO power applied, the so developed resistance is negative and results in bias oscillations. This is evidenced from the unpumped I/V curve in Fig. 4.25. The onset of the vortex creation region is, under idealized conditions known as the Berezinskii-Kosterlitz-Thouless transition and has an electron temperature $T_{KT} < T < T_c$. In Fig. 4.25a we depict the condition of no LO power along the horizontal axis with $R(T) \rightarrow R(T, I_{HEB}^{dc})$.

Since the HEB is contacted on both sides by the antenna (Fig. 4.24), we find that under heating (LO or bias) “hot” electrons in close proximity to the contact pads diffuse out to this equilibrium reservoir. Note that heated electrons can be formed by application of LO power, or by application of a bias voltage. At the extremities of the HEB bridge, near the contact pads, the electron temperature will therefore take on the ambient temperature of the antenna. In the center of the bridge, there will be, due to the poor thermal conductivity of the superconducting film (decoupled electrons/phonons), a localized heating of the electrons. This effect results in a distributed temperature profile in the bridge, as shown in Fig. 4.26, and is governed by coupled differential heat balance Eqs. 6.1 and 6.2. In the center of the bridge the electron temperature rises to just above $T_c$ which causes a breakdown in the coherent superconducting state. It is this mechanism that governs the heterodyne mixing process in hot electron bolometers.

The combined heating effect of LO signal and dc bias voltage brings the device to the optimal operating conditions for mixing (the electron temperature in the center of the bridge is approximately $T_c$). The application of LO signal and dc bias thus gives rise to a number of interacting and dissipative mechanisms. The result is that under normal operating conditions several processes occur simultaneously, not the least of which is the spatial variation of electron temperature across the bridge. It is therefore virtually impossible to learn about the underlying HEB physics based purely on measurements of the device under full operating conditions. The physical processes shown in Fig. 4.25a, as well as material and geometric properties such as the electron-phonon interaction time $\tau_{eph}$, phonon escape time $\tau_{esc}$, and electron/phonon specific heat capacity ratio $c_e/c_{ph}$ (IF bandwidth, Chap. 6) all play a role in determining the heterodyne response of the hot electron bolometer mixer [95].
4.3.1 Electro-thermal Feedback and IF Standing Waves

As we have seen in Sec. 4.3, hot electron bolometers are thermal devices that operate near the superconducting transition temperature. It should not come as a surprise therefore that voltage reflections at the IF port influence the operation of the bolometer. This principle is known as electro-thermal feedback, or self heating, and is demonstrated in Fig. 4.27. In the past, primarily as a matter of convenience, the IF embedding impedance \( Z_L \) has usually been treated as a frequency independent resistance (50 Ω). This is however an over-simplification, and one that can have significant implication on the actual mixer operation.

In Chap. 6 we derive an expression for the conversion gain of the HEB mixer based on the standard lumped element model \([96, 97]\)

\[
\eta(\omega) = \frac{2\alpha^2 p_{lo}}{\chi^2 \cdot P_{dc}} \left| \frac{R_o Z_L}{(R_o + Z_L)^2} \left| \frac{C^2}{\Psi(\omega) + \Gamma_{if} C} \right| \right| .
\]  

(4.102)

Included in the bolometric responsivity (Eq. 6.15) is the complex embedding impedance \( Z_L \), and reflection coefficient

\[
\Gamma_{if} = \frac{R_o - Z_L}{R_o + Z_L}.
\]  

(4.103)

\( p_{lo} \) in Eq. 4.102 is the LO power at the device, and can be estimated from the isothermal technique \([81, 98]\). \( \chi \) describes the ratio of LO power to dc power heating efficiency, \( C \) the self heating parameter \([98, 99]\), \( \Psi(\omega) \) the time dependent modulation of the electron temperature, and \( R_o \) the dc resistance at the operating point of the mixer (Re\[Z_o\]). The parameter \( \Gamma_{if} C \) represents the electro-thermal feedback and is zero for \( Z_L = R_o \).

To demonstrate the electro-thermal feedback, we use as an example the HIFI HEB mixer band 6 & 7 IF design (Kooi et al., 2004 \([100]\)). Shown in Fig. 4.28a is the HEB mixer chip connected via a set of triple wirebonds to an IF circuit board. A 206 mm coaxial cable connects the IF board and bias Tee to a SRON/Kuo-Lian 2.4 – 4.8 GHz InP 2-stage HEMT low noise amplifier (LNA) with 26 dB gain and 5 K noise temperature. Due to lack of space, the scientific requirement of having

![Diagram](Figure 4.27: The principle of electro-thermal feedback in hot electron Bolometers (HEBsH). Voltage reflections from the IF signal chain induce a current in the bolometer, which results in a temporal redistribution of the localized heating in the bridge. The result is a small shift in the mixer operating point and subsequent modulation of the IF output signal. In an actual mixer, the IF embedding impedance is both complex and a function of frequency.)
at least 2.4 GHz of instantaneous IF coverage, and the non-existence of a space-qualified isolator, it was decided not to use an isolator in the IF signal chain. To help minimize standing waves between the HEB mixer and low noise amplifier, extensive electromagnetic field simulations [10] were performed. The 3D model of the HEB mixer chip, CPW-microstrip transition, dc-blocking capacitor, and coaxial transition is shown in Fig. 4.28b.

Based on the lumped element expression of the HEB mixer conversion gain of Eq. 4.102, the effect of IF reflections can now be taken into account. This is shown in Fig. 4.29. It should be noted that the conversion gain model remains linked to a lumped element model, and as such deviations can be expected in extreme conditions. In the left panel of Fig. 4.29 we show \( \Psi(\omega) + \Gamma_{IF} C \). At low IF frequencies \( \Psi(\omega) \sim 1 \). \( \Psi(\omega)^{-1} \) is plotted in Fig. 6.6, and can to a first order be approximated as a single pole roll off, hence the monotonic rise in \( \Psi(\omega) + \Gamma_{IF} C \). IF reflections cause a modulation of the electron temperature \( \Psi(\omega) \) via \( \Gamma_{IF} \). In the right panel of 4.29 we depict \( C/(\Psi(\omega) + \Gamma_{IF} C) \). It is evident that, by design, in the 2.4 – 4.8 GHz IF passband the

---

Figure 4.28: Top) (a) Layout of the 2.4 – 4.8 GHz HIFI band 6, 7 HEB mixer and IF signal chain. Due to space and availability no isolator is used. Bottom) (b) Mixer unit IF board layout. Included are the CPW-microstrip transition, dc blocking capacitor, and 50 Ω coaxial transition.
electro-thermal feedback modulation is minimized. The ripple period is $\sim 300$ MHz, and corresponds to the physical distance of the 206 mm coax cable plus IF circuit board ($\sim 50$ mm).

Fig. 4.30 shows the system performance of the HIFI band 6 & 7 HEB mixers. Panel a of Fig. 4.30 shows the system IF gain with and without isolator. The discovery of large excess receiver noise (not shown) in “HIFI-like” system measurements (no isolator and 200 mm coax cable) motivated a re-design of the mixer unit IF circuit board (Fig. 4.28b, [100]) and an investigation of the HEB mixer IF output impedance (Chap. 6). The embedding impedance presented at the HEB IF output port is shown in Fig. 4.30b, both with and without an isolator. It should be noted that the original HIFI HEB IF passband was specified to operate between 4 – 8 GHz. As shown in Chap. 6, a 4 – 8 GHz passband for NbN films is an almost impossible technical requirement, and a scientific compromise was found with a 2.4 – 4.8 GHz IF passband. Unfortunately, due to ferro-magnetic material properties, cryogenic isolators in this frequency range are essentially non-existent. The proper solution to this dilemma is to integrate a low noise amplifier (MMIC) into the mixer block, similar to what has been done by Morales et al. [101]. Given the constraints of HIFI however, this was not an option, and the redesigned IF circuit board of Fig. 4.28b was implemented instead.

The HEB mixer conversion gain is shown in panel c for three situations; with and without isolator, and based purely on the model without IF reflections (no electro-thermal feedback). This plot should be compared to Fig. 6.5 in which we show actually measured data with a much simpler IF board (Fig. 6.2) and wideband MMIC IF amplifier [102]. Finally, the calculated receiver noise temperature is provided in Fig. 4.30d. In the calculation we assume a 80 % optical transmission, 350 K mixer noise temperature, and a 6 K IF (based on measurements) noise temperature. The result is in good agreement with actual HIFI measured receiver system temperatures.
Figure 4.30: a) IF passband of the IF circuit board and SRON InP 2-stage low noise amplifier. The passband specification is 2.4 – 4.8 GHz. 
b) Complex embedding impedance $Z_L$ with and without (ideal) isolator
c) Calculated mixer gain. The model shows the expected roll of in mixer gain without electro-thermal feedback ($R_o = Z_L$).
d) Calculated receiver noise temperature. Note the slope and standing wave modulation across the IF passband. Before the IF re-design of Fig. 4.28 the large excess noise outside the IF passband, has in fact been observed on occasion in the IF passband.

As a final point of interest we show a measured vs. modeled residual standing wave of HIFI mixer band 6 in Fig. 4.31. The spectrum is obtained by the division of two internal “cold” (4.2 K) load measurements at two different 4 s periods, 1640 s apart in time. HIFI housekeeping telemetry indicates a small change in the HEB mixer current, indicative of a drift in LO signal. This in term results in a slight redistribution of the HEB temperature profile of Fig. 4.26, $T_e$ in Table 6.1, and thus a slight change in the mixer conversion gain. By fitting our model to the measured spectra, and taking the ratio, we obtain a residual very similar to the measurement, providing confidence in the physical model.

4.3.2 Investigation of direct detection

It is often of interest to reduce the LO power requirement of HEB mixers operating in the terahertz frequency regime. A simple way to accomplish this is to downsize the
4.3. HEB MIXERS: INTRODUCTION

Figure 4.31: Measured vs. fitted residual IF standing wave, \( \Delta P/\langle P \rangle \), of HIFI mixer band 6 for two 4 s observations, 1640 s apart on an internal calibration load. The normalized change in HEB current, \( \Delta I_{dc}^{HEB}/\langle I_{dc}^{HEB} \rangle \), is 1.62 %. This corresponds to a \( \Delta P/\langle P \rangle \) of \( \sim 2.5 % \), so that the IF power to dc current conversion ratio \( (\Delta P/\langle P \rangle)/(\Delta I_{dc}^{HEB}/\langle I_{dc}^{HEB} \rangle \cdot \langle P \rangle) \) equals 1.53. Note the good agreement between model and measurement. For additional details we refer to Sec. 4.3.1.

Biasing the described small volume HEB mixers at, or near, the optimal operating point causes a small change in mixer bias current when the input signal is switched between a “hot” (290 K) and “cold” (80 K) load. This change in mixer current is due to direct- or continuum detection, and alters the IF output impedance and mixer gain of the HEB mixer in a small but noticeable manner. The direct detection response thus modulates the standing wave between mixer and LNA (Fig. 4.30), which results in a non-perfect subtraction of the hot and cold IF output signals, similar to what is shown in Fig. 4.31 although at a much lower amplitude. Use of an isolator between the HEB and LNA will reduce the direct detection modulation of the IF signal, however changes in the mixer conversion gain remain present.

The direct detection current in an HEB mixer becomes prominent when the RF power from the calibration load, absorbed in the bridge, is non-negligible compared to the absorbed LO– plus dc power, thus upsetting the intricate heat balance in the bridge. In case of the small area hot electron bolometer, the measured [105] LO power requirement at the input of the cryostat was \( \sim 100 \) nW. This compares well
with the 60 nW LO power requirement obtained from the isothermal technique [95]. The difference between both methods is attributed to a small mismatch between the mixer and local oscillator beam waists [106].

To quantify the direct detection effect we can estimate the effective input power of the loads in the Rayleigh-Jeans limit as

$$P_{RF} = k_B \Delta \nu T_{load}. \quad (4.104)$$

$T_{load}$ is the effective “hot” and “cold” load temperature as described in Fig. 5.3, and $\Delta \nu$ the instantaneous bandwidth, estimated to be 900 GHz in case of the twin-slot antenna of Fig. 4.24. In the experiment the “hot” and “cold” loads are external to the cryostat. Provided an optical transmission of $\sim 90 \%$, $T_{load}$ is estimated, using the Callen & Welton formulism [20, 22], as 296 K and 105 K respectively. For the “hot” load this equates to a radiative load of 3.7 nW, while for the “cold” load 1.3 nW of continuum loading is present on the bridge area. It is the difference in power, 2.4 nW, that changes the thermal equilibrium of the hot electron bolometer, and is responsible for the direct detection effect in the HEB mixer. Given an optimum dc bias of 0.5 mV and 8–10 $\mu$A of current, approximately 5 nW of dc power loading is present as well. The total ($P_{dc} + P_{LO}$) dissipation of our small volume device therefore equals $\sim 65$ nW. Thus we find that switching between the room temperature and liquid nitrogen calibration load changes the power budget in the bolometer bridge by 3.6 %. This is large enough to noticeably alter the bias point of the mixer.
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Figure 4.33: Measured receiver noise temperature on HIFI FM HEB mixer B6L, S/N 10 with and without bandpass filter. The application of a bandpass filter reduced the receiver noise temperature, as referred to the input of the mixer, by $\sim 17\%$. Note also the roll-off in sensitivity across the IF band. This is related to the IF response time of the HEB (Chap. 6). Operating frequency was 1616 GHz.

In Fig. 4.32 we show the normalized direct detection current, defined as $I_{DD}/I_{dc HEB}$ with $I_{DD} = I_{hot} - I_{cold}$, with and without a narrow passband RF metal mesh filter [107], as measured on the HIFI HEB mixer B6, S/N 10 flight model. For this device the bridge area was 1 x 0.15 $\mu$m. The bandpass filter has a 90% transmission in a 200 GHz band around 1.6 THz at 4.2 K. Thus the difference in radiated signal power in the presence of the cooled metal mesh bandpass filter is $\sim 1/5^{th}$ of the signal power without the filter, or about 0.8% of the total 65 nW power budget. The measured reduction in direct detection current in the case of metal mesh filter is $\sim 2.5$. Though diminished, even with the 200 GHz passband filter a small amount of direct detection remains. It is important to observe that $I_{DD}$ is always negative, in agreement with results reported [108, 109, 110, 111]. This indicates that the difference in RF power between the 105 K and 295 K load changes the bias current of the mixer in the same manner as a decrease in $P_{LO}$. As a consequence the hot load output power is evaluated at a slightly lower bias current than the cold load output power. And because the mixer output power is a function of bias current, the Y-factor is influenced by a shift in bias current and hence an error is introduced into the calibration procedure [110]. It is interesting to note that the best sensitivity found for small 1.0 $\mu$m $\times$ 0.1 $\mu$m area devices, once corrected for direct detection, is identical to the best sensitivity (700 K) reported in a spiral-antenna coupled, large area (4.0 $\mu$m $\times$ 0.4 $\mu$m) HEB mixer at the same frequency [79, 95].

In Fig. 4.33 we show the averaged receiver noise with and without a 1.6 THz bandpass filter. Spectrometer integration time was set to 5 s, a value known the be on the edge of the intrinsic total power stability of HEB mixer’s (Fig. 2.6).
bandpass filter has a cold loss of approximately 10%. For the “RF bandpass filter measurement” the LO power had to be increased slightly so as to keep the HEB mixer current at the same level for both measurements. With the narrow passband metal mesh RF filter in place the receiver noise temperature decreased $\sim 17\%$. This is in agreement with the detailed experiment of [110].

### 4.3.2.1 HIFI

In the actual HIFI instrument, the calibration loads are cryogenic (80 K and 4 K). We can thus expect a factor three reduction in the signal input “hot” load radiation temperature, and because a Martin-Pupplet style diplexer is employed to inject the LO signal another factor of $\sim 2$ by the reduction in RF passband. The actual HIFI bridge area is $2.0 \times 0.1 \mu m^2$ (length $\times$ width) with a critical current of 200 $\mu A$ [112]. If approximate linear behavior is assumed, we estimate an HIFI HEB mixer direct detection current of 0.06–0.08 % of the operating current.

To quantify the HEB mixer direct detection in HIFI under calibration conditions (internal “hot” and “cold” load), we have determined the amplitude variation and statistics of the HEB current and IF output power from 35 long duration (> 40 min), 4 s phase differential (chopped) internal load stability measurements [113]. For this we use the HEB current as recorded in the instrument level test (ILT) housekeeping. These data were acquired at $\sim 1$ Hz. In Fig. 4.34 we show as an example a FFT spectrum of the HEB mixer band 6 current during 0.25 Hz chopped load operation. Timing errors in the housekeeping data likely cause the apparent 50 ms shift in time.

Figure 4.34: Direct detection as measured in B6b of the HIFI instrument at 1673.5 GHz. The direct detection current is obtained from Eq. 4.105. In this particular example, $E_{fit}=0.702$ $\mu A/Hz$, and $N=3600$. The from the FFT calculated normalized direct detection current equals 0.1 % with $\langle I_{EB} \rangle = 50$ $\mu A$. The calibration loads are switched at a rate of 1/4 Hz.
A Hanning window is applied in the Fourier transform to take into account the finite data set. To estimate the direct detection current from the energy spectral density (ESD), we first find the power under the FFT peak by fitting a Gaussian to the spectrum. From this we obtain the detection current $I_{DD}$ via

$$I_{DD} = \frac{E_{fit}}{2\sqrt{2NF_{han}}} \, \mu A.$$

(4.105)

$E_{fit}$ is the power under the Gaussian, $N$ the number of data samples (3600 in the case of Fig. 4.34), and $F_{han}$ 1.4657. The factor 2 in Eq. 4.105 accounts for the frequency folding in the FFT, and the $\sqrt{2}$ converts the rms to a peak current value. For all 35 data samples, we derive in this manner a normalized direct detection current $I_{DD}/I_{HEB} = 0.103 \pm 0.043 \%$. This is slightly higher than the crudely estimated value of 0.06–0.08 \%, not a surprise. The change in standing wave amplitude (not shown) due to the direct detection current has a mean value of 0.2 \%. This value goes directly into the calibration error budget. The thus obtained direct detection IF power-to-current conversion ratio $(\Delta P \cdot \langle I_{HEB}^d \rangle) / (\Delta I_{HEB}^d \cdot \langle P \rangle)$ for HIFI equals 2.10, slightly higher than the obtained ratio of 1.53 in Fig. 4.31.

### 4.3.3 Effect on instrument calibration

The astronomical consequences of the direct detection lie in the absolute, or total power, calibration accuracy of the measurement. To properly correct for the direct detection effect the local oscillator power needs to be (slightly) increased when switching from a “hot” to “cold” calibration load. This is needed to ensure that the HEB bias current remains constant. However such a procedure requires a very accurate control of the LO output power, for example via a precise rotation of a polarizing grid in the LO path of the mixer, as in [110]. For the HIFI instrument under discussion such a complication would be prohibitive.

As we have seen, another technique to reduce the direct detection is to minimize the total power difference between the hot and cold load, e.g. reduce the temperature difference between the calibration loads. This is done for HIFI, and results in a 0.2 \% direct detection calibration error. Alternatively, the calibration load brightness temperature may be reduced by including in the optics path a narrow RF bandpass metal mesh filter [107] or by utilizing a waveguide, rather than open-structure antenna design.

### 4.3.4 Parametric stability

As part of the HIFI instrument level test program (ILT), parametric studies of the HEB mixer band 6 & 7 were performed. For the HIFI instrument the LO is subdivided into subbands “a, b”. For example, mixer band 6b refers to HEB mixer band 6, LO subband “b”. In Fig. 4.35 we show a representative subset of the parametric stability tests at a LO frequency of 1666 GHz (B6b) and 1773.5 GHz (B7a). Two situation were examined: Spectroscopic system stability (Appendix A) as a function of LO pump power (HEB current), and spectroscopic system stability as a function of HEB bias voltage.
Figure 4.35: a) Normalized spectroscopic stability as a function of LO pump level. HEB mixer band 6b. 30 μA is slightly over pumped, 40 μA optimally pumped, and 50 μA slightly under pumped. The LO frequency is 1666 GHz. See also the I/V curves of Fig. 4.25b. b) Same as a), except for HEB mixer band 7a and at an LO frequency of 1773.5 GHz. c) Band 6b normalized spectroscopic stability as a function of HEB mixer bias voltage. Optimal sensitivity is ordinarily achieved around 0.5 mV. d) Band 7a normalized spectroscopic stability as a function of HEB mixer bias voltage. At a tradeoff of some sensitivity, a higher bias voltage improves the spectroscopic instrument stability.

In the top panel of Fig. 4.35 (a, b) we show the normalized spectroscopic Allan variance as a function of LO power. For both mixer bands 30 μA is slightly over pumped, 40 μA optimally pumped, and 50 μA on the verge of being under pumped. Slightly over pumping the HEB mixer from a stability point of view appears beneficial. This is understood to be the combined effect of a slight decrease in sensitivity and increase in required LO pump level (see also Fig. 4.25b). Higher LO power levels generally cause the W-band power amplifiers in the LO chain to run more saturated, thereby clipping the amplitude modulated (AM) noise on the LO carrier signal [114]. Consistent also is the trend that lower LO power (HEB current) results in a reduced Allan time, around 10 s in our case. This agrees with the picture that AM noise is present on the LO carrier, and that saturation of the LO chain power amplifiers is highly important.

In the bottom panel of Fig. 4.35 (c, d) we show the normalized spectroscopic Allan
variance as a function of bias voltage for a nominal LO pump level. Again we have a consistent trend with higher HEB bias voltages providing more stable behavior. At the larger (1 mV) bias voltage this is simply related to the sensitivity of the HEB mixer, e.g. the less sensitive the mixer the less sensitive it will also be to AM local oscillator noise. However between 0.4 and 0.6 mV the sensitivity of the mixer is more or less constant and the instability is likely the result of how close the mixer is biased to the (known) instability region [81]. From this discussion it is evident that slightly over pumping the HEB mixer (20 %), while biasing it above the nominal operating voltage (20 %) enhances the mixer stability, and thereby the integration efficiency and baseline quality.

4.4 Summary

In this Chapter we have taken a close look at the fundamental theory of both SIS and HEB mixers. We have seen that the described mixers operate on a completely different physical mechanism (quasi-particle tunneling vs. bolometric effect). SIS mixers suffer from a large intrinsic capacitance, related to the superconducting-insulator-superconducting geometry of barrier. To compensate (or tune out) this large parasitic capacitance imaginative tuning structures have been devised. The geometry of HEB mixers lacks the parasitic capacitance, making it in principle easy to connect to over a very broad RF bandwidth. However, the lack of geometric capacitance can also result in significant direct detection, which in actual application adversely affects absolute calibration accuracy.

Yet another significant difference between SIS and HEB mixers is that the SIS mixers have an upper frequency limit of $4\Delta$, or two times the superconducting energy gap. For niobium this equates to $\sim 1400$ GHz. Hot electron bolometer mixers do not operate on this principle, and in fact are preferably operated above the superconducting energy gap ($2\Delta$) of the material. In this way Cooper pairs are broken and uniform heating may be assumed in the bridge. An additional feature HEB mixers lack is the need to suppress Cooper pair tunneling through a tunnel barrier. This is known as the Josephson effect, and causes large excess noise in the SIS mixer when left unsuppressed (by application of a suitable magnetic field). Despite some of these advantages, HEB mixers generally exhibit considerably higher conversion loss than SIS mixers, which are capable, due to their quantum mechanical nature, of conversion gain. The higher conversion loss of HEB mixers leads to a generally lower mixer and receiver noise temperature. At approximately 1 THz both mixer types become competitive in sensitivity. This is primarily due to the Ohmic loss in the SIS RF matching network.

Because HEB mixers are thermal devices, in contrast to SIS tunnel junctions, the IF response is governed by several time constants: The electron-electron interaction time which is very fast (ps), the electron-phonon interaction time ($\sim 12$ ps for NbN) and the phonon escape time to the surrounding substrate ($\sim 50$ ps for thin film NbN). This time dependence causes the HEB mixer IF response to roll off above several GHz. This is an important drawback of HEB mixers, whose function is primarily in the terahertz frequency regime where astronomical lines have significant Doppler
broadening. A significant effort (and success) has been expanded to improve the IF response of HEB mixers (Chap. 6). SIS mixers do not have an IF frequency limitation, and can in principle be operated at IF frequencies of tenth of GHz. As we have seen in this Chapter, the thermal nature of HEB mixers also results in an electrothermal feedback mechanism, with reflected IF currents changing the heat balance in the HEB bridge. This is a process unknown to SIS mixers.

As a final note, SIS receivers generally have markedly better stability performance than HEB mixers. The improved stability of SIS receivers over HEB receivers is most likely related to the susceptibility of either device to fluctuations in local oscillator power, optical standing waves, and electronic noise. It is certainly the case that HEB mixers, being thermal devices that operate primarily in the terahertz regime, face a much more challenging operating environment than SIS mixers. Chap. 10 is devoted to this subject.
Bibliography


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[54] Virginia Diodes Inc., 321 West Main Street, Charlottesville, VA 22903, USA.


Note that the paper is supposed to report measured electron-phonon time $\tau_{e-ph}$, but the thermal time constant $\tau_{th}$ instead of $\tau_{e-ph}$ was mistakenly plotted in Fig. 4 of this paper according to private communication with one of the authors (G. Gol’tsman). Note also that the $\tau_{e-ph}$ extrapolated for 10 K from $\tau_{e-ph} \approx 500 T^{-1.6} ps K^{1.6}$ was measured separately by K. S. Il’lin, G. N. Gol’tsman, and B. M. Voronov, in Proc. of the 10th Int. Symp. on Space Terahertz Technol., edited by T. W. Crowe and R. M. Weikle, UVa, Charlottesville, USA, (1999), pp. 390.


[100] In 2004 the IF matching network was re-designed by the author in an effort to eliminate excess noise in HIFI mixers bands 6 & 7. It is now understood that the observed excess noise was caused by standing waves between the mixer and low noise amplifier, modulating the temperature balance in the HEB bridge via an electrothermal feedback mechanism.


[107] QMC Instruments Ltd., Cardiff University, Cardiff, U.K.


