Controllability and Observability for Affine Nonlinear Hamiltonian Systems

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The infinite jet of the analytic function $f(q_\tau)-f(q)$ is zero at $\tau=0$, implying that $f(q')=f(q)$.

The input–output behaviors of (Σ) initialized at q and q' are characterized by the noncommutative *generating* power series (cf. [4])

$$g = h|_q + \sum_{\nu\geq 0}\ \sum_{j_0,\cdots,j_\nu=0}^{n} A_{j_0}\cdots A_{j_\nu} h|_q x_{j_\nu}\cdots x_{j_0}$$

and

$$g' = h|_{q'} + \sum_{\nu\geq 0}\ \sum_{j_0,\cdots,j_\nu=0}^{n} A_{j_0}\cdots A_{j_\nu} h|_{q'} x_{j_\nu}\cdots x_{j_0}$$

(the bars $|_q$ and $|_{q'}$ indicate the evaluations at q and q') which are equal. ■

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Controllability and Observability for Affine Nonlinear Hamiltonian Systems

A. J. VAN DER SCHAFT

***Abstract*—It is shown that an affine nonlinear Hamiltonian system is "controllable" if and only if it is "observable," in the sense that strong accessibility implies local weak observability and vice versa.**

Furthermore, it is shown that a nonminimal Hamiltonian system can be reduced to a locally weakly observable and strongly accessible system, in such a way that the reduced system is again Hamiltonian.

I. Introduction and Notation

It can be proven (see [6], [3]) that a linear input–output system, given by an $m\times m$ transfer matrix $G(s)$ which satisfies the condition $G(s)=G^T(-s)$, has a minimal realization of the form

$$\begin{aligned}\dot x &= Ax+Bu \qquad x\in X:=\mathbb{R}^{2n}\\ y&=Cx+Du \qquad u\in\mathbb{R}^m,\ \ y\in\mathbb{R}^m\end{aligned}\tag{1.1}$$

with A, B, C, and D satisfying $A^TJ+JA=0$, $B^TJ=C$, $D=D^T$, and J a symplectic form on X, in suitable coordinates given by

$$\begin{pmatrix}0 & -I_n\\ I_n & 0\end{pmatrix}.$$

We will call (1.1) a *linear Hamiltonian* system. It is easy to prove that such a system is controllable if and only if it is observable. In [6] (see also [2]) a definition was given of a *nonlinear* Hamiltonian system with inputs and outputs. It seems natural to conjecture that for such nonlinear Hamiltonian systems there also exists a relation between controllability and observability. Actually, recently in [7] it is proven that for such systems "some kind of controllability" implies "some kind of observability," and vice versa. We now want to specialize these results to the somewhat restricted class of nonlinear systems of the form

$$\dot x = A(x)+\sum_{i=1}^{m}u_iB_i(x),\quad y_i=C_i(x)\qquad i=1,\cdots,m$$

provided with a Hamiltonian structure.

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In the sequel we will use the following definition of a nonlinear system with inputs and outputs (introduced in [10] and elaborated in [7], [10]). We denote the space of *external variables* (think of inputs *and* outputs) by a smooth manifold W (think of W as $U\times Y$). The state space is given by a smooth manifold M. Finally, there is a bundle B above M (with projection π: $B\to M$) and a smooth function f such that the diagram

$$\begin{tikzcd} B \arrow[rr, "f"] \arrow[dr, "\pi"'] & & TM\times W \arrow[dl, "\pi_M"] \\ & M & \end{tikzcd}\qquad \text{commutes}$$

(π_M is the natural projection of TM on M).

When we denote f: $B\to TM\times W$ by $f=(g,h)$ with g: $B\to TM$ and h: $B\to W$, then in local coordinates this definition comes down to

$$\dot x = g(x,v),\quad w=h(x,v)$$

with x coordinates for M, v coordinates for the fibers of B (to be seen as "dummy" input variables), and w coordinates for W. We will denote the above system by $\Sigma(M,W,B,f)$.

For the definition of a Hamiltonian system we will need some notions from symplectic geometry, for which we refer to [1].

If (M,ω) is a manifold with symplectic form ω, we can also construct a symplectic form on TM, denoted by $\dot\omega$ (see [6]). Given a function H: $M\to\mathbb{R}$ we define the Hamiltonian vector field X_H by $\omega(X_H,-)=dH$. Let F, H: $M\to\mathbb{R}$ be smooth functions which induce, as above, Hamiltonian vector fields X_F, resp. X_H. Then the *Poisson bracket* of F and H denoted by $\{F,H\}$ is again a smooth function given by

$$\{F,H\}:=\omega(X_H,X_F).$$

When we denote the vector space of functions on M by $C(M)$ and the vector space of vector fields on M by $V(M)$, then $C(M)$ equipped with the Poisson bracket is a Lie algebra and the map $H\to X_H$ given by $\omega(X_H,-)=dH$ is a Lie algebra morphism to the Lie algebra of Hamiltonian vector fields $\subset V(M)$, i.e., $X_{\{F,H\}}=[X_F,X_H]$ (cf. [1]).

Finally, given a number of vector fields $X_1,\cdots,X_k$, we will denote by $(X_1,\cdots,X_k)$ the linear subspace of $V(M)$ spanned by $X_1,\cdots,X_k$. Analogously, let $F_1,\cdots,F_k$ be functions on M; then $(F_1,\cdots,F_k)$ is the linear subspace of $C(M)$ spanned by $F_1,\cdots,F_k$.

II. Controllability and Observability

In [6] the following definition for nonlinear Hamiltonian systems was proposed and elaborated.

Definition 2.1: Let (M,ω) be a symplectic manifold, denoting the state space. Let (W,ω^e) be a symplectic manifold denoting the space of external variables (inputs *and* outputs). A system $\Sigma(M,W,B,f)$ is called full Hamiltonian if $f(B)\subset TM\times W$ is a Lagrangian submanifold of the symplectic manifold $(TM\times W, \pi_1^*\dot\omega-\pi_2^*\omega^e)$ (π_1, resp. π_2 denotes the projection of $TM\times W$ on TM, resp. W).

We now want to specialize Def. 2.1 somewhat further to what we will call *affine* Hamiltonian systems.

Definition 2.3 (see, e.g., [5]): An *affine system* is given by a manifold M, together with an affine distribution Δ on M (i.e., $\Delta(x)$ is in every $x\in M$ an affine subspace of T_xM) and a map C: $M\to Y$, where Y is the output manifold.

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Hence, in local coordinates an affine system is represented by

$$\dot{x}=A(x)+\sum_{i=1}^{m}u_iB_i(x) \quad y_j=C_j(x) \qquad j=1,\cdots,p \tag{2.1}$$

with x coordinates for M, $y=(y_1,\cdots,y_p)$ coordinates for Y, and with A and B_i vector fields on M such that $\Delta(x)=A(x)+\operatorname{span}\{B_1(x),\cdots,B_m(x)\}$.

Definition 2.4: An affine system on a symplectic manifold (M,ω) is called *Hamiltonian* if in local coordinates it can be represented by

$$\dot{x}=A(x)+\sum_{i=1}^{m}u_iB_i(x), \quad y_i=C_i(x) \qquad i=1,\cdots,m$$

with A a locally Hamiltonian vector field, i.e., $L_A\omega=0$, and with B_i Hamiltonian vector fields such that $\omega(B_i,-)=dC_i$.

This kind of Hamiltonian system forms a natural subclass of the class of systems given by Def. 2.1., as can be seen from the following theorem which we state without proof.

Theorem 2.5: Let $\Sigma(M,W,B,f)$ be a full Hamiltonian system. Denote f by (g,h) with $g: B\to TM$ and $h: B\to W$. Suppose B is a vector bundle. Suppose also W to be a vector bundle, namely $W=T^*Y$ with Y the output manifold. T^*Y has as a cotangent bundle a natural symplectic form ω^e. Suppose further that $h: B\to T^*Y$ is a bundle morphism, and that h is a linear bijection from the fibers of B onto the fibers of T^*Y. Then we can find vector fields A and B_i, $i=1,\cdots,m$ ($m=$ dimension fiber of B) and a map $C: M\to Y$ such that the system is locally described by

$$\dot{x}=A(x)+\sum_{i=1}^{m}u_iB_i(x), \quad y_i=C_i(x) \qquad i=1,\cdots,m$$

with $(y_1,\cdots,y_m)$ coordinates for Y and $C=(C_1,\cdots,C_m)$, and such that $L_A\omega=0$ and $\omega(B_i,-)=dC_i$, i.e., an affine Hamiltonian system as in Def. 2.4.

We now want to specialize the duality results obtained in [7] for controllability and observability of Hamiltonian systems to affine Hamiltonian systems as above.

First we define strong accessibility for affine systems (see [8]).

Definition 2.6: Let $\dot{x}=A(x)+\sum_{i=1}^{m}u_iB_i(x)$ be an affine system on the state space M. Define $\Gamma_0:=(B_1,\cdots,B_m)$ and $\Gamma:=A+(B_1,\cdots,B_m)$ [an affine subspace of $V(M)$]. Define further $\Gamma_k:=[\Gamma,\Gamma_{k-1}]+\Gamma_{k-1}$, $k\geqslant 1$ [with $+$ denoting the sum of two subspaces of $V(M)$]. Finally, let $K:=\cup_{k\geqslant 0}\Gamma_k$. From the Jacobi identity it follows that K is a Lie subalgebra of $V(M)$. Then the system is called *strongly accessible* if $K(x)=T_xM$ for every $x\in M$, where $K(x)$ is the linear subspace of T_xM spanned by the vector fields in K.

Remark: Throughout we will assume that $\dim K(x)$ is constant.

Local weak observability is defined as follows [4].

Definition 2.7: Let

$$\dot{x}=A(x)+\sum_{i=1}^{m}u_iB_i(x), \quad y_j=C_j(x) \qquad j=1,\cdots,p. \tag{2.2}$$

Define $F_0:=(C_1,\cdots,C_p)$ and $F_k:=L_\Gamma F_{k-1}+F_{k-1}$ (with Γ as in Def. 2.6), $k\geqslant 1$. Then the system is *locally weakly observable* if $G:=\cup_{k\geqslant 0}F_k$ satisfies $dG(x)=T_x^*M$ for every $x\in M$, where $dG(x)$ is the linear subspace of T_x^*M spanned by $dh(x)$ with $h\in G$.

Remark: As above we will assume that $\dim dG(x)$ is constant.

In the case of Hamiltonian systems the situation becomes particularly nice. Let there be given an affine Hamiltonian system as in Def. 2.4. Because $L_A\omega=0$, there exists (locally) an $H: M\to\mathbb{R}$ such that $A=X_H$ (see [1]). Then we can derive the following proposition.

Proposition 2.8: Define $F:=H+(C_1,\cdots,C_m)$ [an affine subspace of $C(M)$]. Then the F_k's defined above satisfy $F_k=\{F,F_{k-1}\}+F_{k-1}$, with $\{\cdot,\cdot\}$ the Poisson bracket.

Proof: Elements of F_k are sums of functions of the form

$$L_{f_1}L_{f_2}L_{f_3}\cdots L_{f_k}C_j \tag{2.3}$$

with $f_l=A$ or $f_l=B_l$ for $l=1,\cdots,m$. The Poisson bracket is defined by the equalities

$$\{N_1,N_2\}=\omega(X_{N_2},X_{N_1})=X_{N_1}(N_2)$$

and therefore, because $A=X_H$ and $B_i=X_{C_i}$, the expressions (2.3) equal

$$\{h_1,\{h_2,\{h_3,\cdots\{h_k,C_j\}\cdots\}$$

with $h_i=H$ or $h_i=C_l$ for $l=1,\cdots,m$. ∎

Everything has now been set up for our final results.

Theorem 2.9: Let there be given an affine Hamiltonian system. Let Γ_k and F_k be defined as above; then they are related as follows. The map $N\xrightarrow{\alpha}X_N$ (defined by $\omega(X_N,-)=dN$) is an isomorphism between F_k (modulo $\mathbb{R}$) and Γ_k.

Proof: It is easy to see that α maps constant functions to the zero vector field. Therefore, we will drop for brevity the suffix (modulo $\mathbb{R}$).

By induction: for $k=0$ it is immediate because

$$\Gamma_0=(B_1,\cdots,B_m) \quad\text{and}\quad F_0=(C_1,\cdots,C_m) \quad\text{and}\quad \omega(B_i,-)=dC_i.$$

Suppose it is true for $k-1$. We will prove it for k. Now $F_k=\{F,F_{k-1}\}+F_{k-1}$. By the induction assumption F_{k-1} is mapped isomorphically onto Γ_{k-1}, and hence we only have to prove that $\{F,F_{k-1}\}$ is mapped under α onto $[\Gamma,\Gamma_{k-1}]$. We have

$$\{F,F_{k-1}\}=\{H+F_0,F_{k-1}\}=\{H,F_{k-1}\}+\{F_0,F_{k-1}\}$$

and

$$[\Gamma,\Gamma_{k-1}]=[A+\Gamma_0,\Gamma_{k-1}]=[A,\Gamma_{k-1}]+[\Gamma_0,\Gamma_{k-1}].$$

Because the map α satisfies $\alpha(\{N_1,N_2\})=[X_{N_1},X_{N_2}]$ (see the Introduction) and, moreover, $A=X_H$, $\alpha(F_0)=\Gamma_0$, and by the induction assumption $\alpha(F_{k-1})=\Gamma_{k-1}$, it easily follows that

$$\alpha(\{H,F_{k-1}\})=[A,\Gamma_{k-1}] \quad\text{and}\quad \alpha(\{F_0,F_{k-1}\})=[\Gamma_0,\Gamma_{k-1}]$$

and therefore $\alpha(\{F,F_{k-1}\})=[\Gamma,\Gamma_{k-1}]$. □

Corollary 2.10: An affine Hamiltonian system is locally weakly observable if and only if it is strongly accessible.

Proof: From Theorem 2.9 it follows that G (see Def. 2.7) is mapped by α isomorphically onto K (see Def. 2.6). Therefore, $dG(x)=T_x^*M$ iff $K(x)=T_xM$.

III. Minimality and Reduction of the State Space

Let us again consider an affine Hamiltonian system (Def. 2.4):

$$\dot{x}=A(x)+\sum_{i=1}^{m}u_iB_i(x), \qquad x\in M, \quad (M,\omega)\text{ symplectic manifold}$$

$$y_i=C_i(x) \qquad i=1,\cdots,m. \tag{3.1}$$

We define the *controllability distribution* D by $D(x):=K(x)$, with K as in Def. 2.6. Further, we define the *observability codistribution* P by $P(x):=dG(x)$, with G as in Def. 2.7. When we define the (involutive) distribution $\ker P:=\{X\in TM\,|\,\alpha(X)=0 \text{ for every } \alpha\in P\}$, then the leaves of the foliation of M induced by this distribution represent the "nonobservable spaces" (in [4] the points on a same leaf are called *strongly indistinguishable*).

Now notice that by Theorem 2.9 we have $\omega(D,-)=P$ and, therefore, when we define the distribution $D^\perp$ by

$$D^\perp(x):=\{X\in T_xM\,|\,\omega_x(X,Y)=0,\text{ for every } Y\in D(x)\}$$

then it is easy to see that $\ker P=D^\perp$. This suggests that when we first restrict the system to its "controllable part," intuitively generated by D, and then factor out by the "nonobservable part," approximately given by $D\cap\ker P$, the reduced state space is generated by $D/D\cap\ker P=D/D\cap D^\perp$ and can be given again a symplectic structure! We will now make this more precise.

Proposition 3.1: Let there be given an affine system (2.2). Suppose there exists an $x_0\in M$, such that $A(x_0)\in D(x_0)$; then an integral manifold Q of D through x_0 satisfies

1) $A(x) \in T_x Q$, for every $x \in Q$

2) the system restricted to Q is strongly accessible.

Proof: It is clear that $[A, D] \subset D$. Therefore, when $A(x_0) \in D(x_0)$ and Q is a leaf of D through x_0, then $A(x) \in D(x)$ for every $x \in Q$ (otherwise A would not leave the leaves invariant). Because $TQ = D$, the system restricted to Q is strongly accessible. □

When we are in the situation of Prop. 3.1, we can speak about a "controllable part." By factoring out the "nonobservable part" of an affine Hamiltonian system, we obtain the following nice situation.

Theorem 3.2: Given an affine Hamiltonian system (3.1), suppose it is not strongly accessible, and assume there exists an $x_0 \in M$ and a $Q \subset M$ such that Prop. 3.1. is satisfied. Then there exists a manifold N and a submersion π: $Q \to N$ such that $\ker \pi_* = D \cap D^{\perp}$. Moreover, N has a symplectic form $\bar{\omega}$ such that $\pi^* \bar{\omega} = \omega$. Furthermore, on N there is defined an affine Hamiltonian system

$$\dot{\bar{x}} = \bar{A}(\bar{x}) + \sum_{i=1}^{m} u_i \bar{B}_i(\bar{x}), \quad y_i = \bar{C}_i(\bar{x}) \qquad i = 1, \cdots, m \tag{3.2}$$

which is locally weakly observable and strongly accessible and which has the same input-output properties as the original system (3.1) restricted to Q.

Proof: First we will show that *locally* Q can be factored out as above. Because $D \cap D^{\perp}$ is an involutive distribution on Q, we can locally factor out Q by the leaves of this distribution and obtain a manifold N and a submersion π: $Q \to N$ such that $\ker \pi_* = D \cap D^{\perp}$. In, for instance, [9], it is proven that N can be given a unique symplectic form $\bar{\omega}$, such that $\pi^* \bar{\omega} = \omega$. Because $[A, D^{\perp}] \subset D^{-}$ and $[B_i, D^{\perp}] \subset D^{-}$, $i = 1, \cdots, m$, the vector fields A and B_i project under π to vector fields $\bar{A}$, resp. $\bar{B}_i$ on N, i.e.

$$\pi_* A = \bar{A}, \quad \pi_* B_i = \bar{B}_i, \qquad i, \cdots, m.$$

Because $D^{\perp} \subset \ker dC_i$, $i = 1, \cdots, m$, there exist functions $\bar{C}_i$ on N such that $\pi^* \bar{C}_i = C_i$, $i = 1, \cdots, m$. The equalities $\omega(B_i, -) = dC_i$ then imply

$$\bar{\omega}(\bar{B}_i, -) = d\bar{C}_i, \qquad i = 1, \cdots, m.$$

Furthermore, we can see that $\omega(A, -) = \pi^*(\bar{\omega}(\bar{A}, -))$ and, therefore, $\bar{\omega}(\bar{A}, -)$ is closed or, equivalently, $L_{\bar{A}} \bar{\omega} = 0$. We now refer to [4, Theorem 3.9] to conclude that the locally defined new system (3.2) is locally weakly observable and strongly accessible and has the same input-output properties as the original system on Q. Moreover, this last theorem also states that because (3.1) is strongly accessible on Q, we can *globally* factor out Q by $D \cap D^{\perp}$, and hence the local constructions above hold globally. □

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Control of a Class of Nonlinear Systems by Decentralized Control

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***Abstract*—This correspondence considers the control of a system composed of several interconnected subsystems. Each subsystem is described by a canonical nonlinear state model, with the restriction that interaction between different subsystems is linear. Under very mild additional conditions a decentralized dither feedback control is constructed which will drive the system state to zero. The dither control method is shown to stabilize this system for all initial conditions.**

I. Introduction

Recently, considerable interest has arisen in the stability of interconnected systems, in particular, methods of stabilization using decentralized state or output feedback. One method is by direct eigenvalue placement as in [1]–[4] and [9]. Such an approach has the advantage of being able to do more than just stabilize the system, and as seen in [9] to accomplish this with reasonable gains. The main disadvantage of eigenvalue placement is its applicability only to linear time invariant systems, or to small displacements of a nonlinear system from equilibrium. Somewhat different eigenvalue methods can be found in [5] and [6].

The literature also contains much research on variable structure or dither controllers [7]–[9]. This approach stabilizes a system through a fast switching global control which forces the original system to "behave" as a second linear time invariant system which can be chosen to be stable.

The method of this correspondence is to combine the ideas of variable structure systems with the methods of eigenvalue placement in a decentralized context. This is done by constructing a decentralized dither controller which will force the original nonlinear interconnected system to behave as a second linear interconnected system which has had its eigenvalues placed in the left half plane by the method of [4].

II. Model Formulation

The system to be considered is composed of N subsystems, where each subsystem is written as an nth-order state model as

$$\dot{x}_i = A_i x_i - B_i u_i + B_i K_i \tag{1}$$

$$u_i = \sum_{j=1}^{N} L_{ij} x_j \tag{2}$$

$$A_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & \vdots & & & \vdots \\ 0 & 0 & 0 & & 1 \\ g_{i1} & g_{i2} & g_{i3} & \cdots & g_{in_i} \end{bmatrix}; \quad B_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

where for each instant of time $x_i \in R^{n_i}$, $u_i \in R^1$, and $L_{ij} = [0, L_{ij}^2, \cdots, L_{ij}^{n_j}] \in R^{1 \times n_j}$, $A_i \in R^{n_i \times n_i}$. The zero in the first entry of L_{ij} is a technical condition needed so that each surface as defined in (4) below can be expressed as a linear combination of system state variables. Moreover, feasible computation of the local dither controller also necessitates the condition. This is always possible to do via dynamic compensation—i.e., an extension of each subsystem state space by one. A composite state model will take the form

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