SUPERMEMBRANES AND ELEVEN-DIMENSIONAL SUPERGRAVITY

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We construct an action for a supermembrane propagating in $d=11$ supergravity background. Using the constraints of $d=11$ curved superspace, we show that the action is invariant under Siegel-type transformations recently generalized by Hughes, Li and Polchinski. The transformation parameter is a world-volume scalar and $d=11$ spacetime spinor. We also discuss the general problem of the coupling of $n$-dimensional extended objects to $d$-dimensional supergravity.

1. Now that we have become accustomed to the notion that strings should replace particles, it is natural to investigate the properties of higher-dimensional extended objects, in particular of membranes since they are the simplest extended objects, and they might describe strings in an appropriate limit.

In 1962 Dirac [1] put forward a theory of an extended electron based on the idea of a relativistic membrane. In 1976, Collins and Tucker [2] studied the classical and quantum mechanics of free relativistic membranes. A year later a locally supersymmetric and reparametrization-invariant action for a spinning membrane was constructed by Howe and Tucker [3]. The action describes anti de Sitter supergravity coupled to a number of scalar multiplets in three dimensions. It is the membrane analog of the Neveu–Schwarz–Ramond formulation of the spinning string theory.


The generalization of the Hughes et al. model to $n$-extended objects propagating in flat $d$-dimensional superspace is evident. All that is required is the existence of a closer super $(n+2)$-form given by

$$ H = E^a E^a E^a \cdots E^a (\gamma_{a_1 a_2 \cdots a_{n-1}})_{a\theta} \,, $$

(1)

where $(E^a, E^a)$ are the basis one-forms in superspace. This form is closed provided that the following $\Gamma$-matrix identity holds:

$$ (\gamma^a)_{a\theta} (\gamma^{a_1 a_2 \cdots a_{n-1}})_{a\theta} = 0 \,. $$

(2)

The purpose of this note is to construct Hughes et al. -type actions describing the propagation of an $n$-extended object in $d$-dimensional curved superspace. We give a general formula for the action and the transformation rules, whose consistency requires, among other things (see below), the existence of a closed $(n+2)$-form in curved superspace. Thus we
expect that the \( n \)-extended objects under consideration can consistently propagate only in \( d \leq 11 \) supergravities whose superspace formulation involves a closed \((n+2)\)-form. We further expect that such forms exist in supergravity theories in which a closed bosonic \((n+2)\)-form occurs. As far as we know, the following possibilities exist (we include the Yang-Mills couplings whenever possible):

The dual formulation of \( d=10, N=1 \) supergravity involves a closed seven-form. Its dimensional reduction on a \((10-d)\)-dimensional torus leads to real closed \((d-3)\)-forms in \( d \)-dimensional supergravities. (These are \( N=1 \) supergravities in \( d=8, 9, 10 \); \( N=2 \) in \( d=7 \) and \( N=2 \) or \( 4 \) in \( d=6 \)) [9]. Apart from these, there is: (i) A real closed four-form in \( d=11, N=1 \) supergravity, (ii) a real closed three-form in non-chiral \( d=10, N=2 \) supergravity, (iii) a complex closed three-form in chiral \( d=10, N=2 \) or \( 4 \) supergravity. Excluding Yang-Mills couplings, as is well known, closed super three-forms exist in \( d=3, 4, 6, \) and \( 10 \).

Considering the case of the membranes, from the above list it follows that the candidate dimensions are 7 and 11. Since the superspace formulation of \( d=7, N=2 \) supergravity is not known at present, we are led to consider the supermembrane propagating in eleven-dimensional spacetime.

Our main result is the construction of an action which describes a consistent coupling of \( d=11 \) supergravity to a supermembrane. In particular the Kalb-Ramond-like third rank antisymmetric tensor field of \( d=11 \) supergravity [10] is needed for the superspace description of \( d=11 \) supergravity [11]. In (3) we have used the notation

\[
E_i^A = (\partial_i Z^M) E_M^A ,
\]

where \( Z^M(\xi) \) are the superspace coordinates, and \( E_M^A(Z) \) is the supervielbein.

Note that the action has a cosmological constant with a fixed magnitude. This is so that the field equation of the metric \( g_{\mu
u} \) gives the embedding equation

\[
g_{\mu\nu} = E_i^a E_j^b \eta_{ab} \equiv T_{ij} .
\]

We require that the action \( S \) is invariant under a fermionic gauge transformation of the form [5]

\[
\delta E^a = 0 , \quad \delta E^a = (1 + \Gamma)^{\alpha\beta} \kappa^{\beta} ,
\]

\[
\delta g_{\mu\nu} = 2 [X_{\mu\nu} - g_{\mu\nu} X^k (n-1)] (n=2 \text{ for membrane}) ,
\]

where the transformation parameter \( \kappa^\alpha(\xi) \) is a 32 component Majorana spinor, and a world-volume scalar, and

\[
\delta E^A = \delta Z^M E_M^A ,
\]

\[
\Gamma^{\alpha\beta} = (1/6\sqrt{-g}) \epsilon^{\alpha\beta\gamma} E_i^a E_j^b E_k^c \epsilon(\gamma_{abc})^{\alpha\beta} .
\]

Here \( \gamma^a(\xi=0, 1, ..., 10) \) are the Dirac matrices in eleven dimensions. \( X_{\mu\nu} \) is a function of \( E_i^a \) which will be determined by the invariance of the action. The choice of \( \delta g_{\mu\nu} \) is due to the fact that, given a variation of the action of the form \( \delta S = T_{ij} X^{ij} \), and writing this variation as

\[
T_{ij} X^{ij} = g_{ij} X^{ij} + (T_{ij} - g_{ij}) X^{ij} ,
\]

the second term on the right-hand side cancels \( (\delta S/\delta g_{ij}) \delta g_{ij} \). Thus we are left with the first term on the right-hand side, which equals the left-hand side upon the use of (5). Effectively, this means that whenever we encounter a variation of the form \( T_{ij} X^{ij} \), we can use eq. (5), provided that we add \( X^{ij} \) to \( \delta g_{ij} \) as in (6).

The matrix \( \Gamma^{\alpha\beta} \) occurring in (8) satisfies the property

\[
\Gamma^{\alpha\beta} \Gamma^\beta\delta = (T^i_{ij} T^j_{ij} T^k_{ij}) \delta^{\alpha\delta} \equiv \Gamma^2 \delta^{\alpha\delta} .
\]

The normalization in (8) is chosen such that upon the use of the equation \( T_{ij} = g_{ij} \), the matrix \( \Gamma^{\alpha\beta} \) satisfies the relation \( \Gamma^{\alpha\beta} \Gamma^\beta\gamma = \delta^{\alpha\gamma} \).
Now using (6) the variation of the action (3) is (we consider a closed supermembrane and therefore discard the surface terms)
\[ \delta S = \int d^2 \xi \left[ \sqrt{-g} \ g^{ij} \left( -\delta E^\beta E_i \gamma^{T} T_{\alpha \beta} \right) E_{j\alpha} \right. \\
+ \sqrt{-g} \ g^{ij} \left( -\delta E^\beta E_i \gamma^{T} T_{\alpha \beta} \right) E_{j\alpha} \\
+ \epsilon^{ijkl} E_i E_j E_k E_L \delta E_{\alpha} H_{\alpha CBA} \\
- \frac{1}{2} \sqrt{-g} \ \delta g^{ij} \left( T^{ij} - \frac{1}{2} g^{ij} T - \frac{1}{2} g^{ij} \right). \] (11)
The torsion two-form \( T^A \) and the four-form field strength \( H \) are defined by (our superspace conventions are those of Howe [12])
\[ T^A = dE^A + E^B \Omega_B^A = \frac{1}{2} E^B E^C T_{CB}^A, \]
\[ H = dB = \frac{1}{2} E^D E^E E^F H_{ABCD}. \] (12)

We now organize the terms in (11) according to the number of one-forms \( E^\alpha \) they contain. Those with three \( E^\alpha \) and two \( E^\alpha \) come only from the Wess-Zumino term. They must vanish separately, and this requires the constraints
\[ H_{\alpha \beta \gamma} = 0. \] (13)
The cancellation of the terms linear in \( E^\alpha \) lead to the constraints
\[ T^A_{\alpha \beta} = \delta \chi_{\alpha \beta}, \] (14)
\[ H_{\alpha \beta \gamma} = -\frac{1}{2} \gamma_{\alpha \beta}, \] (15)
while the cancellation of the terms not containing \( E^\alpha \) require the constraint
\[ \eta_{(a} T^{b)\alpha} = \eta_{ab} A_{\alpha}, \] (16)
\[ H_{\alpha \beta \gamma} = -\frac{1}{2} A_{\beta} \gamma_{\alpha \gamma}. \] (17)

Here \( A_{\alpha} \) is an arbitrary spinor superfield which is vanishing in \( d=11 \) [10].

It is important to realize that in obtaining (14)–(17) we have used the identity
\[ \delta E^\alpha = \Gamma_{\alpha \beta} \delta E^\beta + (1 - \Gamma^2) \kappa^\alpha. \] (18)
Using this identity in the variation of the kinetic term, the terms arising from \( \Gamma_{\alpha \beta} \) in (18) can be shown to cancel similar terms coming from the variation of the Wess-Zumino term, modulo terms which cancel by an appropriate variation of \( g_{ij} \). Using the argument below (8) once. In the remaining terms coming from \( (1 - \Gamma^2) \), we use the argument given below (8) repeatedly to compute further contributions to \( \delta g_{ij} \). Thus we find the result
\[ X_{ij} = -\frac{1}{2} \epsilon^{ijkl} E_k E_l E_i E_j \delta E^\beta E_{j\alpha} \\
+ \frac{1}{2} \kappa^\alpha E_n \delta \chi_{\alpha \beta} E^m E^d g_{ij} \left( T^{k \rho} T^{l \eta} + \delta^{k \rho} T^{l \eta} \right) \\
+ \iota \rightarrow j. \] (19)

In summary, the action (3) is invariant under (6) provided that (13)–(17) hold, and \( X^{ij} \) is given by (19). In addition, the following Bianchi identities must hold:
\[ DT^A = -E^B \wedge R_{B}^A, \quad DH = 0. \] (20)

The generalization of the results of this section to the general case of \( n \)-extended objects in \( d \)-dimensional supergravity is straightforward. The result is given in the appendix.

3. We observe that the superspace constraints of \( d=11 \) supergravity given by Cremmer and Ferrara [10] and Brink and Howe [10] do provide a solution to (13)–(17) and the Bianchi identities (20), with \( A_{\alpha} = 0 \).

In conclusion, we have shown that there exists a consistent coupling of a closed supermembrane to eleven-dimensional supergravity. (Note that it is natural to consider a closed supermembrane in eleven dimensions, since there are no matter multiplets in this dimension).

4. There are several directions in which the present work can be extended. We shall name a few. Firstly, it is of interest to study the quantization of the supermembrane model in eleven dimensions. In particular, the question of whether massless gauge fields can possibly arise is a challenging one. Although usually one encounters difficulties in finding massless excitations of a membrane [13], it is encouraging that, here, we have a spacetime supersymmetric membrane action.

Secondly, it is natural to consider the dimensional reduction of our model from eleven- to ten-dimensional spacetime, and at the same time from three-dimensional world volume to a two-dimensional
world sheet. It would be interesting to see what kind of\( d = 10 \) string theories could possibly emerge in an infinitely thin membrane limit.

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Appendix. In this appendix we construct the action for an \( n \)-extended object propagating in \( d \)-dimensional supergravity background. We also give the transformation rules, and the constraints on the background.

The action is

\[
S = \int d^2 \xi \left[ \frac{1}{4} - \sqrt{-g} g^{\mu \nu} E_\mu E_\nu \eta_{ab} 
+ \epsilon^{a_1 \ldots a_n} E_{a_1} \ldots E_{a_{n+1}} B_{a_{n+1}} \right] 
- \frac{1}{2} (n-1) \sqrt{-g} .
\]  

(A1)

The transformation rules are those in (6), where the matrix \( \Gamma^\alpha_\beta \) is now given by

\[
\Gamma^\alpha_\beta = [\eta' (n+1)! \sqrt{-g} ]
\times \epsilon^{a_1 \ldots a_n} E_{a_1} \ldots E_{a_{n+1}} \eta_{a b} \gamma^{b} .
\]  

(A2)

where \( \eta \) is given by

\[
\eta = (-1)^{(n+1)(n-2)/4} .
\]  

(A3)

Invariance of the action (A1) is ensured by imposing the following set of constraints:

\[
T^a_{\alpha b} = (\gamma^a)_{\alpha b} , \quad \eta_{(a} T^c_{b) \alpha} = \eta_{a b} \gamma^{c} .
\]  

(A4)

\[
H_{a b a_{n+1} \ldots a_1} = (\eta(n)! A_\beta \gamma^{a_{n+1} \ldots a_1})_{a b} .
\]  

(A5)

References