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SUPERSTRING ACTIONS IN $D = 3, 4, 6, 10$ CURVED SUPERSPACE

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The constraints on the supergeometry of $D = 3, 4, 6, 10$ curved superspace, required for the coupling of supergravity to the Green–Schwarz superstring, are determined. We show that these are consistent with *off-shell* supergravity or super-Yang–Mills backgrounds for the superstring for $D = 3, 4, 6$.

1. Introduction. The covariant superstring action of Green and Schwarz [1] can be considered as a two-dimensional σ -model with a Wess–Zumino term and flat D -dimensional superspace as its target manifold [2]. Witten has recently generalized this action for the type I superstring to a curved $D = 10$, $N = 1$ superspace [3] satisfying the superspace field equations of $D = 10$, $N = 1$ supergravity [4], and this result has been extended to the $N = 2$ type IIB $D = 10$ superstring [5].

Classically the Green–Schwarz (G–S) flat superspace string action exists for $D = 3, 4, 6$ and 10. Quantum mechanically, one expects $D = 10$ to be singled out as the unique dimension of space–time allowing a consistent superstring action, although there is always the possibility that sometime in the future a quantum mechanically consistent superstring theory may be found for $D = 6$ (if so, it would presumably have one of the chiral, anomaly free, $D = 6$ supergravity theories [6] as a field theory limit). But, aside from its relevance to a $D = 10$ (or $D = 6$) unified superstring theory of fundamental interactions, the G–S action is also of relevance for $D = 4$ as a description of macroscopic or “cosmic” superstrings interacting with the fields of an effective $N = 1$ supergravity obtained as

the low-energy limit of a $D = 10$ superstring theory after compactification on some six-dimensional compact manifold.

In this paper we investigate the classical consistency of superstring actions in curved superspace, and in the presence of Yang–Mills (Y–M) fields, for $D = 3, 4, 6$ and 10. We first write down an action of the required general form together with a general set of transformation rules for a fermionic gauge invariance generalizing that of the flat superspace action of ref. [1]. We then determine the constraints on the superspace torsion two-form and field strength two- and three-forms required by fermionic gauge invariance, and investigate whether the existing superspace formulations of supergravity in superspace have constraints on the supergeometry consistent with these. The crucial point is the existence of a closed three-form H ($dH = 0$) with a non-vanishing flat superspace limit. This is possible only for $D = 3, 4, 6$ and 10, which is why we are restricted, a priori, to these dimensions of space–time [1]. For $D = 6$ and $D = 10$ supergravity such closed three-forms, H , are known to exist and for $D = 4$, H is known to exist for some sets of off-shell fields. In other cases, one has to check directly that the Bianchi identity $dH = 0$ is satisfied. We have done this for $D = 3$ supergravity.

As yet our results cannot be applied, except for $D = 3$, to superstrings in supergravity and super-Yang–Mills backgrounds *simultaneously* because the supergeometry of these theories have only been worked out for $D = 4$ ^{*1}. One new feature that is expected to arise, at least for $D = 4$ and $D = 10$, is the modification of the Bianchi identity $dH = 0$ to a different identity of the form $dH = \text{Tr}(F^2)$, where F is the Y–M superspace two-form (and also, perhaps, terms of the form $\text{Tr}(R^2)$ with R the curvature two-form). This would still be sufficient to ensure the existence of a closed but *non*-Y–M invariant three-form. It is not the purpose of this paper, however, to investigate which superspace constraints are consistent with this type of modified Bianchi identity, so that, for $D = 4, 6, 10$, we shall discuss separately supergravity and Y–M backgrounds. This means, in particular, that we shall not discuss the heterotic superstring in this work. We should mention that for $D = 6$ one expects two dual formulations of the supergeometry of supergravity plus super-Y–M theory; one involving a modified three-form H' , the other with the standard Y–M invariant three-form H . This is because, for $D = 6$, the dual of a second-rank antisymmetric tensor gauge potential is again a second-rank antisymmetric tensor. This suggests the possibility that in $D = 6$ one may be able to couple supergravity and super-Y–M backgrounds to the superstring simultaneously.

The consistency of the G–S superstring action depends on a generalization of the fermionic gauge invariance [8] of the superparticle action. We show that, in addition to $D = 10$ supergravity, this invariance can be maintained in the presence of the following supergravity backgrounds:

- $D = 3$: 4+4 supergravity,
- $D = 4$: 12+12 new-minimal supergravity,
16+16 supergravity,
- $D = 6$: *on-shell* 12+12 supergravity,
off-shell 48+48 supergravity.

This list is certainly not complete. We have not investigated the coupling of non-minimal $D = 4$ supergravity, nor extended supergravities, for example. But included in this list is the most interesting case for $D < 10$, i.e. the $D = 4$ off-shell 16+16 supergravity multiplet. This version of off-shell supergravity was first

proposed by Galperin et al. [9]; it appears to be the natural version for a description of low-energy supergravity obtained by compactification of the $D = 10$ superstring [10], although it is, of course, a reducible multiplet [11]. In addition, we construct the following couplings of matter supermultiplets (spins ≤ 1) to superstrings:

- $D = 3$: 2+2 Yang–Mills in *curved* superspace,
- $D = 4$: 4+4 Yang–Mills in flat superspace,
4+4 linear multiplet in flat superspace,
- $D = 6$: 8+8 Yang–Mills in flat superspace,
on-shell antisymmetric tensor multiplet in flat superspace,
- $D = 10$: *on-shell* 8+8 Yang–Mills in flat superspace.

One feature that emerges from these constructions is that whereas for $D = 10$ the background fields are constrained to satisfy their field equations, for $D < 10$ they are not. When the background fields can be taken off-shell, one can easily write an action for the combined supergravity (or super-Y–M) and superstring system, yielding the coupled supergravity/superstring equation. The supergravity equations obtained in this way will include the back reaction of the superstring on the supergeometry. This is what one requires for a description of cosmic superstrings moving in an effective $D = 4, N = 1$ supergravity background. It seems that such a dynamical system is not possible for $D = 10$ because of the absence of the back reaction of the superstring in the $D = 10$ supergravity equations. But for $D = 10$ we are presumably not interested in such a system. Rather, we are interested in determining the classically allowed backgrounds of the superstring, which hopefully includes those of the form of $D = 4$ Minkowski space times a compact six-manifold. Presumably the spin-two perturbations about this background are provided by the excitations of the string. It is gratifying, therefore, that precisely in $D = 10$, where a *dynamical* coupling of superstrings to supergravity seems impossible it is also irrelevant, whereas for $D = 4$ where it is essential it is also possible.

2. *Fermionic invariance of superstring actions.* We shall consider a superstring action of the form [3,12]

$$S = \int d^2\xi \left[\frac{1}{2} e^k V g^{ij} E_i^a E_j^b \eta_{ab} + \frac{1}{2} \epsilon^{ij} E_i^B E_j^A B_{AB} + \frac{1}{2} V V_m^i \bar{\psi}^J \gamma^m (\partial_i \delta^{JJ} + E_i^B A_B^{JJ}) \psi^J \right]. \quad (2.1)$$

^{*1} The supergeometry of $D = 10$ Einstein–Yang–Mills has been given recently in ref. [7].

Here $i = 0, 1$ labels the coordinates $\xi^i = (\tau, \sigma)$ of the string world sheet with metric g_{ij} and zweibein V_i^m . We have $g_{ij} = V_i^m V_j^n \eta_{mn}$ where $\eta = \text{diag}(-1, +1)$ and $V = \det V_i^m$. The alternating tensor *density* ϵ^{ij} ($\epsilon^{01} = -1$) satisfies the identity $\epsilon^{ij}\epsilon^{kl} = -V^2(g^{ik}g^{jl} - g^{il}g^{jk})$ and we define $\epsilon_{ij} \equiv V^{-1}g_{ik}g_{jl}\epsilon^{kl}$, which is an anti-symmetric *tensor*. Similarly, we define the two-dimensional Lorentz tensors $\epsilon_{mn} = V_m^i V_n^j \epsilon_{ij}$ and $\epsilon^{mn} = \eta^{mn}\eta^{pq}\epsilon_{pq}$. The two-dimensional gamma matrices γ^m satisfy $\epsilon^{mn}\gamma_n = -\gamma^m\gamma_3$ ($\gamma_3 = \gamma_0\gamma_1$). The coordinates of the string are $\{Z^M(\xi), \psi^I(\xi)\}$. The coordinates Z^M are those of superspace, while the coordinates ψ^I are two-dimensional Majorana–Weyl spinors belonging to the fundamental representation of some Lie group, G . The external fields are the superspace forms

$$E^A = dZ^M E_M^A(Z),$$

$$B = \frac{1}{2}E^B E^A B_{AB}(Z), \quad A^{IJ} = E^A A_A^{IJ}(Z), \quad (2.2)$$

the A^{IJ} belonging to the adjoint map of G . We use the notation

$$E_i^A = \partial_i Z^M E_M^A, \quad (2.3)$$

and as the superspace structure group will be taken to be the Lorentz group (possibly $\times U(1)$) the index A can be split into vector and spinor indices $A \rightarrow (\alpha, a)$. An action of the form (2.1) encompasses both those of refs. [3,5] as well as that of the $D = 10$ flat superspace heterotic superstring. Our superspace conventions are those of Howe [13].

We shall require that the action S be invariant under a fermionic gauge transformation of the form

$$\delta E^a = 0, \quad \delta E^\alpha = 2(\Gamma_a)^\alpha{}_\beta E_i^a V_m^i \eta_\beta^m, \quad (2.4)$$

$$\delta \psi^I = -(\delta E^\alpha) A_\alpha^{IJ} \psi^J, \quad \delta V_i^m = -\frac{1}{2}(g_{ij} + \epsilon_{ij}) m^{\alpha j} \eta_\alpha^m,$$

where $\eta_\alpha^m(\xi)$ is a D -dimensional anticommuting spinor that is also a two-dimensional vector satisfying

$$(\delta^{mn} - \epsilon^{mn}) \eta_n = 0, \quad (2.5)$$

i.e. that is anti-self-dual. The quantity $M^{\alpha j}$ is to be determined by the invariance of the action. In (2.4) we have used the notation

$$\delta E^A \equiv \delta Z^M E_M^A, \quad (2.6)$$

so that specifying δE^A is equivalent to specifying δZ^M . The $(\Gamma_a)^\alpha{}_\beta$ are the D -dimensional gamma matrices $(\Gamma_a)_\alpha{}^\beta$ with one index raised by the charge conjugation

matrix. Although $(\Gamma_a)^{\alpha\beta}$ is symmetric for $D = 3, 4, 10$ and antisymmetric for $D = 6$, we can take the $D = 6$ spinor index to include an $SU(2)$ index, i.e. $\alpha \rightarrow (\hat{\alpha}, i)$ and then $(\Gamma_a)^{\alpha\beta} = (\Gamma_a)^{\beta\alpha} \rightarrow \hat{\Gamma}_a^{\hat{\alpha}\hat{\beta}} \epsilon^{ij}$, where the $(\hat{\Gamma}_a)_{\hat{\alpha}\hat{\beta}}$ are the standard $D = 6$ gamma matrices. With this notation $(\Gamma_a)^{\alpha\beta}$ is symmetric for $D = 3, 4, 6$ and 10.

Now using (2.4) the variation of the action (2.1) is

$$\begin{aligned} \delta S = & \int d^2\xi \left[\frac{1}{2} e^k \delta(V g^{ij}) E_i^a E_j^b \eta_{ab} \right. \\ & + \delta E^\alpha \left(\frac{1}{2} e^k V g^{ij} E_i^a E_j^b \eta_{ab} \Lambda_\alpha - e^k V g^{ij} E_i^B E_j^C T_{B\alpha C} \right. \\ & + \frac{1}{2} \epsilon^{ij} E_i^B E_j^C H_{CB\alpha} - \frac{1}{2} V \bar{\psi}^I \gamma^i \psi^J E_i^B F_{B\alpha}^{IJ} \left. \right) \\ & \left. + \frac{1}{2} \delta(V V_m^i) \bar{\psi}^I \gamma^m (\mathcal{D}_i \psi)^J \right], \quad (2.7) \end{aligned}$$

where $\Lambda_\alpha = D_\alpha k$ and $\mathcal{D}_i$ is the G -covariant derivative appearing in (2.1). The torsion two-form T^A , three-form field strength H and Yang–Mills two-form field strength F^{IJ} are defined by

$$T^A = dE^A + E^B \Omega_B^A = \frac{1}{2} E^B E^C T_{CB}^A,$$

$$H = dB = \frac{1}{6} E^C E^B E^A H_{ABC},$$

$$F^{IJ} = dA^{IJ} + A^{IK} A^{KJ} = \frac{1}{2} E^B E^A F_{AB}^{IJ}. \quad (2.8)$$

Clearly the last term in (2.7) must vanish by itself. Indeed, with δV_i^m as given in (2.4) one can verify that

$$\delta(V V_m^i) \bar{\psi}^I \gamma^m (\mathcal{D}_i \psi)^J = 0, \quad (2.9)$$

provided that the chirality of ψ^I is chosen such that

$$\psi^I = -\gamma_3 \psi^I. \quad (2.10)$$

The remaining terms cancel provided one imposes the constraints

$$T_{\alpha\beta}^a = -i(\Gamma^a)_{\alpha\beta}, \quad \eta_{c(a} T_{b)}^c = \eta_{ab} B_\alpha,$$

$$H_{\alpha\beta\gamma} = 0, \quad H_{\alpha\beta a} = -ie^k (\Gamma_a)_{\alpha\beta},$$

$$H_{\alpha ab} = 2e^k (\Gamma_{ab})_\alpha{}^\beta H_\beta,$$

$$F_{\alpha\beta}^{IJ} = 0, \quad F_{b\alpha}^{IJ} = (\Gamma_b)_{\alpha\beta} W^{\beta IJ}, \quad (2.11)$$

and chooses

$$\begin{aligned} M^{\alpha i} = & 4iE^{\alpha i} - 4E_a^i (\Gamma^a)^{\alpha\beta} H_\beta \\ & - \psi^I \gamma^i \psi^J W^{\alpha IJ} e^{-k}, \quad (2.12) \end{aligned}$$

where H_α and B_α are spinors satisfying the relation

$$\Lambda_\alpha + 2H_\alpha - 2B_\alpha = 0. \quad (2.13)$$

Consistency with the Bianchi identity $dH = 0$ requires that

$$(\Gamma^a)_{\alpha\beta}(\Gamma^a)_{\gamma\delta} + (\Gamma^a)_{\gamma\alpha}(\Gamma^a)_{\beta\delta} + (\Gamma^a)_{\beta\gamma}(\Gamma^a)_{\alpha\delta} = 0, \quad (2.14)$$

which is true only if $D = 3, 4, 6$ and 10 .

This takes care of the $\alpha\beta\gamma\delta$ and $\alpha\beta\gamma a$ components of the $dH = 0$ equation. The remaining terms will give information about H_{abc} . The constraints on the torsion in (2.11) are known to be consistent with the torsion and curvature Bianchi identities. These constraints imply that $T_{\alpha\beta}{}^\gamma \sim \delta_{(\beta}^\gamma B_{\alpha)}$. If one imposes in addition the conventional constraint $T_{ab}{}^c = 0$ the only remaining independent components of the torsion and curvature are to be found in $T_{\alpha\beta}{}^\gamma$ and $T_{ab}{}^\alpha$. These have to be determined, on a case-by-case basis, depending on the dimension D and the choice of B_α . The constraints of F^{IJ} are also known to be consistent with the Bianchi identity $\nabla F^{IJ} = 0$ which implies that all components of F^{IJ} can be expressed in terms of the field strength spinor superfield W_α^{IJ} . Observe that, provided $D = 3, 4, 6$ and 10 , we always have the trivial solution

$$T^a = -\frac{1}{2}iE^\beta E^\alpha (\Gamma^a)_{\alpha\beta}, \quad H = -\frac{1}{2}iE^\beta E^\alpha E^a (\Gamma_a)_{\alpha\beta},$$

$$T^\alpha = 0, \quad F^{IJ} = 0 \quad (2.15)$$

to the constraints (2.11). We shall now proceed to investigate non-trivial solutions for each dimension $D = 3, 4, 6, 10$ in turn.

3. Consistent superstrings in supergravity or $Y-M$ backgrounds. The $D = 3$ Poincaré supergravity multiplet [14] has components $(e_\mu^a, \psi_\mu, \phi)$. The corresponding superspace geometry has $k = B_\alpha = H_\alpha = 0$ and all components of the torsion and the three-form H can be expressed in terms of the superfields $G_{abc} \sim \epsilon_{abc} R$ and $R_{ab}{}^\alpha$ (the gravitino field strength) and their spinor derivatives. The non-zero components of the torsion (apart from $T_{\alpha\beta}{}^a$) are $T_{bc}{}^\alpha \sim (\Gamma^a)_{\alpha\beta} D_\beta G_{abc}$, $R_{bc}{}^\alpha$ and $T_{bc}{}^\alpha \sim (\Gamma^{ba})_{\beta}^\alpha G_{abc}$. The three-form H is given by

$$H = -\frac{1}{2}iE^\beta E^\alpha E^a (\Gamma_a)_{\alpha\beta} + E^c E^b E^a G_{abc}. \quad (3.1)$$

There are three distinct *irreducible* $D = 4, N = 1$ off-

shell Poincaré supergravity multiplets. The “minimal” set has $B_\alpha = 0$. In the absence of matter $k = 0$ too, so that $H_\alpha = 0$ by (2.14). Then H is just the flat superspace three-form. However this H is not closed and cannot therefore be used for a coupling to the string. More interesting is the “new-minimal” set [15] for which $B_\alpha \neq 0$. The components are

$$(e_\mu^a, \psi_\mu, A_\mu, B_{\mu\nu}), \quad (3.2)$$

where A_μ and $B_{\mu\nu}$ are non-propagating gauge fields. The new-minimal supergravity has a $U(1)$ gauge invariance for which A_μ is the gauge field. Because of this it is convenient to include a $U(1)$ factor in the tangent superspace group. In this $U(1)$ -superspace (see ref. [16] for a recent review) $B_\alpha = 0$, and the only independent components of the torsion are G_{abc} and $R_{ab}{}^\alpha$, as in three dimensions. As there is no scalar field in (3.2) $k = 0$, so that $H_\alpha = 0$. In fact, the three-form H has precisely the form of (3.1) but where G_{abc} now contains the field strength of $B_{\mu\nu}$ as its $\vartheta = 0$ component [17].

The third irreducible set of off-shell supergravity fields is the “non-minimal” set, which we shall not discuss. Instead we shall consider the reducible $16+16$ supergravity which can be obtained from the new minimal set by adding a chiral $U(1)$ compensating field. The pseudoscalar of the latter becomes the longitudinal part of the $U(1)$ gauge field A_μ of the former, yielding the components

$$\{e_\mu^a, \psi_\mu, B_{\mu\nu}, \lambda, \sigma\} + \{A_\mu F, G\}. \quad (3.3)$$

The fields in the second bracket are auxiliary and can be eliminated. The remaining fields bear a strong resemblance to those of $D = 10$ supergravity. We now have a scalar field so we expect to find $k \sim \sigma$ and $\Lambda_\alpha \neq 0$ as a consequence. However, it is possible to set $k = 0$ if we allow $B_\alpha \sim D_\alpha \sigma$. Then all components of the torsion can be expressed in terms of σ , G_{abc} and $R_{ab}{}^\alpha$. The three-form H is given by

$$H = (-\frac{1}{2}iE^\beta E^\alpha E^a (\Gamma_a)_{\alpha\beta} - E^b E^a E^\alpha (\Gamma_{ab})_{\alpha}{}^\beta D_\beta \sigma + \frac{1}{6}E^c E^b E^a G_{abc}), \quad (3.4)$$

and the action by (2.1) but still with $k = 0$. One may also consider the flat-space limit in which case $16+16$ supergravity reduces to the linear multiplet. The H given in (3.4) then describes the coupling of a linear multiplet to the superstring.

For $D = 6$ the minimal supergravity multiplet $(e_\mu^a, \psi_\mu, B_{\mu\nu}^-)$ is necessarily on-shell because the field strength $G_{\mu\nu\rho}^-$ for $B_{\mu\nu}^-$ is anti-self-dual. All torsion components can be expressed in terms of the single superfield G_{abc}^- which is anti-self-dual and contains the field strength of $B_{\mu\nu}^-$ as its $\vartheta = 0$ component. The non-zero components of the torsion are $T_{\beta\gamma}^\alpha \sim (\Gamma^{ba})^\alpha_\beta G_{abc}^-$ and $T_{bc}^\alpha \sim (\Gamma^a)^\alpha_\beta D_\beta G_{abc}^-$. The three-form H has the form of (3.1) with G_{abc}^- replacing G_{abc} [18]. The minimal $D = 6$ multiplet can be combined with the on-shell antisymmetric tensor multiplet $(\sigma, \lambda B_{\mu\nu}^+)$, and this can be extended to a $48+48$ component off-shell multiplet. The constraints on the supergeometry have been found [19] (off-shell $N = 2, d = 6$ supergravity in terms of components has been given in ref. [20]) and are consistent with (2.11) with $B_\alpha = 0$ and $k = \sigma$. In this case $\Lambda_\alpha \neq 0$ and $H_\alpha \neq 0$ and the three-form H is given by the same expression as in (3.4). One may also consider the *on-shell* supergravity antisymmetric tensor multiplet, in which case one may go to the flat space limit in which $e_\mu^a = \delta_\mu^a$, $\psi_\mu = B_{\mu\nu}^- = 0$; H is then still given by (3.4) but with G_{abc} replaced by G_{abc}^+ .

Finally we shall consider the $D = 10$ case. The constraints of Nilsson [21] have $B_\alpha \neq 0$ but $T_{ab}^c = 0$. This allows us to take $k = 0$. If, on the other hand, one takes $B_\alpha = 0$ but $T_{ab}^c \neq 0$ one requires $k \neq 0$ and this leads to the action of Witten [3]. Turning now to Y–M backgrounds we observe that in flat superspace the Bianchi identity $\mathcal{D}_F^{IJ}$ implies that

$$F^{IJ} = E^\alpha E^a (\Gamma_a)_{\alpha\beta} W^{\beta IJ} + \frac{1}{2} i E^b E^a (\Gamma_{ab})^\alpha_\beta (\mathcal{D}_\alpha W^\beta)^{IJ}. \quad (3.5)$$

If H and T^A take their flat superspace values the constraints (2.11) are obviously satisfied and (2.13) is trivially satisfied with $k = 0$. The Bianchi identity for F^{IJ} also implies additional constraints on W_β which reduce the component content to the standard off-shell Y–M multiplet for $D = 3, 4, 6$ and to *on-shell* Y–M multiplet for $D = 10$.

For $D = 3$ there is no problem in extending the flat superspace Y–M coupling to curved superspace. One simply takes the $D = 3$ supergravity result but with $W^{\beta IJ} \neq 0$. As there is no antisymmetric tensor component field no problem with modified field strengths arises. In all other cases, one can construct a supersymmetric coupling of Y–M to the superstring but the action will *not* be invariant under the modified

Y–M transformation $\delta' B_{MN} = \text{Tr}(\Lambda F_{MN})$. It would be interesting to see whether one can construct a Y–M invariant action by using the results of ref. [22].

4. Comments. The outstanding problem in the program of superstring couplings to background fields pursued in this paper is that of the heterotic string. As we have explained this depends to a large extent on a better understanding of the relevance of Y–M invariance of the superstring action. It seems possible that a complete understanding of this supergeometry will require the study of superfields defined in a superspace with coordinates (Z^M, ψ^I) rather than just Z^M . It is also unclear how to incorporate $E_8 \times E_8$ couplings to the heterotic superstring, although $SO(32)$ is straightforward as ψ^I can be taken to be a 32-component vector of this group.

There are, of course, many other possible superstring backgrounds including those of other types of matter multiplets. It may be possible to find such couplings by dimensional reduction as has been done for the bosonic string [21]. Also, we have not discussed extended supergravity here, but one expects that the superstring coupling to $N = 4, D = 6$ supergravity will parallel the construction of ref. [5].

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