Part III

Gravitational Lensing
ABSTRACT — 50 deep FORS1 I-band images have been used to perform a galaxy-galaxy lensing study. After having applied the popular Kaiser, Squires and Broadhurst (KSB) method to correct the point spread function and to estimate the ellipticity of each object, the lens and source galaxies have been sorted and the average tangential shear has been computed as a function of the angular distance. To study the dark matter halo of the lens galaxies two models are used, the singular isothermal sphere (SIS) model and the Navarro-Frenk-White (NFW) profile. For both models the redshift distributions of the lens and source galaxies are estimated from the redshift measurements performed during the VVDS-deep Survey. After fitting the SIS model to our average tangential shear profile, we obtained the einstein radius \( \langle r_e \rangle = 0.11 \pm 0.02 \) arc seconds. Assuming that the luminosity evolves with respect to the redshift (\( L \propto (1 + z) \)) and that the cosmological model used is the \( \Lambda \)-CDM universe, we obtained the velocity dispersion \( \langle \sigma_v^* \rangle^{1/2} = 111^{+10}_{-11} \) km/s for a galaxy at a reference luminosity \( L_I^* = 1.12 \times 10^{10} h^{-2} L_\odot \). The best fit of the NFW model yields, after a \( \chi \)-squared minimum analysis using the above conditions for the cosmological model, the luminosity evolution and reference luminosity of the galaxy, a virial dispersion of \( V_{200}^* = 159^{+64}_{-52} \) km/s, a scaling radius of \( r_s^* = 15^{+42}_{-11} h^{-1} \) kpc and a virial mass of \( M_{200}^* = 9.3^{+11.3}_{-9.9} \times 10^{11} h^{-1} M_\odot \). Similar analyses have been performed using the SIS and NFW profiles for a different reference luminosity of galaxies, different cosmological models (CDM and \( \Lambda \)-CDM) and assuming in addition two cases where the luminosity evolves with respect to the redshift and where it does not.
5.1 Introduction

The mass distribution of galaxies has been a fundamental subject of study in cosmology these past decades. A better understanding of this elementary property is crucial for the present-day cosmology as well as probing the quantity of matter that the universe contains. Theories predict that the brightest galaxies should have masses of the order of \( \sim 1 - 2 \times 10^{12} \, M_\odot \) in a radius out to distances of \( \sim 100 - 300 \, h^{-1}\text{kpc} \) (Brainerd et al., 1996b; Navarro et al., 1996) and observations seem to confirm these predictions (Hoekstra et al., 2004, for example). It is currently well established that an extended dark matter halo surrounds the luminous core and disk of all galaxies. The presence of this dark matter halo has been revealed during the study of rotation curves of spiral galaxies (van Albada et al., 1985, for example). This method probes dark matter at small scale (about 30kpc) but dynamical studies of galaxy pairs have also shown that the dark matter halos can extend out to radii of 200kpc (Zaritsky and White, 1994; Brainerd and Specian, 2003). X-ray studies of hot gas in elliptical galaxies (e.g. NGC 4636) confirm this picture and show that the halo can extend to a larger scale (Mushotzky et al., 1994; Loewenstein and Mushotzky, 2003). Unfortunately this method requires very sensitive observations which are available only for few objects.

Gravitational lensing is an alternative approach, that can be used to quantify the mass of galaxies. The mass of galaxies is capable of deflecting light which causes mass concentrations to act as lens galaxies. To study the dark matter halos of galaxies, we make statistical studies of slightly distorted galaxy images which come from objects that are positioned in the background of the lens galaxies. The analysis of these distorted images is also known as Galaxy-Galaxy Lensing (Mellier, 1999; Bartelmann and Schneider, 2001; Schneider, 2005).

Several attempts to measure the distortions in the images of background galaxies have been made. The first unsuccessful one was carried out on photographic plates with poor seeing (Tyson et al., 1984). We had to wait for 12 years, and the advent of CCDs, for successful detections (Brainerd et al., 1996b; dell’Antonio and Tyson, 1996; Griffiths et al., 1996). Since the first detections the techniques to extract a distortion signal have evolved significantly:

- The point spread function is corrected before analysis, (Kaiser et al., 1995; Bonnet and Mellier, 1995; van Waerbeke et al., 1997; Hoekstra et al., 1998; Kuijken, 1999; Kaiser, 2000; Bernstein and Jarvis, 2002; Hirata and Seljak, 2003; Refregier, 2003).

- The size of the images have increased with the development of new detectors.

- New models to characterize the mass distribution of a galaxy have been created (truncated isothermal sphere (TIS) (Brainerd et al., 1996b), pseudo-isothermal elliptical mass distribution (PIEMD) (Kneib et al., 1996), Navarro-Frenk-White profile (NFW) (Navarro et al., 1996) in addition to the standard singular isothermal sphere (SIS) model.

- Maximum likelihood techniques have been introduced to estimate the parameters which characterize lens galaxy halos (Schneider and Rix, 1997; Natarajan and Kneib, 1997, for example).
An important step was made by Fisher et al. (2000) who analyzed a large area (225 square degrees) of imaging data from the Sloan Digital Sky Survey (SDSS) in three bands. A photometric red-shift analysis was carried out to extract the lens and source galaxies which improves significantly the extraction of the distortion signal (Fischer et al., 2000). Currently, it is possible to study the dark matter halo of an average galaxy as a function of its luminosity (McKay et al., 2001) or color (Kleinheinrich et al., 2005b). It is also possible to study the shape of the halo or its maximum radius (Hoekstra et al., 2004).

In this work, we study the dark matter halo properties of I-band selected galaxies using galaxy-galaxy lensing properties. This study has been carried out using 50 deep FORS1 images taken in the I-band. The data have been kindly provided by Yannick Mellier (IAP, France) and have been previously used by Maoli et al. to detect cosmic shear (Maoli et al., 2001). Since we do not have red-shift information about the objects present in these images, we deduce the red-shift distribution of the lens and the source galaxies from spectroscopic data (Le Fèvre et al., 2005). To characterize the dark matter halo of the lens galaxies, the SIS and NFW models are used and the results are compared with those obtained in other studies (Hoekstra et al., 2004; Kleinheinrich et al., 2005b).

This chapter is structured as follows. In section 2, we briefly describe the basics of galaxy-galaxy lensing, the point spread function correction we use as well as the shear estimate. Section 3 describes the models used to characterize the dark matter halo of the lens galaxies. We have made use of the SIS and NFW models throughout the study. In section 4, we describe the data used to perform our galaxy-galaxy lensing study. We also give details on the data reduction procedure. In section 5, we summarize results from the SIS and NFW models.

5.2 Galaxy - Galaxy Lensing Theory

5.2.1 Galaxy-Galaxy Lensing

The theory of General Relativity shows that a massive gravitational potential well, such as galaxy, has an effect on the light coming from galaxies positioned in the background near the line of sight. A foreground object (in our case, a galaxy with its dark matter halo) systematically distorts the images of background galaxies by stretching them tangentially (see Figure 5.1).

The gravitational lensing tools developed over the past decades permit us to express the optical properties of a gravitational lens using two parameters: the convergence \( \kappa \) and the shear \( \gamma \). The first quantity describes the isotropic focusing of a light ray and represents an isotropic magnification while the second one describes the effects on the shape of the source images produced by gravitational forces. Unfortunately, individual galaxies do not have enough mass to produce a visible signal. We need to superpose statistically the signals of many galaxies to observe it. To perform such an analysis we need to extract from the images of each background galaxy its ellipticity. This parameter is composed of the intrinsic ellipticity of the galaxy plus the shear produced by the galaxy in the foreground. The next section discusses the method used to estimate the total ellipticity.
Figure 5.1: Example of the effect of the weak lensing by a gravitational potential (e.g., galaxy, cluster of galaxies). The background galaxies are aligned tangentially after adding a massive object in front of them. In this example the distortion of the sources is emphasized. In reality, in the weak lensing regime, the distortion can be observed only after averaging the ellipticities of many source objects.

5.2.2 Definition of the Ellipticity of an Object

To model the galaxies ellipticities, a method based on the second moments of a brightness distribution within an isophote (curve of constant intensity) was used. Once the second moments are calculated, the estimation of the shape of the galaxies can be performed. The formulation of the second moment is described below:

$$Q_{ij} = \frac{\iint q_l [I(\theta_i, \theta_j)] (\theta_i - \overline{\theta_i}) (\theta_j - \overline{\theta_j}) d\theta_i d\theta_j}{\iint q_l [I(\theta_i, \theta_j)] d\theta_i d\theta_j}$$

(5.1)

where $q_l$ is the limiting isophote of the galaxy inside which the second moments are calculated. $\overline{\theta_i}$ is the galaxy center defined by the barycenter of the brightness distribution of the galaxy, $Q_{ij}$ are the second moments, $\theta$ is an angular coordinate on the sky and $I$ is the light intensity.

The shape of an object is defined by a complex ellipticity. This ellipticity is deduced from the tensor of the second brightness moment and given by:

$$e = \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22}} = e_1 + i e_2$$

(5.2)

First, we explain briefly the geometrical meaning of the ellipticity. A positive or a negative $e_1$ corresponds to an elongation or a compression respectively, of the object along the $x$ axis. For $e_2$, a positive or a negative value corresponds to an elongation or a compression of the object along the axis $y = x$. A graphical representation of $e_1$ and $e_2$ can be found in Figure 5.2.
5.2: Galaxy-Galaxy Lensing Theory

Figure 5.2: Illustration of the ellipticity \( e_1, e_2 \). If \( e_1 > 0 \) \( (e_1 < 0) \), we observe an elongation (compression) of the object along the x-axis in cartesian \((x, y)\) coordinates. The same effects are observed for \( e_2 \) along the axis \( y = x \).

5.2.3 Distortion Matrix

The observed source surface brightness distribution is related to the source surface brightness distribution as follows:

\[
I^{\text{obs}}(\theta) = I^{\text{src}}(\beta(\theta))
\]

where \( I^{\text{src}}(\beta) [I^{\text{obs}}(\theta)] \) is the [observed] surface brightness distribution of an object in the source [lens] plane, \( \beta \) and \( \theta \) are the positions in the source plane and in the lens plane, respectively.

Since the distortion signal is only about 1%, the equation can be linearised. The distorted galaxy images are related to the source galaxy images by:

\[
I^{\text{obs}}(\theta) = I^{\text{src}}[\beta_0 + \bar{A}(\theta_0).(\theta - \theta_0)]
\]

where \( \beta_0 \) and \( \theta_0 \) are the object’s center in the source plane and the lens plane respectively and \( A(\theta_0) \) is the Jacobian matrix (also called distortion matrix) of the lens at the object’s position in the lens plane. The matrix \( \bar{A}(\theta) \) which appears above is defined as follows:

\[
\bar{A}(\theta) = \frac{\partial \beta}{\partial \theta} = \begin{pmatrix}
1 - \kappa & -\gamma_1 \\
-\gamma_2 & 1 - \kappa + \gamma_1
\end{pmatrix} = (1 - \kappa) \begin{pmatrix}
1 - g_1 & -g_2 \\
-g_2 & 1 + g_1
\end{pmatrix}
\]

where \( \gamma_1 \) and \( \gamma_2 \) are the shear coefficients, \( \kappa \) is the local dimensionless surface mass density of the lens and \( g \) is the reduced shear.

The shear coefficients \( \gamma_1 \) and \( \gamma_2 \) have the same geometrical meaning as the ellipticity coefficients \( e_1 \) and \( e_2 \) (see section 5.2.2). The reduced shear, \( g \), in equation (5.5) is defined as:

\[
g(\theta) = \frac{\gamma(\theta)}{1 - \kappa(\theta)}
\]

In weak gravitational lensing regime, we have \( \kappa << 1 \) and \( |\gamma| << 1 \), so we may set \( \gamma \approx g \) (Bartelmann and Schneider, 2001). Using \( \bar{Q}^S = \bar{A} \bar{Q} \bar{A}^T \), which relates the second brightness moment tensor of the source to the observed image of the source, and equation (5.2), it is possible to express \( e^{\text{src}} \), the ellipticity of the source, in term of \( e^{\text{obs}} \), the observed ellipticity \( (e^{\text{obs}} \approx e^{\text{src}} + \gamma) \). Unfortunately this method is not directly applicable to real CCD data because significant noise, such as photon noise due to the sky background and readout noise from CCD electronics, is present in the images. These
perturbations enter the calculation of $Q_{ij}$ in $q(I)$ and bias the measurements. Another problem is the atmospheric turbulence which smears the point spread function as well as other inconveniences which are related to the telescope defects such as imperfect optics and oscillations due to the wind. These perturbations have to be quantified and subtracted from the ellipticity estimation of the objects to access to the real distortion experienced by the galaxy sources in the presence of a potential well. These actions which compensate the defects mentioned above are performed during the Point Spread Function (PSF) correction where the PSF describes the brightness distribution of a point source.

Kaiser et al. (1995) developed a procedure to correct the PSF which takes into account all known defects found in normal observation conditions. They could define relations which link the observed ellipticity to the real ellipticity of an object. The real ellipticity corresponds to the ellipticity of the object measured by a system without defects and noises. This technique, known as KSB technique, is presented in the next section.

5.2.4 KSB

Quadrupole Tensor and Ellipticity

To minimize the effects of the different sources of noises, the weighted function $q(I(\theta))$ in equation (5.1) is replaced by the function $I(\theta)W(\theta)$, where $W$ is a gaussian weighted function, $W(\theta) = \exp\left(-\frac{|\theta - \theta|^2}{2\sigma^2}\right)$. The quadrupole moment tensor becomes:

$$Q_{ij} = \int \int (\theta_i - \bar{\theta}_i)(\theta_j - \bar{\theta}_j)I(\theta_i, \theta_j) \exp\left(-\frac{(\theta_i - \bar{\theta}_i)^2 + (\theta_j - \bar{\theta}_j)^2}{2\sigma^2}\right)d\theta_id\theta_j$$  \hspace{1cm} (5.7)

where $\bar{\theta}_i$ is the galaxy center defined by the barycenter of the brightness distribution of the galaxy, $\theta_i$ and $\theta_j$ are the angular coordinates, $I$ is the light intensity at the angular coordinates and $\sigma$ is a typical scale obtained during the object extraction procedure. $\sigma$ corresponds to the size of the mexican hat filter used to estimate the characteristics of the object at highest significance.

Once the quadrupole moment tensor is defined, the ellipticity can be calculated using equation (5.2). This parameter, affected by the defects outlined in section 5.2.3, is corrected by the procedure described below.

PSF Anisotropic Correction

At the first step of the KSB method, for each object, the atmospheric distortion and the tracking errors of the telescope are corrected. These errors, which correspond to an anisotropic smearing, are compensated using the smear polarisability tensor $P_{sm}$. This $2 \times 2$ shear tensor, obtained from the CCD image, describes the linear response of the ellipticity to a PSF anisotropy and depends on higher order moments of the brightness distribution of the object. The PSF anisotropy is subtracted from the observed ellipticity, $e_{obs}(e_{obs,1}, e_{obs,2})$, to obtain the isotropic ellipticity, $e_{iso}(e_{iso,1}, e_{iso,2})$. This step is performed using the following equation:

$$e_{iso} = e_{obs} - P_{sm}q$$  \hspace{1cm} (5.8)
where \( q \) is the measure of PSF anisotropy. To estimate \( q \) one may use the fact that the isotropic ellipticity of the stars is \( e_{iso}^* = 0 \) before being affected by the atmospheric and telescope distortion because the stars light is not affected by gravitational potential. To estimate \( q \), equation (5.8) is used and \( q \) becomes:

\[
q = (P_{sm}^*)^{-1} e_{obs}^* \tag{5.9}
\]

where \( P_{sm}^* \) is the anisotropic smearing tensor of the stars and \( e_{obs}^* \) is the observed ellipticity of the stars. Both parameters are extracted from the measurements on the CCD image.

Finally, combining equation (5.8) with (5.9) yields the isotropic ellipticity of an object.

\[
e_{iso} = e_{obs} - P_{sm}(P_{sm}^*)^{-1} e_{obs}^* \tag{5.10}
\]

### Isotropic Smearing and Shear Correction

Next the isotropic smearing and the shear are corrected simultaneously. The \( P_{sm} \) tensor is used to correct the isotropic smearing while the \( 2 \times 2 \) shear tensor \( P_{sh} \) is used to compensate the shear produced by a gravitational potential. \( P_{sh} \) describes the linear response of the ellipticity to the shear and depends on higher order moments of the brightness distribution of the object. Like \( P_{sm} \), the coefficients of \( P_{sh} \) are obtained using the CCD image. The intrinsic ellipticity \( e_{int}(e_{int,1}, e_{int,2}) \) is related to the isotropic ellipticity, \( e_{iso} \), as follows:

\[
e_{int} = e_{iso} - P_{sh} g - P_{sm} q_{iso} \tag{5.11}
\]

where \( g \) is the reduced shear defined in section 5.2.3 and \( q_{iso} \) is the measure of the PSF isotropy. The term \( P_{sh} g \) expresses the shift in the intrinsic ellipticity due to lensing. This term subtracted from \( e_{iso} \) gives the ellipticity of an object before being affected by the atmospheric layer. The last term shows how the ellipticity of a sheared object is affected by the isotropic smearing of the seeing disk.

The properties of the stars will be used again to determine the PSF isotropy. Since \( e_{iso}^* = e_{int}^* = 0 \), the \( q_{iso} \) vector can be deduced from equation (5.11) as follows:

\[
q_{iso} = -P_{sh}^*(P_{sm}^*)^{-1} g \tag{5.12}
\]

After re-writing equation (5.11) using equation (5.12) we arrive at:

\[
e_{int} = e_{iso} - P_{sh} g + P_{sh}^* P_{sm}(P_{sm}^*)^{-1} g \tag{5.13}
\]

For convenience we define the pre-seeing shear polarizability tensor \( P_g \) (Luppino and Kaiser, 1997):

\[
P_g = P_{sh} - P_{sh}^* P_{sm}(P_{sm}^*)^{-1} \tag{5.14}
\]

and

\[
e_{int} = e_{iso} - P_g g \tag{5.15}
\]
Shear Estimate

Using equation (5.15), the averaged shear can be determined for a contiguous group of objects. To estimate this parameter we assume that in this group $\varepsilon_{\text{int}} = 0$ and $\bar{g}$ is constant. We also know that the reduced shear, $g$, and the shear, $\gamma$, can be considered to be identical in a weak lensing regime ($\bar{g} \approx \bar{\gamma}$, see section 5.2.3). The averaged shear can then be given by:

$$\bar{\gamma} = (P_g)^{-1} \bar{\varepsilon}_{iso}$$  (5.16)

To perform a galaxy-galaxy lensing analysis we have to work on individual source galaxies. We assume that the source galaxy has a circular shape before being sheared by the lens galaxy. In practice this is not true because each galaxy has an intrinsic ellipticity, but this assumption is applicable because when we average the shear below, the averaged intrinsic ellipticity vanishes ($\varepsilon_{\text{int}} = 0$). With this supposition we can use equation (5.16) to determine for each object a noisy shear estimator, $\tilde{\gamma}_i$ of a galaxy $i$. In reality $\tilde{\gamma}_i$ is a function of the intrinsic ellipticity and the shear.

$$\tilde{\gamma}_i = (P_{g,i})^{-1} e_{iso,i}$$  (5.17)

5.2.5 Shear, Theory and Measurements

Tangential Shear

The average tangential shear for a radius, $r$, can be written using the average local dimensionless surface mass density, $\bar{\kappa}(r)$, within and at the radius $r$:

$$\bar{\gamma}_t(r) = \bar{\kappa}(\leq r) - \bar{\kappa}(r)$$  (5.18)

This expression has been introduced by Miralda-Escude (1991).

The parameter, $\kappa$, is the ratio between the surface mass density of the lens galaxy and the critical surface mass density of the lens galaxy.

$$\kappa(r) = \frac{\Sigma(r)}{\Sigma_c}$$  (5.19)

The critical surface mass density, $\Sigma_c$, introduced above (Schneider et al., 1992) corresponds to a value of the surface mass density above which a gravitational lens can produce multiple images of a background source. $\Sigma_c$ is defined as follows:

$$\Sigma_c = \frac{c^2}{4\pi G D_S D_L D_{LS}}$$  (5.20)

$$\Sigma_c = \frac{c^2}{4\pi G D_L \beta}$$  (5.21)

where $G$ is the gravitational constant, $c$ is the speed of light and $D_S$, $D_L$ and $D_{LS}$ are the angular diameter distances from the observer to the source, from the observer to the lens and from the lens to the source, respectively. The shear signal depends on the position of the observer, the lenses and the sources. $\beta$, which is defined as
Figure 5.3: Illustration of a gravitational lensing event. L is a lens object and S is a source object. The observer, O, sees depending on the configuration of the observer, the lens and the source, a single object in the source plane, E (weak lensing), or multiple objects in C and E (strong lensing). In the case of Galaxy-Galaxy lensing, this drawing is exaggerated. The shift from S to E is very small, the image E is just slightly stretched in the direction tangential to the lens galaxy.

\[ \beta = \frac{D_{LS}}{D_S}, \] characterizes the dependence of the shear signal on the source distance, while its dependence on the lens distance is accounted for by \( D_L \).

From equation (5.18), we can deduce an essential relation for our study. The average tangential shear, \( \gamma_t(r) \), can be expressed as a function of the mass density contrast, \( \Delta \Sigma_t \), and the critical surface mass density (Miralda-Escude, 1991):

\[
\begin{align*}
\gamma_t(r) & = \frac{\Sigma(\leq r) - \Sigma(r)}{\Sigma_c} \\
& \equiv \frac{\Delta \Sigma_t}{\Sigma_c}
\end{align*}
\]  

(5.22)  

(5.23)

where \( \Sigma(\leq r) \) and \( \Sigma(r) \) are the mean surface mass densities within and at radius \( r \), respectively. \( \Delta \Sigma_t = \Sigma(\leq r) - \Sigma(r) \) is the mass density contrast.
Equation (5.23) relates the tangential distortion profile which is drawn out of the FORS I data to the models used to characterize the halos of lens galaxies. Two models will be used to express the density contrast namely, the singular isothermal sphere (SIS) and the NFW model (Navarro et al., 1996). We introduce below the extraction of the average tangential profile.

Shear Measurements

The main problem in a galaxy-galaxy lensing study is the lack of objects around the lens galaxy. Because of the weakness of the shear signal produced by a lens galaxy on background objects (only a maximum of \(\sim 1\%\) of the total ellipticity of a source galaxy), a large number of background objects is necessary to extract the signal (\(\sim 10000\) background objects). Such a large number cannot be reached per one lens galaxy. In the case of our study, only an averaged number of \(\sim 400\) background galaxies has been found per lens galaxy, in a radius of 110 arc-sec. To compensate the lack of sources, the strategy employed consists of combining the shear signals from a number of lens galaxies. This technique permits to obtain the necessary number of source galaxies to extract the averaged shear. However the technique does not allow for characterizing the lens galaxies of our sample individually. We can only describe an averaged lens galaxy.

To extract the mean tangential shear at a defined angular distance \(r_k\), the shear estimator for each source galaxy \(i\), defined in section 5.2.4, is decomposed into tangential and orthogonal shear components with respect to the lens galaxy \(j\). The following equations are used to extract for each pair \(i, j\), these two components from the tensor \(\tilde{\gamma}_i\),

\[
\tilde{\gamma}_{t,i} = -\tilde{\gamma}_{1,i}\cos(2\varphi) - \tilde{\gamma}_{2,i}\sin(2\varphi) \tag{5.24}
\]

\[
\tilde{\gamma}_{x,i} = +\tilde{\gamma}_{1,i}\sin(2\varphi) - \tilde{\gamma}_{2,i}\cos(2\varphi) \tag{5.25}
\]

where \(\tilde{\gamma}_{t,i}\) is the tangential shear estimate, \(\tilde{\gamma}_{x,i}\) is the orthogonal shear estimate from the source galaxy \(i\), \(\tilde{\gamma}_{1,i}\) and \(\tilde{\gamma}_{2,i}\) are the two components of the tensor \(\tilde{\gamma}_i\) and \(\varphi\) is the azimuthal angle with respect to the center of the lens galaxy, \(j\).

In the case of galaxy-galaxy lensing, the shear produced by the lens is present only in the tangential shear estimator \(\tilde{\gamma}_{t,i}\) of the source galaxy. To obtain the average tangential shear at a projected radius \(r_k\), all tangential shear estimators of the source galaxies which are within a thin annulus of thickness \(l\) are averaged. Each lens galaxy is the center of the annulus, the inner circle has a projected radius at \(r_{ak} = r_k - l/2\) and the outer circle has a projected radius at \(r_{bk} = r_k + l/2\) (See Figure 5.4). Assuming that the tangential shear is constant in this annulus and the averaged intrinsic ellipticity is null (\(\tilde{\epsilon}_{int} = 0\)), we have:

\[
\overline{\gamma}_t(r_k) = \frac{1}{N_l} \sum_{j=1}^{N_l} \left( \frac{1}{N_{s,j}} \sum_{i=1}^{N_{s,j}} \tilde{\gamma}_{t,j,i} \right) = \overline{\gamma}_t(r_k) \tag{5.26}
\]

\[
\overline{\gamma}_x(r_k) = \overline{\gamma}_x(r_k) \tag{5.27}
\]

where \(\overline{\gamma}_t(r_k)\) is the average tangential shear estimator at projected radius \(r_k\) which gives an estimate for the average tangential shear \(\overline{\gamma}_t(r_k)\). \(N_l\) is the number of lens galaxies and
5.3: Models to Study the Dark Matter Halos Properties

Figure 5.4: Illustration of the selection of the source galaxies around a lens galaxy. This manipulation is repeated for all the lens galaxies. The observer Ob. sees the $j^{th}$ lens galaxy at red-shift $z_{lj}$ and the source galaxies $i$ at red-shift $z_{sji}$.

$N_{s,j}$ is the number of source galaxies used around the lens galaxy $j$. $\tilde{\gamma}_{t,j,i}$ is the tangential shear estimator of the source galaxy $i$ with respect to the lens galaxy $j$. $\overline{\gamma}_x(r_k)$ is the average orthogonal shear estimator of the average orthogonal shear $\gamma_x(r_k)$ and should be close to zero for a lensing event.

The mean tangential shear profile can now be estimated using different annuli. This manipulation gives an ensemble of pairs $(r_k, \overline{\gamma}_x(r_k))$. For extraction of the dark matter halo properties, two models discussed below, make use of this profile.

5.3 Models to Study the Dark Matter Halos Properties of Lens Galaxies

Two models of dark matter halos are used in our analysis, the singular isothermal sphere (SIS) model and the Navarro, Frenk and White (NFW) model. The SIS model is a simple model regularly used to characterize dark matter halos and thus, this model is employed as a baseline for all comparisons.

According the collisionless cold dark matter (CDM) simulations, the mass density for halos follows a specific profile, the NFW profile (Navarro et al., 1996). The NFW model, probably more realistic than the SIS model, is exploited in our study to determine the characteristics of the dark matter halos.

5.3.1 The SIS Model

The Singular Isothermal Sphere model makes use of a semi realistic model to describe the lens galaxies. SIS is defined by the mass density profile $\rho(r)$ and the surface mass density $\Sigma(r)$. The SIS model is characterized by the stellar velocity dispersion $\sigma_v$ which corresponds to the stellar velocity in elliptical galaxies.
\[ \rho(r) = \frac{\sigma_v^2}{2\pi G r^2} \]  
(5.28)

\[ \Sigma(r) = \frac{\sigma_v^2}{2G r} \]  
(5.29)

### The Tangential Distortion Profile

Combining equation (5.28) and (5.29) with equation (5.21) and taking into consideration that the mass density contrast \( \Delta \Sigma_c \) is equal to the mass surface density itself for the case of an isothermal mass profile \( (\Sigma \propto r^{-1}) \), the tangential distortion can be expressed in terms of the Einstein radius, \( r_e \), and the radius, \( r \),

\[ \gamma_t(r) = \frac{r_e}{2r} \]  
(5.30)

The Einstein radius is expressed as follows,

\[ r_e = 4\pi \frac{\sigma_v^2}{c^2} \beta \]  
(5.31)

### The Velocity Dispersion and Averaged Velocity Dispersion

Equation (5.31) gives the stellar velocity dispersion of a galaxy which follows the mass density profile from the SIS model. Equation (5.31) becomes:

\[ \sigma_v = 186.4 \sqrt{\frac{r_e}{\beta}} \]  
(5.32)

where \( r_e \) is in arc seconds and \( \sigma_v \) is in kilometers per second.

The best fit of the singular isothermal sphere model (equation 5.30) to the tangential distortion profile gives the average Einstein radius, \( \langle r_e \rangle \). The average velocity dispersion \( \langle \sigma_v^2 \rangle^{1/2} \) can then be deduced by using the following equation,

\[ \langle \sigma_v^2 \rangle^{1/2} = 186.4 \sqrt{\frac{\langle r_e \rangle}{\langle \beta \rangle}} \]  
(5.33)

In the next section the determination of \( \langle \beta \rangle \) is discussed.

### Determination of \( \langle \beta \rangle \)

We defined in the section 5.2.5 the \( \beta \) parameter (see equation 5.21) as

\[ \beta = \frac{D_{LS}}{D_S} \]  
(5.34)

where \( D_{LS} \) is the angular diameter distance from the lens to the source and \( D_S \) is the angular diameter distance from the observer to the source (see Figure 5.3).
5.3: Models to Study the Dark Matter Halos Properties

Figure 5.5: This plot shows the dimensionless angular diameter distance as a function of the red-shift for two cosmological flat models, CDM and Λ-CDM, also called Einstein-De Sitter and Λ-dominated universe.

This parameter, \( \beta \), can also be defined theoretically using the red-shift of the lens, \( z_l \), and the source, \( z_s \). The angular diameter distances are defined as follows,

\[
D_S = \frac{R_0 S_k(r_s)}{1 + z_s} \quad (5.35)
\]
\[
D_{LS} = \frac{R_0 S_k(r_s - r_l)}{1 + z_s} \quad (5.36)
\]

where \( R_0 S_k(r) \) is the effective distance and is given by,

\[
R_0 S_k(r) = \frac{c}{H_0} \int_0^z \frac{1}{\left[ (1 - \Omega) (1 + z)^2 + \Omega_v + \Omega_m (1 + z)^3 + \Omega_r (1 + z)^4 \right]^{\frac{1}{2}}} dz \quad (5.37)
\]

where \( \Omega_m \) is the pressureless matter parameter, \( \Omega_v \) is the vacuum parameter, \( \Omega_r \) is the radiation parameter and \( \Omega \) is the sum of \( \Omega_m, \Omega_v \) and \( \Omega_r \) (Peacock, 1999), \( H_0 \) is the Hubble constant and \( c \) is the speed of the light.

In the case of flat models (e.g. \( \Omega = 1 \) CDM and \( \Lambda \)-CDM), \( R_0 S_k(r_s - r_l) = R_0 S_k(r_s) - R_0 S_k(r_l) \) and \( \beta \) becomes,

\[
\beta = 1 - \frac{R_0 S_k(r_l)}{R_0 S_k(r_s)} \quad (5.38)
\]

The angular diameter distance, \( D_S \), and \( \beta \) versus the source red-shift are illustrated in Figure 5.5 and 5.6, respectively.

In the CDM universe model (\( \Omega_m = 1, \Omega_r = 0, \Omega_v = 0 \)), \( \beta \) can be re-written as follows,

\[
\beta(z_l, z_s) = \max \left( 0, 1 - \frac{1 - (1 + z_l)^{-\frac{1}{2}}}{1 - (1 + z_s)^{-\frac{1}{2}}} \right) \quad (5.39)
\]
chapter 5: Galaxy Dark Matter Halo from 50 FORS1 Images

Figure 5.6: Illustration of the $\beta$ parameter at different lens red-shifts of the source, $z_s$, for two cosmological flat models, CDM and $\Lambda$-CDM. The solid lines represent the CDM model, while the dash lines the $\Lambda$-CDM model. The points, squares and triangles are for the three different fixed red-shifts of the lens $z_l = 0.2, 0.6, 1.$

Our study requires obtaining an estimate for $\langle \beta \rangle$. To achieve this two different approaches are invoked. The first approach consists of measuring the red-shift of each object and then using equation (5.39) we determine $\beta$ and $\langle \beta \rangle$, respectively. The second approach which is used in our study makes use of the density profiles of lens and source galaxies to estimate $\langle \beta \rangle$ when the red-shift information is missing.

Other Parameters

The averaged Einstein radius of the characteristic lens galaxy is also introduced:

$$\langle r_e^* \rangle = \frac{4\pi c^2}{\sigma_v^*} \langle \beta \rangle$$ (5.40)

as well as the asymptotic circular velocity which corresponds to the stellar velocity in spiral galaxies:

$$V_c^* = \sqrt{2} \sigma_v^*$$ (5.41)

5.3.2 The NFW Model

One physically more motivated form for the dark halos is given by the Navarro, Frenk and White (NFW hereafter) model. From N-body simulations of halo collapse in expanding CDM universes, Navarro, Frenk and White found that the mass density profiles of haloes vary from $r^{-1}$ in the central regions to $r^{-3}$ beyond a characteristic radius $r_s$ (Navarro et al., 1996). This model produces a more realistic estimate of halo mass density profiles than the SIS model for the simulation of large structures such as galaxy groups, clusters of galaxies and super clusters (Wright and Brainerd, 2000). On the other hand the inner slope of CDM halos is still being debated. There are claims that the NFW profile may not
be able to correctly model the mass density profile at smaller scales (Ghigna et al., 1998; Moore et al., 1998). However Ricotti (2003) results have shown that the NFW profile can describe massive galaxies. In the next sections the NFW model is outlined. From galaxies following the NFW density profile we deduce the shear produced by them (Bartelmann, 1996; Wright and Brainerd, 2000).

**Description of the NFW Model**

The NFW mass density profile takes the form,

\[ \rho_{NFW}(r) = \frac{\delta_c \rho_c}{r/r_s(1 + r/r_s)^2} \]  

(5.42)

where \( \delta_c \) is a characteristic over-density or a density contrast of the halo, \( \rho_c \) is the critical mass density at halo’s red shift and \( r_s \) is the scale radius or characteristic radius of the halo. These three parameters are expressed as follows:

\[ \delta_c = \frac{200}{3} \frac{c^3}{\ln(1 + c) - c/(c + 1)} \]  

(5.43)

\[ \rho_c = \frac{3H^2(z)}{8\pi G} \]  

(5.44)

\[ r_s = \frac{r_{200}}{c} \]  

(5.45)

where \( c \) is the dimensionless concentration parameter, \( H(z) \) is the Hubble constant at red-shift \( z \) and \( r_{200} \) is the virial radius, radius within which the average mass density is 200 times the critical mass density \( \rho_c \), and \( G \) is the gravitational constant.

We define,

\[ M_{200} = \frac{800\pi}{3} \rho_c r_{200}^3 \]  

(5.46)

as the virial mass, and the corresponding rotation velocity, \( V_{200} \) at \( r_{200} \) is,

\[ V_{200}^2 = \frac{GM_{200}}{r_{200}} \]  

(5.47)

An additional equation relates the maximum rotation velocity, \( V_{max} \) to the scale radius, \( r_s \),

\[ V_{max}^2 = \frac{\pi G \rho_c \delta_c}{3} r_s^3 \]  

(5.48)

**The Distortion \( \gamma_{NFW} \)**

The shear is expressed using equation (5.22). The surface mass density terms in this equation are calculated by integrating the NFW mass density profile along the line of sight (Bartelmann, 1996; Wright and Brainerd, 2000).

\[ \gamma_{NFW}(x) = \frac{\Sigma_{NFW}(x) - \Sigma_{NFW}(x)}{\Sigma_c} \]  

(5.49)
where \( x = D_L \theta / r_s \) is the dimensionless radial distance measured in units of scale radius, \( r_s \). It corresponds to the distance between the lens and the source projected on the lens plane. \( \Sigma_{NFW}(x) \) and \( \Sigma_{NFW}(x) \) are the mean surface mass density within radius \( x \) and the surface mass density at \( x \), respectively. These last two terms are written as

\[
\Sigma_{NFW}(x) = 2 \int_0^\infty \rho_{NFW}(x, z)dz
\]

\[
\Sigma_{NFW}(x) = \frac{2}{x^2} \int_0^x x' \Sigma_{NFW}(x')dx'
\]

The shear is then given by

\[
\gamma_{NFW}(x) = \frac{r_s \delta_c \rho_c}{\Sigma_c} g(x)
\]

where \( g(x) \) is defined as

\[
g(x) = \begin{cases} 
8 \frac{\text{arctanh} \sqrt{(1-x)/(1+x)}}{x^2\sqrt{1-x^2}} + 4 \frac{x}{x^2} \ln \left( \frac{x}{2} \right) \cdots & x < 1 \\
\frac{10}{3} + 4 \ln \left( \frac{1}{2} \right) & x = 1 \\
8 \frac{\text{arctan} \sqrt{(x-1)/(1+x)}}{x^2\sqrt{x^2-1}} + 4 \frac{x}{x^2} \ln \left( \frac{x}{2} \right) \cdots & x > 1 
\end{cases}
\]

5.4 Data Analysis, PSF Correction and Distortion Profile

5.4.1 Data

The set of images for which we performed a galaxy-galaxy lensing study, have been kindly provided by Yannick Mellier (Institut d’Astrophysique de Paris, France). They consist of 50 FORS I fields randomly distributed over more than 1000 deg\(^2\) and represent a total field of view of 0.64 deg\(^2\). The FORS I instrument was mounted on the VLT/UT1 (ANTU) at the Paranal Observatory. It was equipped with a 2048 \( \times \) 2048 thinned CCD backside illuminated. Each field covers 6.8 \( \times \) 6.8 min\(^2\) with a resolution of 0.2 arc-second per pixel (24 \( \times \) 24\( \mu \)m). Each image is a composition of 6 \( \times \) 6 minutes exposures taken with the filter I which offers the best image quality. With this setup the expected average red-shift of the source galaxies is \( \bar{z}_s \approx 1 \). The data have been processed by the TERAPIX data center and this work yields 50 images with an excellent image quality and a seeing lower than 0.8 arc-seconds (Maoli et al., 2001).
5.4.2 Stars and Galaxies Detection

To extract objects (galaxies and stars) from a complete image a modified version of the IMCAT objects extractor is used. The original code was written by Nick Kaiser\(^1\) and modified by Henk Hoekstra and Ole Moeller. The extracted objects constitute a catalog. For each object in the catalog the following characteristics are determined, namely, the position of the objects, the significant threshold, the first estimation of the half light radius, the magnitude, and the gaussian radius. The significant threshold is a parameter calculated by an IMCAT sub-routine to quantify the pertinence of the estimations of the other parameters (this parameter is called \(\nu\) in the IMCAT formalism). In the second step the shear and smear polarizability tensors of the objects are determined. During the second step the half-light radius and the magnitude are refined. This procedure yields totally 50 catalogs.

Star Extraction

Next, for each catalog a plot of the objects magnitude versus the half-light radius, \(r_h\), is generated (see Figure 5.7) and used to select the stars. The brightest objects deduced from such a plot which constitute 20% of all objects, are extracted and a histogram of the selected objects is drawn as a function of the half light-radius, \(r_h\). The peak in the histogram corresponds to the number of the stars in the complete image. To select the minimum and maximum half-light radius inside within the stars are located, the mean, \(\bar{r}_h\), and the standard deviation of \(\bar{r}_h\), \(\sigma_{r_h}\), are calculated for the first peak in the histogram. The minimum and maximum radius correspond to \(r_{h,\text{min}}(= \bar{r}_h - 5\sigma_{r_h})\) and \(r_{h,\text{max}}(= \bar{r}_h + 5\sigma_{r_h})\), respectively. The thresholds for the stars magnitude are defined as follows. The maximum magnitude corresponds to the magnitude of the brightest objects and the minimum magnitude is 19.2. This last value has been fixed after an analysis of the data to exclude saturated stars. Typically, the magnitude of the stars is within 19.2 and 22.8.

Galaxy Extraction

The pre-selection of galaxies is based on determining the magnitude of the objects as a function of the objects radius (see Figure 5.7). All objects with a half-light radius larger than the maximum half-light radius defined in the previous section (Stars Extraction), \(r_{h,\text{max}}\), are considered to be galaxies. To refine the selection, a statistical study has been performed on the complete set of catalogs of pre-selected galaxies. This work has shown that the majority of these objects has a half-light radius below fifteen pixels, which value is used as a maximum limit. Objects with a larger radius are disregarded. Moreover, the study has shown that the vast majorities (3\(\sigma\)) of the diagonal elements of the tensors \(P_{sh}\) and \(P_{sm}\) are between 0 and 2, and, 0 and 1, respectively. All objects which do not appear within these thresholds are not considered. All objects with a significance threshold below 5 are not selected. Finally, we remove all objects with an ellipticity \(e > 1\).

After applying all these criteria for selection to the object catalogs, we are left with 50 new, "cleaned" catalogs on which we can perform our study.

\(^1\)http://www.ifa.hawaii.edu/~kaiser/
Figure 5.7: Example of a (radius, magnitude) plot which is generated for each image. This plot is used to select the stars, foreground and background galaxies. The selected stars are denoted within the box.

Figure 5.8: Uncorrected star ellipticities are shown in one of the 50 VLT FORS1 images to illustrate the Point Spread Function (PSF). On the bottom right, the stick with a cross represents the ellipticity of an object with a norm equal to 0.05.

5.4.3 Point Spread Function Correction and Shear Estimation

The ellipticities obtained in the galaxy catalog are affected by the PSF (see Figure 5.8 for example). Atmospheric perturbations and defects of the instrument smear and shear the images. The procedure described in section 5.2.4 is used to correct these defects. To
apply the necessary corrections, we need to estimate the PSF at the galaxy position. The stars $l$ are used to realize this manipulation. A second order polynomial fit is performed across the images for the following star parameters, \((e^*_{l})_{uv}, (P^*_{sm,l})_{uv}\) and \((P^*_{sh,l})_{uv}\) \((u\text{ and } v\text{ are indexes in the tensors})\), which allows us to calculate an estimate of the star parameters at the galaxy position, \(e^*_i, P^*_{sm,i}\) and \(P^*_{sh,i}\), and then to calculate \(e^*_iso,i\) and \(P^*_g,i\) for all galaxies $i$. Using the isotropic ellipticity, the pre-seeing shear polarizability tensor and equation (5.17), we can extract the (reduced) shear from the measured ellipticity of the galaxy $i$,

\[
\tilde{\gamma}_i = (P^*_g,i)^{-1}(e^*_{obs,i} - P^*_{sm,i}(P^*_{sm,i})^{-1}e^*_{obs,i})
\]

(5.54)

An extra step is needed before calculating $\tilde{\gamma}_i$, namely, $P^*_g,i$ has to be smoothed. The diagonal elements of this tensor can sometimes be very small when the source size is smaller than the PSF. $P_g$ can also be strongly affected by noise when the object is very faint. The inverse of $P^*_g,i$ becomes in that case very large and mainly amplifies the noise. To attenuate this source of disturbance, $P^*_g,i$ is smoothed before taking its inverse. To perform this we follow the procedure used by Kaiser et al. (1998), Hoekstra et al. (1998) and Van Waerbeke et al. (2000) which consists of smoothing $P_g$ in the basis (objects size, magnitude). For each $P_g$ the parameters of the nearest neighbors objects are used to calculate a smoothed $P^*_g,i, P^*_g,smo$ (see Figure 5.9, for example). Then the shear of the object $i$ can be determined:

\[
\tilde{\gamma}_i = (P^*_g,smo)^{-1}(e^*_{obs,i} - P^*_{sm,i}(P^*_{sm,i})^{-1}e^*_{obs,i})
\]

(5.55)
5.4.4 Object Selection

To perform our galaxy-galaxy lensing study on the FORS1 data, the catalogs obtained after a data reduction (see section 5.4.2) have to be sorted into foreground lens and background source samples. All galaxies with an AB magnitude between 18 and 22.7 are considered to be lenses. The source galaxies are objects with an AB magnitude between 23.2 and 26.5. The selection of the objects is performed simply on the basis of their apparent AB magnitude because this data lack photometric redshift or spectroscopic information. The magnitude thresholds are chosen from statistical studies.

5.4.5 Catalog of Pairs

The two objects catalogs created are used to build a final catalog (pairs catalog) in which the pairs (lens, source) of galaxies are present. This new catalog contains, among other parameters, the angular distance between the lens and the source, the tangential and orthogonal ellipticities of the source. The last two parameters are deduced from equations 5.24 and 5.25.

The pairs catalog is built taking into consideration two extra requirements. First we impose a maximum limit to the distortion $\gamma$. The distortion calculated during the PSF correction with the tensor of second order $P_g$ may become very large because the values of the tensor elements are determined less accurately since they are affected by a significant noise. Although we attempted to reduce this effect by smoothing the $P_g$ tensor during the PSF correction, we still encountered it for some objects. Since it is unlikely that the lower accuracy of the tensor elements, caused by the noise, reflects a realistic distortion, those objects and all information related to them are disregarded. Our final catalog contains only objects with $\sqrt{g_1^2 + g_2^2} \leq 0.9$.

This complete procedure yields a total number of $\sim 1,030,000$ pairs for our study.

5.4.6 Tangential and Orthogonal Shear Profile

The pairs catalog is now used to extract the average tangential and orthogonal shear, $\overline{\gamma_t}(r_k)$ and $\overline{\gamma_x}(r_k)$. The procedure explained in section 5.2.5 is followed and it yields the results plotted in figures 5.10 and 5.11 for the average tangential and orthogonal shear profile respectively. The error bars are obtained using random realizations of galaxy catalogs which procedure consists of random rotations of each galaxy in every catalog, followed by a new calculation of the average tangential and orthogonal shears. As a consequence a multiple number of catalogs is generated for each initial catalog. 200 random realizations are performed and the standard deviation of $\overline{\gamma_t}(r_k)$ and $\overline{\gamma_x}(r_k)$ is obtained.

In theory, the complete distortion of the source image produced by a gravitational potential appears in the tangential shear profile while no image distortion is recorded in the orthogonal shear profile. Since this behavior is observed in our data, we can conclude that we detect an image distortion effect caused by gravitational lensing. We can now model the observed shear profile in the context of SIS and NFW models.
5.5 Results

5.5.1 SIS Profile

The SIS model is characterized by the following mass density profile, $\rho(r) = \sigma_v^2/(2\pi Gr^2)$ where $r$ is the radial distance from the center of the lens galaxy to its periphery and $\sigma_v$ is the velocity dispersion. From this mass density profile we deduce the average tangential shear profile, $\gamma_t(r) = r_e/(2r)$ (see Section 5.3.1 for more detail). Since we do not have in our data red-shift information for the objects, a finer study is not possible. Therefore we concentrate our effort on the description of an average lens galaxy. The best fit of the SIS model to our tangential shear data is obtained and it gives the ensemble average Einstein radius $r_e = 0.11 \pm 0.02\"$.

To calculate the average velocity dispersion, $\langle \sigma_v^2 \rangle^{1/2}$, equation (5.33) is exploited. At this stage of our study, the term, $\langle \beta \rangle$, in this equation is unknown. One possibility to
estimate this term is to use the red-shift information about our objects by employing equation (5.39). The average of all the $\beta$'s would give us the expected parameter $\langle \beta \rangle$. Unfortunately we cannot use this approach because the only information at our disposal, concerning the extracted objects, is deduced from the optical images of the VLT FORS1 instrument using only the I filter.

To estimate $\langle \beta \rangle$, we use the red-shift measurements performed during the VVDS-deep Survey (Le Fèvre et al., 2005). Le Fèvre et al. measured the red-shift distribution within a magnitude range for $I_{\text{AB}}$ between 18 and 24 (see Table 5.1). Based on their work we model the red-shift distribution of lens and source galaxies and using this distribution we calculate $\langle \beta \rangle$.

Definition of $\langle \beta \rangle$

We need to estimate $\langle \beta \rangle$ for both CDM and $\Lambda$-CDM universes. To determine the average $\langle \beta \rangle$, the following equation will be used:

$$\langle \beta \rangle = \frac{\int \int F(z_l, z_s) \beta(z_l, z_s) dz_l dz_s}{\int \int F(z_l, z_s) dz_l dz_s}$$

(5.56)

where $\beta(z_l, z_s)$ has been defined for both CDM and $\Lambda$-CDM universes (see section 5.3.1) and $F(z_l, z_s)$ is the total red-shift distribution of galaxies.

Definition of $F(z_l, z_s)$

Assuming that the lens and source red-shift distributions are independent we can write:

$$F(z_l, z_s) = F_{\text{lens}}(z_l) F_{\text{source}}(z_s)$$

(5.57)

where $F_{\text{lens}}(z_l)$ and $F_{\text{source}}(z_s)$ are the red-shift distributions of the lens and source objects. These two distributions are calculated as follows.

$F_{\text{lens}}(z_l)$ and $F_{\text{source}}(z_s)$ Distributions

For each magnitude interval $I_{AB}$ in Table (5.1), we extrapolate a gaussian distribution $F_i(z)$ (see Figure 5.14). The ensemble of these gaussian distributions is used to express the red-shift distributions $F_{\text{lens}}(z_l)$ and $F_{\text{source}}(z_s)$ (see Figure 5.12). For our study, we consider the magnitude of the lens objects to be between the 18 and 23. The data in the first ten lines of Table 5.2 are used to express the lens red-shift distribution, $F_{\text{lens}}(z_l)$. We can write it as follows:

$$F_{\text{lens}}(z_l) = \sum_{i=1}^{10} w_{li} \cdot F_i(z_l)$$

(5.58)

where $w_{li}$ is a weight function which corresponds to the number of objects in the interval $i$ divided by the total number of objects used to estimate $F_{\text{lens}}(z_l)$. The same reasoning is applied for the source objects. The magnitude of these objects is above 23. We use the last seven lines of table (5.2) to obtain the source red-shift distribution.

$$F_{\text{source}}(z_s) = \sum_{i=11}^{17} w_{si} \cdot F_i(z_s)$$

(5.59)
where $w_{si}$ is a weight function which corresponds to the number of objects in the interval $i$ divided by the total number of objects used to estimate $F_{\text{source}}(z_s)$.

We describe how to estimate the gaussian distributions $F_i(z)$ in which the $F_{\text{lens}}(z_l)$ and $F_{\text{source}}(z_s)$ distributions are expressed.

**Definition of $F_i(z)$**

Before expressing $F_{\text{lens}}(z_l)$ and $F_{\text{source}}(z_s)$ in terms of $F_i(z)$, we need to estimate the red-shift distribution for each magnitude interval $I_{AB}$. We assume that the distributions $F_i(z)$ are gaussians which are characterized by their mean, $\hat{z}_i$, and the standard deviation $\hat{\sigma}_i$. To estimate the pair ($\hat{z}_i$, $\hat{\sigma}_i$) for each magnitude interval, the following approach is employed.

Following Le Fèvre et al we have for each $I_{AB}$ magnitude interval the median, the 1st and the 3rd quartile of the red-shift distribution (See Table 5.1). A linear square fit is performed on these three sets of data (See Figure 5.13). The obtained fitted results for the median, 1st and the 3rd quartile of the red-shift distribution give for each magnitude interval an estimate of those terms. We realize the fitting procedure in order to obtain better symmetrical values and also to extrapolate these parameters up to the 26.5 magnitude. The linear square fits give us:

\[
q_1(I_{AB}) = 0.08I_{AB} - 1.37 \tag{5.60}
\]
\[
z_m(I_{AB}) = 0.13I_{AB} - 2.21 \tag{5.61}
\]
\[
q_3(I_{AB}) = 0.19I_{AB} - 3.18 \tag{5.62}
\]

For each magnitude interval we deduce the center of the red-shift distribution, $\hat{z}_i$, as follows:

\[
\hat{z}_i = z_m(I_{AB}^{c,i}) \tag{5.63}
\]
Figure 5.13: Illustration of the O. Le Fèvre data. The median, 1st and 3rd quartile of the redshift distribution are plotted versus object’s magnitude. The solid lines are the best fits.

Table 5.1: Reproduction of the data from Le Fèvre et al. Median, 1st and 3rd quartile of the red-shift distribution versus object’s magnitude interval.
Table 5.2: Estimation of the median red-shift $z_m$ and the standard deviation $\sigma$ for the gaussian red-shift distribution.

<table>
<thead>
<tr>
<th>$I_{AB}$ Range</th>
<th>Median estimation</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.0-18.5</td>
<td>0.18</td>
<td>0.08</td>
</tr>
<tr>
<td>18.5-19.0</td>
<td>0.25</td>
<td>0.12</td>
</tr>
<tr>
<td>19.0-19.5</td>
<td>0.32</td>
<td>0.16</td>
</tr>
<tr>
<td>19.5-20.0</td>
<td>0.38</td>
<td>0.20</td>
</tr>
<tr>
<td>20.0-20.5</td>
<td>0.45</td>
<td>0.24</td>
</tr>
<tr>
<td>20.5-21.0</td>
<td>0.51</td>
<td>0.27</td>
</tr>
<tr>
<td>21.0-21.5</td>
<td>0.58</td>
<td>0.31</td>
</tr>
<tr>
<td>21.5-22.0</td>
<td>0.64</td>
<td>0.35</td>
</tr>
<tr>
<td>22.0-22.5</td>
<td>0.71</td>
<td>0.39</td>
</tr>
<tr>
<td>22.5-23.0</td>
<td>0.77</td>
<td>0.43</td>
</tr>
<tr>
<td>23.0-23.5</td>
<td>0.84</td>
<td>0.47</td>
</tr>
<tr>
<td>23.5-24.0</td>
<td>0.91</td>
<td>0.51</td>
</tr>
<tr>
<td>24.0-24.5</td>
<td>0.97</td>
<td>0.55</td>
</tr>
<tr>
<td>24.5-25.0</td>
<td>1.04</td>
<td>0.59</td>
</tr>
<tr>
<td>25.0-25.5</td>
<td>1.10</td>
<td>0.62</td>
</tr>
<tr>
<td>25.5-26.0</td>
<td>1.17</td>
<td>0.66</td>
</tr>
<tr>
<td>26.0-26.5</td>
<td>1.23</td>
<td>0.70</td>
</tr>
</tbody>
</table>

where $I_{AB}^{c,i}$ is the center of the magnitude interval $i$ (see Table 5.1) The standard deviation, $\hat{\sigma}_i$, is given by:

$$\hat{\sigma}_i = \frac{q_3(I_{AB}^{c,i}) - q_1(I_{AB}^{c,i})}{2 \cdot 0.674}$$ (5.64)

Then we can obtain the gaussian distribution as follows (see Figure (5.14)):

$$F_i(z) = \frac{1}{\hat{\sigma}_i \sqrt{2\pi}} e^{-\frac{(z-z_i)^2}{2\hat{\sigma}_i^2}}$$ (5.65)

**Determination of $\langle \beta \rangle$**

We finally calculate $\langle \beta \rangle$ as

$$\langle \beta \rangle \approx \frac{\sum_{i=0}^{200} \sum_{j=0}^{200} f_{i,j} \beta_{i,j}}{\sum_{i=0}^{200} \sum_{j=0}^{200} f_{i,j}}$$ (5.66)

where $f_{i,j} = F(i\Delta z, j\Delta z)$ and $\beta_{i,j} = \beta(i\Delta z, j\Delta z)$ with $\Delta z = 0.01$. $\beta_{i,j}$ is calculated for both CDM and $\Lambda$-CDM universes. A graphic representation of the matrix $[f_{i,j}]$ and the two matrices $[\beta_{i,j}]$ for the two cosmological models can be found in Figure (5.15), Figure (5.16) and Figure (5.17), respectively. This procedure yields $\langle \beta \rangle \approx 0.27$ for the CDM model and $\langle \beta \rangle \approx 0.3$ for the $\Lambda$-CDM model, respectively.
Figure 5.14: Illustration of the gaussian red-shift distribution for each interval $I_{AB}$ in Table 5.2. These gaussians distribution are used to create the lens and source red-shift distributions.

Figure 5.15: Illustration of the estimate of the lens and source red-shift distribution, $F(z_l, z_s)$. 
5.5: Results

Figure 5.16: $\beta$ as a function of $z_{\text{lens}}$ and $z_{\text{source}}$ for the CDM model.

Figure 5.17: $\beta$ as a function of $z_{\text{lens}}$ and $z_{\text{source}}$ for the $\Lambda$-CDM model.

Correcting for Scaling Relations

The average velocity dispersion, $\langle \sigma_v^2 \rangle^{1/2}$, can now be determined for the CDM and $\Lambda$-CDM universe models. Using equation (5.32), we obtain $\langle \sigma_v^2 \rangle^{1/2} = 120^{+11}_{-12} \text{ km/s}$ for the CDM model and $\langle \sigma_v^2 \rangle^{1/2} = 113^{+11}_{-12} \text{ km/s}$ for $\Lambda$-CDM. Unfortunately these values characterize the average lens galaxy of our sample of lens galaxies and therefore it is difficult to compare the results to those, yielded by other studies. Instead we have to estimate the velocity dispersion of an average characteristic galaxy ($\sigma_v^* \rangle$ defined by a $L^\ast$ luminosity. This velocity dispersion is deduced following three steps.

1. We determine the velocity dispersion-luminosity relation.
2. We define the luminosity-magnitude relation.
3. The results from step one and two are employed to express the Einstein radius.
Finally we define the velocity dispersion of an average characteristic galaxy.

Faber-Jackson / Tully-Fischer relation

To relate the velocity dispersion, \( \sigma_v \), which characterizes the properties of a galaxy halo, to the observable galaxy luminosity, the Faber-Jackson / Tully-Fischer relation is used (\( L \propto \sigma_v^n \)). To compensate the red-shift evolution of the characteristic luminosity, the following model, \( L^*(z_l) = L^*(1 + z_l)^\alpha \), is employed where \( L^* \) refers to the characteristic luminosity extrapolated at zero red-shift. Using these assumptions we express the velocity dispersion of a lens galaxy, \( \sigma_v \), as a function of its apparent luminosity, \( L(z_l) \), (1 + \( z_l \))^\( \alpha \), the luminosity (\( L^* \)) and velocity dispersion (\( \sigma_v^* \)) of an average characteristic galaxy. We obtain:

\[
\sigma_v(z_l) = \left( \frac{L(z_l)}{L^*} \right)^{1/n} (1 + z_l)^{-\alpha/n} \sigma_v^*(z_l)
\]

The Luminosity-Magnitude Relation: \( L(z_l)/L^* \)

Next we express the ratio \( L(z_l)/L^* \) as a function of the lens galaxy magnitude \( m \), the absolute magnitude of an average characteristic galaxy, \( M^* \), and the red-shift of the lens galaxy, \( z_l \). This manipulation permits us to correlate directly the velocity dispersion to the magnitude of the galaxies. By definition we have,

\[
m - m^* = -2.5 \log \left( \frac{L(z_l)}{L^*} \right) \quad (5.68)
\]

\[
m^* = M^* + 2.5 \log \left( \frac{d_L}{10 \text{pc}} \right)^2 + K(z) \quad (5.69)
\]

where \( d_L \) is the luminosity distance \( (d_L = (1 + z_l)^2 D_L) \), \( m^* \) is the magnitude of the average characteristic galaxy and \( K(z) \) is a correction factor. Since the absolute magnitude, \( M^* \) is affected by a shift of the spectrum in frequency, the correction factor is introduced to compensate this effect (Peacock, 1999). \( K(z) \) is defined as follows,

\[
K(z) = 2.5(\lambda - 1) \log (1 + z) \quad (5.70)
\]

where \( \lambda \approx 2 \) in the I-band.

The angular diameter distance introduced above, \( D_L = (1 + z_l)^{-1} R_0 S_k (r_l) \) (where \( R_0 S_k (r) \) is the effective distance, see Peacock, 1999), is expressed within matter-dominated Friedmann models. In the CDM universe model, \( \Omega_m = \Omega = 1 \) and \( D_L \) becomes,

\[
D_L = \frac{2c'}{H_0} \frac{1}{1 + z_l} \left( 1 - \frac{1}{\sqrt{1 + z_l}} \right) \quad (5.71)
\]

where \( c' \) is the speed of the light in km/s and \( H_0 \) is the Hubble constant in km/s/Mpc. To be consistent with the units below,

\[
\frac{2c'}{H_0} \text{[Mpc]} \equiv \frac{20c}{h} \text{[pc]} \quad (5.72)
\]

where \( c \) is the speed of the light in m/s and \( h = H_0/(100 \text{km/s/Mpc}) \).
Then, the angular diameter distance becomes,

\[ D_L[\text{pc}] = \frac{20c}{h} \frac{1}{1 + z_l} \left( 1 - \frac{1}{\sqrt{1 + z_l}} \right) \] (5.73)

and the ratio \( L(z_l)/L^* \) can be written as follows,

\[ \frac{L(z_l)}{L^*} = 10^{-0.4(m - M^*)} \left[ \frac{2c}{h} \left( 1 - \frac{1}{\sqrt{1 + z_l}} \right) \right]^2 (1 + z_l)^{\lambda + 1} \] (5.74)

**Expression for \( r_e \)**

The Einstein radius, \( r_e = 4\pi\sigma_v^2\beta(z_l, z_s)/c^2 \), can be re-written using equations (5.73 and 5.74) above. In this manner \( r_e \) is expressed also as a function of the velocity dispersion of an average characteristic galaxy, \( \sigma_v^* \). We have,

\[ r_e(z_l) = \frac{4\pi}{c^2} \sigma_v^2 \beta(z_l, z_s) \Gamma(z_l, m_l) \] (5.75)

where \( \Gamma(z_l, m_l) \) is the luminosity correction factor of the velocity dispersion of the characteristic lens galaxy (\( \sigma_v^* \)) and it is defined as follows:

\[ \Gamma(z_l, m_l) = 10^{-0.8(m - M^*)/n} \left[ \frac{2c}{h} \left( 1 - \frac{1}{\sqrt{1 + z_l}} \right) \right]^{4/n} (1 + z_l)^{(\lambda + 1 - \alpha)/n} \] (5.76)

The average Einstein radius can now be deduced as follows:

\[ \langle r_e \rangle = \frac{4\pi}{c^2} \langle \sigma_v^* \rangle \langle \beta(z_l, z_s) \rangle \Gamma(z_l, m_l) \] (5.77)

where

\[ \langle \Gamma_{\beta} \rangle = \langle \beta(z_l, z_s) \Gamma(z_l, m_l) \rangle = \frac{\int \int \int F(z_l, z_s, m_l) \beta(z_l, z_s) \Gamma(z_l, m_l) dz_l dz_s dm_l}{\int \int \int F(z_l, z_s, m_l) dz_l dz_s dm_l} \] (5.78)

and

\[ F(z_l, z_s, m_l) = F_l(z_l, m_l) F_s(z_s) \] (5.79)

where \( F_l(z_l, m_l) \) is the red-shift distribution of the lens galaxies at a given magnitude, \( m_l \) and \( F_s(z_s) \) is the red-shift distribution of the source galaxies.

In the derivations above, the \( \langle \Gamma_{\beta} \rangle \) parameter has been calculated for the CDM cosmological model. A similar work has been performed for the \( \Lambda \)-CDM model. The angular diameter distance \( D_L \) introduced in equation (5.71) is simply replaced by the general form (equation 5.35) where \( D_L \) is expressed as a function of \( \Omega_m \) and \( \Omega_\text{v} \). To calculate \( \langle \Gamma_{\beta} \rangle \), several parameters are necessary, namely namely the absolute magnitude of the average characteristic galaxy at a red-shift 0 (we choose three different values for comparison with other studies), the lens and source distributions, \( F_l \) and \( F_s \) (they are deduced from O. Le Fèvre et al.), the K-correction factor and in addition a luminosity evolution model has to be taken into consideration (\( \text{L}^*(z_l) = \text{L}^* (1 + z_l)^\alpha \) where \( \alpha = \)
1 or 0 assuming that we either have or not such an evolution. The results for \( \langle \Gamma\beta \rangle \) are summarized in Table 5.3. We have now the necessary information to calculate \( \sigma^*_v \). To estimate \( \sigma^*_v \), an equation similar to equation (5.33) is introduced:

\[
\langle \sigma^*_v \rangle^{1/2} = 186.4 \sqrt{\frac{\langle r_e \rangle}{\langle \Gamma\beta \rangle}} \tag{5.80}
\]

where the average Einstein radius, \( \langle r_e \rangle \), is in arcsec and the constant 186.4 is in kilometer per second.

Discussion

The tangential shear signal contained in our data is extracted for radii between 0 and 2’ (see Figure 5.10). These angular distances correspond to an interval between 0 and \( \sim 460h^{-1}\)kpc at a median red-shift of the lenses \( (z_m \approx 0.6, \text{see Figure 5.10}) \). The SIS model is fitted to our data. The best-fit obtained gives us an Einstein radius \( \langle r_e \rangle = 0.11 \pm 0.02" \). From this Einstein radius we can deduce, using \( \langle \beta \rangle \), the average velocity dispersion, \( \langle \sigma^2 \rangle^{1/2} \) of our sample of lens galaxies. As it is shown in the section above, \( \langle \beta \rangle \) depends on the geometry of the universe. We have chosen two cases, the CDM and \( \Lambda \)-CDM universe models. In the first case we determine \( \langle \beta \rangle = 0.268 \) and in the second case \( \langle \beta \rangle = 0.299 \). We can now calculate the average velocity dispersion, for the CDM universe, \( \langle \sigma^2 \rangle^{1/2} = 120^{+11}_{-11}\)km/s and for the \( \Lambda \)-CDM universe, \( \langle \sigma^2 \rangle^{1/2} = 113^{+10}_{-11}\)km/s. A direct comparison of these velocities with velocities yielded by other studies is not possible yet, because the former characterize the average lens galaxy of our sample of lens galaxies. To carry out such a comparison we need to define the velocity dispersion of the galaxies at a reference luminosity \( L^* \). First we assume that the velocity dispersion of a galaxy is proportional to \( L^n \) (\( n = 0.25 \)). Second, we expect that the luminosity of the galaxies evolves with the red-shift as follows, \( L \propto (1+z)^\alpha \). With these hypotheses the Einstein radius can be re-written as \( \langle r_e \rangle = 4\pi \langle \sigma^*_v \rangle^{2} \langle \Gamma\beta \rangle / c^2 \), where the correction factor, \( \langle \Gamma\beta \rangle \), is calculated following the step in section 5.5.1. We have estimated this parameter for different cases:

1. The analysis is performed using two models of the universe, CDM and \( \Lambda \)-CDM.

2. We suppose that the luminosity of our galaxies can evolve with respect to the red-shift following \( L \propto (1+z)^\alpha \). Assuming that the luminosity evolves, we use \( \alpha = 1 \) (Gabash et al., 2006), and compare with the no-evolution model, \( \alpha = 0 \).

3. Finally, we use three different values for the luminosity \( L^*_I \) of the average characteristic galaxy in the I-band:

   - \( L^*_I = 1.00 \times 10^{10}h^{-2}L_\odot (M_{IAB} = -21.06 + 5\log h) \)
   - \( L^*_I = 1.12 \times 10^{10}h^{-2}L_\odot (M_{IAB} = -21.18 + 5\log h) \)
   - \( L^*_I = 1.27 \times 10^{10}h^{-2}L_\odot (M_{IAB} = -21.32 + 5\log h) \)

The luminosity \( L^*_I = 1.00 \times 10^{10}h^{-2}L_\odot \) is often used as a reference in publications. The second luminosity \( L^*_I \) is given by O. Ilbert et al in the I-band for an average characteristic galaxy in the following z interval [0.05-0.2] (Ilbert et al., 2005). In the
<table>
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<th>$&lt; r_e &gt;$ [arcsec]</th>
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<th>$&lt; \sigma_v^2 &gt;^{1/2}$ [km/s]</th>
<th>Luminosity evolution $\alpha$</th>
<th>Luminosity of the $L^*$ galaxy ($\ast \times 10^{10} h^{-2} L_\odot$)</th>
<th>$&lt; \Gamma_\beta &gt;$</th>
<th>$&lt; \sigma^* &gt;$ [km/s]</th>
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Table 5.3: Average and characteristic velocity dispersion for different cosmological models. Column (a): Einstein radius estimated from our computations using a SIS model (Figure 5.10). (b) models of the universe, CDM with $\Omega_m = 1.$ and $\Omega_\Lambda = 0.$ and $\Lambda$-CDM with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7.$ (c) estimate of $\beta = D_{LS}/D_S$ for both universe models. (d) estimate of the average velocity dispersion of the galaxies in our sample for the CDM and $\Lambda$-CDM models. (e) Value of $\alpha$ in our luminosity evolution model ($\alpha = 0$ no evolution). (f) Galaxies at a reference luminosity $L^*$. (g) estimate of the $< \Gamma_\beta >$ parameter (see section 5.5.1). (h) calculation of the characteristic velocity dispersion of the $L^*$ galaxies.
present study we use this luminosity as a reference. The last value is the estimate of the luminosity in the I-band of an average characteristic galaxy which has a luminosity $L_B^* = 1.00 \times 10^{10} h^{-2} L_\odot$ in the B-band. The parameters deduced using the last luminosity $L_I^*$ are compared with the results from Hoekstra et al. (2004) and Kleinheinrich et al (2005).

For the CDM and $\Lambda$-CDM cosmological models, the $\langle \Gamma_\beta \rangle$ parameter is calculated and $\langle \sigma_v^2 \rangle^{1/2}$ is deduced using equation (5.80). The results are listed in Table 5.3.

If the luminosity does not evolve, we obtain a characteristic luminosity $L_I^* = 1.12 \times 10^{10} h^{-2} L_\odot$, a characteristic velocity dispersion $\langle \sigma_v^2 \rangle^{1/2} = 102^{+09}_{-10} (116^{+11}_{-12})$ km/s for the $\Lambda$-CDM (CDM) model. If the luminosity evolves, we have $\langle \sigma_v^2 \rangle^{1/2} = 111^{+10}_{-11} (126^{+11}_{-12})$ km/s for the same two models of the universe.

We compare our results with those obtained by Hoekstra, Yee and Gladders (2004) and Kleinheinrich et al (2005). Hoekstra et al (2004) used $R_C$-band imaging data from the Red-Sequence Cluster Survey. The red-shift distribution of their lens galaxies yielded a median red-shift $z = 0.35$. We work in the I-band and according to our estimates the red-shift distribution of our sample of galaxies is at a median red-shift $z = 0.6$. As a result of that study Kleinheinrich et al. (2005) determined a red-shift interval in which their lens galaxies are distributed ($z \in [0.2; 0.7]$). Hoekstra et al (2004) found a velocity dispersion $\langle \sigma_v^2 \rangle^{1/2} = 140 \pm 4$ km/s for an average characteristic galaxy with $L_B^* = 10^{10} h^{-2} L_\odot$, assuming that the luminosity does not evolve with the red-shift. If the luminosity evolves, they found $\langle \sigma_v^2 \rangle^{1/2} = 150 \pm 4$ km/s. In both cases the Einstein radius is $\langle r_e \rangle = 0.14 \pm 0.01 \arcsec$. Kleinheinrich et al (2005) obtained $\langle \sigma_v^2 \rangle^{1/2} = 138^{+18}_{-22}$ km/s for a characteristic luminosity $L_B^* = 1.25 \times 10^{10} h^{-2} L_\odot$ using the $\Lambda$-CDM model. $L_B^*$ is the estimate of $L_B^*$, used by Hoekstra, in the r-band, without accounting for an evolution of the luminosity with the red-shift. In the present study we work in the I-band and the estimate of $L_B^*$ in this band is $L_I^* = 1.27 \times 10^{10} h^{-2} L_\odot$. If we do not consider a luminosity evolution for $L_B^* = 1.27 \times 10^{10} h^{-2} L_\odot$ and if we work within the $\Lambda$-CDM model we calculate $\langle \sigma_v^2 \rangle^{1/2} = 106^{+10}_{-11}$ km/s. If, in addition, the luminosity evolves, the velocity dispersion becomes $\langle \sigma_v^2 \rangle^{1/2} = 115^{+10}_{-11}$ km/s. Our results are in agreement with those of Hoekstra et al. (2004) and Kleinheinrich et al. (2005).

### 5.5.2 NFW Profile

The NFW model is characterized by a density profile given by $\rho(r) = \delta_c \rho_c / (r/r_s(1 + r/r_s)^2)$ where $\delta_c$ is the density contrast. $\delta_c$ is related to the concentration $c$ via equation (5.43). $r_s$ is the characteristic scale radius where the profile of the density changes from a quantity proportional to $r^{-1}$ to another, proportional to $r^{-3}$. For the purpose of this study the following parameter has been introduced, $r_{200} = c r_s$. It corresponds to the radius within which the mean density is 200 times the mean density of the universe. With these assumptions, the tangential shear profile, produced by a lens galaxy that follows the NFW density profile, can be defined and compared to the tangential shear profile extracted from our data. Such an analysis requires the knowledge of few parameters. First we assume that the maximum rotation velocity in a galaxy and the luminosity of the galaxy are related, $V_{max} \propto L^\zeta$. We adopt a similar equation to relate the mass
and the luminosity, $M \propto L^\nu$. Finally we assume that the density contrast, $\delta_c$, and the concentration $c$ are related by, $\delta_c \propto c^\omega$. Knowing $\zeta$, $\iota$ and $\varrho$, we can deduce the scaling relation between $r_s$ and the luminosity, $r_s \propto L^\nu$. A similar formalism can be applied to $\delta_c$, $c$ and $V_{200}$. We have $\delta_c \propto L^\tau$, $c \propto L^\omega$ and $V_{200} \propto L^\eta$ where $\nu = (\zeta\varrho - 6\iota)/[3(\varrho - 2)]$, $\tau = 2(3\varrho\zeta - \varrho\iota)/[3(\varrho - 2)]$, $\omega = 2(3\zeta - \iota)/[3(\varrho - 2)]$ and $\eta = \zeta/3$. To determine these parameters ($\nu$, $\tau$, $\omega$ and $\eta$), the equations presented in section 5.3.2 are used. $r_s$ and $V_{200}$ have been chosen to be free parameters in our galaxy sample, we express $V_{200}$ and $r_s$ as a function of $V^*_{200}$ and $r^*_s$, respectively. The latter two parameters characterize an average galaxy with a luminosity $L^*$ and they will be our free parameters for the $\chi^2$ analysis.

Next, we employ scaling relations for $r_s$, $V_{200}$, $\delta_c$ and $c$. We also assume that the luminosity of the lens galaxies evolves with the red-shift ($L(z) = L^*(1 + z)^\alpha$). We have the following expressions,

$$r_s = r^*_s \left( \frac{L(z)}{L^*} \right)^\nu (1 + z)^{-\alpha \nu}$$
$$V_{200} = V^*_{200} \left( \frac{L(z)}{L^*} \right)^\eta (1 + z)^{-\alpha \eta}$$
$$\delta_c = \delta^*_c \left( \frac{L(z)}{L^*} \right)^\tau (1 + z)^{-\alpha \tau}$$
$$c = c^* \left( \frac{L(z)}{L^*} \right)^\omega (1 + z)^{-\alpha \omega}$$

We need now to express the NFW shear profile (equation 5.52) in terms of $V^*_{200}$ and $r^*_s$. The critical surface mass density, $\Sigma_c$, is given by (Schneider et al. 1992):

$$\Sigma_c = \frac{c'^2}{4\pi GD_L \beta}$$

where $G$ is the gravitational constant, $c'$ is the speed of the light, $\beta$ is the ratio between the angular diameter distance from the lens to the source, and the angular diameter distance from the observer to the source. $D_L$ is the angular diameter distance from the observer to the lens.

The shear, $\gamma_{NFW}$, in the Einstein-de Sitter model (CDM model)

Next, the $\Omega = 1$ CDM model is used to define the angular diameter distances. We have $D_L$ in pc:

$$D_L = \frac{20c'_o}{h} \frac{1}{1 + z_l} \left( 1 - \frac{1}{\sqrt{1 + z_l}} \right)$$

where $c'_o$ is the speed of the light in m/s, $h$ is the dimensionless Hubble constant and $z_l$ is the red-shift of the lens.

Using equations (5.45) and (5.46), we can express the numerator $r_s\delta_c \rho_c$ of equation 5.52 as follows,

$$r_s\delta_c \rho_c = \frac{3\delta_c}{800\pi Gc^2} \frac{V^2_{200}}{r_s}$$
Then the shear becomes,

\[ \gamma_{\text{NFW}}(x) = \frac{3\delta_c \beta}{10 c'_0 h^2 \frac{r_s}{r_s}} \frac{V_{200}^2}{r_s} \frac{1}{1 + z_l} \left( 1 - \frac{1}{\sqrt{1 + z_l}} \right) g(x) \] (5.88)

Furthermore we can express \( \gamma_{\text{NFW}} \) as a function of, amongst other parameters, the scale radius, \( r_s^* \), and the rotation velocity in the galaxy at virial radius, \( V_{200}^* \) of an average characteristic galaxy (see equations 5.81 and 5.82). We obtain:

\[ \gamma_{\text{NFW}}(x) = \frac{3 \beta \delta_c^*}{10 c'_0 h c^*} \frac{V_{200}^*}{r_s^*} \left( 1 - \frac{1}{\sqrt{1 + z_l}} \right) \left( \frac{L(z_l)}{L^*} \right)^{2\eta + \tau - 2\omega} \]
\[ \cdots \cdots (1 + z)^{\alpha(2\omega + \tau - 2\eta) - 1} g(x) \] (5.89)

where \( x = D_L \theta/r_s \) by definition. Employing equation (5.81) and (5.86), we can re-write \( x \) as follows:

\[ x = \frac{20 c'_0 \theta}{h} \frac{1}{r_s^*} \frac{1}{1 + z_l} \left( 1 - \frac{1}{\sqrt{1 + z_l}} \right) \left( \frac{L(z_l)}{L^*} \right)^{-\alpha} \] (5.90)

It was shown in section 5.5.1 that the ratio \( L(z_l)/L^* \) is given by:

\[ \frac{L(z_l)}{L^*} = 10^{-0.4(m - M^*)} \left[ \frac{2 c}{h} \left( 1 - \frac{1}{\sqrt{1 + z_l}} \right) \right]^2 (1 + z_l)^{\lambda + 1} \] (5.91)

and if we assume in addition that we work in an Einstein-De Sitter universe (\( \Omega = 1 \)) the expression for the estimate of \( \beta \) takes the following form:

\[ \beta(z_l, z_s) = \max \left( 0, 1 - \frac{1 - (1 + z_l)^{-1/2}}{1 - (1 + z_s)^{-1/2}} \right) \] (5.92)

The shear, \( \gamma_{\text{NFW}} \), in the \( \Lambda \)-CDM model

In the case of the \( \Lambda \)-CDM model we use the general form of the angular diameter distances, \( D_L, D_S \) and \( D_{LS} \) defined in Section 5.3.1 (equations 5.35 and 5.36). The integration of these parameters is performed numerically. Following the same principle as in the section above, we can express the shear, \( \gamma_{\text{NFW}} \), as a function of two free parameters: the scale radius, \( r_s^* \), and the rotation velocity in the galaxy at virial radius, \( V_{200}^* \).

Estimate of parameters which characterize the galaxies using the NFW profile

By making use of the relations above, we can determine the average shear produced by a group of lens galaxies on an ensemble of source galaxies for a given pair \( (r_s^*, V_{200}^*) \) for both the \( \Omega = 1 \) CDM and \( \Lambda \)-CDM models. The average shear, \( \overline{\gamma}(x) \), is calculated using a simulated Monte-Carlo catalogue in which the lens and source galaxies follow the red-shift distributions defined in section 5.5.1. A \( \chi^2 \) minimization analysis is used to estimate the best profile. The \( \chi^2 \) of the average shear, \( \overline{\gamma}_{\text{NFW}}(x) \), versus \( \overline{\gamma}(x) \) determined from the real data, is obtained for different \( r_s^* \) and \( V_{200}^* \). The \( \Delta \chi^2 \) difference is calculated within
the $V_{200}^* - r_s^*$ basis and it corresponds to the difference of $\chi^2$ and $\chi_{\text{min}}^2$ at any point. The contours which correspond to the 68%, 95% and 99% confidence level, respectively are plotted (e.g. Figure 5.18). The calculation of the errors for $r_s^*, M_{200}^*$ and $c^*$ is based on the values extracted at 68% confidence level. Once $V_{200}^*$ and $r_s^*$ are estimated we can deduce $c^*$, $r_{200}^*$ and $M_{200}^*$ using the following relations,

$$r_{200}^* [h^{-1}\text{kpc}] = V_{200}^* [\text{km/s}]$$  \hspace{1cm} (5.93)

$$M_{200}^*[h^{-1}\text{M}_\odot] = 2.324 \times 10^5 \left(V_{200}^* [\text{km/s}]\right)^3$$  \hspace{1cm} (5.94)

$$c^* = r_{200}^*/r_s^*$$  \hspace{1cm} (5.95)

Results

We assume that each galaxy follows a mass density profile described by the NFW model. Since we know the red-shift distribution of the lens and source galaxies we can build a catalog of pairs (lens-source galaxies) using the Monte-Carlo method. This catalog allows us to predict the average shear produced by lens galaxies in the optical images of the source galaxies at different radii around the lens galaxies. To estimate the best fit of the theoretical shear profile, two steps are undertaken. First, we express the theoretical shear for two cosmological models ($\Omega = 1$ CDM and $\Lambda$-CDM), $\gamma_{\text{NFW}}^{th}$, as a function of various parameters characterizing an average characteristic galaxy. To realize this step we use the scaling relations defined in section 5.5.2 (equations 5.81 - 5.84). The second step consists of adjusting our theoretical shear profile to the one determined from the 50 FORS I images by retaining two of the parameters free. These free parameters are the virial rotation velocity, $V_{200}^*$, and the characteristic radius, $r_s^*$. A $\chi^2$ minimization analysis is performed to find the theoretical shear, $\gamma_{\text{NFW}}^{th}$, as a function of $V_{200}^*$ and $r_s^*$ which fits best our data. Once these two parameters are known, the average characteristic galaxy can be fully characterized.
Table 5.4: Estimate of parameters which characterize a galaxy as a reference luminosity. The first four columns contain fixed parameters used to estimate the remaining parameters. Column (a) is the power factor in the relation between the rotation velocity at a fixed radius, and the luminosity, \( r \propto L \). Column (b) is the power factor in the relation between the rotation velocity at a fixed radius, and the luminosity, \( r \propto L^\star \). Column (c) is the power factor in the relation between the rotation velocity at a fixed radius, and the luminosity, \( r \propto L^{\star + \delta} \). Column (d) is the power factor in the relation between the rotation velocity at a fixed radius, and the luminosity, \( r \propto L^{\star + \delta + \epsilon} \). Column (e) is the power factor in the relation between the rotation velocity at a fixed radius, and the luminosity, \( r \propto L^{\star + \delta + \epsilon + \zeta} \). Column (f) is the power factor in the relation between the rotation velocity at a fixed radius, and the luminosity, \( r \propto L^{\star + \delta + \epsilon + \zeta + \eta} \). Column (g) is the power factor in the relation between the rotation velocity at a fixed radius, and the luminosity, \( r \propto L^{\star + \delta + \epsilon + \zeta + \eta + \chi} \). Column (h) is the power factor in the relation between the rotation velocity at a fixed radius, and the luminosity, \( r \propto L^{\star + \delta + \epsilon + \zeta + \eta + \chi + \nu} \). Column (i) is the power factor in the relation between the rotation velocity at a fixed radius, and the luminosity, \( r \propto L^{\star + \delta + \epsilon + \zeta + \eta + \chi + \nu + \tau} \). Column (j) is the power factor in the relation between the rotation velocity at a fixed radius, and the luminosity, \( r \propto L^{\star + \delta + \epsilon + \zeta + \eta + \chi + \nu + \tau + \rho} \). Column (k) is the power factor in the relation between the rotation velocity at a fixed radius, and the luminosity, \( r \propto L^{\star + \delta + \epsilon + \zeta + \eta + \chi + \nu + \tau + \rho + \omega} \). Column (l) is the power factor in the relation between the rotation velocity at a fixed radius, and the luminosity, \( r \propto L^{\star + \delta + \epsilon + \zeta + \eta + \chi + \nu + \tau + \rho + \omega + \chi} \). Column (m) is the power factor in the relation between the rotation velocity at a fixed radius, and the luminosity, \( r \propto L^{\star + \delta + \epsilon + \zeta + \eta + \chi + \nu + \tau + \rho + \omega + \chi + \nu} \). Column (n) is the power factor in the relation between the rotation velocity at a fixed radius, and the luminosity, \( r \propto L^{\star + \delta + \epsilon + \zeta + \eta + \chi + \nu + \tau + \rho + \omega + \chi + \nu + \tau} \).
Table 5.5: Estimate of parameters which characterize a galaxy at a reference luminosity $L^*$ using the NFW profile and the $\Lambda$-CDM model. The legend of this table is the same as that of Table 5.4.

<table>
<thead>
<tr>
<th>$\zeta$</th>
<th>$\alpha$</th>
<th>$\iota$</th>
<th>$L^*$</th>
<th>$\omega$</th>
<th>$\tau$</th>
<th>$\eta$</th>
<th>$\nu$</th>
<th>$\chi^2$</th>
<th>$V_{200}^*$</th>
<th>$r_s^*$</th>
<th>$c^*$</th>
<th>$r_{200}^*$</th>
<th>$M_{200}^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
<td>(f)</td>
<td>(g)</td>
<td>(h)</td>
<td>(i)</td>
<td>(j)</td>
<td>(k)</td>
<td>(l)</td>
<td>(m)</td>
<td>(n)</td>
</tr>
<tr>
<td>0</td>
<td>1.12</td>
<td>-0.41</td>
<td>-1.13</td>
<td>0.4</td>
<td>0.81</td>
<td></td>
<td></td>
<td>5.1</td>
<td>$115^{+42}_{-33}$</td>
<td>$07^{+15}_{-04}$</td>
<td>$16^{+41}_{-14}$</td>
<td>$115^{+42}_{-33}$</td>
<td>0.35 +0.39</td>
</tr>
<tr>
<td>0.25</td>
<td>1.4</td>
<td>-0.6</td>
<td>-1.63</td>
<td>0.47</td>
<td>1.06</td>
<td></td>
<td></td>
<td>4.9</td>
<td>$115^{+57}_{-35}$</td>
<td>$06^{+19}_{-03}$</td>
<td>$19^{+70}_{-15}$</td>
<td>$115^{+57}_{-35}$</td>
<td>0.35 +0.53</td>
</tr>
<tr>
<td>1</td>
<td>1.12</td>
<td>-0.41</td>
<td>-1.13</td>
<td>0.4</td>
<td>0.81</td>
<td></td>
<td></td>
<td>5.0</td>
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<td>$151^{+48}_{-46}$</td>
<td>0.80 +0.76</td>
</tr>
<tr>
<td>1.4</td>
<td>1.27</td>
<td>-0.6</td>
<td>-1.63</td>
<td>0.47</td>
<td>1.06</td>
<td></td>
<td></td>
<td>4.9</td>
<td>$162^{+72}_{-53}$</td>
<td>$15^{+51}_{-11}$</td>
<td>$11^{+34}_{-11}$</td>
<td>$162^{+72}_{-53}$</td>
<td>0.99 +13.2</td>
</tr>
</tbody>
</table>

Table 5.5: Estimate of parameters which characterize a galaxy at a reference luminosity $L^*$ using the NFW profile and the $\Lambda$-CDM model. The legend of this table is the same as that of Table 5.4.
To realize the galaxy-galaxy lensing study few parameters that appear above have to be fixed, namely ζ and ι. In section 5.5.2, it has been shown that the power factors in the scaling relations for $r_s$, $V_{200}$, $\delta_c$ and $c$ are themselves functions of those other two parameters, namely the power factor $\zeta$ in the relation $V_{\text{max}} \propto L^\zeta$ which expresses the maximum velocity in terms of the luminosity and the power factor $\iota$ in the relation $M \propto L^\iota$ which expresses the mass within a radius of 200 kpc as a function of the luminosity.

- We assume that the maximum rotation velocity is proportional to the luminosity to the power of 0.25 ($\zeta = 0.25$ in the relation, $V_{\text{max}} \propto L^\zeta$). This value is commonly accepted in the I-band.

- The second assumption concerns the power factor of the virial mass ($\iota$). We use two values, $\iota = 1.2$ and $1.4$. The use of the lower limit, 1.2, is motivated by the work of Kleinheinrich et al. (2005b) in which the power factor is 1.14 and 1.16 in R-band. Based on their work we suppose that in the I-band $\iota$ can take a value not lower than 1.2. The upper limit comes from the study of Gulzik and Seljak (2002) in which they obtain $\iota = 1.4 \pm 0.15$ in the I-band.

Using these two parameters, $\zeta$ and $\iota$, the power factors of $r_s$, $V_{200}$, $\delta_c$ and $c$ can be deduced (see table 5.4).

- In our study we take into consideration the possibility that the luminosity of the galaxies evolves with the red-shift as follows $L \propto L_o(1 + z)^\alpha$ (Hoekstra et al., 2004) and hence the data are reduced assuming two cases. In the first case the luminosity does not evolve as a function of the red-shift ($\alpha = 0$) while in the second case we suppose that the luminosity evolves with the red-shift ($\alpha = 1$).

- Finally we carry out our study for an average characteristic galaxy for which $L^*$ takes three different values.

\[
\begin{align*}
L_1^* &= 1.12 \times 10^{10} h^{-2} L_\odot \quad (M_{IAB} = -21.18 + 5 \log h) \\
L_2^* &= 1.27 \times 10^{10} h^{-2} L_\odot \quad (M_{IAB} = -21.32 + 5 \log h) \\
L_3^* &= 2.05 \times 10^{10} h^{-2} L_\odot \quad (M_{IAB} = -21.84 + 5 \log h)
\end{align*}
\]

The first value is used as a reference in this study. This value is obtained by O. Ilbert who has studied the evolution of the galaxy luminosity with the red-shift up to $z = 2$, using the data from the VIMOS-VLT Survey (Ilbert et al., 2005). The second luminosity value is used to facilitate the comparison of our results with those of Hoekstra et al. (2004) who have worked with a luminosity $L_{IAB}^* = 10^{10} h^{-2}$ in the B-band (Hoekstra et al., 2004). We convert the luminosity used by those authors from the B to the I-band, $L_I^* = 1.27 \times 10^{10} h^{-2} L_\odot$. The last luminosity value chosen for an average characteristic galaxy is the value obtained by J. Gulzik in the I-band $L_I^* = 2.05 \times 10^{10} h^{-2} L_\odot$ (Guzik and Seljak, 2002).

For each combination of parameters given above, a $\chi^2$ minimization analysis is performed to extract $V_{200}^*$ and $r_s^*$. From $V_{200}^*$ and $r_s^*$ we deduce the concentration, $c^*$, the virial radius, $R_{200}^*$, and the virial mass, $M_{200}^*$. The results are listed in Table 5.4 for the Einstein-de Sitter model and in Table 5.5 for the Λ-CDM model.
To analyze our data, we distinguish two cases. First we assume that the maximum velocity, \( V_{\text{max}} \), scales proportional to \( L_I^{0.25} \) and the luminosity does not evolve with the red-shift \( (\alpha = 0 \text{ in } L = L_0 (1 + z)^\alpha) \). In that case, for two different values of the power factor, \( [\iota = 1.4 (1.2)] \) in the relation \( M_{200} \propto L^\iota \), a \( \chi^2 \) minimization analysis is carried out which yields \( V_{200}^* = 142^{+64}_{-50} (140^{+50}_{-35}) \, \text{km/s} \) and \( r_s^* = 10^{+27}_{-07} (10^{+19}_{-05}) \, h^{-1}\text{kpc} \). Since the factor \( \iota \) has a small impact on our estimates, the following values are given only for \( \iota = 1.4 \). Knowing \( V_{200}^* \) and \( r_s^* \) we deduce the concentration \( c^* = 14^{+45}_{-14} \), the virial radius, \( r_{200}^* = 142^{+64}_{-50} \, h^{-1}\text{kpc} \) and the virial mass \( M_{200}^* = 6.6^{+9.6}_{-6.6} \times 10^{11} \, h^{-1}\text{kpc} \) for the Einstein-de Sitter model. For the \( \Lambda \)-CDM model we obtain \( V_{200}^* = 115^{+57}_{-35} \, \text{km/s} \) and \( r_s^* = 06^{+19}_{-03} h^{-1}\text{kpc} \), \( c^* = 19^{+70}_{-15} \), \( r_{200}^* = 115^{+57}_{-35} \, h^{-1}\text{kpc} \) and \( M_{200}^* = 3.5^{+5.5}_{-3.2} \times 10^{11} \, h^{-1}\text{kpc} \).

To compare our results with those of Hoekstra et al. (2004) and Kleinheinrich et al. (2005) who have performed a similar analysis, we need to use an average characteristic galaxy with a luminosity \( L_I^* = 1.27 \times 10^{10} h^{-2} L_\odot \) where \( L_I^* \) is the luminosity of a galaxy in the I-band which has a luminosity \( L_B^* = 10^{10} h^{-2} L_\odot \) in the B-band. Both authors have estimated the necessary parameters based on \( L_B^* \). Hoekstra et al. (2004) have found without accounting for luminosity evolution, \( V_{200} = 162 \pm 5 \, \text{km/s} \), \( r_s^* = 16.2 \pm 3.6 \, h^{-1}\text{kpc} \) and \( M_{200}^* = 8.4 \pm 0.7 \times 10^{11} \, h^{-1}\text{kpc} \) and Kleinheinrich et al. (2005) have obtained for the \( \Lambda \)-CDM model, \( M_{200}^* = 8.0^{+3.9}_{-3.0} \times 10^{11} \, h^{-1}\text{kpc} \). Our computations are in agreement with their results. Using this \( L_I^* \) luminosity we determine \( V_{200}^* = 152^{+64}_{-36} \, \text{km/s} \), \( r_s^* = 12^{+31}_{-09} \, h^{-1}\text{kpc} \) and \( M_{200}^* = 8.2^{+10.0}_{-8.2} \times 10^{11} \, h^{-1}\text{kpc} \) for the Einstein-de Sitter model and \( V_{200} = 126^{+51}_{-43} \, \text{km/s} \), \( r_s^* = 09^{+30}_{-20} \, h^{-1}\text{kpc} \) and \( M_{200} = 4.6^{+5.6}_{-4.0} \times 10^{11} \, h^{-1}\text{kpc} \) for the \( \Lambda \)-CDM model, respectively. A similar work has been done on the data obtained by Guzik et al who have found for \( L_I^* = 2.05 \pm 0.08 \times 10^{10} h^{-2} L_\odot \) and \( \iota = 1.4 \), \( M^* = 7.16^{+1.53}_{-0.35} \times 10^{11} \, h^{-1}\text{M}_\odot \). Our values for the \( L_I^* \) luminosity and factor \( \iota \) result into \( M_{200}^* = 14.9^{+16.4}_{-14.9} \times 10^{11} \, h^{-1}\text{M}_\odot \) for the \( \Omega = 1 \) CDM model and \( M_{200}^* = 6.5^{+8.9}_{-6.0} \times 10^{11} \, h^{-1}\text{M}_\odot \) for the \( \Lambda \)-CDM model. The mass appears to be twice as large compared to that quantity obtained by Guzik et al. for the Einstein-de Sitter model and it is in accordance for the \( \Lambda \)-CDM model. Note however that at fixed \( \iota = 1 \), Guzik et al. have determined \( M^* = 15. \times 10^{11} \, h^{-1}\text{M}_\odot \) in the i’-band, but their fit occurs to be worse when using this value of \( \iota \).

Next, we assume that the luminosity evolves with the red-shift \( (\alpha = 1) \). The relation between the maximum velocity and the luminosity remains unchanged. Upon these conditions we obtain for the Einstein-de Sitter model, \( V_{200} = 184^{+72}_{-52} \, \text{km/s} \) and \( r_s^* = 17^{+46}_{-12} \, h^{-1}\text{kpc} \) and for the \( \Lambda \)-CDM model we have \( V_{200}^* = 159^{+64}_{-52} \, \text{km/s} \) and \( r_s^* = 15^{+42}_{-11} \, h^{-1}\text{kpc} \). The results are reported for \( \iota = 1.4 \) because this parameter does not have a significant impact on the estimate of \( V_{200} \) and \( r_s^* \). Furthermore, we deduce \( c^* = 11^{+34}_{-11} \), \( r_{200}^* = 184^{+72}_{-52} \, h^{-1}\text{kpc} \) and \( M_{200}^* = 14.5^{+17.0}_{-12.3} \times 10^{11} \, h^{-1}\text{kpc} \) for the Einstein-de Sitter model and \( c^* = 11^{+34}_{-11} \), \( r_{200}^* = 159^{+62}_{-52} \, h^{-1}\text{kpc} \) and \( M_{200}^* = 9.3^{+11.3}_{-0.91} \times 10^{11} \, h^{-1}\text{kpc} \) for the \( \Lambda \)-CDM model, respectively. Hoekstra et al. (2004) have also performed this analysis assuming that the luminosity evolves proportional to \( (1 + z) \). They have found for \( V_{200}^* = 176^{+5}_{-4} \, \text{km/s} \) and for \( r_s^* = 17.2^{+3.8}_{-3.1} \, h^{-1}\text{kpc} \). Employing their luminosity value we obtain, \( V_{200} = 191^{+77}_{-65} \, \text{km/s} \) and \( r_s^* = 18^{+13}_{-11} \, h^{-1}\text{kpc} \) for the \( \Omega = 1 \) CDM model and \( V_{200}^* = 169^{+72}_{-53} \, \text{km/s} \) and \( r_s^* = 15^{+51}_{-11} \, h^{-1}\text{kpc} \) for the \( \Lambda \)-CDM model, respectively. Our results are in agreement with their calculations.

Although the results discussed above are in good agreement with other calculations available, we still observe rather large error bars. Additional considerations show that
the $\chi^2$ minimization analysis is very sensitive to the scattering of the average tangential shear, $\bar{\gamma}_t$, with respect to the model employed. Hoekstra et al. (2004) have obtained a very good constraint of $V_{200}^*$ and $r_s^*$. Their theoretical model fits better their average tangential shear profile compared to the shear profile obtained from our analysis. This is due to the large sky area within which they performed the survey. Their data covers 45.5 squared degree which area provides them with $\sim 1.5 \times 10^6$ source galaxies while in our study only 0.64 deg$^2$ area is probed which contains $\sim 2.38 \times 10^4$ background galaxies. We clearly see here the importance of a large survey on the study of the galaxy-galaxy lensing. From another point of view our analysis shows that, even if the data are noisy, such analysis can be carried out successfully. We also have to keep in mind that the red-shift information is crucial to constrain better the parameters we have determined (Schneider and Rix, 1997; Kleinheinrich et al., 2005a). The lack of enough information, in our case, contributes significantly to the large error bars we obtain.

5.6 Conclusions

In this chapter, a galaxy-galaxy lensing study has been performed on 50 FORS1 deep optical images. All 50 images are combinations of 6 images with 6 minutes exposure time taken through an I-band filter. The total sky area covered by this survey is around 0.64 deg$^2$. Because of the very good quality of the images and very good seeing ($<0.6''$), a galaxy-galaxy lensing signal is expected to be found in these data. To perform the galaxy-galaxy lensing analysis, a preliminary work has to be done which consists of extracting the objects and applying a point spread function correction to them. The IMCAT object extractor is used to create for each image a catalog in which the position, size and magnitude of the objects are listed. A second procedure from the IMCAT package refines the measurements of the objects magnitude and the size and calculates the components of the ellipticity, smear and shear polarizability tensors, $e_{ij}$, $(P_{sm})_{ij}$ and $(P_{sh})_{ij}$. Using these parameters, we follow the standard KSB procedure to correct the PSF, which consists initially of extracting catalogs of stars and galaxies from the main catalogs. The stars are selected using the diagram (radius, magnitude), see figure 5.7, for example. The galaxies are objects with a radius larger than the maximum radius that is used as a threshold to select the stars. From the stars parameters we estimate the PSF correction that has to be applied to the ellipticity of the galaxies to correct the defects caused by the atmosphere and the telescope. Next, we extract from each galaxy an estimate of the shear.

Furthermore we need to make a distinction between the lens and source galaxies to perform a galaxy-galaxy lensing analysis. After a statistical study, we concluded that the foreground galaxies are objects with an AB magnitude between 18 and 22.7 while the magnitudes for the background galaxies vary between 23.2 and 26.5. For each lens galaxy we select background galaxies which are at an angular distance inferior to 120 arcsecs from the lens galaxy. The shear ($\tilde{\gamma}_1$, $\tilde{\gamma}_2$) components of the source galaxies are converted to ($\tilde{\gamma}_t$, $\tilde{\gamma}_x$) also called tangential and orthogonal shear. In a weak galaxy-galaxy lensing study the shear signal is present only in the tangential shear. The problem is that $\tilde{\gamma}_t$ is very small compared to the intrinsic ellipticity of the source galaxies and it is usually strongly affected by noise. Therefore, we need to combine the shears from a large number of source galaxies to observe a real shear signal. We succeeded in extracting a significant average shear signal produced by the lens galaxies on source galaxies (see figure 5.10 and
The average tangential shear signal is however still affected by noise and hence, the error bars appear to be rather large. The poor accuracy is due to the small area covered by the observations as well as the small field of view and the lack of red-shift information. Nevertheless the analyses were carried out successfully and the results have shown that the applied procedure for extracting the average shear signal is rather robust.

To study the dark matter halo properties of the selected lens galaxies, we fit two models to the average shear signal, the singular isothermal sphere (SIS) model and the NFW model. We obtain an Einstein radius of \( r_e = 0.11 \pm 0.02'' \) corresponding to an average lens velocity dispersion of \( \langle \sigma_v \rangle^{1/2} = 120^{+11}_{-12} \text{ km/s} \) in the CDM model and \( \langle \sigma_v \rangle^{1/2} = 113^{+10}_{-11} \text{ km/s} \) in \( \Lambda \) CDM, respectively. By adopting scaling relations, we express \( \langle \sigma_v \rangle^{1/2} \) in terms of the velocity dispersion of a characteristic galaxy at a reference luminosity \( L_I^* = 1.12 \times 10^{10} h^{-2} L_\odot \) (Ilbert et al., 2005). Assuming that the luminosity does not evolve as a function of the red-shift, we determined \( \langle \sigma_v^* \rangle^{1/2} = 116^{+12}_{-11} \text{ km/s} \) in a \( \Omega = 1 \) CDM universe and \( \langle \sigma_v^* \rangle^{1/2} = 102^{+09}_{-10} \text{ km/s} \) in a \( \Lambda \) CDM universe. If the luminosity evolves, we obtained, for the \( \Omega = 1 \) CDM model \( \langle \sigma_v^* \rangle^{1/2} = 126^{+11}_{-12} \text{ km/s} \) and for the \( \Lambda \) CDM model \( \langle \sigma_v^* \rangle^{1/2} = 111^{+10}_{-11} \text{ km/s} \).

The second model used to study the dark matter halo of our galaxy sample is the NFW model. Knowing the red-shift distribution of the foreground and background galaxies, a catalog of pairs (lens-source galaxies) is created employing the Monte-Carlo method. Using this catalog we estimated the theoretical shear produced by the lens galaxies. A chi-square minimization method is used to obtain the best fit rotation velocity at virial radius, \( V_{200}^* \), and the characteristic scale radius, \( r_s^* \). We assumed that \( V_{\max}^* \propto L^{0.25}, M_{200}^* \propto L^{1.4} \) and the luminosity does not evolve \( (\alpha = 0 \text{ in } L(z) = L(1+z)^\alpha) \). Under these conditions the \( \chi^2 \) analysis yielded for the virial velocity dispersion \( V_{200}^* = 142^{+64}_{-56} \text{ km/s} \) and for the scaling radius \( r_s^* = 10^{+27}_{-07} h^{-1} \text{ kpc} \) at \( L_I^* = 1.12 \times 10^{10} h^{-2} L_\odot \) for the Einstein-de Sitter universe. For the \( \Lambda \) CDM model we obtain \( V_{200}^* = 115^{+57}_{-35} \text{ km/s} \) and \( r_s^* = 06^{+19}_{-03} h^{-1} \text{ kpc} \). In addition if the luminosity evolves \( (\alpha = 1) \), we measured \( V_{200}^* = 184^{+72}_{-52} \text{ km/s} \) and \( r_s^* = 17^{+46}_{-12} h^{-1} \text{ kpc} \) for the Einstein-de Sitter model and \( V_{200}^* = 159^{+64}_{-52} \text{ km/s} \) and \( r_s^* = 15^{+42}_{-11} h^{-1} \text{ kpc} \) for the \( \Lambda \) CDM model. From these values, \( c^*, r_s^*, M_{200}^* \) were deduced.

To conclude, we extracted successfully a galaxy-galaxy lensing signal from 50 uncorrelated FORS1 VLT fields. From these images, we were able to characterize the dark matter halos of I-band selected galaxies and found similar results compared to those obtained by other surveys. Although the results are affected by noise, we have demonstrated that the procedure is robust and we expect that analyses of surveys such as KiDs, with the OmegaCam VST telescope system, will improve drastically the accuracy of the results and hence it will give important constraints on the dark matter halos of observed lens galaxies.