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Extruders
Leon P.B.M. Janssen

6.1 Introduction

Although there are a variety of different types of extruders, a main division that can be made is between single-screw extruders and twin-screw extruders. The most important difference between these two types of machines is the transport mechanism. A single-screw extruder consists of one screw rotating in a closely fitting barrel; the transport mechanism is based on friction between the material and the walls of the channel. If the material slips at the barrel wall, it is easy to envisage that it will rotate with the screw without being pushed forward. This makes these types of machines strongly dependent on the frictional forces at the wall and the properties of the material processed. Therefore, in the processing of natural polymers, single-screw extruders are less suitable for extrusion of mixtures with high water or high fat contents. However, many starch processes are possible in this (inexpensive) type of machine.

An exceptional type of single-screw extruder is the pin extruder. In this extruder the screw has interrupted flights and it rotates in a barrel with stationary pins. The effect is twofold: because of the rotation of the screw a good mixing action can be achieved, and because of the increased resistance against slip the throughput is more stable than in ordinary single-screw extruders. However, the pressure built up in this type of machine is rather poor. In some special types the screw not only rotates but also oscillates in an axial direction.

A twin-screw extruder consists of two screws, placed in an “8”-shaped barrel. In the case of intermeshing extruders the flights of one screw protrude into the channel of the other screw. Because of this, the polymer cannot rotate with the screw, irrespective of the rheological characteristics of the material. This indicates the most important advantage of intermeshing twin-screw extruders: the transport action depends on the characteristics of the material to a much lesser degree than in the case of a single-screw extruder.

Figure 6.1 represents the main types of extruders.
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- **The single-screw extruder (a)** is the most common machine, if no problems in transport are envisioned. The working characteristics of this relatively inexpensive machine are strongly dependent on the material properties. Very short single-screw extruders with high rotation rates are often used in the starch industry for gelatinization purposes. The barrel can be equipped with grooves to increase the friction and to prevent the material from rotating with the screw.

- **Pin extruder and co-kneaders (b)** have one single screw while the barrel is equipped with kneading pins and the screw flight is interrupted at the pin locations. In co-kneaders the screw rotates and oscillates, giving a very good mixing action. In both types of extruders different pin geometries can provide different mixing actions. The pins can also be used for monitoring the temperature or as injection points. An important feature is that the pins prevent the material from rotating with the screw and therefore ensure a more stable operation than can be provided in an ordinary single-screw extruder.

- **Tangential twin-screw extruders (c and d)** are not closely intermeshing; they can be envisaged as a parallel connection of two single-screw extruders with

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![Figure 6.1 Different types of extruders: a) single-screw; b) co-kneader; c) non-intermeshing, mixing mode; d) non-intermeshing, transport mode; e) counter-rotating, closely intermeshing; f) co-rotating, closely intermeshing; g) conical counter-rotating; h) self-wiping, co-rotating.](image)
mutual interaction. A model based on three parallel plates is often used to describe this type of twin-screw extruder. This model shows a strong resemblance to the two-parallel-plate model that is used for single-screw extruders. This type of machine has clear advantages if very elastic materials have to be extruded. The screws can be arranged in two different ways: a mixing mode (c) or a transport mode (d). All commercial tangential twin-screw extruders are counter-rotating.

- **The closely intermeshing twin-screw extruders, both counter-rotating (e) and co-rotating (f)** can best be modeled as series of C-shaped chambers. Because of the rotation of the screws these chambers transport the material from hopper to die, while interactions between the chambers occur through leakage flows. In general these leakage gaps are larger in co-rotating machines than in counter-rotating ones. Because of the large resistance to back-flow through the narrow gaps, these extruders have strong positive conveying characters and their stabilities are high.

- **The screws and the barrel of a closely intermeshing twin-screw extruder can also be conical (g)** This has advantages for the feeding process if the material has a low bulk density. While it is passing through the extruder the chambers gradually decrease in size and compress the material. Moreover, conical screws provide a larger space for the bearings of the screws and the screws can easily be removed from the barrel.

- **Depending on the exact geometry, self-cleaning co-rotating twin-screw extruders (h)** can be described in two ways. They can be modeled as series of C-shaped chambers with very wide leakage gaps or, more commonly, they can be considered to be constructed as continuous channels with some flow restrictions at regular intervals. This type of machinery imposes high shear forces on the material, and special shearing elements to increase shear even further are common. Both screw configurations with two or three lobes or thread starts per screw exist. Two-lobe screws possess a better conveying capacity and provide a higher throughput, whereas three-lobe screws have greater mechanical strength and higher shear rates in the channel. Most modern machines have two-lobe screws.

Closely intermeshing extruders, both co-rotating and counter-rotating, have in general deeply cut channels and narrow leakage gaps. Their rotational speeds are usually low, due to the large shear forces on the material in the gaps, so the average shear level imposed on the material in the chambers is low. Self-wiping extruders, on the other hand, have shallow channels (especially if equipped with triple-flighted screws). They operate with high rotational speeds and the transport efficiency (the amount of material transported per revolution) is lower than in closely intermeshing machines. In these machines the material is subjected to high average shear forces and the viscous dissipation is much larger than in closely intermeshing machines.

Conical twin-screw extruders have the advantage that the space provided for the bearing is large. Moreover, by moving the screws axially it is possible to
compensate for wear of the screws. A disadvantage of machines of this type is that screw elements with different geometries are not easily interchangeable.

Tangential extruders are often used in situations in which the elasticity of the material would pose problems in the leakage gaps. Extruders of this type are mainly found in the rubber industry.

In general it can be concluded that, because of the large shear forces in the channel and the simple application of mixing elements, self-wiping extruders are often used in processes in which high shear is desirable. Their high shearing action is particularly convenient if intensive mixing or devolatilization is required. When working with thermoplastic starches the low average shear in closely intermeshing extruders is an advantage because degradation of the starch results in deterioration of mechanical properties. Moreover, because of the low shear levels the heat generated by viscous dissipation is low, allowing for good temperature control. On the other side, self-wiping extruders generally display better mixing action and higher throughput at comparable screw diameters.

6.2
Single-Screw Extruders

Single-screw extruders can be used in processes in which no slip occurs at the wall and in which high shear during the process is not a problem. Their working mechanism and modeling is well covered in the literature and an extensive description can be found in various books such as those by Rauwendaal [1] and by Tadmor and Klein [2]. The basis of the fluid flow in single-screw extruders can also be generalized to other types of extruders, so here we introduce a simple analysis based on Newtonian behavior of the liquid. For this analysis the channel of the screw is simplified into a flat plane geometry. The single-screw extruder consists of a barrel containing a rotating screw. As a first step in the simplification we will keep the screw stationary and let the barrel rotate. The next step is to unwind the screw channel into a straight trough. Figure 6.2 shows the results. The rotation of the screw can now be transformed into the movement of a plate over the channel. The velocity of this plate is of course the circumferential velocity of the screw and equals \( \pi ND \), whereas its direction relative to the channel equals the screw angle (\( \phi \)). \( N \) is the rotation rate of the screw and \( D \) is the screw diameter.

![Figure 6.2 The channel in a single-screw extruder.](image-url)
The velocity profile in the down-channel direction can, under the approximation of a Newtonian rheology, easily be calculated and, after integration, the throughput follows:

\[ Q = AN - B \frac{\Delta p}{\eta} \]  

(6.1)

where \(N\) is the screw rotation rate, \(\Delta p\) is the pressure difference over the zone and \(\eta\) is the viscosity. \(A\) and \(B\) are parameters that depend only on the screw geometry as shown in Section 6.8.1. The interesting aspect of this equation is that the influence of the rotation rate (the drag flow) and the influence of the pressure (the pressure flow) are uncoupled (Figure 6.3).

If the flow across the flights of the screw can be neglected (which is generally the case for the hydrodynamics in single-screw extruders) the velocity profile in the direction transverse to the channel is given by:

\[ v_x = 3U_y \left( \frac{2y}{H} - \frac{y}{H} \right) \]  

(6.2)

The profile of which, in combination with the down-channel direction, shows a helical path of the fluid elements through the channel.

The cross-channel flow forms a circulatory flow as sketched in Figure 6.4. The center of circulation, where \(v_x\) equals zero, lies at \(2/3\) of the channel height. This cross-channel profile must of course be combined with the down-channel profile; the cross-channel flow should be superimposed on the flow in the channel direction. As a result the polymer elements follow a helical path through the channel. The center of rotation of this helical flow lies at \(2/3\) of the channel height. Particles in this location follow a “straight” line through the channel without being interchanged with particles at other locations. Therefore these fluid elements will never

Figure 6.3 Superposition of drag flow and pressure flow in a single-screw extruder.

Figure 6.4 Rotating flow in the cross–channel direction.
approach the wall closer than 1/3 of the channel depth if no mixing elements are used in the screw design. This will appear to be particularly important when considering heat transfer and thermal homogenization in processes with viscous dissipation.

6.3 Pin Extruders

A particular type of single-screw extruder that can be used for starch processing is the pin extruder. An extruder of this type may have two different designs: one with stationary pins and a simple rotating screw and interruptions of the flights, called the expander, and one with a rotating and axially oscillating screw, generally known as a co-kneader. The latter type of machine is also often used in the rubber industry and in processes in which intensive mixing is required. Though discovered in 1945 and commonly used in industry, its application is far ahead of theoretical understanding.

The co-kneader also consists of a single screw with interrupted flights [3]. A schematic representation is given in Figure 6.5. Its working principle is based on the rotation and axial oscillation of the screw, causing transportation and mixing. The mixing is enhanced by stationary pins in the barrel. During one passage of the pin, the material is subjected both to high shear stress and to reorientation. The dispersive mixing process is promoted by the local weaving action of the pins and screw flights, in which the distributive mixing is enhanced by the reorientation that is introduced by the pins. Figure 6.6 displays an unrolled screw and barrel. The trajectories of the kneading pins are visualized in relation to the rotating screw. It is clear that all surfaces are subjected to wiping actions.

The expander extruder is a relatively simple machine. The function of the pins is twofold: to provide a more stable throughput and to improve mixing. Because of the stationary pins, slip at the wall is prevented and the material cannot rotate with the screw. This provides a more stable throughput. The mixing is increased
relative to an ordinary single-screw extruder. Pin extruders provide both distribu-
tive and dispersive mixing. The distributive mixing is caused by the geometry of
the screw flights. The interruptions in the flights divide material in the screw
channel into two streams. After the following interruption of the screw flight the
streams recombine partially and the material is divided again. The kneading pins
also provide a rearrangement of the stream lines. Because of these two effects the
distributive mixing of a pin extruder is very good and the distributive mixing
process requires relatively low energy.

Through the rotating and oscillating movement of the screw in a co-kneader,
the screw flights slide along the pins and barrel. This results in high shear stresses
and disperse mixing. It also leads to good self-cleaning properties, which is much
less the case in expander extruders.

The temperature can be controlled by the thermostatted barrel and screw. The
large area/volume ratio, together with the radial mixing, contribute to good heat
transfer capacities. These characteristics make pin extruders well-suited for proc-
esses in which good dispersive mixing is required, although a disadvantage is that
the relatively high shear can easy give rise to degradation of the starch. Because
of the large interruptions of the flights it is not possible to use this machine for
pressure build-up. If, at the end of the line, pressure is needed, a normal single-
screw extruder is generally placed behind the pin extruder. The mixing mecha-
nism in the co-kneader is rather complex.

6.4 Closely Intermeshing Twin-Screw Extruders

If the chambers of a closely intermeshing twin-screw extruder are fully filled with
material, the maximum throughput of a zone \(Q_{th}\) can be written as the number
of C-shaped chambers (Figure 6.7) that is transported per unit of time, multiplied
by the volume of one such chamber:

\[
Q_{th} = 2mNV
\]  

(6.3)
Here $N$ is the rotation rate of the screws, $m$ is the number of thread starts per screw, and $V$ is the volume of a single chamber. In reality the output of a twin-screw extruder is, of course, smaller than the theoretical throughput, because the chambers are not completely closed. Four different kinds of leakage flows can be distinguished [4] (see Figure 6.8):

- Leakage ($Q_f$) through the gap between the flights and the barrel wall. This leakage shows clear parallels with the leakage that occurs in a single-screw extruder. The gap through which this leakage flows is called the flight leak.

- Leakage ($Q_c$) between the flight of one screw and the bottom of the channel of the other screw. Because the flow through this gap resembles the flow in a calender this leak is called the calender leak.

- Leakage ($Q_t$) through the gap between the sides of the flights, which is called the tetrahedron leak. In principle, this leak is the only leak that leads from one screw to the other. In closely intermeshing counter-rotating twin-screw extruders, this gap is generally very narrow. In self-cleaning counter-rotating twin-screw extruders this gap is very wide and the major part of the material passes through this gap regularly.
• Leakage \( Q_s \) through the gap between the sides of the flights, normal to the plane through the two screw axes. This leak is called the side leak. In its behavior this leak most resembles the calender leak.

The throughput through the different leakage gaps consists partly of drag flow and partly of pressure flow. The pressure flow, in its turn, is a consequence of the internal pressure build-up in the chambers and of the pressure that is built up at the die, resulting for the leakages in:

\[
Q_f = A_f N + B_f \frac{\Delta P}{\mu} \quad Q_s = A_s N + B_s \frac{\Delta P}{\mu} \\
Q_c = A_c N + B_c \frac{\Delta P}{\mu} \quad Q_t = A_t N + B_t \frac{\Delta P}{\mu}
\]

(6.4)

here the subscripts f, s, c, and t stand for flight, side, calendar, and side gap. The total amount of leakage through a section of the extruder is the sum of the individual leakage flows. For a screw with \( m \) thread starts this can be written as [4]:

\[
\Sigma Q_i = 2Q_f + Q_s + 2m(Q_c + Q_t) = AN + B \frac{\Delta P}{\mu}
\]

(6.5)

in which \( \Delta P \) is the pressure drop between two consecutive chambers, \( \mu \) the viscosity, and \( N \) the rotation rate of the screw. Numerical values for \( A \) and \( B \) and for the chamber volume \( (V) \) follow from the geometrical parameters of the screws only and are given in Section 6.8.2 of this chapter. The real throughput of a pumping zone in a twin-screw extruder, completely filled with polymer, can now be determined easily.

\[
Q = Q_{th} - \Sigma Q_i = (2mV - A)N - B \frac{\Delta P}{\mu}
\]

(6.6)

For Newtonian liquids the use of dimensionless numbers for throughput and pressure drop leads to simple relations:

\[
Q^* = \frac{Q}{2mNV} \\
P^* = \frac{\Delta P}{N\mu}
\]

(6.7)

Here \( \Delta P \) is the pressure drop per chamber, caused by the pressure in front of the die, and \( \mu \) is the Newtonian viscosity. These two dimensionless groups for throughput and pressure are very powerful. When these groups are used, the characteristics for the completely filled pumping zone of twin-screw extruders are straight lines that are independent of viscosity and speed of rotation of the screws. Figure 6.9 presents an example of these lines and their dependence on the size of the
calender gap. As these lines are only dependent on geometrical parameters, they keep their validity when the pumping zone of a twin-screw extruder is scaled up geometrically. If, for instance, the screw diameter, the pitch, the chamber height, and the different leakage gaps are all enlarged by the same factor, the lines do not change.

### 6.4.1 The Different Zones

If a twin-screw extruder is stopped and opened, several zones can be distinguished clearly (Figure 6.10). Depending on whether the extruder is fed with a solid or a liquid material, two different situations occur. In the case of a solid feed (which is generally the case in starch processing) the chambers near the feed hopper are more or less filled with solids. This material plasticizes, and a zone with only partly filled chambers can be seen. At the end of the screw, close to the die, the chambers are completely filled with material.

If the extruder is fed with a liquid, as in some candy processes, the first part does not necessarily need to be partly empty. However, as will be explained later, for reasons of stability it is advisable to create a zone in which the chambers are not fully filled.

In particular, the fully filled zone is very important for proper functioning of the extruder. In this zone the pressure is built up, mixing and kneading mainly take place, and the major influence of viscous dissipation also occurs. In order to explain the existence of the fully filled zone, we need to realize that the different
zones in a twin-screw extruder cannot be viewed separately, but are interconnected. This can be shown by the throughput.

The actual throughput of a twin-screw extruder is determined by the feeding zone. What comes into the extruder here will also have to leave the extruder at the other end. Because the chambers are only partially filled no pressure can be built up in this zone and the leakage flows will be limited to the drag component only. Under normal circumstances, the throughput of this zone is therefore independent of pressure at the die end of the extruder.

In the last part of the extruder, where the pressure that is needed for squeezing the polymer through the die is built up, the chambers are fully filled with material, a pressure gradient is present, and considerable leakage flows are dependent on this gradient.

The effect of the throughput is as follows:

As derived, the actual throughput through the completely filled zone is given by:

$$Q = 2mNV - \Sigma Q_j$$

(6.8)

but the real throughput is determined by the feeding zone:

$$Q = 2m, NV, \varepsilon$$

(6.9)

in which $\varepsilon$ is the degree of filling of the chambers in this zone and the index $v$ indicates that volume and number of thread starts of the screws relate to the geometry in this zone. For reasons of continuity, the difference between theoretical throughput and real throughput should equal the sum of the leakage flows:

$$\Sigma Q_j = 2N (mV - m, V, \varepsilon)$$

(6.10)

Because the degree of filling ($\varepsilon$) does not depend on the final pressure and because all other parameters in the right-hand term of this equation are also independent of pressure it becomes clear that the sum of the leakage flows in a twin-screw extruder must be independent of pressure. However, if the equation for the leakage flows is taken into consideration:

$$\Sigma Q_j = AN + B \frac{\Delta P}{\mu}$$

(6.11)

the pressure drop per chamber is fixed and dependent on viscosity and geometry only. If geometry and viscosity were not to change, the pressure gradient would be constant over the whole completely filled zone. In addition, a fixed pressure is built up in the die of the extruder, depending on throughput, viscosity, and die geometry. Therefore, within reason, there will be a point in the extruder at which the actual pressure becomes zero. Between this transition point of partly and fully filled chambers and the die there exist pressure gradients, there are leakage flows, and the chambers are completely filled with polymer. Between this transition point
and the feed hopper, there is no difference in pressure between consecutive chambers, the leakage flows are zero or consist only of drag components, and the chambers are only partly filled with material.

The length of the completely filled zone is an important factor, and for good process control, knowledge of the different parameters that influence the length of the completely filled zone is indispensable. Table 6.1 gives this influence schematically.

- If, for instance, for the simplified case that the extruder is filled with an isoviscous liquid, the resistance of the die is doubled, the pressure in front of the die will also double, because the output remains constant. However, the leakage flow is not influenced by the die pressure, so the pressure gradient in the extruder remains constant. Ergo, the completely filled length increases as indicated in Figure 6.11.

- If the rotation speed is doubled, while the specific throughput (throughput per revolution) is kept constant, the throughput will also double and consequently
the die pressure will do so too. Because the leakage flows also double, the pressure gradient will double and the length of the completely filled zone will remain the same (Figure 6.11).

If the viscosity is not constant, the ability to build up pressure is proportional to the viscosity. Changes in consistency—by gelatinization of the starch or by mixing with polyols, for instance—influence the local pressure gradient and therefore the filled length. In addition, the influence of the temperature in this scheme is based on a change in the local viscosity and, because of that, on pressure drop.

6.4.2 Co-Rotating Versus Counter-Rotating Closely Intermeshing Extruders

Both co-rotating and counter-rotating twin-screw extruders can be modeled by a C-shaped chamber model. However, there are some differences. For reasons of construction (the screws must fit into each other) the tetrahedron gap in a co-rotating machine is generally bigger than in a counter-rotating machine. Moreover, the drag flow in the tetrahedron gap in a co-rotating extruder is parallel to the direction of this gap. Whereas in a counter-rotating extruder the direction of internal pressure generation favors the flow through the calender gap, in co-rotating machines this pressure generation favors the tetrahedron flow (Figure 6.12). Because the tetrahedron leakage is the only leakage connecting the chambers on one screw with the chambers on the other screw, the mixing between material on the different screws is better in co-rotating machines. In counter-rotating extruders both drag flow and internal pressure generation favor the calender leak. Because the material is elongated and kneaded in the calender gap it can be concluded that counter-rotating extruders favor a good kneading action.

6.5 Self-Wiping Twin-Screw Extruders

An important difference between closely intermeshing and self-wiping twin-screw extruders is the way the screws fit into each other. In self-wiping extruders the screw geometry is such that in the plane through both screw axes there is a very close fit between the two screws (Figure 6.13). This requires a special geometry...
with, as a consequence, a very large tetrahedron gap between the “chambers”. Because of this special character of most self-wiping extruders, the C-shaped chamber concept has to be abandoned; a model based on continuous channels can better be used. Therefore, the self-wiping extruder acts more as a drag pump than as a displacement pump. Figure 6.14 shows, analogously to single-screw extruders, the unwound channel. Each time when the material changes screw during its transport through a channel there exists a certain flow restriction. This can give an extra pressure build-up, which is, however, in general very minute, and for practical purposes this effect is often neglected.

Screws of self-wiping extruders consist of one, two, or three thread starts. With increasing numbers of thread starts the distance between the screw axes has to increase and as a consequence the maximum channel depth decreases, which in turn influences the maximum throughput per screw rotation. For this reason extruders with four or more thread starts are not common. Because there exist hardly any parallel planes close to each other in the geometry of a self-wiping machine, their rotational speed can be chosen to be much higher than for closely intermeshing twin-screw extruders. Combined with the shallow channels this leads to a high average shear level, which is in practice five to ten times higher than in closely intermeshing machines. The shear levels can be increased further by the use of so-called mixing or kneading elements. Changes in the angle between these elements determine the kneading action, as will be seen later.

6.5.1
Screw Geometry

Because the screws have to fit closely in the plane through the axes, the degrees of freedom in screw geometry are very limited. Because of the requirement of close
fitting in a cross section perpendicular to the axes, the surfaces of the screws must always (nearly) touch. This is shown in Figure 6.15. The channel depth as a function of the angle $\psi$ can be written in its most elementary form as:

$$H(\psi) = R(1 + \cos \psi) - \sqrt{c^2 - R^2 \sin^2 \psi}$$

(6.12)

where $c$ is the center line distance [5]. More complicated geometries exist, with use of different radii of curvature to obtain geometries with a larger flight width or deeper channels. The cross section of the screw elements is the same as the cross section of the kneading elements. If extra pressure build-up is required, elements with a narrower pitch (pumping elements) are used. Nearly all self-wiping extruders are based on screws with separate screw elements and, depending on process requirements, different screw lay outs can be constructed.

6.5.2 Transporting Elements

The transporting elements in a self-wiping extruder, similarly to single-screw extruders, can be modeled by a channel with a plate moving over it and their working is again basically by drag flow. The throughput can therefore be expressed as:

$$Q = \left( m - \frac{1}{2} \right) ANf_{ds} - (2m - 1) B \frac{\Delta P}{\mu} f_{ps}$$

(6.13)

where $A$ and $B$ are geometrical parameters and $m$ is the number of tread starts per screw. The most common value for $m$ is two or three, extruders with double tread starts ($m = 2$) having a larger throughput and triple start screws ($m = 3$) exerting a higher shear on the material. The correction factors $f_{ds}$ and $f_{ps}$ account for the curvature in the channel.

It can be seen that the throughput can also be divided into a drag component and a pressure component—not interrelated under the assumption of Newtonian rheology—in the case of a self-wiping transport element.
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6.5.3 Elements for Pressure Build-Up

Through the construction of screws with a large overlap in the intermeshing region, the pressure build-up abilities can be increased considerably. In the limiting case the self-wiping profile will be lost and C-shaped chambers will emerge. Figure 6.16 shows these so-called pressure build-up elements. The chosen pitch is generally smaller than in transport elements. A model based on C-shaped chambers leads to good results for this type of elements.

6.5.4 Kneading Elements

The third type of elements present in self-wiping twin-screw extruders is the kneading element (Figure 6.17). These elements consist of kneading discs that have the same cross sectional shape as the transport elements. The angle between the individual kneading discs, the so-called stagger angle, determines the kneading action. Stagger angles of 30, 60, 90, 120, and 150° are frequently used; the last two angles (120 and 150°) are also sometimes referred to as −60 and −30°. As a general rule it can be stated that the larger the angle between the discs (30, 60, 90, 120,
150), the larger the kneading action, but this also results, of course, in a larger energy dissipation in the element. In addition, the pressure drop over the kneading element is dependent on the stagger angle. At moderate throughputs a kneading element with a stagger angle up to 60° can still build up some pressure; in all other situations pressure is needed to transport the polymer through the element. This dependence on the stagger angle can be understood if we regard the kneading element as an interrupted screw flight.

At angles of 30 and 60° there exists a certain pumping action that depends on the rotational rate. At larger throughputs, the pressure drop over the element will decrease or, as stated, even become negative. In kneading elements with a stagger angle of 90° the pumping action is absent. This implies that the pressure needed to pump the polymer through the kneading element is independent of the rotation rate but proportional to the throughput. At stagger angles larger than 90° the kneading element acts as a screw element with reversed pitch. The transport elements in front of the kneading element have to pump against the reversed pumping action of the kneading element, and high shear levels and large energy dissipation will be attained. Figure 6.18 shows an example of the pressure drop over the kneading elements as a function of the stagger angle at two different rotation rates at large and at small throughput. At increasing throughput the curves will be shifted upward, resulting in higher pressure drops at large stagger angles and disappearance of the pressure generation at low stagger angles. For reactive extrusion the kneading elements have two functions: they improve the mixing and, as will be seen in the next paragraph, they increase the filled length in front of the element and therefore the hold-up in the extruder-reactor.

6.5.5
The Fully Filled Length

Similarly to the situation in closely intermeshing twin-screw extruders, in self-wiping extruders it is possible to distinguish different regions where the screws
are either fully filled with material or only partially filled. Again, in the fully filled region pressure is built up, whereas in the partially filled zone the pressure gradient equals zero. This implies that a fully filled zone must be present before the die and before pressure-consuming kneading elements. The lengths of these zones before the die ($L_d$) and before a kneading section ($L_m$) can be simplified to:

$$L_d = \frac{p}{\mu(AN-BQ)}$$

$$L_m = \frac{A'Q+B'N}{C'N-2Q}$$

The above equations can be used to quantify the influence of different extrusion parameters on the filled length. It is striking that the filled length in front of a kneading section is independent of viscosity. This is due to the simplification we have made in assuming an iso-viscous process: if the viscosity changes, a relative viscosity factor has to be introduced. The fact that the filled length is independent of the absolute value of the viscosity can be understood if we realize that both the pressure-generating capabilities of the transport elements and the pressure drop over the kneading elements are proportional to the viscosity: the absolute viscosity therefore has no influence on the length of the fully filled zone. Moreover, the equations show that if we change the rotational speed and the throughput in the same way—or, in other words, if we keep the relative throughput constant—the fully filled length does not change. If the throughput increases at constant rotation speed the filled length will increase, and if the rotation rate increases at constant throughput the filled length decreases. Finally, an increase in the stagger angle in kneading elements (increasing $B'$) will also increase the filled length. Application of kneading elements with a more severe kneading action (larger stagger angles) will not only increase the kneading action in the elements themselves, but will also result in a larger filled length, resulting in extra mixing. Figure 6.19 shows an example of the pressure profile in a screw, consisting of transport elements, followed by pressure build-up elements, a kneading zone, transport elements, and finally pressure build-up elements before the die. At loca-

![Figure 6.19](image-url)
tion A the material is pressureless and the channel is not necessarily fully filled. This location can be used for, for instance, devolatilization or for an easy feeding of an extra component. However, it should be realized that at increasing throughput the filling of the extruder increases and the pressureless zone will disappear.

6.6 Non-Intermeshing Twin-Screw Extruders

Non-intermeshing twin-screw extruders can be seen as a first approximation as two parallel single-screw extruders (Figure 6.20). This would be accurate if the open area that connects the two barrel halves (the apex area) were zero. Because of the existence of the apex area, the screws interact with each other and the throughput of a non-intermeshing twin-screw extruder will be less that that of two single-screw extruders. The screws can be arranged in two different ways (Figure 6.21): in the staggered configuration mixing is enhanced, but pressure build-up abilities are low, whereas in the matched configuration a better pressure build-up is achieved at the cost of some mixing abilities.

The conveying action of this type of machine can be derived from a three-parallel-plate model, in which the outside plates represent the screw surfaces and the middle plate (with zero thickness) represents the barrel. The apex is accounted for by parallel slots in the middle plate, perpendicular to the circumferential

Figure 6.20 Cross section through a non-intermeshing twin-screw extruder with staggered configuration.
velocity. This model leads (again) to a throughput that can be divided into a term dependent on the rotation speed (the drag flow) and a term dependent on the pressure difference (the pressure flow), leading to:

\[ Q = AN - B \frac{\Delta p}{\mu} \]  

(6.15)

6.7 Influence of Low-Viscosity Monomeric Feed

In the simplified case of an iso-viscous process the pressure built up along the filled length appeared to be constant. In reactive extrusion, however, the material is generally far from iso-viscous, which influences the filled length considerably. Figure 6.22 shows the influence of increasing viscosity on the pressure built up in the extruder. For a simple screw with uniform geometry this pressure build-up is uniform in the simplified case of an iso-viscous material (a). For reactive extrusion the viscosity increases in the direction of the die and the low viscosity in the filling region results in poor local pressure build-up abilities (b). This makes the extruder more sensitive to disturbances, and small fluctuations in pressure may
result in large changes in the fully-filled length and therefore in residence time. Moreover, overfilling of the extruder can easily occur [9].

A low-viscosity feed stream also influences the working of the feed zone. In this partially filled zone, material with low viscosity is not likely to be dragged properly along the channels of the screw but tends to flow on the bottom of the extruder because gravitational forces predominate rather than viscous forces. A dimensionless parameter that relates gravitational and viscous forces is the Jeffreys number:

\[ Je = \frac{\rho g D}{\mu N} \]  

where \( g \) is the gravitational acceleration. This Jeffreys number equals the ratio between the Reynolds number and the Froude number. De Graaf et al. [10] described experiments in which the pattern of the fluid in the chambers was correlated to the Jeffreys number and to the degree of fill. Three different patterns could be distinguished (Figure 6.23):

- at low degrees of filling and high Jeffreys numbers the material remained basically at the bottom of the channel and was not carried over the screws,
- at low degrees of filling and low Jeffreys number the material collected at the pushing flight, and
- at high degrees of filling the material sticks to both flights.

6.8 The Mathematics of Extrusion

Very sophisticated numerical models for different types of extruders exist, and computational fluid dynamics nowadays plays an important role in extruder modeling. Nevertheless, the different extruders can also be described with relatively
simple models. Extensive description of these models can be found, mainly in the literature for extrusion of synthetic polymers \([1, 2, 4]\). Nevertheless, many features of these theories can also be used for starch processing, and therefore some simple mathematical models with general validity are presented here.

### 6.8.1 Single-Screw Extruders

As stated before, the rotation of the screw can be transformed into the movement of a plate over a flat channel. The velocity of this plate is of course the circumferential velocity of the screw and equals \(\pi ND\), whereas its direction relative to the channel equals the screw angle \(\phi\). \(N\) is the rotation rate of the screw and \(D\) is the screw diameter.

The movement of the plate has a component in the down-channel direction and a component in the cross-channel direction; both drag the liquid along and introduce flow profiles with components parallel and perpendicular to the direction of the channel.

\[
U_z = \pi ND \cos \phi \\
U_x = \pi ND \sin \phi
\]

The flow in the down-channel direction can be calculated from a force balance:

\[
v_z = \pi ND \cos \phi \frac{y}{H} - \frac{1}{2\mu} \frac{dP}{dz} \left( \frac{y}{H} - \left( \frac{y}{H} \right)^2 \right)
\]

where \(dP/dz\) denotes the pressure gradient in the down-channel direction. A closer look at this equation reveals that the right-hand side consists of two terms. The first part (apart from geometric parameters) only depends on the rotational speed and the second part is a unique function of the pressure, so the effects of screw rotation and pressure can be separated. This is shown in Figure 6.3. The actual flow profile is a superposition of the (linear) drag flow and the (parabolic) pressure flow. However, strictly speaking, this separation is only valid for Newtonian liquids.

From the velocity profile in the down-channel direction the throughput of the pump zone of a single-screw extruder can be obtained by integration:

\[
Q_v = W \int_0^H v(z) dy = \frac{WH}{2} \pi ND \cos \phi - \frac{H^3 W}{12\mu} \frac{dP}{dz}
\]

The effects of drag flow and pressure flow can also be separated in this equation; the drag flow is proportional to the rotational speed of the screws and the pressure flow is proportional to the ratio of pressure gradient and viscosity.

This equation is often written as:
\[ Q = \frac{1}{2} \pi^2 ND^2 H (1 - a) \sin \theta \cos \theta \]  
(6.20)

\( \theta \) is the flight angle and \( a \) is the throttle coefficient, which signifies the ratio between the pressure flow and the drag flow:

\[ a = \frac{H^2 \Delta P \tan \theta}{6\mu(\pi ND)L} \]  
(6.21)

In the analyses above the width of the flight is neglected; it can be shown that this simplification has no significant influence on the final outcome.

An analogous calculation can also be set up for the transverse direction of the channel, and if the flow across the flights of the screw can be neglected (which is generally the case for the hydrodynamics in single-screw extruders), the velocity profile in the direction transverse to the channel can easily be calculated.

\[ v_x = 3U_s \frac{y}{H} \left( \frac{2}{3} - \frac{y}{H} \right) \]  
(6.22)

This profile, in combination with the down-channel direction, shows a helical path of the fluid elements through the channel.

The cross-channel flow produces a circulatory flow as sketched in Figure 6.4. The center of circulation, where \( v_x \) equals zero, lies at 2/3 of the channel height. This cross-channel profile must of course be combined with the down-channel profile; the cross-channel flow should be superimposed on the flow in the channel direction. As a result the polymer elements follow a helical path through the channel. The center of rotation of this helical flow lies at 2/3 of the channel height. Particles in this location follow a “straight” line through the channel without being interchanged with particles at other locations. Therefore these fluid elements will never approach the wall closer than 1/3 of the channel depth if no mixing elements are used in the screw design. This will appear to be particularly important when considering heat transfer and thermal homogenization for reactions in which large amounts of reaction heat are released.

6.8.2

**Correction Factors**

Strictly speaking, Equations 6.18 and 6.19 are only valid for straight extruder channels of infinite width. To account for the curvature of the channel and the finite width, correction factors can be used and Equation 6.19 can be written as:

\[ Q_s = \frac{WH}{2} \pi ND \cos \phi \cdot f_a - \frac{H^3 W}{12\mu} \frac{dP}{dz} \cdot f_p \]  
(6.23)
The correction factors follow from an analytical solution of a two-dimensional stress balance:

\[ f_d = \frac{16W}{\pi^3 H} \sum_{i=1,3,5} \frac{1}{i^3} \tanh \left( \frac{i\pi H}{2W} \right) \]

\[ f_p = 1 - \frac{192H}{\pi^3 W} \sum_{i=1,3,5} \frac{1}{i^3} \tanh \left( \frac{i\pi W}{2H} \right) \]

(6.24)

For practical purposes \((H/W < 0.6)\) these factors can conveniently be approximated by:

\[ f_d = 1 - 0.57 \frac{H}{W} \]

\[ f_p = 1 - 0.62 \frac{H}{W} \]

(6.25)

Basically these correction factors can be used not only for single-screw extruders but for all types of extrusion processes.

### 6.8.3 Counter-Rotating Closely Intermeshing Twin-Screw Extruder

The equations for the different leakage gaps are mathematically simple [4]. The variables are defined in Figure 6.15. For the tetrahedron gap in a counter-rotating twin-screw extruder an empirical equation exists:

\[ Q_t = 0.0054 \left( \frac{H}{R} \right)^{1.8} \left[ \psi + 2 \left( \frac{6 + \sigma \tan \psi}{H} \right) \right]^{2/3} \Delta PR \pi \mu \]

(6.26)

The derivation of the flight leakage is similar to that for the flight leakage in a single-screw extruder:

\[ Q_f = (2\pi - \alpha) R \left\{ \frac{NB\delta}{2} + \frac{\delta^3}{6B} \left[ 3\mu \frac{NB}{H^2} \left( \frac{B}{m} - B \right) + \Delta P \right] \right\} \]

(6.27)

The equation for the calender leakage follows the classical derivation for the throughput of a two-roll calender, with the only difference being that the velocities of the two calender surfaces are different because of the different radii:

\[ Q_c = \frac{4B}{3m} \left\{ \frac{2m\Delta P\sigma^3}{6\pi\mu(2R-H)\sigma/2} + N\pi(2R-H)\sigma \right\} \]

(6.28)
For the side leak a semiempirical equation is used:

\[
Q_s = \pi N (2R - H)(H - \sigma)(\varepsilon + \sigma \tan \psi) + \frac{m \Delta P (H - \sigma)(\varepsilon + \sigma \tan \psi)^3 \cos \psi^2}{12 \mu R \sin \frac{\alpha}{2} \left[ 1 - 0.63 \frac{\varepsilon + \sigma \tan \psi}{H - \sigma} \cos^2 \psi + 0.052 \left( \frac{\varepsilon + \sigma \tan \psi}{H - \sigma} \cos^2 \psi \right)^2 \right]^5
\]  

(6.29)

From the equations above it can simply be deduced that every leakage flow has two components: one drag component, proportional to the rotational speed, and one pressure component, proportional to the pressure difference between two consecutive chambers and inversely proportional to the viscosity. The proportionality factors are only dependent on geometrical parameters. The only exception is the tetrahedron gap, which only depends on pressure differences. The total amount of leakage can now be written as:

\[
\sum Q_i = AN + B \frac{\Delta P}{\mu}
\]  

(6.30)

in which, as can be seen from the equations, A and B are constants that only depend on the geometry of the screws.

The volume of a C-shaped chamber can also easily be determined by straightforward calculations. For this we subtract the volume of the screw over the length of one pitch from the free volume of the barrel over the same length. The volume of the barrel follows from Figure 6.15:

\[
V_R = \pi R H^2 - \left( \left( \pi - \frac{\alpha}{2} \right) R^2 + \left( R - \frac{H}{2} \right) \sqrt{RH - \frac{H^2}{4}} \right) S
\]  

(6.31)

The volume of the screw root is:

\[
V_2 = \pi (R - H)^2 S
\]  

(6.32)

and the volume of the flight is:

\[
V_j = \int_{R - H}^{R} b(r) * 2\pi r dr
\]  

(6.33)

For a screw with straight-sided flights this yields:

\[
b(r) = B + 2(R - r) \tan \psi
\]  

(6.34)

and the integral can be written as:

\[
V_j = 2\pi \left( \left( RH - \frac{H^2}{2} \right) B + \left( RH^2 - \frac{2}{3} H^3 \right) \tan \psi \right)
\]  

(6.35)
The total volume of one chamber can now be calculated from:

\[ V = \frac{V_i - V_j - mV_i}{m} \quad (6.36) \]

### 6.8.4 Self-Wiping Twin-Screw Extruders

Because the working of self-wiping twin-screw extruders is mainly based on drag in an open channel, the equations derived for single-screw extrusion can be used. Combination of drag flow and pressure flow in one single channel leads to an equation for the throughput per channel:

\[ Q = \frac{1}{2} WH_0 U_z f_{ds} - \frac{WH_0^3}{12\mu} \frac{dP}{dz} f_{ps} \quad (6.37) \]

Here \( f_{ps} \) and \( f_{ds} \) are correction factors for the channel geometry and \( H_0 \) is the maximum channel depth. In cases involving rectangular screw channels the correction factors can be calculated analytically; for the complex geometries of self-wiping extruders they can be approximated or can be calculated numerically [6]. The number of parallel channels in a screw with \( m \) thread starts equals \( 2m-1 \). This leads to a throughput for a self-wiping twin-screw extruder of [5, 6]:

\[ Q = \left( m - \frac{1}{2} \right) WH_0 U_z f_{ds} - (2m-1) \frac{WH_0^3}{12\mu} \frac{dP}{dz} f_{ps} \quad (6.38) \]

Apart from this, an extra pressure build-up will occur in the intermeshing region. This can be expressed by means of a correction factor \( \kappa \), a function of the relative flow area in the intermeshing region, leading to the final equation:

\[ Q = \left( m - \frac{1}{2} \right) WH_0 U_z f_{ds} - (2m-1)\kappa \frac{WH_0^3}{12\mu} \frac{dP}{dz} f_{ps} \quad (6.39) \]

In practical situations this factor \( \kappa \) is close to one and its influence is often neglected. For shallow channels the shape factors \( f_{ds} \) and \( f_{ps} \) can be approximated by [7]:

\[ f_{ds} = \int_{-w/2}^{w/2} \frac{H(x)}{WH_0} \, dx \]

and

\[ f_{ps} = \int_{-w/2}^{w/2} \frac{H(x)^3}{WH_0^3} \, dx \quad (6.40) \]

The pressure gradient in the fully filled zones can be calculated from the equation for the throughput and equals for an iso-viscous process:
As a consequence the fully filled length before the die in axial direction is:

\[ L_f = Z \sin \phi = \frac{\kappa W H_0 f_{ds} P_{\text{fs}}}{12 \mu \left\{ \frac{W H_0}{2} (\pi N D \cos \phi) f_{ps} - \frac{Q}{2 m - 1} \right\} \sin \phi} \quad (6.42) \]

For kneading elements the pressure drop can be written as:

\[ \Delta P = \mu \{ A Q + \xi B N \} \quad (6.43) \]

in which A and B are geometrical constants and \( \xi \) denotes the dependence of pressure on the stagger angle. \( \xi \) is negative for angles smaller that \( 90^\circ \), zero for \( 90^\circ \) and it increases with increasing stagger angle. If the pressure is zero after the kneading element, indicating a partially filled zone in that region, an expression for the filled length in front of the kneading element can be obtained:

\[ Z = \frac{(2 m - 1) (A Q + \xi B N)}{(2 m - 1) W H_0 (\pi N D \cos \phi) f_{ps} - 2 Q} \frac{\kappa W H_0^3 f_{ds}}{6} \quad (6.44) \]

obtained from:

\[ L_f = Z \sin \phi \quad (6.45) \]

6.8.5 Non-Intermeshing Twin-Screw Extruders

The mathematical description of the conveying action of non-intermeshing twin-screw extruders consists of a three-parallel-plate model, similar to the two-parallel-plate model used in single-screw extrusion. The outside plates represent the screw surfaces and the middle plate (with zero thickness) represents the barrel. The apex is accounted for by parallel slots in the middle plate, perpendicular to the circumferential velocity. Using this model Kaplan and Tadmor [8] derived an expression for the throughput for Newtonian flows:

\[ Q = W H U z f_{dn} - \frac{W H_0^3}{6 \mu} \frac{d P}{d z} f_{pn} \quad (6.46) \]

This throughput is of course twice the throughput for a single-screw extruder, except for the correction factors, which can be written in their simplest forms as:
where \( f \) is the ratio between the uninterrupted barrel circumference and the total barrel circumference, or:

\[
f = \alpha / \pi
\]

More accurate descriptions of the correction factors are based on a model in which the middle plate has a final thickness [10]:

\[
f_{dn} = \frac{f \cdot f_d \cdot f_p (2 + W_d/H)^3}{f (2 + W_d/H)^3 + 2 f_p (1 - f)}
\]

\[
f_{pm} = \frac{f_p (2 + W_d/H)^3}{f (2 + W_d/H)^3 + 2 f_p (1 - f)}
\]

\( W_a \) is the apex width and \( f_p \) and \( f_d \) are the shape correction factors, as defined for single-screw extruders (Eq. 6.24 and Eq. 6.25).

### 6.9 Concluding Remarks

Various types of extruders exist, all with their specific characteristics. For starch processing the consistency of the material changes from a solid to a highly viscous dough with addition of some polyols. This requires good pumping abilities, preferably independent of viscosity and slip. If the slip is limited, single-screw extruders can do a good job, especially if the barrel wall has grooves to increase friction and to prevent slip. Generally a more stable process can be obtained in (more expensive) twin-screw extruders.

A special type of single-screw extruder is the pin extruder. This type of extruder is provided with mixing pins through the barrel wall and the flights of the screw are interrupted. This extruder provides a very good mixing action and good transporting characteristics, but poor abilities to build up pressure. Especially when extensive micro-mixing is required this extruder is a good alternative, although care has to be taken that no significant degradation occurs.

Twin-screw extruders can be divided into different classes: non-intermeshing (counter-rotating), closely intermeshing (counter- or co-rotating), and self-wiping extruders (co-rotating). The last two types in particular have good pumping capabilities. Specific for twin-screw extruders is the occurrence of a fully filled zone and a zone where the screws are only partially filled with material. The operating
Symbol List

\( \alpha \) \hspace{1cm} \text{apex angle} \hspace{1cm} –
\( \psi \) \hspace{1cm} \text{angle} \hspace{1cm} –
\( \delta \) \hspace{1cm} \text{degree of chamber or channel filling} \hspace{1cm} –
\( \kappa \) \hspace{1cm} \text{flight gap width} \hspace{1cm} \text{m}
\( \sigma \) \hspace{1cm} \text{calender gap width} \hspace{1cm} \text{m}
\( \xi \) \hspace{1cm} \text{pressure factor for kneading elements} \hspace{1cm} –
\( \phi \) \hspace{1cm} \text{screw angle} \hspace{1cm} –
\( \varepsilon \) \hspace{1cm} \text{tetrhedron width at the channel bottom} \hspace{1cm} \text{m}
\( \mu \) \hspace{1cm} \text{viscosity} \hspace{1cm} \text{Pa}.\text{s}
\( \Delta P \) \hspace{1cm} \text{pressure difference} \hspace{1cm} \text{Pa}
\( \Sigma Q \) \hspace{1cm} \text{total of leakage flows} \hspace{1cm} \text{m}^3\text{s}^{-1}
\( A \) \hspace{1cm} \text{geometry parameter} \hspace{1cm} \text{m}^3
\( B \) \hspace{1cm} \text{chamber width} \hspace{1cm} \text{m}
\( B \) \hspace{1cm} \text{geometrical parameter} \hspace{1cm} \text{m}^3
\( c \) \hspace{1cm} \text{distance between screw axes} \hspace{1cm} \text{m}
\( D \) \hspace{1cm} \text{screw diameter} \hspace{1cm} \text{m}
\( f_d \) \hspace{1cm} \text{drag flow correction factor single-screw extruder} \hspace{1cm} –
\( f_p \) \hspace{1cm} \text{pressure flow correction factor single-screw extruder} \hspace{1cm} –
\( f_{ds} \) \hspace{1cm} \text{drag flow correction factor self-wiping extruder} \hspace{1cm} –
\( f_{ps} \) \hspace{1cm} \text{pressure flow correction factor self-wiping extruder} \hspace{1cm} –
\( f_{dn} \) \hspace{1cm} \text{drag flow correction factor non-intermeshing extruder} \hspace{1cm} –
\( f_{pn} \) \hspace{1cm} \text{pressure flow correction factor non-intermeshing extruder} \hspace{1cm} –
\( g \) \hspace{1cm} \text{gravitational acceleration} \hspace{1cm} \text{m}s^{-2}
\( H \) \hspace{1cm} \text{channel depth} \hspace{1cm} \text{m}
\( H_o \) \hspace{1cm} \text{maximum channel depth} \hspace{1cm} \text{m}
\( J_e \) \hspace{1cm} \text{Jeffreys number} \hspace{1cm} –
\( L_f \) \hspace{1cm} \text{filled length in axial direction} \hspace{1cm} \text{m}
\( m \) \hspace{1cm} \text{number of thread starts of one screw} \hspace{1cm} –
\( N \) \hspace{1cm} \text{rotation rate of the screws} \hspace{1cm} \text{s}^{-1}
\( P \) \hspace{1cm} \text{pressure} \hspace{1cm} \text{Pa}
\( P^* \) \hspace{1cm} \text{dimensionless pressure} \hspace{1cm} –
\( Q^* \) \hspace{1cm} \text{dimensionless throughput} \hspace{1cm} –
\( Q_f, Q_c, Q_t, Q_s \) \hspace{1cm} \text{leakage flow through the flight gap, calender gap, tetrhedron gap and side gap} \hspace{1cm} \text{m}^3\text{s}^{-1}
\( Q_l \) \hspace{1cm} \text{leakage flow} \hspace{1cm} \text{m}^3\text{s}^{-1}
\( R \) \hspace{1cm} \text{screw radius} \hspace{1cm} \text{m}
\( S \) \hspace{1cm} \text{pitch of the screw} \hspace{1cm} \text{m}


$U_x$ wall velocity in the cross-channel direction  \( \text{m s}^{-1} \)

$U_z$ wall velocity in the down-channel direction  \( \text{m s}^{-1} \)

$v$ local velocity  \( \text{m s}^{-1} \)

$V$ volume of a C-shaped chamber  \( \text{m}^3 \)

$W$ width of the channel  \( \text{m} \)

$y$ height coordinate in the screw channel  \( \text{m} \)

$z$ down-channel coordinate  \( \text{m} \)

$Z$ length of the extruder channel  \( \text{m} \)

References


5 Booy, M.L. (1978) *Polymer Engineering and Science*, 18, 973


