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Chapter 2

Quantification of ventricular volume load in the context of a bidirectional cavopulmonary shunt: A theoretical treatise


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Abstract

**Background** Functional univentricular hearts are currently palliated by a staged procedure of which the bidirectional cavopulmonary shunt is usually the second stage. In addition to this stage, a calibrated amount of additional pulmonary blood flow may be preserved to promote pulmonary artery growth and increase the length of the interval preceding the total cavopulmonary connection. However, additional pulmonary blood flow can be deleterious for ventricular functioning and development as it increases functional ventricular volume load.

**Methods** Using the Fick principle we devised a theoretic framework to estimate the ventricular volume loading caused by additional pulmonary and collateral aortopulmonary flow. To use this framework, blood samples need to be taken intraoperatively from the aorta, pulmonary veins, and inferior caval vein to determine oxygen saturations. The oxygen saturation samples have to be taken sequentially with and without additional pulmonary blood flow.

**Results** The objective of this paper is to provide a theoretic framework to estimate the ventricular volume loading caused by collateral aortopulmonary flow and additional pulmonary blood flow in the context of a bidirectional cavopulmonary shunt in the staged palliation of univentricular hemodynamics. The formulas have not yet systematically been applied in vivo.

**Conclusions** The added volume loading of the ventricle caused by additional pulmonary blood flow can theoretically be estimated using the newly devised formulas so as to calibrate ventricular volume loading to a desired level intraoperatively.
Introduction

Functional univentricular hearts are currently palliated by a staged procedure of which the bidirectional cavopulmonary shunt (BCPS) is usually the second stage. The first procedure aims at regulating pulmonary blood flow shortly after birth in order to balance pulmonary and systemic flows, which is often necessary. Only patients with a naturally adequate pulmonary stenosis need no surgery to achieve this balance. The second procedure is the BCPS at which additional pulmonary blood flow (APBF) can consist of a banded pulmonary artery or an aortopulmonary shunt, such as a Blalock-Taussig shunt. Advantages of APBF include higher oxygen saturations and a potentially longer interval preceding the total cavopulmonary connection enabling the use of larger extracardiac conduits that may be beneficial after somatic growth [1,2]. Nevertheless, as APBF can be deleterious for ventricular functioning and development by adding disproportionally to the ventricular volume load [3,4], quantification is desired. Conceptually, after a BCPS the functional univentricular volume load is limited to, and equal to, the combined caval flows from the inferior vena cava (IVC) and the superior vena cava (SVC), then pulmonary flow. The collateral flow and APBF are then additional flows that are very hard to quantify, particularly during an operation.

In this treatise we describe a theory that aids in estimating the added volume load on the functionally univentricular ventricle caused by these additional flows. This method may be employed in the catheterization laboratory as well as in the operating theatre.

Material and methods

Volumetric balances of intracardially mixing blood volumes can be calculated by taking blood samples, determining their oxygen saturations (to use as the indicator substance), and performing the calculations appropriate to the situation. The Fick principle is the basis for these calculations in which measured concentrations of marker substances are used to calculate flow proportions. For this paper, oxygen saturation (S) is the obvious choice as it is easy and readily available in the intraoperative setting. Systemic (aortic) oxygen saturation (S_Ao) is determined by 2 different volumes of blood entering the heart with 2 different saturations: the pulmonary venous (S_PV) and inferior caval blood (S_{ivc}). The Fick principle states that the quantity (Q) and the oxygen saturation (merely the indicator substance) of each source need to be multiplied. The results of
each source are then added and divided by the multiplication of quantity and saturation of the resulting mixture. Of note, a preload volume consisting of joint SVC and IVC flows is considered to be normal for a normal ventricle in this theory. Limitation of that statement is that the unknown amount of collateral lung flow can be larger than imagined, thus contributing as an unknown proportion to ventricular volume load. Unknown is also whether the ventricular anatomic volume (optimal preload) in the setting of a “univentricular” heart malformation is equal to, or possibly larger than that of a normal left ventricle. In addition there may be a substantial individual variability, possibly also related to different types of univentricular anatomy.

In this paper textual statements are abbreviated as formulas where appropriate. Of note, with “flow volume” (\( Q \)) the volume per unit of time is meant (for example, cm\(^3\)/sec), not “flow velocity” (V) which means the linear velocity (cm/sec). For readability in the following “flow” should be read as “flow volume” (\( \dot{Q} \)), while “flow velocity” (V) will be indicated when appropriate.

### Results

**Bidirectional Cavopulmonary Shunt**

Before a BCPS is constructed surgically, the univentricular preload consists of the flow (\( \dot{Q} \)) exiting jointly from the SVC and the IVC through the right atrium and from the pulmonary veins. Performing a BCPS alters this situation so that all SVC flow is redirected into the pulmonary arteries, to which other inflow is traditionally cut off. The SVC then becomes the sole blood source for the lungs, in addition to an unknown amount of bronchial and other systemic-to-pulmonary collateral flow. As the SVC now bypasses the atrium by constructing the BCPS and other pulmonary flow is abolished, ventricular flow volume (preload) is substantially reduced. In the situation after the BCPS total pulmonary flow (\( \dot{Q}_P \)) is then equal to SVC flow (\( \dot{Q}_{SVC} \)) plus collateral flow (\( \dot{Q}_{COLL} \)) (formula 1a; appendix 1). In a BCPS the systemic-to-pulmonary collateral flow (\( \dot{Q}_{COLL} \)) thus consists of total lung flow (\( \dot{Q}_P \)) minus superior vena cava flow (\( \dot{Q}_{SVC} \)) (formula 1b). The resulting aortic oxygen saturation depends on the mixture in the heart and thus on the ratio between \( \dot{Q}_P \) and \( \dot{Q}_{IVC} \) as these are the sole venous entries into the heart adding up to systemic flow (\( \dot{Q}_S \)) (formula 2a). This implies that \( \dot{Q}_{IVC} \) equals \( \dot{Q}_S \) minus \( \dot{Q}_P \) (formula 2b). In small infants, \( \dot{Q}_{SVC} \) is thought to be more or less equal to \( \dot{Q}_{IVC} \),
but with somatic growth and changing body proportions the relative contribution of the SVC is assumed to diminish. In clinical practice the exact superior to inferior caval flow ratio ($\dot{Q}_{\text{SVC}}/\dot{Q}_{\text{IVC}}$) is usually unknown. Summarizing above formulas into one creating a ratio of pulmonary-to-systemic flow ($\dot{Q}_p/\dot{Q}_s$) is shown as formula 3.

**Inferior Caval flow: An Unknown Constant**

After the BCPS pulmonary flow ($\dot{Q}_p$) consists of at least $\dot{Q}_{\text{SVC}}$ and $\dot{Q}_{\text{COLL}}$ in addition to any additional pulmonary blood flow ($\dot{Q}_{\text{APBF}}$) that could be left surgically. As it is the contribution of $\dot{Q}_{\text{APBF}}$ as part of $\dot{Q}_p$ that we wish to quantify in its contribution to ventricular stroke volume, we chose $\dot{Q}_p$ to be the numerator in the equation to be used. For the denominator we can use either $\dot{Q}_s$ or $\dot{Q}_{\text{IVC}}$. With APBF the $\dot{Q}_p$ rises and so does ventricular flow $\dot{Q}_v$, but not $\dot{Q}_s$ as $\dot{Q}_{\text{APBF}}$ is ejected from the ventricle into the pulmonary but not the systemic circulation. The major advantage of using $\dot{Q}_{\text{IVC}}$ is that we find it conceptually easier to vary $\dot{Q}_p$ and relate it to $\dot{Q}_{\text{IVC}}$ which is an unknown constant in the mix resulting in $\dot{Q}_s$.

1. **Ratio of Superior to Inferior Vena Cava flow by Echo Doppler** ($\dot{Q}_{\text{SVC}}/\dot{Q}_{\text{IVC}}$)

In the absence of direct quantification we could measure $\dot{Q}_{\text{SVC}}$ and $\dot{Q}_{\text{IVC}}$ by echocardiography and Doppler. We would then have to measure the diameter of the caval veins by echo and multiply them by Doppler measured flow velocities in the respective vessels. However, in addition to variable oxygen saturations, the diameters of caval veins vary and usually do not have a circular cross section. Furthermore, they can collapse even more during the ventilatory and cardiac cycle when the patient is in the supine position. Nonetheless, the mean cross-sectional area (CSA) can be measured echocardiographically even if it is not circular, while the mean flow velocity (V) can be measured with Doppler investigation. When the CSAs are then multiplied by flow velocities, the flow volumes can be calculated. The ratio of the CSAs multiplied by the flow velocities is then equal to the ratio of the caval flow volumes and then indicates the caval flow ratio derived by a different methodology as compared with oxymetry (formula 4). Since this is still a ratio, $\dot{Q}_{\text{SVC}}$ can be expressed again as a ratio of $\dot{Q}_{\text{IVC}}$. This method can be performed either in the cath lab, with echocardiography, or with magnetic resonance imaging just as well as in the operating theatre.
To summarize, $Q_S$ or $Q_{IVC}$ are used as unknown constants, while $Q_P$ consists of 2 sources that cannot be influenced: $Q_{SVC}$ and $Q_{COLL}$, and 1 that can be influenced: $Q_{APBF}$. All flows are then expressed as proportions of $Q_S$ or $Q_{IVC}$, permitting in the end to calculate increase in ventricular stroke volume caused by APBF. With this method it is possible to quantify ventricular volume load in the following paragraphs, first dealing with the 3 sources of pulmonary blood flow and ending with ventricular stroke volume.

2. Ratio of Pulmonary to Inferior Vena Cava flow by Oxymetry ($\frac{Q_P}{Q_{IVC}}$)

When temporarily disregarding $Q_{COLL}$, there is no other source for the pulmonary circulation than the SVC and $Q_{SVC}$ is then equal to $Q_P$. When, for example, $Q_{SVC}$ is equal to $Q_{IVC}$, the $Q_P/Q_S$ ratio is then 0.5 as shown in formula 5. Usually, however, $Q_{SVC}$ is not equal to $Q_{IVC}$ and the Fick principle determines that $S_{Ao}$ is then dependent on $Q_P/Q_S$ and thus on $Q_{SVC}/Q_{IVC}$, mixed IVC saturation ($S_{IVC}$) and pulmonary venous saturation ($S_{PV}$) (formula 6). Solving formula 6 to derive $Q_P/Q_{IVC}$ and $Q_P/Q_S$ from oxygen saturations measured in aorta, pulmonary vein, and IVC involves a simple arithmetic process shown in formula 7. The $Q_P/Q_{IVC}$ and $Q_P/Q_S$ can then be calculated according to the Fick principle, with the resulting formulas 8a and 8b. For these equations the saturations are measured where $Q_P$ is expressed as a proportion of $Q_{IVC}$ or $Q_S$. How this works in practice is shown in a hypothetical example (example 1; appendix 2).

In the following formulas exact flow volumes in the aorta, SVC, and IVC are presumed to be unknown as is usually the case during an operation. Oxygen saturations, however, can easily be sampled and determined quickly.

3. Ratio of Collateral Pulmonary to Inferior Vena Cava flow ($\frac{Q_{COLL}}{Q_{IVC}}$)

Systemic-to-pulmonary collateral flow through bronchial or other arteries is a second source of pulmonary blood flow in addition to $Q_{SVC}$, adding to ventricular volume load. When quantifying $Q_{COLL}$, $Q_{SVC}$ has to be subtracted from $Q_P$ (formula 1b) and thus $Q_{SVC}$ has to be quantified separately. The ideal method to distinguish $Q_{SVC}$ from $Q_{COLL}$ would be to measure caval flow volumes directly during operation or in the cath lab, which may be possible but to our knowledge has not yet been reported.

In section 2 we had calculated $Q_P/Q_{IVC}$ through oxymetry by formula 8, which includes the up to this time disregarded $Q_{COLL}$. We can now determine $Q_{COLL}$ by subtracting $Q_{SVC}$
from $Q_p$, when both are expressed as a proportion of $Q_{IVC}$ (formula 9). How this could work is illustrated in example 2. $Q_{COLL}$, being the second source of $Q_p$, results in a larger $Q_p$ than with $Q_{SVC}$ alone and thus causes a higher $S_{Ao}$ than with $Q_{SVC}$ alone, but at the cost of higher ventricular load ($Q_v$).

4. Ratio of Additional Pulmonary to Inferior Vena Cava flow ($\dot{Q}_{APBF}/\dot{Q}_{IVC}$)

The $\dot{Q}_{APBF}$ is ventricular blood that is ejected into the pulmonary circulation (presumably after complete mixing) through a banded or stenotic connection of the ventricle to the pulmonary arteries. A Blalock-Taussig or comparable shunt left in place can constitute APBF as well. The completeness of the mixing might, however, not be total in individual cases. In contrast to $Q_{COLL}$, $Q_{APBF}$ is the only variable source of $Q_p$ that can be influenced surgically with some ease. When APBF is present, $Q_p$ consists of 3 sources as is shown in formula 10. As APBF increases $Q_p$, $S_{Ao}$ rises as a result. Presuming equal systemic oxygen extraction, $S_{IVC}$ rises with the same percentage points. To calculate $\dot{Q}_{APBF}$ oxygen saturations in the IVC ($S_{IVC}$), pulmonary veins ($S_{PV}$) and aorta ($S_{Ao}$) have to be measured with and without APBF (formulas 8 and 11) as shown in example 3.

5. Ventricular Stroke Volume ($\dot{Q}_{SV}$)

To calculate the extra volume load on the ventricle, ventricular stroke volume ($\dot{Q}_{SV}$) needs to be calculated. The total flow volume into and through the heart ($Q_V$) has then to be divided by the beating frequency (f) to obtain stroke volume. It is essential to consider that in the presence of APBF $Q_V$ is greater than $Q_S$ by definition, as some volume is ejected by the ventricle into the pulmonary circulation and not into the systemic circulation. Strictly speaking this does not hold true for the rare case with an intact aortopulmonary shunt, because then blood is ejected into the systemic circulation first; whereupon it is diverted to the pulmonary circulation. In clinical practice it should be easy to measure $S_{Ao}$, $S_{PV}$, and $S_{IVC}$ without and with APBF. Then $Q_V$ can be calculated with and without APBF (example 3), where $Q_S$ with $\dot{Q}_{APBF}$ is divided by $Q_S$ without $\dot{Q}_{APBF}$, after which $Q_{SV}$ can be calculated using formula 12. Because numerator and denominator are both expressed as proportions of $Q_{IVC}$ or $Q_S$, this element disappears in the division and the resulting figure is then a proportion of $Q_V$ without APBF. For example, if formula 13 shows that $\Delta Q_V = 0.395$ then the additional load caused by APBF on the single ventricle is 39.5% of the total ventricular load.
Figure 1 demonstrates how formula 2 and 8a can work easily in clinical practice, the operating theatre, or the cath lab to calculate $\frac{Q_V}{Q_{IVC}}$.

![Table to facilitate the calculation of $\frac{Q_V}{Q_{IVC}}$.](image)

**Figure 1** - Table to facilitate the calculation of $\frac{Q_V}{Q_{IVC}}$. Oxygen saturations measured in the aorta ($S_{Ao}$), pulmonary veins ($S_{PV}$) and inferior vena cava ($S_{IVC}$) are used to calculate the $\frac{Q_P}{Q_{IVC}}$ ratio by $(S_{Ao} - S_{IVC}) / (S_{PV} - S_{Ao})$. The horizontal axis shows $(S_{Ao} - S_{IVC})$. The vertical axis shows $(S_{PV} - S_{Ao})$. At the intersection the corresponding $\frac{Q_P}{Q_{IVC}}$ can be found considering that $Q_V = Q_P + Q_{IVC}$ thus $\frac{Q_V}{Q_{IVC}} = \frac{Q_P}{Q_{IVC}} + 1$. By repeating the measurements and calculations in two different situations the difference in $\frac{Q_V}{Q_{IVC}}$ can be calculated.
Comment

By calculating differentiated pulmonary flows as proportions of the IVC flow, this framework of formulas enables us to calculate ventricular volume loading intraoperatively without having to measure cardiac output. The results of these measurements and calculations then enable us theoretically to balance pulmonary flow and thus ventricular volume loading, taking into account the particular anatomy of the heart under investigation and the operation. This knowledge is a step forward because up to this time ventricular volume loading could only be seen echocardiographically during operation and could not be quantified, except for eyeballing. Assessing such images by just “eye-balling” them is subjective and at best uncertain.

Using \( Q_{IVC} \) or \( Q_S \) as unknown constants used as denominator to quantify other flows in this setting is defendable because the source of \( Q_P \) is \( Q_{SVC} \) and \( Q_{COLL} \), while \( Q_{SVC} \) can only be determined by a different method (section 2), so there are no other practical choices. Why it works is because when APBF is applied, \( Q_P \) rises but not \( Q_S \) and subsequent systemic venous return (formulas 2 and 10). The \( S_{IVC} \) will rise with APBF for one reason only, because \( Q_P/Q_{IVC} \) rises and thus the mixture entering the aorta has a higher oxygen saturation. Systemic oxygen extraction remaining the same, \( S_{IVC} \) will rise proportionally. All flows can thus be expressed as proportions of \( Q_{IVC} \) or \( Q_S \), permitting in the end to calculate increase in ventricular stroke volume caused by APBF because then \( Q_{IVC} \) or \( Q_S \) is lost in the last division. \( Q_{IVC} \), however, has our personal preference because intuitively it is easier to deal with the constant factor of the mix in the denominator, than the mix itself.

Whatever the method, the physiology under investigation is subject to various other variable influences such as anesthesia. The principles of this physiology have been described thoroughly by Santamore and colleagues in 1998 [5]. Of note, aortopulmonary collateral flow was not taken into account in their paper.

Volume overloading of a single ventricle has always been of particular concern in patients on a “Fontan pathway.” The object of surgical strategies is usually to “unload” the ventricle as soon as is possible. Theoretically, ventricular preload is normal after a BCPS, when collateral pulmonary blood flow is disregarded. The additional volume load then imposed by uncalibrated APBF has therefore been contentious and sometimes worrisome. The theory discussed in this treatise, using oxymetry, allows for the calibration of the additional volume load. A potential drawback of this method is the IVC oxygen saturation sample, as various venous blood sources in the IVC usually have
different oxygen saturations. The reason for this is the incomplete mixing of hepatic and other IVC blood in the IVC because of its short length, while mixing within the atrium potentially includes some pulmonary venous flow.

Another unknown factor is the anatomic volume of the single ventricle. In assuming that combined IVC and SVC flow volumes (and possibly the collateral flow) represent a normal preload for a normal right (or left) ventricle, this might not hold true for a single ventricle, the volume of which can be influenced by a range of morphologies. The sizes of these ventricles of univentricular physiology have been described as generally larger than normal, although reliable quantification of this notion has not to our knowledge been published. If this notion proves to be true, then combined IVC and SVC flow volumes may well be less than normal for these particular ventricular sizes, at least in individual cases.

Pulmonary collateral blood flow is the third source of pulmonary blood flow that needs to be considered. Surgical observation has it that the amount of collateral blood flow can be substantial. This collateral flow can be so much that in some cases it permits surgeons to construct a BCPS without employing extracorporeal circulation and without any other source of pulmonary blood flow. By measuring the cross-sectional areas of the caval veins and the flow velocities therein, and by calculating the ratio of the flow volumes, the actual flow ratio through the caval veins can be calculated. In a BCPS the SVC flow entering the heart as pulmonary blood flow, having an oxygen saturation of approximately 100%, marks this very flow. The difference then between the ratio of pulmonary venous blood flow and SVC flow, both expressed as a ratio of IVC flow, results in the pulmonary collateral blood flow also expressed as a ratio of IVC flow. This latter figure has to be taken into account when estimating ventricular preload. This collateral flow is less effective than normal systemic flow because of its higher oxygen saturation when entering the pulmonary circulation. If the amount of collateral flow is considerable, it also adds considerably to ventricular stroke volume and counteracts ventricular unloading. Thus, if collateral flow is large a case could be made for diminishing it either surgically or in the cath lab at any time.

Working with ratios of the CSAs of the caval veins precludes having to relate to expected diameters for body size or age. There is a paucity of data on caval morphometry. Only one paper reported on the computed tomographic scan measured size distribution of the IVC, but no mention of the SVC [6]. Another paper reported on SVC size distribution as measured on “venograms” [7]. Different methodology makes these studies hard to compare. When, in the future, we have more reliable anatomic volume measurements
of the single ventricle, probably through MRI studies, hopefully ventricular preload can be matched better to ventricular anatomic volume so as to preclude ventricular overloading.

**Limitations**

The obvious limitation of this theoretical treatise is that in practice it might be difficult to employ in the operating theatre particularly because cardiac output is notoriously difficult to measure reliably during operation. The only indirect indication of cardiac output changes might be the mixed oxygen saturation of the caval veins. Furthermore, circumstances before and after extracorporeal circulation (if employed) can differ substantially, for example hemoglobin concentration. Nonetheless, ceteris paribus, this theoretical treatise provides the framework within which these phenomena could be quantified. Another limitation is that measurements taken at rest do not reflect nor predict the hemodynamic situation during exercise. Obviously the intraoperative situation precludes measurements during exercise, but this would be extremely interesting, if at all possible, in a cath lab.

**Conclusions**

These measurements and calculations provide a tool for estimating various sources of pulmonary blood flow in the operating theatre as well as in the cath lab in the setting of a bidirectional cavopulmonary shunt.

*We would like to acknowledge the eminent support of Prof W.G. Zijlstra.*
References


### Appendices - Abbreviations and Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>APBF</td>
<td>Additional Pulmonary Blood Flow</td>
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<tr>
<td>BCPS</td>
<td>Bidirectional Cavo-Pulmonary Shunt</td>
</tr>
<tr>
<td>CSA</td>
<td>Cross-Sectional Area</td>
</tr>
<tr>
<td>IVC</td>
<td>Inferior Vena Cava</td>
</tr>
<tr>
<td>$\dot{Q}$</td>
<td>Flow volume per time unit / cardiac cycle</td>
</tr>
<tr>
<td>$\dot{Q}_{APBF}$</td>
<td>Additional Pulmonary Blood Flow volume per time unit</td>
</tr>
<tr>
<td>$\dot{Q}_{coll}$</td>
<td>Collateral blood flow volume per time unit</td>
</tr>
<tr>
<td>$\dot{Q}_{IVC}$</td>
<td>Flow volume per time unit in the Inferior Vena Cava</td>
</tr>
<tr>
<td>$\dot{Q}_P$</td>
<td>Total Pulmonary flow volume per time unit</td>
</tr>
<tr>
<td>$\dot{Q}_S$</td>
<td>Total Systemic flow volume per time unit</td>
</tr>
<tr>
<td>$\dot{Q}_{SV}$</td>
<td>Ventricular Stroke Volume per time unit</td>
</tr>
<tr>
<td>$\dot{Q}_{SVC}$</td>
<td>Flow volume per time unit in the Superior Vena Cava</td>
</tr>
<tr>
<td>$\dot{Q}_V$</td>
<td>Total Ventricular flow volume per time unit</td>
</tr>
<tr>
<td>$S$</td>
<td>Oxygen Saturation</td>
</tr>
<tr>
<td>$S_{IVC}$</td>
<td>Oxygen Saturation of blood in Inferior Vena Cava</td>
</tr>
<tr>
<td>$S_{PV}$</td>
<td>Pulmonary Venous blood oxygen Saturation</td>
</tr>
<tr>
<td>$S_{SVC}$</td>
<td>Oxygen Saturation of blood in Superior Vena Cava</td>
</tr>
<tr>
<td>SVC</td>
<td>Superior Vena Cava</td>
</tr>
<tr>
<td>$V_{IVC}$</td>
<td>Flow velocity in the Inferior Vena Cava</td>
</tr>
<tr>
<td>$V_{SVC}$</td>
<td>Flow velocity in the Superior Vena Cava</td>
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Appendix 1 - Formulas

1. Pulmonary flow volume after BCPS equals superior vena cava flow volume plus any collateral systemic-to-pulmonary flow volume.

\[ \dot{Q}_P = \dot{Q}_{SVC} + \dot{Q}_{COLL} \]  

(1a)

Thus:

\[ \dot{Q}_{COLL} = \dot{Q}_P - \dot{Q}_{SVC} \]  

(1b)

2. Both systemic and ventricular flow volume after BCPS equals pulmonary flow volume plus inferior vena caval flow volume in the absence of APBF.

\[ \dot{Q}_S = \dot{Q}_V = \dot{Q}_P + \dot{Q}_{IVC} \]  

(2a)

Thus:

\[ \dot{Q}_{IVC} = \dot{Q}_S - \dot{Q}_P \]  

(2b)

3. Using formulas (1a) and (2a): pulmonary to systemic flow volume ratio can be written as:

\[ \frac{\dot{Q}_P}{\dot{Q}_S} = \frac{\dot{Q}_P}{\dot{Q}_P + \dot{Q}_{IVC}} = \frac{\dot{Q}_{SVC} + \dot{Q}_{COLL}}{\dot{Q}_P + \dot{Q}_{IVC}} \]  

(3)

4. Superior vena cava flow volume (as proportion of \( \dot{Q}_{IVC} \)) separately from pulmonary flow volume can be determined by measuring cross sectional area of the caval veins and their flow velocities and using this flow volume.

\[ \frac{\dot{Q}_{SVC}}{\dot{Q}_{IVC}} = \frac{V_{SVC} \times CSA_{SVC}}{V_{IVC} \times CSA_{IVC}} \]  

(4)

5. \( \dot{Q}_P / \dot{Q}_S \) is equal to \( \dot{Q}_{SVC} / (\dot{Q}_{SVC} + \dot{Q}_{IVC}) \) in BCPS and when caval flows are equal, the result is 0.5.

\[ \frac{\dot{Q}_P}{\dot{Q}_S} = \frac{\dot{Q}_{SVC}}{\dot{Q}_P + \dot{Q}_{IVC}} = \frac{\dot{Q}_{SVC}}{\dot{Q}_{SVC} + \dot{Q}_{IVC}} = \frac{1}{1 + 1} = 0.5 \]  

(5)
6. The Fick principle applied to the situation after BCPS determines that flow volumes entering and exiting the heart multiplied by respective oxygen saturations are equal and can be written as follows:

\[ Sao \times Q_s = S_{PV} \times Q_p + S_{IVC} \times Q_{IVC} \]  

(6)

7. Solving formula (6) when \( Q_s \) is replaced by \( (Q_p + Q_{IVC}) \) according to formula (2a) can be done in the following 3 steps, resulting in formulas (8a) and (8b) when \( Q_{IVC} \) is replaced by \( (Q_s - Q_p) \):

\[ Sao \times (Q_p + Q_{IVC}) = S_{PV} \times Q_p + S_{IVC} \times Q_{IVC} \]  

(7a)

\[ Sao \times Q_p + Sao \times Q_{IVC} = S_{PV} \times Q_p + S_{IVC} \times Q_{IVC} \]  

(7b)

\[ (Sao - S_{IVC}) \times Q_{IVC} = (S_{PV} - Sao) \times Q_p \]  

(7c)

Which results in (8a):

8. Pulmonary flow volume as a proportion of \( Q_{IVC} \) can thus be determined by taking 3 oxygen saturation samples and using this formula:

\[ \frac{Q_p}{Q_{IVC}} = \frac{Sao - S_{IVC}}{S_{PV} - Sao} \]  

(8a)

Performing the same arithmetic as in formula (7) but replacing \( Q_{IVC} \) by \( (Q_s - Q_p) \) according to formula (2b) into formula (6) results in the \( Q_p/Q_s \) ratio in the following formula:

\[ \frac{Q_p}{Q_s} = \frac{Sao - S_{IVC}}{S_{PV} - S_{IVC}} \]  

(8b)

This formula is shown in an effort to be complete. As it is our preference to use \( Q_{IVC} \) as the denominator instead of \( Q_s \) this formula is not used in subsequent reasoning, calculations and examples.

9. Collateral flow volume as proportion of \( Q_{IVC} \) can be determined by subtracting \( Q_{SV} \) from \( Q_p \) according to formula (1b):

\[ \frac{Q_{COLL}}{Q_{IVC}} = \frac{Sao - S_{SV}}{S_{PV} - Sao} - \frac{Q_{SV}}{Q_{IVC}} \]  

(9)
10. Pulmonary flow volume with APBF then has 3 sources and can be written as:

\[ Q_p = Q_{SVC} + Q_{COLL} + Q_{APBF} \]  

(10)

11. APBF flow volume (as proportion of \( Q_{IVC} \)) can be determined by taking 3 oxygen samples in 2 different situations, with APBF and without APBF:

\[ Q_{APBF} = Q_p \text{ (with APBF)} - Q_p \text{ (without APBF)} \]  

(11)

12. Stroke volume can be determined by adding all sources of blood and dividing it by heart rate (f):

\[ Q_{SV} = \frac{Q_{SVC} + Q_{COLL} + Q_{APBF} + Q_{IVC}}{f} \]  

(12)

13. Proportional addition to ventricular flow or stroke volume can be calculated by dividing APBF flow volume by systemic flow volume without APBF. Because both numerator and denominator are expressed as proportions of \( Q_{IVC} \) (not shown), this factor disappears in the division as \( Q_{IVC}/Q_{IVC} = 1 \).

\[ \Delta Q_{SV} = \frac{Q_{APBF}}{Q_p \text{ (without APBF)} + 1} \]  

(13)
Appendix 2 - Examples

Example 1

In this example the following oxygen saturation samples have been taken intraoperatively in a patient after BCPS has been completed. The findings are: 
$S_{IVC} = 50\%; S_{PV} = 100\%$ and $Sao = 70\%$.
Entering these values into formula (8a) then results in:

$$\frac{\dot{Q}_P}{\dot{Q}_{IVC}} = \frac{Sao - S_{IVC}}{S_{PV} - Sao} = \frac{70 - 50}{100 - 70} = \frac{20}{30} = 0.67$$ (8a)

Indicating that $\dot{Q}_P = 0.67 \times \dot{Q}_{IVC}$, and formula (2a) determines that:
$\dot{Q}_V = \dot{Q}_{IVC} + \dot{Q}_P$ then $\dot{Q}_P = 1.67 \times \dot{Q}_{IVC}$.

Example 2

In this example the oxygen saturations are the same as in example 1. The flow velocities in SVC and IVC are measured echocardiographically and prove to be equal: 0.5 m/sec.
The cross-sectional areas (CSA) of the SVC = 314 mm$^2$ and IVC = 706.5 mm$^2$. In this example SVC and IVC are presumed to be circular and to have a diameter of 10 mm and 15 mm respectively.

$$\frac{\dot{Q}_{COLL}}{\dot{Q}_{IVC}} = \frac{70 - 50}{100 - 70} - \frac{0.5 \times 314}{0.5 \times 706.5} = 0.67 - 0.44 = 0.23$$ (9)

The interpretation of this example is that: 
$\dot{Q}_P$ is 0.67 x $\dot{Q}_{IVC}$, while $\dot{Q}_{SVC}$ is 0.44 x $\dot{Q}_{IVC}$, thus $\dot{Q}_{COLL}$ is 0.23 x $\dot{Q}_{IVC}$.

Example 3

We start with the calculation of example 1 where $\dot{Q}_P / \dot{Q}_{IVC} = 0.67$ and $\dot{Q}_V / \dot{Q}_{IVC} = 1.67$.
When APBF is added and $Sao$ rises to 85%, we assume $S_{IVC}$ rises with the same percentage points to 65%. Formula (8a) is used again in this new situation.

$$\frac{\dot{Q}_P}{\dot{Q}_{IVC}} = \frac{Sao - S_{IVC}}{S_{PV} - Sao} = \frac{85 - 65}{100 - 85} = \frac{20}{15} = 1.33$$ (8a)
As in example 1:
\[ \dot{Q}_V = \dot{Q}_{IVC} + \dot{Q}_P = 2.33 \times \dot{Q}_{IVC}, \] also shown in figure 1, application of formula (11) then leads to \( \dot{Q}_{APBF} \).

\[ \dot{Q}_{APBF} = \dot{Q}_P \text{ (with APBF)} - \dot{Q}_P \text{ (without APBF)} = 1.33 - 0.67 = 0.66 \times \dot{Q}_{IVC} \quad (11) \]

\( \Delta \dot{Q}_V \) is then calculated as follows using the measurements in the previous examples:

\[ \Delta \dot{Q}_{SV} = \frac{\dot{Q}_{APBF}}{(\dot{Q}_P \text{ (without APBF)} + \dot{Q}_{IVC})} = \frac{0.66}{0.67 + 1} = 0.395 \quad (13) \]

Or:

\[ \Delta \dot{Q}_{SV} = \frac{\dot{Q}_P \text{ (with APBF)}}{\dot{Q}_P \text{ (without APBF)}} - 1 = \frac{2.33}{1.67} - 1 = 0.395 \]