Spin-polarized electron injection into semiconductors has been a field of growing interest during the last years [1-4]. The injection and detection of a spin-polarized current in a semiconducting material could combine magnetic storage of information with electronic readout in a single semiconductor device, yielding many obvious advantages. However, up to now experiments for spin-injection from ferromagnetic metals into semiconductors have only shown effects of less than 1% [5,6], which sometimes are difficult to separate from stray-field-induced Hall- or magnetoresistance-effects [2]. In contrast, spin-injection from magnetic semiconductors has already been demonstrated successfully [7,8] using an optical detection method.

Typically, the experiments on spin-injection from a ferromagnetic contact are performed using a device with a simple injector-detector geometry, where a ferromagnetic metal contact is used to inject spin polarized carriers into a two dimensional electron gas (2DEG) [3]. A spin-polarization of the current is expected from the different conductivities resulting from the different densities of states) for spin-up and spin-down electrons in the ferromagnet. For the full device, this should result in a conductance which depends on the relative magnetization of the two contacts [1].

A simple linear-response model for transport across a ferromagnetic/normal metal interface, which nonetheless incorporates the detailed behaviour of the electrochemical potentials for both spin directions was first introduced by van Son et al. [9]. Based on a more detailed (Boltzmann) approach, the model was developed further by Valet and Fert for all metal multilayers and GMR [10]. Furthermore, it was applied by Jedema et al. to superconductor-ferromagnet junctions [11]. For the interface between a ferromagnetic and a normal metal, van Son et al. obtain a splitting of the electrochemical potentials for spinup and spin down electrons in the region of the interface. The model was applied only to a single contact and its boundary resistance [9]. We now apply a similar model to a system in which the material properties differ considerably.

Our theory is based on the assumption that spin-scattering occurs on a much slower timescale than other electron scattering events [12]. Under this assumption, two electrochemical potentials $\mu_\uparrow$ and $\mu_\downarrow$, which need not be equal, can be defined for both spin directions at any point in the device [3]. If the current flow is one dimensional in the $x$-direction, the electrochemical potentials are connected to the current via the conductivity $\sigma$, the diffusion constant $D$, and the spin-flip time constant $\tau_{sf}$ by Ohm’s law and the diffusion equation, as follows:

\[
\frac{\partial \mu_\uparrow}{\partial x} = -\frac{e}{\sigma_{\uparrow}} j_{\uparrow}\] (1a)

\[
\frac{\mu_\uparrow - \mu_\downarrow}{\tau_{sf}} = D \frac{\partial^2 (\mu_\uparrow - \mu_\downarrow)}{\partial x^2} \] (1b)

where $D$ is a weighted average of the different diffusion constants for both spin directions [3]. Without loss of generality, we assume a perfect interface without spin scattering or interface resistance, in a way that the electrochemical potentials $\mu_{\uparrow,\downarrow}$ and the current densities $j_{\uparrow,\downarrow}$ are continuous.

Starting from these equations, straightforward algebra leads to a splitting of the electrochemical potentials at the boundary of the two materials, which is proportional to the total current density at the interface. The difference $(\mu_\uparrow - \mu_\downarrow)$ between the electrochemical potentials
decays exponentially inside the materials, approaching zero difference at $\pm\infty$.

$$\mu_1(\pm\infty) = \mu_1(\pm\infty) \quad (2)$$

A typical lengthscale for the decay of $(\mu_1 - \mu_1)$ is the spin-flip length $\lambda = \sqrt{D_{\alpha\beta}}$ of the material. In a semiconductor, the spin-flip length $\lambda_{sc}$ can exceed its ferromagnetic counterpart $\lambda_{fm}$ by several orders of magnitude. In the limit of infinite $\lambda_{sc}$, this leads to a splitting of the electrochemical potentials for both spin directions as assumed above. In addition, these boundary conditions imply that the exponential part of $\mu$ must behave as $c \exp(x/\lambda_{fm})$ in region 1 and as $c \exp(-(x-x_0)/\lambda_{fm})$ in region 3.

In the semiconductor we set $\tau_{sf} = \infty$, based on the assumption that the spin-flip length $\lambda_{sf}$ is several orders of magnitude longer than in the ferromagnet and much larger than the spacing between the two contacts. This is correct for several material systems, as semiconductor spin-flip lengths up to 100$\mu$m have already been demonstrated [3]. In this limit, we can write the electrochemical potentials for spin-up and spin-down in the semiconductor as

$$\mu_{2i,1}(x) = \mu_{1i,1}(0) + \gamma_{1i} \cdot x, \quad \gamma_{1i} = \text{constant} \quad (5)$$

While the conductivities of both spin-channels in the ferromagnet are different, they have to be equal in the two dimensional electron gas. This is because in the 2DEG, the density of states at the Fermi energy is constant, and in the diffusive regime the conductivity is proportional to the density of states at the Fermi-energy. Each spin channel will thus exhibit half the total conductivity of the semiconductor ($\sigma_{2i,1} = \sigma_{sc}/2$).

If we combine equation 1 and 2 and solve in region 1 at the boundary $x = 0$ and in region 3 at $x = x_0$ we are in a position to sketch the band bending in the overall device. From symmetry considerations and the fact that $j_{z\uparrow}$ and $j_{z\downarrow}$ remain constant through the semiconductor (no spin-flip) we have

$$\mu_{1i,1}(0) - \mu_{1i,1}(0) = \pm(\mu_{3i,1}(x_0) - \mu_{3i,1}(x_0)) \quad (6)$$

where the $+(-)$ sign refers to parallel (antiparallel) magnetization, respectively. This yields $c_{3i} = -c_{3i}$ and $c_{3i} = -c_{3i}$ in the expression for $\mu_{1i,1}$ in Eq. 2 for the parallel case, which is shown schematically in Fig. (1b). The antisymmetric splitting of the electrochemical potentials at the interfaces leads to a different slope and a crossing of the electrochemical potentials at $x = x_0/2$. We thus obtain a different voltage drop for the two spin directions over the semiconductor, which leads to a spin polarization of the current. In the antiparallel case where the minority spins on the left couple to the majority spins on
the right the solution is $c_{11} = -c_{31}$ and $c_{11} = -c_{31}$ with $j_1 = j_1$. A schematic drawing is shown in Fig. (1c). The splitting is symmetric and the current is unpolarized.

The physics of this result may readily be understood from the resistor model (Fig. 1a). For parallel (antiparallel) magnetization we have $R_{11} + R_{33} \neq R_{11} + R_{33}$ ($R_{11} + R_{33} = R_{11} + R_{33}$), respectively. Since the voltage across the complete device is identical for both spin channels, this results either in a different (parallel) or an identical (antiparallel) voltage drop over $R_{SC1}$ and $R_{SC1}$.

For parallel magnetization ($\beta_1 = \beta_3 = \beta$) the finite spin-polarization of the current density in the semiconductor can be calculated explicitly by using the continuity of $j_{i1,1}$ at the interfaces under the boundary condition of charge conservation for $(j_i + j_i)$ and may be expressed as:

$$\alpha_2 = \beta \frac{\lambda_{fm}}{\sigma_{fm}} \frac{\sigma_{sc}}{x_0} \frac{2}{(2\lambda_{fm} x_0/\sigma_{fm} + 1) - \beta^2}$$

where $\alpha_2$ is evaluated at $x = 0$ and constant throughout the semiconductor, because above we have set $\tau_{sf} = \infty$ in the semiconductor.

For a typical ferromagnet, $\alpha_2$ is dominated by $\lambda_{fm}/\sigma_{fm} / (x_0/\sigma_{sc})$ where $x_0/\sigma_{sc}$ and $\lambda_{fm}/\sigma_{fm}$ are the resistance of the semiconductor and the relevant part of the resistance of the ferromagnet, respectively. The maximum obtainable value for $\alpha_2$ is $\beta$.

However, this maximum can only be obtained in certain limiting cases, i.e., $x_0 \rightarrow 0$, $\sigma_{sc}/\sigma_{fm} \rightarrow \infty$, or $\lambda_{fm} \rightarrow \infty$, which are far away from a real-life situation. If, e.g., we insert some typical values for a spin injection device ($\beta = 60\%$, $x_0 = 1 \mu m$, $\lambda_{fm} = 10$ nm, and $\sigma_{fm} = 10^4 \sigma_{sc}$), we obtain $\alpha \approx 0.002\%$. The dependence of $\alpha_2$ on the various parameters is shown graphically in Figs. (2a) and (2b) where $\alpha_2$ is plotted over $x_0$ and $\lambda_{fm}$, respectively, for three different values of $\beta$. Apparently, even for $\beta > 80\%$, $\lambda_{fm}$ must be larger than 100 nm or $x_0$ well below 10 nm in order to obtain significant (i.e. $> 1\%$) current polarization. The dependence of $\alpha_2$ on $\beta$ is shown in Fig. (3a) for three different ratios $\sigma_{fm}/\sigma_{sc}$. Even for a ratio of 10, $\alpha_2$ is smaller than $1\%$ for $\beta < 98\%$, where the other parameters correspond to a realistic device.

By calculating the electrochemical potential throughout the device we may also obtain $R_{par}$ and $R_{anti}$ which we define as the total resistance in the parallel or antiparallel configuration, respectively. The resistance is calculated for a device with ferromagnetic contacts of the thickness $\lambda_{fm}$, because only this is the lengthscale on which spin dependent resistance changes will occur. In a typical experimental setup, the difference in resistance $\Delta R = (R_{anti} - R_{par})$ between the antiparallel and the parallel configuration will be measured. To estimate the magnitude of the magnetoresistance effect, we calculate $\Delta R/R_{par}$ and we readily find

$$\Delta R \approx \frac{\lambda_{fm}^2}{1 - \beta^2} \frac{\sigma_{sc}^2}{x_0^2} \frac{4}{(2\lambda_{fm} x_0/\sigma_{fm} + 1) - \beta^2}$$

(8)

Now, for metallic ferromagnets, $\Delta R/R_{par}$ is dominated by $\lambda_{fm}^2/x_0^2\sigma_{sc}^2$ and is approx. $\alpha_2^2$. In the limit of $x_0 \rightarrow 0$, $\sigma_{sc}/\sigma_{fm} \rightarrow \infty$, or $\lambda_{fm} \rightarrow \infty$, we again obtain a maximum which is now given by

$$\Delta R \approx \frac{\beta^2}{(\beta - 1)(\beta + 1)}$$

(9)

Fig (3b) shows the dependence of $\alpha_2$ and $\Delta R/R_{par}$ on $\beta$, for a realistic set of parameters. Obviously, the change in resistance will be difficult to detect in a standard experimental setup.

We have thus shown, that, in the diffusive transport regime, for typical ferromagnets only a current with small spin-polarization can be injected into a semiconductor 2DEG with long spin-flip length even if the conductivities of semiconductor and ferromagnet are equal (Fig (3a)). This situation is dramatically exacerbated when ferromagnetic metals are used; in this case the spin-polarization in the semiconductor is negligible.

Evidently, for efficient spin-injection one needs a contact where the spin-polarization is almost 100%. One example of such a contact has already been demonstrated: the giant Zeeman-splitting in a semimagnetic semiconductor can be utilized to force all current-carrying electrons to align their spin to the lower Zeeman level. Other promising routes are ferromagnetic semiconductors or the so called Heusler compounds or other half-metallic ferromagnets. Experiments in the ballistic transport regime (where $\sigma_{sc}$ has to be replaced by the Sharvin contact resistance) may circumvent part of the problem outlined above. However, a splitting of the electrochemical potentials in the ferromagnets, necessary to obtain spin-injection, will again only be possible if the resistance of the ferromagnet is of comparable magnitude to the contact resistance.

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FIG. 1. (a) Simplified resistor model for a device consisting of a semiconductor (SC) with two ferromagnetic contacts (FM) 1 and 3. The two independent spin channels are represented by the resistors $R_{1\uparrow}$, $R_{SC\uparrow}$, and $R_{3\uparrow}$. (b) and (c) show the electrochemical potentials in the three different regions for parallel (b) and antiparallel (c) magnetization of the ferromagnets. The solid lines show the potentials for spin-up and spin-down electrons, the dotted line for $\mu_0$ (undisturbed case). For parallel magnetization (b), the slopes of the electrochemical potentials in the semiconductor are different for both spin orientations. They cross in the middle between the contacts. Because the conductivity of both spin channels is equal, this results in a (small) spin-polarization of the current in the semiconductor. In the antiparallel case (c), the slopes of the electrochemical potentials in the semiconductor are equal for both spin orientations, resulting in unpolarized current flow. (Note that the slope of $\mu$ in the metals is exaggerated).

FIG. 2. Dependence of $\alpha_2$ on $\lambda_{fm}$ (a) and $x_0$ (b), respectively for $\sigma_{fm} = 100 \sigma_{sc}$ and three different values of $\beta$. In fig. (a), $x_0$ is 1 $\mu$m. Note that $\alpha_2$ is only in the range of % for $\beta \approx 100\%$ or $\lambda_{fm}$ in the $\mu$m-range. In fig. (b) we have $\lambda_{fm} = 10$ nm and again, we see that for a contact spacing of more than 10 nm, $\alpha_2$ will be below 1% if a standard ferromagnetic metal ($\beta < 80\%$) is used.

FIG. 3. Dependence of $\alpha_2$ and $\Delta R/R$ on $\beta$. In (a) $\alpha_2$ is plotted over $\beta$ for different ratios $\sigma_{fm}/\sigma_{sc}$. For a ratio of 100, $\alpha_2$ is well below 0.1% for $\beta < 99\%$. In (b), again $\alpha_2$ is plotted versus $\beta$ with $\sigma_{fm}/\sigma_{sc} = 100$, with the corresponding values for $\Delta R/R$ on a logarithmic scale. For $\beta$ between 0 and 90%, $\Delta R/R$ is smaller than $10^{-7}$ and thus difficult to detect in the experiment.
Fig. 1 Schmidt et al.
\[ \alpha_2 = \frac{\lambda_m}{n} \]

**Fig. 2** Schmidt et al.
Fig. 3  Schmidt et al.