Controllability and Observability for Affine Nonlinear Hamiltonian Systems

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and

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Hence, in local coordinates an affine system is represented by
\[ \dot{x} = A(x) + \sum_{i=1}^{m} u_i B_i(x), \quad y_i = C_i(x) \quad j = 1, \ldots, p \]  
(2.1)
with \( x \) coordinates for \( M \), \( y = (y_1, \ldots, y_p) \) coordinates for \( Y \), and with \( A \) and \( B \) vector fields on \( M \) such that \( \Delta(x) = A(x) + \sum_{i=1}^{m} B_i(x) \).

**Definition 2.4:** An affine system on a symplectic manifold \((M, \omega)\) is called Hamiltonian if in local coordinates it can be represented by
\[ \dot{x} = A(x) + \sum_{i=1}^{m} u_i B_i(x), \quad y_i = C_i(x) \quad i = 1, \ldots, m \]
with \( A \) a locally Hamiltonian vector field, i.e., \( L_{A(x)} \omega = 0 \), and with \( B_i \) Hamiltonian vector fields such that \( \omega(B_i, -) = dC_i \).

This kind of Hamiltonian system forms a natural subclass of the class of systems given by Def. 2.1., as can be seen from the following theorem with which we state without proof.

**Theorem 2.5:** Let \( \Sigma(M, W, f, B, f) \) be a full Hamiltonian system. Denote \( f \) by \((g, h)\) with \( g : = TM \) and \( h : = W \). Suppose \( B \) is a vector bundle. Suppose also \( W \) to be a vector bundle, namely \( W = TM + \gamma Y \) with \( Y \) the output manifold. \( TM + \gamma Y \) has as a coadjoint a natural symplectic form \( \omega \). Suppose further that \( h : B \to TM + \gamma Y \) is a bundle morphism, and that \( h \) is a linear bijection from the fibers of \( B \) onto the fibers of \( TM + \gamma Y \). Then we can find vector fields \( A \) and \( B_i, i = 1, \ldots, m \) (where \( m = \text{dimension fiber of } B \)) and a map \( C : M \to Y \) such that the system is locally described by
\[ \dot{x} = A(x) + \sum_{i=1}^{m} u_i B_i(x), \quad y_i = C_i(x) \quad i = 1, \ldots, m \]
with \((y_1, \ldots, y_m)\) coordinates for \( Y \) and \( C = (C_1, \ldots, C_m) \), and such that \( L_{A(x)} \omega = 0 \) and \( \omega(B_i, -) = dC_i \), i.e., an affine Hamiltonian system as in Def. 2.4.

We now want to specialize the duality results obtained in [7] for controllability and observability of Hamiltonian systems to affine Hamiltonian systems as above.

First we define accessibility for affine systems (see [8]).

**Definition 2.6:** Let \( \dot{x} = A(x) + \sum_{i=1}^{m} u_i B_i(x) \) be an affine system on the state space. Define \( \Gamma_0 = (B_1, \ldots, B_m) \) and \( \Gamma_r = (A + (B_1, \ldots, B_m)) \) an affine subspace of \( V(M) \). Define further \( \Gamma_{k-1} = \left[ \Gamma_k, \Gamma_{k-1} \right] - \Gamma_{k-1} \), \( k > 1 \) [with + denoting the sum of two subspaces of \( V(M) \)]. Finally, let \( K : = \bigcup_{k=1}^{\infty} \Gamma_k \). From the Jacobi identity it follows that \( K \) is a Lie subalgebra of \( V(M) \). Then the system is called strongly accessible if \( K(x) = T_x M \) for all \( x \in M \). where \( K(x) \) is the linear subspace of \( T_x M \) spanned by the vector fields in \( K \).

**Remark:** Throughout we will assume that \( \dim K(x) \) is constant.

Local weak observability is defined as follows [4].

**Definition 2.7:** Let
\[ \dot{x} = A(x) + \sum_{i=1}^{m} u_i B_i(x), \quad y_i = C_i(x) \quad j = 1, \ldots, p \]  
(2.2)
Define \( F_k = (C_1, \ldots, C_m) \) and \( F_k : = L_{A(x)} B_{k-1} + B_k \) (with \( \Gamma \) as in Def. 2.6), \( k \geq 1 \). Then the system is locally weakly observable if \( G : = \bigcup_{k=0}^{\infty} F_k \) satisfies \( dG(x) = T^*_x M \) for all \( x \in M \), where \( dG(x) \) is the linear subspace of \( T^*_x M \) spanned by \( dA(x) \) with \( h \in G \).

**Remark:** As above we will assume that \( \dim dG(x) \) is constant.

In the case of Hamiltonian systems the situation becomes particularly nice. Let there be given an affine Hamiltonian system as in Def. 2.4. Because \( L_{A(x)} \omega = 0 \), there exists (locally) an \( H : M \to \mathbb{R} \) such that \( A(x) = \dot{x} \). (see [1]). Then we can derive the following proposition.

**Proposition 2.8:** Define \( F : = H + (C_1, \ldots, C_m) \) an affine subspace of \( C(M) \). Then the \( F_k \) defined above satisfy \( F_k = (F_k, F_k-1) + F_k-1 \), with \( \cdot, \cdot \) the Poisson bracket.

**Proof:** Elements of \( F_k \) are sums of functions of the form
\[ f_{1} f_{2} \cdots f_{k} \]  
(2.3)
with \( f_i = A_i \) or \( f_i = B_i \) for \( i = 1, \ldots, m \). The Poisson bracket is defined by the equalities
\[ \{N_1, N_2\} = \omega(X_{N_1}, X_{N_2}) = \omega(X_{N_2}, X_{N_1}) \]
and therefore, because \( A = X_H \) and \( B_i = X_{C_i} \), the expressions (2.3) equal
\[ \{h_1, h_2, \ldots, h_k, c_j\} \]
with \( h_i = H \) or \( h_i = C_i \) for \( i = 1, \ldots, m \).

Everywhere has now been set up for our final results.

**Theorem 2.9:** Let there be given an affine Hamiltonian system. Let \( \Gamma_0 \) and \( \Gamma_1 \) be defined as above; then they are related as follows. The map \( N \to X_N \) (defined by \( \omega(X_N, -) = dN \)) is an isomorphism between \( F_1 \) (modulo \( \mathbb{R} \)) and \( \Gamma_1 \).

**Proof:** It is easy to see that \( \alpha \) maps constant functions to the zero vector field. Therefore, we will drop for brevity the suffix (modulo \( \mathbb{R} \)). By induction: for \( k = 0 \) it is immediate because
\[ \Gamma_0 = (B_1, \ldots, B_m) \]  
and \( F_k = (C_1, \ldots, C_m) \) and \( \omega(B_i, -) = dC_i \).

Suppose it is true for \( k - 1 \). We will prove it for \( k \). Now \( F_k = (F_k, F_k-1) + F_k-1 \). By the induction assumption \( F_k-1 \) is mapped isomorphically onto \( \Gamma_{k-1} \), and hence we only have to prove that \( (F_k, F_k-1) \) is mapped under \( \alpha \) into \( [\Gamma_k, \Gamma_{k-1}] \). We have
\[ \{F_k, F_k-1\} = \{H + F_k, F_k-1\} = \{F_k, F_k-1\} \]
and
\[ [\Gamma_k, \Gamma_{k-1}] = [A + \Gamma_0, \Gamma_{k-1}] = [A, \Gamma_{k-1}] + [\Gamma_0, \Gamma_{k-1}] \]
Because the map \( \alpha \) satisfies \( \alpha(({N_1, N_2}) = [X_N, Y] \) (see the Introduction) and, moreover, \( A = X_H \), \( a(F_0) = F_0 \), and by the induction assumption \( a(F_k-1) = \Gamma_k-1 \), it easily follows that
\[ \alpha((H, F_k-1)) = [A, \Gamma_{k-1}] \quad \text{and} \quad \alpha((F_0, F_k-1)) = [\Gamma_0, \Gamma_{k-1}] \].

**Corollary 2.10:** An affine Hamiltonian system is locally weakly observable if and only if it is strongly accessible.

**Proof:** From Theorem 2.9 it follows that \( G \) (see Def. 2.7) is mapped by \( \alpha \) isomorphically onto \( K \) (see Def. 2.6). Therefore, \( dG(x) = T^*_x M \) if \( K(x) = T_x M \).
Control of a Class of Nonlinear Systems by Decentralized Control

S. RICHTER, S. LEFEBVRE, AND R. DeCARLO

Abstract—This correspondence considers the control of a system composed of several interconnected subsystems. Each subsystem is described by a canonical nonlinear state model, with the restriction that interaction between different subsystems is linear. Under very mild additional conditions a decentralized dither feedback control is constructed which will drive the system state to zero. The dither control method is shown to stabilize this system for all initial conditions.

I. INTRODUCTION

Recently, considerable interest has arisen in the stability of interconnected systems, in particular, methods of stabilization using decentralized state or output feedback. One method is by direct eigenvalue placement as in [1]-[4] and [9]. Such an approach has the advantage of being able to do more than just stabilize the system, and as seen in [9] to accomplish this with reasonable gains. The main disadvantage of eigenvalue placement is its applicability only to linear time invariant systems, or to small displacements of a nonlinear system from equilibrium. Somewhat different eigenvalue methods can be found in [5] and [6].

The literature also contains much research on variable structure or dither controllers [7]-[9]. This approach stabilizes a system through a fast switching global control which forces the original system to "behave" as a second linear time invariant system which can be chosen to be stable.

The method of this correspondence is to combine the ideas of variable structure systems with the methods of eigenvalue placement in a decentralized context. This is done by constructing a decentralized dither controller which will force the original nonlinear interconnected system to "behave" as a second linear interconnected system which has had its eigenvalues placed in the left half plane by the method of [4].

II. MODEL FORMULATION

The system to be considered is composed of $N$ subsystems, where each subsystem is written as an $n$th-order state model as

\[ \dot{x}_j = A_j x_j + B_j u_j + B_j K_j \]

\[ u_j = \sum_{1}^{n} I_{j,r} x_r \]

where for each instant of time $x_r \in \mathbb{R}^n$, $u_j \in \mathbb{R}^l$, and $L_{j,r} = [0, I_{j,1}, \ldots, I_{j,n}] \in \mathbb{R}^{n \times l}$. The zero in the first entry of $L_{j,r}$ is a technical condition needed so that each surface as defined in (4) below can be expressed as a linear combination of system state variables. Moreover, feasible computation of the local dither controller also necessitates the condition. This is always possible to do via dynamic compensation—i.e., an extension of each subsystem state space by one. A composite state model will take the form

\[ A_j = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

\[ B_j = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} \]

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REFERENCES