CHAPTER 5

Product Mix Variability with Correlated Demands

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Abstract

In food processing, market demands are increasingly important, resulting in regular introductions of new products, or special offers. Often, such an introduction or promotional effort affects demand of other products or packaging types. Here we study the effect of such correlated demand. More specifically, the aim of this paper is to study the effect of product mix variability and correlated demand in a two-stage food production system. Results from a simulation study show that increasing correlation on the product level results in an increase in average lead times. A slightly smaller effect is seen for correlation on the package level. Similar results are found for average waste. Increased variability amplifies these effects.

5.1 Introduction

The food industry is becoming a more and more competitive environment where manufacturers have to cope with short due dates imposed by the high market pressure, specifically from large retailers (Dobson et al., 2001; Rundh, 2005). In the food-processing industry, these due dates are especially important, as they are closely related to the best-before dates on the final consumer products. Other distinctive characteristics (see e.g., Nakhla, 1995; Akkerman and Van Donk, 2006a) are the perishability of products and the high quality

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demands. Also, a divergent product structure is common. Only a few raw materials (often agricultural) are processed and packaged to generate a multitude of end products (see e.g., Fransoo and Rutten, 1994). Often, these end products are customer-specific (e.g., mainly in package type, but sometimes also in product type).

Food production mostly consists of two stages: processing and packaging (Van Dam et al., 1993). Between these stages, intermediate storage is present (generally in the form of tanks or silos) to decouple the two stages. To deal with the short lead times and with customer-specific packages for end products, manufacturers often have make-to-order strategies on the packaging level. This puts additional pressure on the production system, as it becomes partly make-to-order (packaging stage), and partly make-to-stock (processing stage). This mixed MTO/MTS situation is related to the customer-order decoupling point (CODP) concept (see also Van Donk, 2001; Olhager, 2003; Soman et al., 2004; Wikner and Rudberg, 2005), which in this paper is located at the intermediate storage tanks between the processing and packaging stage.

The intermediate storage in these industries is normally constrained in capacity and time. Capacity is not only constrained by a limited number of tanks, but also because quality demands do not allow concurrent use of a tank (i.e., only one batch can be stored in a tank at a certain time). Time constraints result from perishability of the basic food product, which restricts the time until packaging (see also Akkerman et al., 2006). These storage constraints lead to dependency between the two stages, and often make scheduling a complicated matter in these industries (see also Van Dam et al., 1993).

In the literature, production scheduling of batch plants has been extensively studied in operations management and chemical engineering, mostly using mathematical modeling techniques such as MILP (see e.g., Kondili et al., 1993; Pinto and Grossmann, 1998; Rajaram and Karmarkar, 2004). Most of these approaches become very difficult (i.e., computationally intensive) when considering limited intermediate storage. Furthermore, there is an important difference between batch plants and the type of food production system discussed in this paper: In most cases in the literature, the batches go through all production steps. Here, batches produced in the processing stage are used as inputs for the packaging stage. There are some papers that do treat such systems, like Méndez and Cerdá (2002), who study a make-and-pack facility and develop a MILP formulation, but unfortunately consider unlimited intermediate storage.

In food production, the processing stage commonly involves batch pro-
cesses that produce various product types (recipes), while the packaging stage usually involves several lines to accommodate multiple package types (e.g., 1/4, 1/2, and 1 liter). These different product and package types result in a product mix with two dimensions. Due to volatile market behaviour in the food sector, the shares in the product mix change regularly—in both the product dimension and package dimension. For instance, new low-fat products are added to the mix, products can be on special offer, or customers (temporarily) buy more large family-sized packages. These changes cause shifts of the workload between packaging lines (package dimension), and cause changing storage tank usage (product dimension).

From the literature, we know that more variability in individual product demand results in a higher variance of the total demand (e.g., Ross, 1997), which in turn can have consequences like lost sales (Andreou, 1990), increasing flow times (Jensen et al., 1999), and increasing safety stocks (Vaughan, 2003). In the two-stage food production system studied in this paper, variability in the product mix causes short-term imbalance in the volumes for the product types and/or package types, which is likely to (i) influence the blocking effects caused by occupied tanks and/or packaging lines, and (ii) affect the amount of waste due to perished product.

Concerning the variability in the product mix, the situation can be even more complicated due to dependency between demands for various product-package combinations. For example, it is well-known that promotional activities within one retail chain affect the turnover of similar products in other chains, resulting in correlations between demands. Also, seasonal demands and new product introductions can result in products which have demand that is positively or negatively correlated with the demand for other products. In the literature, some papers address the issue of correlated demand. The main results are that the effects of variability in demand are stronger when demand is also correlated and that performance is negatively affected if correlations are ignored (see e.g., Zhang, 1997; Vaughan, 2003; Ma et al., 2004).

For the two-stage food production system, the product mix variability can be correlated on two dimension (products and packages), which has not been addressed before in the literature. Also, the interaction between an order-driven packaging stage, a forecast-driven processing stage, and limited capacity intermediate storage facilities is not at all clear from the literature. Although some papers discuss these types of production systems, they mostly concern mathematical optimization approaches (like MILP), which do not
aim at understanding the basic behaviour and interactions of such systems.

The aim of this paper is therefore to study the effects of product mix variability with correlated demand between product types and package types on the performance of a two-stage food production system with limited intermediate storage. We consider this to be explorative research, and we perform simulation studies to investigate the primary effects.

5.2 Production system

Figure 5.1 illustrates the production environment studied in this paper. In the first stage, a batch process creates $N$ basic food products from (agricultural) raw materials. In the intermediate storage stage, $K$ storage tanks are available to store the basic food products (with $K \leq N$). Here, quality and traceability requirements restrict batches to concurrent storage. In the packaging stage, the basic food products are packaged in $M$ different package sizes or types. More specifically, there are $J$ packaging lines available that can each package all basic food products in one or more package types. Due to technological constraints (e.g., piping), only one packaging line can be connected to a specific storage tank at the same time.

The main characteristic of this production system is the fact that at the processing and intermediate storage level, scheduling is product-oriented, while at the packaging level, it is package-oriented. This results in totally different viewpoints on production scheduling and control. It makes it difficult to create one sequence for the whole production system, as batches are formed on different characteristics (e.g., based on color in the processing stage and based
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on packaging material in the packaging stage) and often preferred sequences exist for each of the two stages. Also, separate sequences cannot be developed independently due to various dependencies resulting from the capacity and time constraints.

5.2.1 Product mix

We assume equal probabilities for all possible product-package combinations. The variability in the product mix expresses itself in the average order sizes for each product-package combination. These vary from week to week. All orders are considered to have customer-specific packaging demands, which have to be considered in the packaging stage.

As mentioned in the introduction, the weekly average order sizes for the various end products are not necessarily independent. Dependencies are possible on product and package level, and this can cause unequal distribution among product types and package type. These dependencies can occur through both positive and negative correlations. An increase for product $i$ in package $j$ could have several reasons (and effects):

- A random increase in the demand for product $i$ in package $j$, without affecting other products and packages (no correlation on product level or package level).

- A general increase in demand for product $i$, in which case demand for the same product in other packages would also increase (positive correlation on the product level). E.g., this could be the result of a short-term increased interest in low-fat products.

- A general increase in demand for package $j$, which also affects the demand for other products in the same package (positive correlation on the package level). E.g., this could be the case for products in small packages, which are more popular in holiday seasons.

- A specific increase in the demand for product $i$ in package $j$. This could be at the expense of other products or packages (negative correlation on product level, package level, or both). E.g., this could be the result of promotional activities.

This list is not exhaustive, but such effects can make it hard to continuously realize short lead times. For instance, as shown by Ma et al. (2004), overall production variability increases with an increasing correlation between the
products. If this increased variability occurs on the packaging level, it could mean significant shifts of the workload between packaging lines.

5.2.2 Production scheduling

The control of the two stages is not fully integrated due to the presence of the intermediate storage tanks. The packaging stage extracts its necessary intermediate product from the tanks. In the batch processing stage, the supply of intermediate product is replenished.

Because of short due dates, the scheduling of the processing stage has to be forecast-driven to safeguard product availability. This is implemented through a runout time procedure. Every time the batch processor becomes idle, the next product to produce is the product with the minimum runout time. This runout time for a product $i$ is calculated by:

$$ RO_i = \frac{I_i - O_i}{D_i}, $$

where $I_i$ is the inventory level of product $i$ in the intermediate storage tanks, $O_i$ is the amount of product $i$ needed for waiting orders, and $D_i$ the average demand arriving per time unit. Due to the fact that two batches cannot be stored concurrently in the same tank, the batch processor can become blocked if all storage tanks are already in use.

The packaging stage, however, is totally customer-order-driven. Every arriving order is customer-specific and requires a specific combination of product type and package type. Sequencing of these order is performed by using an earliest-due-date (EDD) rule. If a packaging line becomes idle, it will continue with the order with the earliest due date from a list of possible orders. These possible orders are determined by their package requirements (it can be produced on the specific line), and their product requirements (there is intermediate product available in a tank that is not currently supplying another packaging line).

We assume that the setup times involved in the packaging stage are negligible in relation to the packaging times. For the processing stage, setup or cleaning times are included in the batch processing times. This can be done because the processing stage is a batch process that takes a certain fixed amount of time before the product can be transported to an intermediate storage tank (for instance, mixing, fermentation).

The performance of the production system is mainly calculated in average lead times. However, due to the perishability of the intermediate product, the amount of waste is also an important performance measure.
5.3 Product mix variability

End products are distinguished by product type \((i = 1, \ldots, N)\) and package type \((j = 1, \ldots, M)\), which creates a total of \(N \times M\) possible end products. The demand is based on average order sizes \(d_{ij}\). We assume that every arriving order can be for any of the possible end products (with equal probabilities). This means the average number or orders is the same for all end products. The average order sizes for these orders are not necessarily the same.

The product mix variability between different periods (we use weeks) expresses itself in varying average order sizes. This is modelled through periodically (weekly) changing average demand order sizes \(d_{ij}\). Every week, a new set of \(d_{ij}\)’s is generated from a multivariate normal distribution.

5.3.1 Modelling demand correlations

To model possible dependencies between products and packages, we introduce the parameters \(\rho_{\text{prod}}\) and \(\rho_{\text{pack}}\), representing the correlation on product level, and the correlation on package level:

- \(\rho_{\text{prod}}\): correlation coefficient for the average order sizes of all end products that have the same product type.
- \(\rho_{\text{pack}}\): correlation coefficient for the average order sizes of all end products that have the same package type.

These correlation coefficients can be used to construct a covariance matrix for the generation of new average order sizes.

For example, if \(N = M = 2\), the weekly average order sizes \(d_{ij}\) are generated from a multivariate normal distribution with means \(\delta_{ij}\) (combined in \(\Delta\)) and (co)variance matrix \(\Sigma\):

\[
\Delta = \begin{bmatrix}
\delta_{11} \\
\delta_{21} \\
\delta_{12} \\
\delta_{22}
\end{bmatrix}, \quad \Sigma = \begin{bmatrix}
\sigma^2 & \rho_{\text{pack}}\sigma^2 & \rho_{\text{prod}}\sigma^2 & 0 \\
\rho_{\text{pack}}\sigma^2 & \sigma^2 & 0 & \rho_{\text{prod}}\sigma^2 \\
\rho_{\text{prod}}\sigma^2 & 0 & \sigma^2 & \rho_{\text{pack}}\sigma^2 \\
0 & \rho_{\text{prod}}\sigma^2 & \rho_{\text{pack}}\sigma^2 & \sigma^2
\end{bmatrix}, \quad (5.2)
\]

where \(\sigma^2\) is the variance for each of the \(\delta_{ij}\).

For \(\Sigma\) to be a valid covariance matrix, it has to be positive semidefinite (e.g., Lindgren, 1993). This sets a number of constraints on the correlation coefficients. These constraints can be derived from \(\Sigma\) by calculating its eigenvalues. These eigenvalues have to be nonnegative, for \(\Sigma\) to be positive semidefinite. The eigenvalues are the roots of the characteristic polynomial resulting
from taking the determinant of $\Sigma - \lambda I$, where $I$ is the identity matrix (construction of the matrix $\Sigma$ is similar to the $2 \times 2$ case in equation (5.2)).

For example, if $N = M = 3$, the $9 \times 9$ matrix $\Sigma$ leads to a characteristic polynomial of degree 9 in $\lambda$, with four unique roots (the original nine contain doubles)\(^2\). These four different roots are the eigenvalues of $\Sigma$:

\[
\begin{align*}
\lambda_1 &= \sigma^2(1 + 2\rho_{\text{pack}} + 2\rho_{\text{prod}}) \\
\lambda_2 &= \sigma^2(1 + 2\rho_{\text{pack}} - \rho_{\text{prod}}) \\
\lambda_3 &= \sigma^2(1 - \rho_{\text{pack}} + 2\rho_{\text{prod}}) \\
\lambda_4 &= \sigma^2(1 - \rho_{\text{pack}} - \rho_{\text{prod}})
\end{align*}
\]

(5.3)

For these eigenvalues to be nonnegative, four restrictions exist for $\rho_{\text{prod}}$ and $\rho_{\text{pack}}$ (the fifth restriction, $\sigma^2 \geq 0$, is obviously redundant). These four restrictions define a domain in which we can vary the two correlation coefficients in our simulation study. The resulting domain is shown in Figure 5.2.

5.3.2 Effects of product mix variability with correlations

From the discussion in Section 5.1, we know that we can expect certain effects of variability in the (correlated) product mix. Among other things, the var-
ability of total demand is affected, safety stocks requirements change, and fulfillment rates are influenced. All these effects are related to an increasing imbalance in the production system. In our study, this imbalance will likely express itself in blocking and starvation effects on the packaging lines and intermediate storage tanks. These effects influence the amount of waste at the intermediate storage and the lead time performance of the system.

However, more interesting than the variability of demand is the possible correlation between the demands for the individual end products. As was discussed in section 5.2.1, these correlations can exist between product types and between package types and can be positive and negative.

Considering lead times, correlated demand could impact the imbalance in the utilization of the packaging lines and intermediate storage tanks. Positive correlations on the package level could strengthen utilization imbalances on the packaging lines, while negative correlations could smoothen these imbalances. Similarly, correlations on the product level could influence the utilization imbalance between storage tanks. This leads us to the following hypothesis:

\[ \text{(H1)} \]

\( (a) \) Increased positive (negative) correlation on the product level results in longer (shorter) lead times.

\( (b) \) Increased positive (negative) correlation on the package level results in longer (shorter) lead times.

Considering waste, the imbalance between intermediate storage tanks will likely affect the storage time of the product. Imbalances between packaging lines are not expected to influence the waste at the intermediate storage. This results in the following hypothesis:

\[ \text{(H2)} \]

\( (a) \) Increased positive (negative) correlation on the product level results in more (less) waste at the intermediate storage.

\( (b) \) Correlation on the package level does not affect waste at the intermediate storage.

From previous literature (e.g., Andreou, 1990; Jensen et al., 1999), we know that increased demand variability negatively affects performance. There is no reason to expect that this is any different in the two-stage food manufacturing system we study in this paper. Demand correlations are obviously related to variations in demand, and based on Ma et al. (2004), we expect that overall variability increases with correlated demand.

However, to what degree the level of variability and correlated demand interact is a different matter. We formulate the following hypothesis:
(H3) Increased variability leads to an increase of the effects of correlated demand.

These hypotheses are the basis for the numerical experiments discussed in the remainder of this paper.

5.4 Numerical experiments

5.4.1 Experimental design and parameter settings

In the experiments, we initially focus on a situation with $N = K = 3$ and $M = J = 3$. This means that we have 3 basic products that can be stored in 3 tanks. Furthermore, these products can each be packaged in 3 different package types (each on a separate packaging line). This creates a total of 9 end products. We chose this configuration to have a minimum amount of interaction between different products and packages, while still having a reasonably simple system.

Customer orders arrive according to an exponential distribution with $\lambda = 0.06$ orders (per minute). This results in a Poisson process with an interarrival time of 16.67 minutes. With equal probabilities, an arriving order can be any product type $i$, and package type $j$. The order size is sampled from a normal distribution with mean $d_{ij}$ and coefficient of variance $CoV_d = 0.2$ (i.e., the variance depends on $d_{ij}$). As described in Section 5.3, the parameters $d_{ij}$ get assigned new values every week, to model the variability in the product mix. These $d_{ij}$ are sampled from a multivariate normal distribution, as was outlined in section 5.3.1. The overall mean demands $\delta_{ij}$ are set to 50, with a coefficient of variance $CoV_\delta = 0.2$. The normal distributions are truncated to exclude possible negative values, although this seldomly occurs.

Processing times of the batch process are normally distributed with mean $\mu_b = 300$ (minutes), and $\sigma_b^2 = 60$. The batch processor operates with a fixed batch size $B = 1000$. For the packaging lines, we also assume normally distributed packaging times with mean $\mu_p = 0.24$ (minute per single package), and $\sigma_p^2 = 0.06$. Here again, the distributions are truncated to exclude possible negative values. In this case, the truncation causes a slight increase in the averages (mainly for the packaging times) but this effect does not impact the results presented in this paper.

The main parameters in our experiments are the parameters underlying the product mix variability and the dependencies on product and package level: $\rho_{prod}$ and $\rho_{pack}$. They can be varied in the domain specified in Section
5.3.1 to study the effect of these dependencies on lead times and waste.

The simulation experiments were performed in MATLAB. As we consider an empty system at the beginning of each week, the simulation has a run length of one week. Before the experiments were conducted, the necessary number of replications was determined using a 95% confidence interval for the average lead time. The number of replications was chosen in such a way as to result in relative half-widths of the confidence interval below 5%.

5.4.2 Experimental results

In the previous paragraph, we determined that \( \rho_{\text{prod}} \) and \( \rho_{\text{pack}} \) could be varied in the domain determined in Section 5.3.1. For \( M = N = 3 \), this means the individual correlations can be varied from \(-0.5\) to \(1\), if the other correlation is assumed to be zero. The first experiments we undertook had these configurations.

Correlated demands

Concerning the performance of the system, average lead times and average waste was calculated for each configuration. For correlation on the product level (\( \rho_{\text{prod}} \in \{-0.5, 1\} \)) the results for average lead times and average waste are shown in Figure 5.3(a) and (c). It can be seen that higher correlation on the product level results in longer lead times and more waste. This also means that negative correlations have a positive effect on performance. These effects are likely caused by the amount of imbalance between workloads between storage tanks, resulting in an increasing amount of starvation on the packaging lines due to product unavailability. This confirms hypothesis H1(a) and H2(a).

For correlation on the package level (\( \rho_{\text{pack}} \in \{-0.5, 1\} \)), we see a very similar behaviour as with correlation on the product level. As shown in Figure 5.3(b) and (d), average lead times and average waste also increases with correlation on the package level. Here, imbalance between packaging lines is experienced, which has the demonstrated effects. This confirms hypothesis H1(b). In hypothesis H2, we did not expect increasing waste from an increasing correlation on the package level. As it turns out (See Figure 5.3(d)), there is also a significant result of increasing correlation on the package level, which rejects hypothesis H2(b).

The results discussed until now concern the effect of either correlation on the product level or on the package level (while the other was kept zero). To
study the interactions between correlations on the product and package level, we extended the experimental design to include all possible combinations of correlations on both levels (which were identified in Figure 5.2). For the effect of the correlation coefficients on average lead time, the results are shown in Figure 5.4.

The 3D graph in Figure 5.4 suggests that there is no proof for interaction effects between the two correlations. This means that the combined effect of correlation on both the product and the package dimension is just the sum of the individual effects. Experiments with the performance criterion waste show similar results.

### Demand variability

The final hypothesis stated in Section 5.3.2 concerned the interaction between correlated demand and demand variability. To study this interaction, we repeated the experiments in the previous section for a number of values for the coefficient of variance of weekly demand averages ($CoV_\delta = \{0.1, 0.2, 0.3\}$). The results for average lead time are shown in Figure 5.5.
Figure 5.4. Effects of $\rho_{\text{prod}}$ and $\rho_{\text{pack}}$ on average lead time.

Figure 5.5. Effects of $\rho_{\text{prod}}$ and $\rho_{\text{pack}}$ on average lead time for different demand variability.

The experimental results show that the effect of demand correlation indeed increases with an increasing coefficient of variance for the average weekly demands, confirming hypothesis H3. Reasoning from a situation where there is no correlation, this also suggests that the effects of variance decrease in the case of negative correlations. Results for the performance criterium waste show a similar result, and are therefore omitted. Concerning the underlying blocking and starvation effects, it turns out that a higher coefficient of variance results in a higher overall level of starvation on the packaging lines.
5.5 Conclusions and further research

This paper studies the effects of product mix variability with dependency between product types and package types in a two-stage food production with intermediate storage. A simulation study is performed to investigate these effects in an explorative way. Dependency between product types and package types is modeled by defining a correlation coefficient for each of these dimensions. In this way, it easily translates to reality, and it also creates useful modelling possibilities.

The paper shows that increasing correlation on either the product level or package level increases average lead times. The same result is found for average waste. For both performance measures the effects of correlation on the product level are slightly larger than on the package level. The paper also shows that increased variability results in an increase in the effects of correlated demand. This also means that negative correlations can reduce the impact of demand variability.

These results have several practical implications. First, the need to include information on correlated demands in managerial decision-making depends on the level of variability. For instance, when a firm has to cope with highly volatile demands, it becomes very interesting to include information on correlated demands in decisions involving the product mix (e.g., order acceptance, adding new products, assigning products to plants). Secondly, when considering demand correlation in decision making, the effects of correlation on the product or package level are about the same, in terms of average lead times and average waste.

A limitation of our study can be found in the system configuration. Although we believe the results we found are relatively generic, other system configurations might show stronger (or weaker) effects of correlated demand and variability of demand. For instance, the product mix is currently chosen symmetric, which is a very specific configuration. In practice, a large part of the demand is often concentrated in only a small part of the product mix. Although this will also be resembled in the available capacity, it will be an interesting opportunity for further research. Other configuration aspects, like the size of the production system and the number of products, can also be interesting topics for further study.

A second limitation is the use of fairly myopic scheduling procedures in both the batch processing and the packaging stage. Therefore, another opportunity for further research is the development and analysis of more intel-
5.5. Conclusions and further research

Intelligent scheduling procedures. For the batch processing stage, a good forecast is important to keep providing the packaging stage with enough (and correct) products. Under low demand variability, a simple cyclic schedule would suffice and is normally better in terms of setups. But with more demand variability or positive correlations on the product level, it could be necessary to abandon cyclic production. This would likely result in a procedure that combines cyclic schedules and runout time rules in an intelligent way. For the scheduling in the packaging stage, better sequencing rules (combining e.g., due dates, processing times, shelf life, ...) could likely be developed to increase performance.