Supermembranes and eleven-dimensional supergravity

Bergshoeff, E.; Sezgin, E.; Townsend, P.K.

Published in:
Physics Letters B

DOI:
10.1016/0370-2693(87)91272-X

SUPERMEMBRANES AND ELEVEN-DIMENSIONAL SUPERGRAVITY

E. BERGSHOEFF[1], E. SEZGIN
International Centre for Theoretical Physics, I-34100 Trieste, Italy

and

P.K. TOWNSEND
DAMPT, University of Cambridge, Silver Street, Cambridge CB3 9EW, UK

Received 6 February 1987

We construct an action for a supermembrane propagating in $d=11$ supergravity background. Using the constraints of $d=11$ curved superspace, we show that the action is invariant under Siegel-type transformations recently generalized by Hughes, Li and Polchinski. The transformation parameter is a world-volume scalar and $d=11$ spacetime spinor. We also discuss the general problem of the coupling of n-dimensional extended objects to d-dimensional supergravity.

1. Now that we have become accustomed to the notion that strings should replace particles, it is natural to investigate the properties of higher-dimensional extended objects, in particular of membranes since they are the simplest extended objects, and they might describe strings in an appropriate limit.

In 1962 Dirac [1] put forward a theory of an extended electron based on the idea of a relativistic membrane. In 1976, Collins and Tucker [2] studied the classical and quantum mechanics of free relativistic membranes. A year later a locally supersymmetric and reparametrization-invariant action for a spinning membrane was constructed by Howe and Tucker [3]. The action describes anti de Sitter supergravity coupled to a number of scalar multiplets in three dimensions. It is the membrane analog of the Neveu–Schwarz–Ramond formulation of the spinning string theory.

More progress towards the construction of a membrane theory was made by Sugamoto [4] in 1983. More recently, Hughes, Li and Polchinski [5] have constructed a Green–Schwarz-type action for a three-extended object propagating in flat six-dimensional spacetime. The consistency of the action requires the existence of a closed superspace five-form, in analogy with the Henneaux and Mezincescu [6] construction for the Green–Schwarz superstring action [7], where a closed superspace three-form is required. The novel feature of the theory of Hughes et al. is that the parameter of the Siegel-type transformation [8] is a scalar rather than a vector on the world volume.

The generalization of the Hughes et al. model to n-extended objects propagating in flat d-dimensional superspace is evident. All that is required is the existence of a closer super $(n+2)$-form given by

$$H=E^{\beta}E^{\alpha}E^{a_n}\dots E^{a_1}(\gamma_{a_1\dots a_n})_{\alpha\beta}\,, \tag{1}$$

where (E^{α}, E^{a}) are the basis one-forms in superspace. This form is closed provided that the following Γ-matrix identity holds:

$$(\gamma^{a})_{(\alpha\beta}(\gamma^{aa_1\dots a_{n-1}})_{\gamma\delta)}=0\,. \tag{2}$$

The purpose of this note is to construct Hughes et al. -type actions describing the propagation of an n-extended object in d-dimensional *curved* superspace. We give a general formula for the action and the transformation rules, whose consistency requires, among other things (see below), the existence of a closed $(n+2)$-form in curved superspace. Thus we

[1] Supported in part by INFN, Sezione di Trieste, Trieste, Italy.

0370-2693/87/$ 03.50 © Elsevier Science Publishers B.V.
(North-Holland Physics Publishing Division)

expect that the n-extended objects under consideration can consistently propagate only in $d \leqslant 11$ supergravities whose superspace formulation involves a closed $(n+2)$-form. We further expect that such forms exist in supergravity theories in which a closed *bosonic* $(n+2)$-form occurs. As far as we know, the following possibilites exist (we include the Yang–Mills couplings whenever possible):

The dual formulation of $d=10$, $N=1$ supergravity involves a closed seven-form. Its dimensional reduction on a $(10-d)$-dimensional torus leads to real closed $(d-3)$-forms in d-dimensional supergravities. (These are $N=1$ supergravities in $d=8$, 9, 10; $N=2$ in $d=7$ and $N=2$ or 4 in $d=6$) [9]. Apart from these, there is: (i) *A real closed four-form in $d=11$, $N=1$* supergravity, (ii) a real closed three-form in non-chiral $d=10$, $N=2$ supergravity, (iii) a complex closed three-form in chiral $d=10$, $N=2$ supergravity.

Excluding Yang–Mills coupling, as is well known, closed super three-forms exist in $d=3$, 4, 6, and 10.

Considering the case of the membranes, from the above list it follows that the candidate dimensions are 7 and 11. Since the superspace formulation of $d=7$, $N=2$ supergravity is not known at present, we are led to consider the supermembrane propagating in eleven-dimensional spacetime.

Our main result is the construction of an action which describes a consistent coupling of $d=11$ supergravity to a supermembrane. In particular the Kalb–Ramond-like third rank antisymmetic tensor field of $d=11$ supergravity couples to the supermembane via a Wess–Zumino term.

In the following we focus our attention on the description of the supermembane action in $d=11$. The extension to the case of n-extended objects is given in the appendix.

2. We propose the following action for a supermembrane coupled to $d=11$ supergravity:

$$S=\int d^2\xi\,\left(\tfrac{1}{2}\sqrt{-g}\,g^{ij}E_i{}^aE_j{}^b\eta_{ab}\right.$$
$$\left.+\epsilon^{ijk}E_i{}^AE_j{}^BE_k{}^CB_{CBA}-\tfrac{1}{2}\sqrt{-g}\right)\,. \tag{3}$$

Here $i=0, 1, 2$ labels the coordinates $\xi^i=(\tau,\sigma,\rho)$ of the world volume with metric g_{ij} and signature $(-, +, +)$. The super three-form B is needed for the superspace description of $d=11$ supergravity [10]. For the Levi-Civita symbol ϵ^{ijk} we use the same conventions as in ref. [11]. In (3) we have used the notation

$$E_i{}^A=(\partial_iZ^M)E_M{}^A\,, \tag{4}$$

where $Z^M(\xi)$ are the superspace coordinates, and $E_M{}^A(Z)$ is the supervielbein.

Note that the action has a cosmological constant with a fixed magnitude. This is so that the field equation of the metric g_{ij} gives the embedding equation

$$g_{ij}=E_i{}^aE_j{}^b\eta_{ab}\equiv T_{ij}\,. \tag{5}$$

We require that the action S is invariant under a fermionic gauge transformation of the form [5]

$$\delta E^a=0,\quad \delta E^\alpha=(1+\Gamma)^\alpha{}_\beta\kappa^\beta\,,$$

$$\delta g_{ij}=2[X_{ij}-g_{ij}X^k{}_k/(n-1)]$$
$$(n=2\text{ for membrane})\,, \tag{6}$$

where the transformation parameter $\kappa^\alpha(\xi)$ is a 32 component Majorana spinor, and a world-volume scalar, and

$$\delta E^A=\delta Z^ME_M{}^A, \tag{7}$$

$$\Gamma^\alpha{}_\beta=(1/6\sqrt{-g})\epsilon^{ijk}E_i{}^aE_j{}^bE_k{}^c(\gamma_{abc})^\alpha{}_\beta\,. \tag{8}$$

Here $\gamma^a(a=0, 1, ..., 10)$ are the Dirac matrices in eleven dimensions. X_{ij} is a function of $E_i{}^A$ which will be determined by the invariance of the action. The choice of δg_{ij} is due to the fact that, given a variation of the action of the form $\delta S=T_{ij}X^{ij}$, and writing this variation as

$$T_{ij}X^{ij}=g_{ij}X^{ij}+(T_{ij}-g_{ij})X^{ij}\,, \tag{9}$$

the second term on the right-hand side cancels $(\delta S/\delta g_{ij})\delta g_{ij}$. Thus we are left with the first term on the right-hand side, which equals the left-hand side upon the use of (5). Effectively, this means that whenever we encounter a variation of the form $T_{ij}X^{ij}$, we can use eq. (5), provided that we add X^{ij} to δg_{ij} as in (6).

The matrix $\Gamma^\alpha{}_\beta$ occurring in (8) satisfies the property

$$\Gamma^\alpha{}_\beta\Gamma^\beta{}_\delta=(T^i{}_{[i}T^j{}_jT^k{}_{k]})\delta^\alpha{}_\delta\equiv\Gamma^2\delta^\alpha{}_\delta\,. \tag{10}$$

The normalization in (8) is chosen such that upon the use of the equation $T_{ij}=g_{ij}$, the matrix $\Gamma^\alpha{}_\beta$ satisfies the relation $\Gamma^\alpha{}_\beta\Gamma^\beta{}_\gamma=\delta^\alpha{}_\gamma$.

Now using (6) the variation of the action (3) is (we consider a closed supermembrane and therefore discard the surface terms)

$$\delta S=\int d^2\xi\,[\sqrt{-g}\,g^{ij}(-\delta E^\beta E_i{}^\gamma T^a{}_{\gamma\beta})E_{ja}$$

$$+\sqrt{-g}\,g^{ij}(-\delta E^\beta E_i{}^c T^a{}_{c\beta})E_{ja}$$

$$+\epsilon^{ijk}E_i{}^A E_j{}^B E_k{}^C \delta E^\alpha H_{\alpha CBA}$$

$$-\tfrac{1}{2}\sqrt{-g}\,\delta g^{ij}(T^{ij}-\tfrac{1}{2}g^{ij}T-\tfrac{1}{2}g^{ij})]\,. \tag{11}$$

The torsion two-form T^A and the four-form field strength H are defined by (our superspace conventions are those of Howe [12])

$$T^A=dE^A+E^B\Omega_B{}^A=\tfrac{1}{2}E^B E^C T_{CB}{}^A\,,$$

$$H=dB=\tfrac{1}{24}E^D E^C E^B E^A H_{ABCD}\,. \tag{12}$$

We now organize the terms in (11) according to the number of one-forms E^α they contain. Those with three E^α and two E^α come only from the Wess–Zumino term. They must vanish seperately, and this requires the constaints

$$H_{\alpha\beta\gamma\delta}=H_{\alpha\beta\gamma d}=0\,. \tag{13}$$

The cancellation of the terms linear in E^α lead to the constraints

$$T^a{}_{\alpha\beta}=(\gamma^a)_{\alpha\beta}\,, \tag{14}$$

$$H_{\alpha\beta ab}=-\tfrac{1}{6}(\gamma_{ab})_{\alpha\beta}\,, \tag{15}$$

while the cancellation of the terms not containing E^α require the constraint

$$\eta_{c(a}T^c{}_{b)\alpha}=\eta_{ab}\Lambda_\alpha\,, \tag{16}$$

$$H_{\alpha abc}=-\tfrac{1}{2}\Lambda_\beta(\gamma_{abc})^\beta{}_\alpha\,. \tag{17}$$

Here Λ_α is an arbitrary spinor superfield which is vanishing in $d=11$ [10].

It is important to realize that in obtaining (14)–(17) we have used the identity

$$\delta E^\alpha=\Gamma^\alpha{}_\beta\delta E^\beta+(1-\Gamma^2)\kappa^\alpha\,. \tag{18}$$

Using this identity in the variation of the kinetic term, the terms arising from $\Gamma^\alpha{}_\beta$ in (18) can be shown to cancel similar terms coming from the variation of the Wess–Zumino term, modulo terms which cancel by an appropriate variation of g_{ij}. [Using the argument below (8) once.] In the remaining terms coming from $(1-\Gamma^2)$, we use the argument given below (8) repeatedly to compute further contributions to δg_{ij}. Thus we find the result

$$X_{ij}=-\tfrac{1}{4}\epsilon_i{}^{kl}E_k{}^a E_l{}^b(\gamma_{ab})_{\alpha\beta}\delta E^\beta E_j{}^\alpha$$

$$+\tfrac{1}{2}\kappa^\beta E_n{}^\alpha(\gamma^d)_{\alpha\beta}E^{nd}g_{i[j}(T^k{}_k T^l{}_{l]}+\delta^k{}_k T^l{}_{l]})$$

$$+i\leftrightarrow j\,. \tag{19}$$

In summary, the action (3) is invariant under (6) provided that (13)–(17) hold, and X^{ij} is given by (19). In addition, the following Bianchi identities must hold:

$$DT^A=-E^B\wedge R_B{}^A\,,\quad DH=0\,. \tag{20}$$

The generalization of the results of this section to the general case of n-extended objects in d-dimensional supergravity is straightforward. The result is given in the appendix.

3. We observe that the superspace constraints of $d=11$ supergravity given by Cremmer and Ferrara [10] and Brink and Howe [10] do provide a solution to (13)–(17) *and* the Bianchi identities (20), with $\Lambda_\alpha=0$.

In conclusion, we have shown that there exists a consistent coupling of a closed supermembrane to eleven-dimensional supergravity. (Note that it is natural to consider a *closed* supermembrane in eleven dimensions, since there are no matter multiplets in this dimension).

4. There are several directions in which the present work can be extended. We shall name a few.

Firstly, it is of interest to study the quantization of the supermembrane model in eleven dimensions. In particular, the question of whether massless gauge fields can possibly arise is a challenging one. Although usually one encounters difficulties in finding massless excitations of a membrane [13], it is encouraging that, here, we have a spacetime supersymmetric membrane action.

Secondly, it is natural to consider the dimensional reduction of our model from eleven- to ten-dimensional spacetime, *and* at the same time from three-dimensional world volume to a two-dimensional world sheet. It would be interesting to see what kind of $d=10$ string theories could possibly emerge in an infinitely thin membrane limit.

P.K.T. would like to thank Professor Abdus Salam for his kind hospitality and ICTP in Trieste where this work was carried out.

Appendix. In this appendix we construct the action for an n-extended object propagating in d-dimensional supergravity background. We also give the transformation rules, and the constraints on the background.

The action is

$$S=\int \mathrm{d}^2\xi\ [\tfrac{1}{2}\sqrt{-g}\, g^{ij}E_i{}^a E_j{}^b \eta_{ab}$$
$$+\epsilon^{i_1\ldots i_n}E_{i_1}{}^{A_1}\ldots E_{i_{n+1}}{}^{A_{n+1}}B_{A_{n+1}\ldots A_1}$$
$$-\tfrac{1}{2}(n-1)\sqrt{-g}]\ . \qquad \text{(A1)}$$

The transformation rules are those in (6), where the matrix $\Gamma^\alpha{}_\beta$ is now given by

$$\Gamma^\alpha{}_\beta=[\eta/(n+1)!\sqrt{-g}]$$
$$\times\epsilon^{i_1\ldots i_{n+1}}E_{i_1}{}^{a_1}\ldots E_{i_{n+1}}{}^{a_{n+1}}(\gamma_{a_1\ldots a_{n+1}})^\alpha{}_\beta\ , \qquad \text{(A2)}$$

where η is given by

$$\eta=(-1)^{(n+1)(n-2)/4}\ . \qquad \text{(A3)}$$

Invariance of the action (A1) is ensured by imposing the following set of constraints:

$$T^a{}_{\alpha\beta}=(\gamma^a)_{\alpha\beta}\ ,\quad \eta_{c(a}T^c{}_{b)\alpha}=\eta_{ab}\Lambda_\alpha\ , \qquad \text{(A4)}$$

$$H_{\alpha a_{n+1}\ldots a_1}=(\eta/n!)\Lambda_\beta(\gamma_{a_1\ldots a_{n+1}})^\beta{}_\alpha\ ,$$

$$H_{\alpha\beta a_n\ldots a_1}=[\eta(-1)^n/(n+1)!](\gamma_{a_1\ldots a_n})_{\alpha\beta}\ , \qquad \text{(A5)}$$

$$H_{\alpha\beta\gamma A_1\ldots A_{n-1}}=0\ , \qquad \text{(A5cont'd)}$$

and by taking X_{ij} occurring in (6) to be

$$X_{ij}=(-\eta/2n!)$$
$$\times\epsilon_i{}^{k_1\ldots k_n}E_{k_1}{}^{a_1}\ldots E_{k_n}{}^{a_n}(\gamma_{a_1\ldots a_n})_{\alpha\beta}\delta E^\beta E_j{}^\alpha$$
$$+\tfrac{1}{2}\kappa^\beta[E_n{}^\alpha(\gamma^a)_{\alpha\beta}E^{na}+(n+1)\Lambda_\beta]$$
$$\times g_{i[j}(T^{k_1}{}_{k_1}\ldots T^{k_n}{}_{k_n]}+\delta^{k_1}{}_{k_1}T^{k_2}{}_{k_2}\ldots T^{k_n}{}_{k_n]}$$
$$+\delta^{k_1}{}_{k_1}\ldots\delta^{k_{n-1}}{}_{k_{n-1}}T^{k_n}{}_{k_n]})$$
$$-\tfrac{1}{2}\delta E^\beta\Lambda_\beta g_{ij}+(i\leftrightarrow j)\ . \qquad \text{(A6)}$$

References

[1] P.A.M. Dirac, Proc. R. Soc. A 268 (1962) 57.
[2] P.A. Collins and R.W. Tucker, Nucl. Phys. B 112 (1976) 150.
[3] P.S. Howe and R.W. Tucker, J. Phys. A 10 (1977) L155.
[4] A. Sugamoto, Nucl. Phys. B 215 (1983) 381.
[5] J. Hughes, J. Liu and J. Polchinski, Supermembranes, University of Texas preprint (1986).
[6] M. Henneaux and L. Mezincescu, Phys. Lett. B 152 (1985) 340.
[7] M.B. Green and J.H. Schwarz, Nucl. Phys. B 243 (1984) 285.
[8] W. Siegel, Phys. Lett. B 128 (1983) 397.
[9] See e.g. A. Salam and E. Sezgin, eds., Supergravity in diverse dimensions, Vol. 1 (World Scientific, Singapore, 1987).
[10] E. Cremmer and S. Ferrara, Phys. Lett. B 91 (1980) 61; L. Brink and P. Howe, Phys. Lett. B 91 (1980) 384.
[11] E. Bergshoeff, E. Sezgin and P.K. Townsend, Phys. Lett. B 169 (1986) 191.
[12] P.S. Howe, Nucl. Phys. B 199 (1982) 309.
[13] K. Kikkawa and M. Yamasaki, Can the membrane be a unification model?, Osaka University preprint (August 1986).