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ONE VERSUS MANY INTENDED APPLICATIONS
REPLY TO SJOERD ZWART

I am grateful to Sjoerd Zwart for the fact that he elaborated for the present purposes a point of view already present in Zwart (1998/2001), that may be representative for many logicians dealing with problems in the philosophy of science. His main claim is that I am mixing up what he calls “logical models” and “physical models.” From my point of view, however, the main divergence is that between the (dominant) model theoretic and the structuralist perspective, according to which the target of theorizing is one particular “intended application” and a set of “intended applications,” respectively. Although the suggested model theoretic perspective may be dominant, I would like to leave room for the possibility that an alternative model theory will be further developed and become respected, not to replace the present dominant one but as an alternative that is more suitable for certain purposes. However, contrary to Zwart’s suggestion, I do not see this alternative as a non-Tarskian move in some deep sense. Starting from Tarski’s basic definition, which is that of “truth in a structure” (Hodges 1986), there are at least two coherent ways of defining that a theory is true or false, depending on whether one has one or more than one intended application in mind. In this reply I will refrain from trying to give a response to every detail of Zwart’s account. Instead, by focusing on some of his main points and by starting the elaboration of the ICR approach to domain revision, I wish to show that the suggested alternative model theory makes sense. Moreover, readers will be able to form their own opinion about whether Zwart’s exposition of the dominant approach applied to a couple of examples, in particular in Section 4, is illuminating or, which is my opinion, functions as a Procrustean bed, even for examples that do not seem to be very representative of scientific theories and domains, let alone for examples that are representative, such as theories about planetary systems.

Since Zwart in his Section 2 gives an adequate summary of some crucial elements of my approach in ICR, I start immediately by responding to the four problems set out in his Section 3.1 and his questioning of my definition of “strong falsehoods” in Section 3.2.

Like Niiniluoto, Zwart objects to my earlier talking about “possible worlds” that are compatible in some sense, which is indeed problematic as long as one has one target “possible world” in mind. To avoid this connotation, I systematically talk in ICR about ‘conceptual possibilities’ instead. This is a general term, which may in specific contexts refer to possible kinds, states, systems, etc.

By calling it the “ICR-paradox of logical strength,” the second problem may seem more serious. Representing the “realized possibilities” at $t$ and $t'$, later than $t$, by $R(t)$ and $R(t')$, and assuming that some (in a certain sense) new experiments have been performed between $t$ and $t'$, that is, some new possibilities have been realized, it follows that $R(t)$ is a proper subset of $R(t')$. The paradoxical air arises from the fact that, although the evidence has increased in some sense, the linguistic representation of $R(t')$ is weaker than that of $R(t)$. The point is, of course, that these linguistic representations merely represent the theories of $R(t)$ and $R(t')$, respectively, in the logical sense of the set of sentences that are true on all their respective members. On the other hand, the evidence has increased in the sense that the relevant claim “$R(t')$ is a subset of $T$” is stronger than “$R(t)$ is a subset of $T$” (where $T$ represents the target set of nomic possibilities). I fail to see this as a paradox, let alone an interesting one.

Zwart’s third problem amounts to the fact that “the linguistic representation of the experiments [R(t)] logically implies (!) the strongest accepted universal empirical law [S(t)]” (p. 383), due to the fact that $R(t)$ is a subset of $S(t)$. Again there is no problem if we look carefully. Indeed, the linguistic representation of $R(t)$ logically entails the linguistic representation of $S(t)$. However, the claim associated with $R(t)$, viz. “$R(t)$ is a subset of $T$”, does not at all entail the claim associated with $S(t)$, i.e., that it represents a law, viz. “$T$ is a subset of $S(t)$”.

Following Ruttkamp (2002, and her contribution to the companion volume), I like to call the first observation “the problem of overdetermination” of theories by data. As soon as we have performed some experiments, represented by $R(t)$, the linguistic representation of the latter entails an enormous number of theories. Only some of them will be true, in the sense of true of all members of $T$, and, under certain conditions, just one will be the strongest. The task of further experimenting and theorizing is to zero in on true ones or the strongest true one.

Although my distinction between the sets of conceptual possibilities $R(t)$ and $S(t)$ and the claims associated with them seems perfectly clear in ICR, a symbolic distinction may be useful for some specific purposes. With thanks to Roberto
Festa, I mention the following plausible symbolization of the claims. The claims associated with $R(t)$ and $S(t)$ may be denoted, in a transparent way, by $r(t)$ and $s(t)$. Hence, $r(t)$ and $s(t)$ amount to, respectively:

$$r(t) = R(t) \subseteq T \quad s(t) = T \subseteq S(t)$$

If one likes one can also express the data available at $t$ by:

$$d(t) = r(t) \& s(t)$$

From the above explicit definitions and from the following “temporal assumption” about the monotonic nature of data:

$$R(t) \subset R(t') \& S(t') \subset S(t)$$

it immediately follows that $r(t')$ implies $r(t)$ (but not vice versa) and that $s(t')$ implies $s(t)$ (but not vice versa). Hence $d(t')$ implies $d(t)$ (but not vice versa). Of course, as Zwart will not dispute, it also immediately follows from the definitions that $r(t)$ does not imply $s(t)$.

Although essentially terminological, I find the fourth problem mentioned by Zwart, that is, my non-standard definition of a theory being false, the most interesting one. It stimulated me to explicitly talk about an “alternative model theory” in the introduction of this reply (and in the introduction of Part III of the ICR synopsis in this volume), for it enables me to indicate clearly where the dominant and the alternative model theory deviate. For further motivation of the following, see also the section “Some reflections on the definition of a ‘false theory’” in my reply to Burger and Heidema, which was written before the present reply.

I assume Tarski’s definition of a sentence, e.g. a (finitely) axiomatized theory, being either true or false in (or on) a structure of the relevant language. From the point of view of one target intended application, that is, one target structure, say $t$, it is plausible to call a theory true or false depending on whether it is true or false on that structure. Since every sentence gets a truth-value on a structure and assuming that the linguistic representation of $t$, that is, the strongest true theory, is axiomatizable, it will be a complete theory. As long as the target structure is, for some reason or other, not yet clearly determined, but only known to belong to a certain set of structures, say $Z$, it is plausible to call a theory true when it is true on all $Z$-structures, false when it is false on all $Z$-structures, and indeterminate otherwise. By zeroing in from $Z$ to $t$, all true/false-judgements remain the same, and the judgements ‘indeterminate’ can be replaced by either ‘true’ or ‘false’.

However, when the target is a fixed set of structures, $T$, these definitions are no longer plausible. Then it becomes plausible to call a theory true (“as a hypothesis,” ICR, p. 184) when it is true on all $T$-structures, and false otherwise. The reason for the latter is that there apparently exists a counterexample to the claim that the theory is true on all $T$-structures. Note that this definition not only makes sense for theories in the empirical sciences, e.g. about planetary systems, but also for certain kinds of mathematical claims. For example, in group theory
one says that a purported theorem is true when it has been proved on the basis of the definition of a group and false when an appropriate (type) of counterexample has been provided. Of course, there is nothing mysterious about the fact that, according to this definition, a theory and its negation can both be false. However, a theory and its negation cannot both be true.

This brings me immediately to the notion of “strong falsehoods.” The rationale of this terminology also presupposes the alternative perspective. As a matter of fact there are two plausible definitions. Although the first is even more plausible than the second, I restricted attention in ICR to the second because that permitted an adapted “consequence reading” of the relevant truthlikeness clause. But let me start with the first definition. A theory is strongly false in the first sense (sf1) if it is not merely false in the sense of being false on some $T$-structure, but being false on all $T$-structures. A theory is strongly false in the second sense (sf2) if it is non-tautological and true on all non-$T$-structures or, equivalently, not merely false (in the sense of being false on some $T$-structure), but in addition true on all non-$T$-structures. From the dominant perspective both definitions make little (new) sense. For the first holds that, whether or not the target structure is already fully fixed, all and only false theories, that is, false in the dominant sense, would become sf1. In other words, sf1 coincides with false in the dominant sense. For the second holds that the definition is empty for a fixed target structure; and only indeterminate theories become sf2, viz. those theories that are only false on some $Z$-structures. Of course, as Zwart clearly illustrates, if you mix up two sets of definitions it is plausible that you may get queer results. For example, sf2-theories are not false in the dominant sense, but indeterminate; however, as indicated, they are not merely indeterminate.

**Truth Approximation by Revision of the Domain of Intended Applications**

I would like to take the opportunity to elaborate a point that was put forward by Sjoerd Zwart earlier (1998/2001). Let me start by quoting from ICR (p. 207).

Finally, variable domains can also be taken into account, where the main changes concern extensions and restrictions. We will not study this issue, but see (Zwart 1998[2001], Ch. 2-4) for some illuminating elaborations in this connection, among other things, the way in which strengthening/weakening of a theory and extending/reducing its domain interact.

More specifically, I propose a coherent set of definitions of ‘more truthlikeness’, ‘empirical progress’ and ‘truth approximation’ due to a revision of the domain of intended applications. This set of definitions seems to be the natural counterpart of the basic definitions of similar notions in ICR as far as theory revision is concerned. Regarding theory revision, there will be some overlap with Zwart’s contribution.
Assuming the distinction between a vocabulary $V$ and a subvocabulary $V_d$ as “domain vocabulary,” a distinction that was only made explicit in the last chapter of ICR, we may also assume that the domain of intended applications can be represented as a well-defined subset $D$ of the set of conceptual possibilities (potential models) $Mp(V_d)$ generated by $V_d$. Leaving out the distinction between theoretical and observational terms, the target set of structures $T(D)$ is the set of nomic possibilities corresponding to $D$ within the set of conceptual possibilities $Mp(V)$ generated by $V$.

For subsets $X$ and $Y$ of $Mp(V)$ representing theories the basic definition for “$Y$ is at least as close to $T$ as $X$” in ICR (cf. (2) in Zwart’s contribution; see also Miller’s evaluation of the basic definition, his own definition and some other explications of verisimilitude) amounts to

(a) $Y \subseteq T \subseteq X$  
(b) $T \subseteq Y \subseteq T \subseteq X$

With a plausible strengthening to capture ‘closer to’, viz. by requiring a proper subset relation at least once, this results in ‘more truthlikeness due to theory revision’, keeping (the vocabularies and) the domain fixed. Note that this definition already makes formal sense without the specific interpretation of $X$, $Y$ and $T$.

However, we may also consider what happens when we change the domain and fix the theory. The following definition is then plausible. For domains $D_1$ and $D_2$, and $T(D_1)=T_1$ and $T(D_2)=T_2$, “$X$ is at least as close to $T_2$ as $T_1$” iff “$T_2$ is at least as close to $X$ as $T_1$”

(a') $T_2 \subseteq X \subseteq T_1 \subseteq X$
(b') $X \subseteq T_2 \subseteq X \subseteq T_1$

again with a plausible strengthening to capture ‘closer to’, now resulting in ‘more truthlikeness due to domain revision’.

When drawing plausible pictures, with $X$ fixed, $T_1$ replacing $T$ and $T_2$ replacing $Y$, it is interesting to see that the revisions go in the opposite direction: whereas $Y$ approaches $T$, starting from $X$, $T_2$ approaches $X$, starting from $T_1$. The divergent graphic representation of the proper subset requirements reflects this in particular. The same switch of direction is relevant for design research. Whereas a new drug may be better for a certain disease than an old one, a certain drug may be better for another disease than the original target disease, a phenomenon which was nicely captured by the title of a study by Rein Vos (1991): “Drugs looking for diseases.”

Let us turn to the notion of empirical progress. In ICR the basic definition of ‘empirical progress by theory revision’ is based on the subsets $R$ and $S$ of $Mp(V)$, representing the set of realized possibilities of $D$ (at a certain moment) and the strongest law about $D$ (inductively) based on $R$, respectively. The minimal
condition for empirical progress, viz. “Y is at least as successful wrt R/S as X”, amounts (in)formally to the following two conditions:

(a-R)  \( R \cap X \subseteq R \cap Y \): all established examples of X are examples of Y

(b-S)  \( S \cup X \supseteq S \cup Y \): all established successes of X are successes of Y

Whereas the paraphrase of the first formal clause will be clear enough, that of the second may need some explication. A superset of \( S \cup X \) represents a law, for it is entailed by \( S \), which is also entailed by \( X \), hence a success of \( X \). The formal clause guarantees that such a success is also a success of \( Y \).

Crucial in ICR is the Success Theorem, according to which “being closer to the truth” entails “being at least as successful,” assuming the correct data hypotheses (CD):

\( R \subseteq T \subseteq S \). This theorem provides several reasons (ICR, p. 62) for claiming that empirical progress by theory revision is functional for truth approximation (by theory revision).

Turning to domain revision, let \( R_1 \) and \( R_2 \) represent the relevant sets of realized possibilities of the corresponding domains \( D_1 \) and \( D_2 \), and \( S_1 \) and \( S_2 \) the corresponding strongest laws. Now it is plausible to define “\( X \) is at least as successful wrt \( R_2/S_2 \) than wrt \( R_1/S_1 \)” or “\( X \) is at least as successful wrt \( D_2 \) than wrt \( D_1 \)” (as far as the available data are concerned) iff

(a'-R)  \( R_1 \cap X \subseteq R \cap Y \): all established \( R_1 \)-examples of \( X \) are \( R_2 \)-examples

(b'-S)  \( S_1 \cup X \supseteq S \cup Y \): all established \( S_1 \)-successes of \( X \) are \( S_2 \)-successes

As a counterpart to the Success Theorem, regarding theory revision, we would now like to prove the following

Success Theorem, regarding domain revision:

“\( X \) is closer to \( T_2 \) than to \( T_1 \)” entails

“\( X \) is at least as successful wrt \( R_2/S_2 \) than wrt \( R_1/S_1 \)”

To prove this theorem we again need, of course, the relevant CD-hypotheses, viz. \( R_1 \subseteq T_1 \subseteq S_1 \) and \( R_2 \subseteq T_2 \subseteq S_2 \). But we need some more, again plausible, relational assumptions. If a realized possibility of \( D_1 \) also is one of \( D_2 \), it is recognized as such. Formally: \( R_1 \cap T_2 \subseteq R_2 \), and of course vice versa: \( R_2 \cap T_1 \subseteq R_1 \). Similarly, a law entailed by \( S_1 \) that also holds for \( D_2 \), is recognized as such, that is, it is also entailed by \( S_2 \). Formally: \( S_1 \cup T_2 \supseteq S_2 \), and vice versa \( S_2 \cup T_1 \supseteq S_1 \).

It is easy to check that the latter assumptions are implied when we start from sets \( R \) and \( S \) relating to a domain \( D \) covering \( D_1 \) and \( D_2 \), and define \( R_1 = R \cap T_1 \), \( R_2 = R \cap T_2 \) and \( S_1 = S \cup T_1 \), \( S_2 = S \cup T_2 \). These definitions amount to the assumptions that, for example, \( R_1 \) captures all realized possibilities of \( D_1 \), and \( S_1 \) the strongest established law about \( D_1 \).

The consequence of the theorem is that ‘empirical progress by domain revision’ is functional for ‘truth approximation by domain revision’ in a similar way as the corresponding type of theory revision.
Turning to the quote from ICR at the beginning of this subsection, the result of all this is that a theory may indeed come closer to the (relevant) truth by either theory revision in the form of strengthening or weakening a theory and by domain revision in the form of extending or reducing the domain. Moreover, a theory may come closer to the (relevant) truth by a combination of such revisions. Similar conclusions apply for the possibility for a theory to become more successful. Due to the two success theorems the latter will be functional for truth approximation, but there is, of course, no guarantee.

REFERENCES


