Chapter 4

An economic model to compare the profitability of pay-per-use and fixed-fee licensing

4.1 Introduction

Packaged software is commercial software that is produced by a software vendor and sold on the market as a commodity item. Examples include Enterprise Resource Planning (ERP) software, Customer Relationship Management (CRM) software, and Human Resource Management (HRM) software. Several strategies for the pricing of this type of software have been developed. This chapter aims at comparing the profitability of two of them: pay-per-use and fixed-fee licensing.

When pay-per-use licensing is applied, pricing is based on the total amount of use of the software, measured in units of use such as the number of users or the number of transactions. Named-user licenses and floating licenses are well-known examples: named-user licenses impose customers to assign a separate license to each user, whereas floating licenses can be shared among a group of users but constrain the number of users that can work with the application concurrently (Bontis and Chung, 2000; Murtojarvi et al., 2007).

In contrast, when fixed-fee licensing is applied, customers obtain the rights to use a particular version of the software package by paying an amount that is independent of product usage. The campus license is a typical example as it allows all faculty, supporting staff, and students of the participating college or university to use the software unlimitedly, while the sales price is usually determined by the total number of faculty rather than the actual number of users.

To compare the profitability of pay-per-use and fixed-fee licensing, an economic model of a monopoly producer of packaged software is created. Although a software vendor generally competes with other vendors to attract new customers, it has a monopoly position in the market of product
upgrades and extensions, which can be explained as follows. Packaged software generally has a modular structure, allowing customers to extend their systems over time by adding new modules (Olson and Saetre, 2007). Because of compatibility requirements, customers have large cost of switching from one software vendor to another: all existing modules have to be replaced by comparable modules from the new software vendor. These switching costs result in lock-in and give a software vendor its monopoly position over existing customers (Burnham et al., 2003; Klemperer, 1995; Shapiro and Varian, 1999).

This chapter contributes to the literature on software pricing and renting by taking into account that customers can develop the required software in-house, rather than obtaining it from the market. In-house development of customized extensions is a common solution for customers to tailor packaged software to their specific requirements, without switching from the software vendor chosen. It can be seen as a likely alternative choice for the customer, especially when the customer would have to pay large amounts of licensing fees if he or she were to obtain the required upgrades and extensions from the market. The possibility of in-house development therefore imposes an additional constraint on the software vendor’s profit maximizing problem: the price that the software vendor charges for using the software cannot exceed the customer’s in-house development cost.

The remainder of the chapter is organized as follows. First, related work on software pricing and renting is discussed. Then, a formal presentation of our model is provided. Next, the software vendor’s optimal licensing strategy is determined for the case when in-house development is equally expensive for each customer. Subsequently, the assumption of a constant in-house development cost is relaxed by letting it vary among customers. Finally, the chapter concludes with a brief summary and suggestions for future research.

## 4.2 Related work

Software vendors can choose between two alternatives in delivering their products to the market: selling and renting (Cusumano, 2007). A software product is sold if the customer obtains the perpetual rights to use a particular version of the product. A separate maintenance agreement is usually required to receive upgraded versions. On the other hand, when the customer obtains the rights to use the software product for a predefined period, such as one year, the product is rented. Upgrades are generally included, but the
customer has to renew his contract if he wants to continue using the software after the expiry date. In addition, software vendors can either express their prices as a function of the amount of use of the software or apply a pricing strategy that is independent of product usage (Sundararajan, 2004). When combined, the two trade-offs yield the four generic strategies shown in Table 4.1.

Table 4.1: Four generic strategies for the pricing of packaged software.

<table>
<thead>
<tr>
<th></th>
<th>Fixed-fee pricing</th>
<th>Usage-based pricing</th>
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</thead>
<tbody>
<tr>
<td>Selling</td>
<td>Fixed-fee licensing</td>
<td>Pay-per-use licensing</td>
</tr>
<tr>
<td>Renting</td>
<td>Fixed-fee subscription</td>
<td>Usage-based subscription</td>
</tr>
</tbody>
</table>

Based on these four generic strategies, related work can be divided into two categories: (i) papers that compare fixed-fee licensing and subscription (first column in Table 4.1), and (ii) papers that compare fixed-fee and pay-per-use licensing (first row in Table 4.1).

4.2.1 Papers that compare fixed-fee licensing and subscription

Choudhary et al. (1998) develop a two-period model of a monopoly firm selling and renting packaged software. In the first period, the firm sells and rents the initial version of the software product; in the second period, the firm sells the upgraded version only. Customers who buy the upgraded version enjoy positive network externalities, i.e. their valuation of the product depends positively on how many other users there are (Katz and Shapiro, 1985), because of bug reports and requests for additional features by first-period customers. The authors show that compared to the case when the product is sold only, the introduction of the rental product in the first period leads to an increase in profit. They also find that as network effects become stronger, the firm reduces its prices in the first period in order to expand the size of its network.

Zhang and Seidmann (2003) study the optimal policy of a monopoly software vendor who provides packaged software to the market through licenses, subscriptions, and sales of upgrades. In their model, customers are heterogeneous in their quality preference but homogeneous in their sensitivity to network externality. It is shown that when there exists uncertainty with regard to product innovations and upgrades, it is optimal to adopt a hybrid
strategy rather than pure selling or renting.

Choudhary (2007) explores how the differences between fixed-fee licensing and subscription affect a software vendor’s investment in product development. In his model, differences in the level of investment translate into differences in software quality, which in turn affects the software vendor’s profit. The results show that compared to software licensing, the software vendor invests more in product development when software renting is applied, which results in higher software quality, larger profit, and higher social welfare.

### 4.2.2 Papers that compare fixed-fee and pay-per-use licensing

Gurnani and Karlapalem (2001) use a monopoly pricing model to examine the optimal pricing strategies for fixed-fee and pay-per-use licensing of packaged software disseminated over the Internet. Customers are assumed to be homogeneous in marginal value of software use but heterogeneous in level of use. In addition, the authors assume that customers incur inconvenience costs when pay-per-use licensing is applied. The results show that compared to the case when fixed-fee licensing is offered only, offering both types of licensing concurrently increases the software vendor’s profit.

Jiang et al. (2006) compare fixed-fee and pay-per-use licensing in a monopoly market where customers are heterogeneous along three dimensions: marginal value of software use, level of use, and honesty type. Similar to Gurnani and Karlapalem (2001), inconvenience costs apply to customers that choose pay-per-use licensing. The authors show that if the proportion of dishonest users in the user population is relatively low, the software vendor will make higher profits by offering fixed-fee licensing. On the other hand, pay-per-use licensing is optimal in markets with a relatively high piracy rate.

This chapter compares the profitability of fixed-fee and pay-per-use licensing in a monopoly market where customers are homogeneous in marginal value of software use but heterogeneous in level of use. Table 4.2 shows that our model differs from the models presented by Gurnani and Karlapalem (2001) and Jiang et al. (2006) in two aspects. First, customers are given the possibility to develop the required software in-house, rather than purchasing it from the software vendor. Empirical data show that in-house development can be a viable alternative, especially when organizations have to pay large amounts of licensing fees if the software is obtained from the market (Thibodeau, 2005). Second, customers do not incur inconvenience costs when
pay-per-use licensing is applied. The next section provides a formal description of our model.

Table 4.2: An overview of the similarities and differences between our model and the models of Gurnani and Karlapalem (2001) and Jiang et al. (2006).

<table>
<thead>
<tr>
<th></th>
<th>Gurnani and Karlapalem</th>
<th>Jiang et al.</th>
<th>Our model</th>
</tr>
</thead>
<tbody>
<tr>
<td>In-house development</td>
<td>no</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Customers incur inconvenience costs when pay-per-use licensing is applied</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Software piracy</td>
<td>no</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Customers heterogeneous in marginal value of software use</td>
<td>no</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Customers heterogeneous in level of use</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

4.3 Model

The model considers a monopoly producer of packaged software. The market consists of \( N \) potential customers who are heterogeneous in level of use. Let \( \theta_i \in [\theta_L, \theta_H] \) denote the level of use of the \( i \)th customer, which is measured in units of use such as the number of (concurrent) users or the number of transactions. Throughout this chapter, we assume that the value of \( N \) is sufficiently large to approximate the distribution of levels of use in the customer population by using a continuous distribution with density function \( f_\theta(\theta) \) and cumulative distribution function \( F_\theta(\theta) \), where \( F_\theta(\theta) = \int_{\theta_L}^{\theta} f_\theta(\theta) d\theta \) denotes the fraction of customers with level of use less than or equal to \( \theta \). The software vendor knows how levels of use are distributed across customers (i.e. it knows \( f_\theta(\theta) \) and \( F_\theta(\theta) \)) but cannot directly observe the level of use of an individual customer. Because of this, the software vendor cannot apply first-degree price discrimination (Varian, 1992).

The value that customers derive from using the software can be compared to the price that is charged by the software vendor through the use of so-called reservation prices (Varian, 1992). Let \( r_i(\theta_i) = v\theta_i \) denote the reservation price of the \( i \)th customer, which is defined as the maximum price that customer \( i \) is willing to pay for the software: at any price lower than or equal
Chapter 4. Pay-per-use versus fixed-fee licensing

to \( r_i(\theta_i) \), the value that the customer derives from using the software is sufficiently large to make obtaining the software package economically feasible, whereas at any price greater than \( r_i(\theta_i) \), the customer is better off by spending his budget on some of the other goods that are available to him. The parameter \( \upsilon \) (referred to as the marginal value of software use) is assumed to be constant across customers.

In addition to buying a copy from the software vendor, customers can develop the required software in-house (or outsource it to an external organization at the same cost). Let \( p(\theta) \) denote the price that the software vendor charges for using the software, and let \( c_i \) denote the cost of in-house development for the \( i \)th customer. Compared to its initial development cost, the marginal cost of reproducing a copy of the software package is negligible, i.e. the software vendor produces against zero unit cost.

Customer \( i \) obtains a copy from the software vendor if (i) he derives a nonnegative surplus from purchasing the software and (ii) the value of this surplus exceeds the net surplus from in-house development. Formally, these constraints can be written as:

\[
\begin{align*}
  p(\theta_i) & \leq \upsilon \theta_i, \\
  p(\theta_i) & \leq c_i.
\end{align*}
\]

The first constraint is the participation constraint: it says that the price that the software vendor charges for using the software cannot exceed the customer’s reservation price. The second constraint, the so-called incentive compatibility constraint, says that this price must be less than the customer’s in-house development cost.

To compare the profitability of fixed-fee and pay-per-use licensing, two additional assumptions have to be made. First, we assume that every customer pays the same price for using the software when fixed-fee licensing is applied, i.e. \( p(\theta) = p_f \). The constraints above can then be formulated as:

\[
\begin{align*}
  p_f & \leq \upsilon \theta_i, \\
  p_f & \leq c_i.
\end{align*}
\]

Second, we assume that under pay-per-use licensing, payments to the software vendor depend linearly on the amount of use of the software, i.e. \( p(\theta) = p_u \theta \). Constraints (4.1) and (4.2) can then be written as:

\[
\begin{align*}
  p_u & \leq \upsilon, \\
  p_u \theta_i & \leq c_i.
\end{align*}
\]
4.4 Analysis for constant in-house development cost

We start our analysis by assuming that in-house development is equally expensive for each customer, which implies that \( c_i = \bar{c} \) for all \( i \in \{1, \ldots, N\} \). Section 4.4.1 considers the case when both types of licensing are offered concurrently, whereas Section 4.4.2 considers the case when the software vendor explicitly chooses between offering fixed-fee and pay-per-use licensing.

4.4.1 Offering both licensing variants concurrently

Customers who can choose between fixed-fee and pay-per-use licensing prefer the variant that gives them the highest net surplus. This causes customers to self-select into two different groups (occasional and frequent users) according to their levels of use and is referred to as second-degree price discrimination (Varian, 1992).

Offering both licensing variants concurrently allows the software vendor to sell its product at different prices to customers for which the participation constraint is binding (occasional users) and customers for which the incentive compatibility constraint is binding (frequent users), according to how much they are willing to pay for it. It follows from Equations (4.1) and (4.2) that the participation constraint is binding for all customers with level of use \( \theta \leq \frac{\bar{c}}{\upsilon} \). Because of this, the software vendor can sell to each of these customers at a price that equals his reservation price by setting \( p_u = \upsilon \). The same equations show that the incentive compatibility constraint is binding for all customers with level of use \( \theta \geq \frac{\bar{c}}{\upsilon} \). The software vendor can therefore sell to these customers at a price that equals their in-house development cost by setting \( p_f = \bar{c} \).

To conclude, when both licensing variants are offered concurrently, the software vendor can charge each customer his maximum willingness-to-pay by setting \( p_f = \bar{c} \) and \( p_u = \upsilon \), which results in a total profit of

\[
\pi_m = N \int_{\theta_L}^{\bar{c}/\upsilon} \upsilon \theta f_\theta(\theta) d\theta + N \int_{\bar{c}/\upsilon}^{\bar{c}} \bar{c} f_\theta(\theta) d\theta = N \{ \int_{\theta_L}^{\bar{c}/\upsilon} \upsilon \theta f_\theta(\theta) d\theta + \bar{c} (1 - F_\theta(\bar{c}/\upsilon)) \}.
\]

4.4.2 Offering one licensing variant only

The previous section assumed that the software vendor offers its product under two different licensing variants. Such an approach to providing pack-
aged software can generally be applied when the software vendor sells a new product (or a next generation of one of its current products) to its existing customer base. In many situations, however, a software vendor must first compete with other software vendors to attract and lock in customers before it can exploit its monopoly position.

As an example, consider a two-period model in which a software vendor enters the market with a new product in the first period and sells an extension of this product in the second period. In the first period, the software vendor must charge a low price to attract customers who know they are going to be exploited in the second period, where they have switching costs as a result of their first-period purchase (Klemperer, 1995). Competition will force the software vendor to break-even or even loose money in the first period, which explains why many software vendors offer the basic version of their software products for free (Shapiro and Varian, 1999). In the second period, the software vendor sells an extension to its existing customer base. Now, the software vendor is locked into a specific type of software license as legal and technical complexities restrict it to only offer the licensing variant that is consistent with a customer’s first-period purchase. The software vendor’s optimal strategy is therefore to enter the market with the licensing variant that maximizes its profits in the second period, where it can exploit its monopoly position. The analysis performed in this section supports software vendors in making an explicit choice between offering fixed-fee and pay-per-use licensing.

**Problem specification**

When fixed-fee licensing is applied, it follows from (4.3) that every customer with level of use $\theta \geq \frac{p_f}{\upsilon}$ gets a positive surplus from purchasing the software. The profit maximizing problem of the software vendor under fixed-fee licensing can therefore be written as

$$\max_{p_f} \pi_f(p_f)$$

s.t. $p_f \leq \bar{c}$,  \hspace{1cm} (4.7)

where the function $\pi_f(p_f) = \int_{\bar{c}}^{\theta_H} p_f \phi(\theta) d\theta = p_f (1 - F_\theta(p_f/\upsilon))$ returns the software vendor’s average profit per customer for a given fixed-fee price $p_f$ (multiplying $\pi_f(p_f)$ by $N$ yields the software vendor’s total profit).

Similarly, when pay-per-use licensing is applied, it follows from (4.6) that every customer with level of use $\theta > \frac{c}{p_u}$ prefers to develop the required
4.4. Analysis for constant in-house development cost

software in-house as compared to obtaining it from the market. The profit maximizing problem of the software vendor under pay-per-use licensing can therefore be formulated as

\[
\max_{p_u} \pi_u(p_u) \quad \text{s.t.} \quad p_u \leq v,
\]

where the function \( \pi_u(p_u) = \int_{\bar{c}}^{p_u} p_u \theta f_\theta(\theta) d\theta = p_u \int_{\bar{c}}^{p_u} \theta f_\theta(\theta) d\theta \) returns the software vendor’s average profit per customer for a given pay-per-use price \( p_u \).

Let \( p_u^*(\bar{c}) \) and \( p_f^*(\bar{c}) \) be the optimal solutions to the problems (4.7) and (4.8), respectively. From the analysis in Section 4.4.1, we know that \( \pi_u(p_u^*(\bar{c})) > \pi_f(p_f^*(\bar{c})) \) for relatively large values of \( \bar{c} \), whereas \( \pi_f(p_f^*(\bar{c})) > \pi_u(p_u^*(\bar{c})) \) for relatively small values of \( \bar{c} \). To explain why, first consider the case when \( \bar{c} > \upsilon \theta H \), which implies that the participation constraint is binding for all customers. Because of this, the software vendor can charge each customer his maximum willingness-to-pay by applying pay-per-use licensing (if the software vendor were to apply fixed-fee licensing, it would only be able to sell to one type of customer at this price: all customers with a reservation price less than \( p_f \) would not obtain the software package, whereas all customers with a reservation price exceeding \( p_f \) would derive a positive surplus from purchasing the software). Next, consider the case when \( \bar{c} < \upsilon \theta L \), which implies that the incentive compatibility constraint is binding for all customers. Now, the software vendor can charge each customer his maximum willingness-to-pay by applying fixed-fee licensing (if the software vendor were to apply pay-per-use licensing, it would again only be able to sell to one type of customer at this price). Since pay-per-use licensing is optimal if the cost of in-house development exceeds the reservation price of the customer with the highest level of use, and fixed-fee licensing is optimal if this cost is less than the reservation price of the customer with the lowest level of use, we conclude that the functions \( \pi_f(p_f^*(\bar{c})) \) and \( \pi_u(p_u^*(\bar{c})) \) intersect at least once. The next two subsections show how to determine the value \( \bar{c}^* \in [\upsilon \theta L, \upsilon \theta H] \) of this intersect for the case when the levels of use in the customer population are uniformly and beta distributed, respectively.

**Uniform distribution**

In this section, the software vendor’s profits under fixed-fee and pay-per-use licensing are compared for the case when the levels of use in the customer
population are uniformly distributed. The density function of the uniform distribution is

$$f_\theta(\theta) = \begin{cases} 
\frac{1}{\theta_H - \theta_L} & \text{if } \theta_L \leq \theta \leq \theta_H \\
0 & \text{otherwise.}
\end{cases} \quad (4.9)$$

In order to determine \( p_f^* \), we consider two mutually exclusive cases. The optimal solution for the software vendor is the maximum of these two cases. First, we consider the case when \( p_f \leq \nu \theta_L \), which implies that each customer gets a positive surplus from purchasing the software. Substituting (4.9) into (4.7) and adding the constraint \( p_f \leq \nu \theta_L \) yields

$$\max_{p_f} \quad p_f$$
$$\text{s.t.} \quad p_f \leq \bar{c}$$
$$p_f \leq \nu \theta_L.$$ 

It is straightforward to see that the optimal solution is \( p_f^* = \bar{c} \) if \( \bar{c} < \nu \theta_L \) and \( p_f^* = \nu \theta_L \) if \( \bar{c} \geq \nu \theta_L \). In the second case, we assume that \( p_f \geq \nu \theta_L \). Now, depending on the fixed-fee price \( p_f \), there may be customers who do not get a positive surplus from purchasing the software. Substituting (4.9) into (4.7) and adding the constraint \( p_f \geq \nu \theta_L \) yields

$$\max_{p_f} \quad \frac{p_f \theta_H - \frac{1}{2}(p_f)^2}{\theta_H - \theta_L}$$
$$\text{s.t.} \quad p_f \leq \bar{c}$$
$$p_f \geq \nu \theta_L.$$ 

The problem has no solution if \( \bar{c} < \nu \theta_L \). The case when \( \bar{c} \geq \nu \theta_L \) is different. Now, the optimal solution depends on the ratio of \( \theta_L \) and \( \theta_H \). When \( \frac{1}{2} \theta_H \leq \theta_L \), \( p_f^* = \nu \theta_L \). On the other hand, \( \frac{1}{2} \theta_H > \theta_L \) implies that \( p_f^* = \bar{c} \) if \( \bar{c} < \frac{1}{2} \nu \theta_H \) and \( p_f^* = \frac{1}{2} \nu \theta_H \) if \( \bar{c} \geq \frac{1}{2} \nu \theta_H \). Taking the maximum of the two mutually exclusive cases yields the results summarized in Proposition 4.1.

**Proposition 4.1**: The software vendor’s optimal price under fixed-fee licensing is

$$p_f^*(\bar{c}) = \begin{cases} 
\bar{c} & \text{if } \bar{c} < \nu \theta_L, \\
\nu \theta_L & \text{if } \bar{c} \geq \nu \theta_L \text{ and } \frac{1}{2} \theta_H \leq \theta_L, \\
\bar{c} & \text{if } \nu \theta_L \leq \bar{c} < \frac{1}{2} \nu \theta_H \text{ and } \frac{1}{2} \theta_H > \theta_L, \\
\frac{1}{2} \nu \theta_H & \text{if } \bar{c} \geq \frac{1}{2} \nu \theta_H \text{ and } \frac{1}{2} \theta_H > \theta_L.
\end{cases}$$
The corresponding average profit per customer is equal to

\[
\pi_f(p^*_f(\bar{c})) = \begin{cases} 
\bar{c} & \text{if } \bar{c} < v \theta_L, \\
v \theta_L & \text{if } \bar{c} \geq v \theta_L \text{ and } \frac{1}{2} \theta_H \leq \theta_L, \\
\frac{\theta_H - \bar{c} / v}{\theta_H - \theta_L} \bar{c} & \text{if } v \theta_L \leq \bar{c} < \frac{1}{2} v \theta_H \text{ and } \frac{1}{2} \theta_H > \theta_L, \\
\frac{1}{4(\theta_H - \theta_L)} v & \text{if } \bar{c} \geq \frac{1}{2} v \theta_H \text{ and } \frac{1}{2} \theta_H > \theta_L.
\end{cases}
\]

The optimal value of \( p_u \) can be determined in a similar way. First, consider the case when \( p_u \leq \frac{\bar{c}}{\theta_H} \), which implies that all customers prefer to buy the software from the market as compared to developing it in-house. Substituting (4.9) into (4.8) and adding the constraint \( p_u \leq \frac{\bar{c}}{\theta_H} \) yields

\[
\max_{p_u} \frac{\theta_L + \theta_H}{2} p_u \\
\text{s.t.} \quad p_u \leq v \\
p_u \leq \frac{\bar{c}}{\theta_H}.
\]

The reader can verify easily that the optimal solution is \( p^*_u = \frac{\bar{c}}{\theta_H} \) if \( \bar{c} \leq v \theta_H \) and \( p^*_u = v \) if \( \bar{c} > v \theta_H \). Next, consider the case when \( p_u \geq \frac{\bar{c}}{\theta_H} \). Now, depending on the pay-per-use price \( p_u \), there may be customers who prefer to develop the required software in-house as compared to obtaining it from the software vendor. Substituting (4.9) into (4.8) and adding the constraint \( p_u \geq \frac{\bar{c}}{\theta_H} \) yields

\[
\max_{p_u} \frac{\bar{c}^2 / p_u - (\theta_L)^2 p_u}{2(\theta_H - \theta_L)} \\
\text{s.t.} \quad p_u \leq v \\
p_u \geq \frac{\bar{c}}{\theta_H}.
\]

If \( \bar{c} \leq v \theta_H \), the optimal solution is \( p^*_u = \frac{\bar{c}}{\theta_H} \). The problem has no solution when \( \bar{c} > v \theta_H \). Proposition 4.2 summarizes the results that are obtained by combining the two mutually exclusive cases.

**Proposition 4.2:** The software vendor’s optimal price under pay-per-use licensing is

\[
p_u^*(\bar{c}) = \begin{cases} 
\frac{\bar{c}}{\theta_H} & \text{if } \bar{c} \leq v \theta_H, \\
v & \text{if } \bar{c} > v \theta_H.
\end{cases}
\]

The corresponding average profit per customer is equal to

\[
\pi_u(p^*_u(\bar{c})) = \begin{cases} 
\frac{\theta_H + \theta_L}{2 \theta_H} \bar{c} & \text{if } \bar{c} \leq v \theta_H, \\
\frac{1}{2}(\theta_H + \theta_L) v & \text{if } \bar{c} > v \theta_H.
\end{cases}
\]
Chapter 4. Pay-per-use versus fixed-fee licensing

When the software vendor’s profits under fixed-fee and pay-per-use licensing are compared, it follows that pay-per-use licensing is more profitable than fixed-fee licensing if in-house development is relatively expensive for customers, whereas fixed-fee licensing is optimal if the cost of in-house development drops below a certain threshold value. Propositions 4.3 and 4.4 can be used to determine this threshold value analytically for the case when \( \frac{1}{2} \theta_H \leq \theta_L \) and for the case when \( \frac{1}{2} \theta_H > \theta_L \), respectively. For proofs of these propositions, the reader is referred to Appendix A.

**Case 1:** \( \frac{1}{2} \theta_H \leq \theta_L \)

**Proposition 4.3:** The software vendor finds pay-per-use licensing more profitable than fixed-fee licensing if \( \bar{c} \geq \frac{2 \theta_L}{(\theta_L/\theta_H) + 1} \), whereas fixed-fee licensing is optimal if \( \bar{c} \leq \frac{2 \theta_L}{(\theta_L/\theta_H) + 1} \).

**Case 2:** \( \frac{1}{2} \theta_H > \theta_L \)

**Proposition 4.4:** The software vendor finds pay-per-use licensing more profitable than fixed-fee licensing if \( \bar{c} \geq \frac{(\theta_H)^3}{2((\theta_H)^2-(\theta_L)^2)} \), whereas fixed-fee licensing is optimal if \( \bar{c} \leq \frac{(\theta_H)^3}{2((\theta_H)^2-(\theta_L)^2)} \).

**Beta distribution**

This section generalizes the results from the previous section by considering the case when the levels of use in the customer population are beta distributed. The beta distribution is a continuous distribution defined on an interval with a minimum and maximum value and is parameterized by two parameters, denoted by \( \alpha_\theta \) and \( \beta_\theta \). Depending on the value of these parameters, the beta density function can take on different shapes, including the U-shape, the triangle shape, and the bell shape. For \( \alpha_\theta = \beta_\theta = 1 \), the beta distribution reduces to the uniform distribution.

The density function of the beta distribution is

\[
f_\theta(\theta; \alpha_\theta, \beta_\theta) = \begin{cases} 
C(\frac{\theta - \theta_L}{\theta_H - \theta_L})^{\alpha_\theta - 1}(1 - \frac{\theta - \theta_L}{\theta_H - \theta_L})^{\beta_\theta - 1} & \text{if } \theta_L \leq \theta \leq \theta_H \\
0 & \text{otherwise} 
\end{cases}
\]

where \( C = \frac{1}{\theta_H - \theta_L} \Gamma(\alpha_\theta + \beta_\theta) \Gamma(\alpha_\theta) \Gamma(\beta_\theta) \). Substituting (4.10) into (4.7) to obtain the soft-
ware vendor’s profit maximizing problem under fixed-fee licensing yields

\[
\begin{align*}
\max_{p_f} & \quad p_f (1 - F_{\theta}(\frac{p_f}{v} ; \alpha_{\theta}, \beta_{\theta})) \\
\text{s.t.} & \quad p_f \leq \bar{c},
\end{align*}
\]

(4.11)

where \( F_{\theta}(\theta; \alpha_{\theta}, \beta_{\theta}) \) denotes the cumulative distribution function of the beta distribution. Similarly, by substituting (4.10) into (4.8), it follows (after rewriting) that the profit maximizing problem of the software vendor under pay-per-use licensing can be written as

\[
\begin{align*}
\max_{p_u} & \quad p_u \{ \theta_L F_{\theta}(\frac{p_u}{\theta_{\theta}} ; \alpha_{\theta}, \beta_{\theta}) + \\
& \quad (\theta_H - \theta_L) \frac{\Gamma(\alpha_{\theta} + 1) \Gamma(\alpha_{\theta} + \beta_{\theta})}{\Gamma(\alpha_{\theta} + \beta_{\theta} + 1) \Gamma(\alpha_{\theta})} F_{\theta}(\frac{p_u}{\theta_{\theta}} ; \alpha_{\theta} + 1 ; \beta_{\theta}) \} \\
\text{s.t.} & \quad p_u \leq v.
\end{align*}
\]

(4.12)

In the previous section, we have shown that when levels of use are uniformly distributed across customers, the software vendor prefers pay-per-use licensing over fixed-fee licensing for relatively large values of \( \bar{c} \), whereas fixed-fee licensing is optimal if \( \bar{c} \) drops below a certain threshold value \( \bar{c}^* \in [v \theta_L, v \theta_H] \). The same is true for the more general case when levels of use are beta distributed across customers. Although the properties of the beta distribution–\( F_{\theta}(\theta; \alpha_{\theta}, \beta_{\theta}) \) does not exist in closed form, in general (Law and Kelton, 2000)–restrict us from determining the value \( \bar{c}^* \) for which the software vendor is indifferent between offering fixed-fee and pay-per-use licensing analytically, we can approximate it numerically (within an error margin of \( \delta \)) by applying the procedure outlined in Figure 4.1. At each iteration, the algorithm requires us to determine the function values \( \pi_f(p_f(m)) \) and \( \pi_u(p_u(m)) \) at the midpoint \( m = \frac{a + b}{2} \) of the interval \([a, b]\). If \( \pi_f(p_f(m)) \geq \pi_u(p_u(m)) \), \( \bar{c}^* \) cannot be contained in the subinterval \([a, m]\), so this portion of the search interval is discarded. Similarly, \( \pi_f(p_f(m)) \leq \pi_u(p_u(m)) \) implies that the subinterval \((m, b]\) can be discarded since it cannot contain \( \bar{c}^* \). The process of narrowing the search interval is continued until \( \bar{c}^* \) has been isolated as accurately as required (i.e. within an error margin of \( \delta \) from its true value).

Table 4.3 summarizes the results of a numerical study that has been performed with a program written in Matlab for the parameter values \( \theta_L = 10, \theta_H = 100, v = 1000 \), and \( \delta = 0.01 \). A graphical representation of the density functions corresponding to the four different combinations of the shape parameters \( \alpha_{\theta} \) and \( \beta_{\theta} \) is given in Figure 4.2. To solve the problems (4.11) and (4.12), we relied on Matlab’s implementation of the bound-constrained optimization algorithm, which is based on golden section search and parabolic interpolation (Forsythe et al., 1977; Nocedal and Wright, 1999).
Chapter 4. Pay-per-use versus fixed-fee licensing

Figure 4.1: A procedure for determining the value $c^*$ for which the software vendor is indifferent between offering fixed-fee and pay-per-use licensing.

Step 1: (Initialization) Define an initial interval $[q, v]$ such that $q = \frac{L}{v-q}$ and $v = q + \frac{L}{q-v}$.

Step 2: Compute the midpoint $m = \frac{a + b}{2}$.

Step 3: Determine the values of $\pi_f(p_f(m))$ and $\pi_u(p_u(m))$ by solving the software vendor's profit-maximizing problems under fixed-fee and pay-per-use licensing, respectively, for the case when $c = m$.

Step 4: Narrow the search interval by applying one of the following rules:

(a) If $\pi_f(p_f(m)) \geq \pi_u(p_u(m))$, update the left endpoint of the interval $[q, v]$ according to $a := \frac{a + b}{2}$.

(b) If $\pi_f(p_f(m)) \leq \pi_u(p_u(m))$, update the right endpoint of the interval $[q, v]$ according to $b := \frac{a + b}{2}$.

Step 5: Assign the value $\frac{a + b}{2}$ to the threshold value $c^*$.
4.5 Analysis for variable in-house development cost

In the previous section, it was assumed that in-house development is equally expensive for each customer. In reality, however, this cost will differ significantly, depending on the quality of a customer’s in-house solution. In this section, we therefore relax the assumption of a constant in-house development cost by letting it vary among customers. Let \( f_c(c) \) and \( F_c(c) \) denote the density function and the cumulative distribution function of the distribution of in-house development costs across customers, respectively. It is assumed that the distribution of the cost of in-house development is independent of the distribution of the level of use, which implies that the density function of their joint distribution factors into the product of the density functions of the two marginal distributions: \( f(c, \theta) = f_c(c) f_{\theta}(\theta) \).

When fixed-fee licensing is applied, it follows from Equations (4.3) and (4.4) that every customer with (i) level of use \( \theta \geq \frac{p_f}{v} \) and (ii) in-house development cost \( c \geq p_f \) obtains the software package from the market. The profit maximizing problem of the software vendor under fixed-fee licensing can therefore be written as

\[
\max_{p_f} \quad p_f [1 - F_{\theta}(\frac{p_f}{v})][1 - F_c(p_f)].
\tag{4.13}
\]

Similarly, when pay-per-use licensing is applied, it follows from Equation (4.6) that every customer with in-house development cost \( c \geq p_u \theta \) prefers to obtain a copy from the software vendor as compared to developing the required software in-house. The profit maximizing problem of the software vendor under pay-per-use licensing can therefore be formulated as

\[
\max_{p_u} \quad p_u \int_{\theta_L}^{\theta_H} \theta f_{\theta}(\theta) [1 - F_c(p_u \theta)] d\theta \quad \text{s.t.} \quad p_u \leq v.
\tag{4.14}
\]
Chapter 4. Pay-per-use versus fixed-fee licensing

Figure 4.2: Density function of the beta distribution for four different combinations of the shape parameters $\alpha_\theta$ and $\beta_\theta$.

The first step in solving problems (4.13) and (4.14) is to specify how levels of use and in-house development costs are distributed across customers. In many real-life situations, we expect to find that the distribution of the cost of in-house development is skewed to the right (i.e. most of the distribution’s mass is located at the left of its mean) as most customers will settle for a low-cost solution, while only a few require a high-quality implementation in terms of performance, scalability, etc. The gamma distribution, a continuous distribution defined on the interval $[0, \infty)$ and parameterized through its shape parameter $\alpha_c$ and its scale parameter $\beta_c$, provides a flexible set of density functions that conform to such a shape. In the analysis below, it is therefore assumed that in-house development costs are gamma distributed across customers. The levels of use in the customer population are assumed to be beta distributed, just as in the previous section.

Let $\mu_c = \frac{1}{N} \sum_{i=1}^{N} c_i$ and $\sigma_c = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (c_i - \mu_c)^2}$ be the mean and standard deviation of the distribution of the cost of in-house development, respectively: the parameter $\mu_c$ reflects its average value in the customer popu-
4.5. Analysis for variable in-house development cost

lation, whereas $\sigma_c$ is a measure of how dispersed the in-house development costs of individual customers are. To fit the gamma distribution to these market characteristics, we must express the parameters $\alpha_c$ and $\beta_c$ in terms of the population mean and standard deviation. This yields:

\[
\alpha_c = \frac{\mu_c^2}{\sigma_c^2}, \quad (4.15)
\]

\[
\beta_c = \frac{\sigma_c^2}{\mu_c}. \quad (4.16)
\]

Based on Equations (4.15) and (4.16), the profitability of fixed-fee and pay-per-use licensing can be compared for any combination of $\mu_c$ and $\sigma_c$. Figures 4.3, 4.4, 4.5, and 4.6 summarize the results of a numerical study that has been performed in Matlab (again for the parameter values $v = 1000$, $\theta_L = 10$, and $\theta_H = 100$). The figures show that the relative attractiveness of fixed-fee and pay-per-use licensing depends on the value of the population mean and standard deviation: fixed-fee licensing is optimal for all combinations of $\mu_c$ and $\sigma_c$ in the black region of the figure, whereas pay-per-use licensing is optimal for all combinations of $\mu_c$ and $\sigma_c$ in the grey region. By distinguishing between “small”, “medium”, and “large” values of $\mu_c$, we can draw the following conclusions:

a) For small values of $\mu_c$, there exists a threshold value $\bar{\sigma}_c$ such that (i) fixed-fee licensing is optimal if $\sigma_c \leq \bar{\sigma}_c$ and (ii) pay-per-use licensing is optimal if $\sigma_c \geq \bar{\sigma}_c$.

b) For medium values of $\mu_c$, there exists an interval $[\sigma_c^L, \sigma_c^H]$ such that (i) fixed-fee licensing is optimal if $\sigma_c \in [\sigma_c^L, \sigma_c^H]$ and (ii) pay-per-use licensing is optimal if $\sigma_c \notin [\sigma_c^L, \sigma_c^H]$.

c) For large values of $\mu_c$, pay-per-use licensing is more attractive than fixed-fee licensing, independent of the value of $\sigma_c$.

For a constant in-house development cost, it holds that pay-per-use licensing is more profitable than fixed-fee licensing for relatively large values of $\mu_c = \bar{\mu}_c$, whereas fixed-fee licensing is optimal if $\mu_c$ drops below a certain threshold value. The analysis performed in this section shows that when the assumption of a constant in-house development cost is relaxed, pay-per-use licensing is still optimal for relatively large values of $\mu_c$ (say $\forall \mu_c \geq \bar{\mu}_c$). Depending on the value of the standard deviation $\sigma_c$, it may happen, however, that fixed-fee licensing is no longer the preferred licensing strategy for small and medium values of $\mu_c$. 
Figure 4.3: Fixed-fee versus pay-per-use licensing for the case when the levels of use in the customer population are beta(1,1) distributed.

Figure 4.4: Fixed-fee versus pay-per-use licensing for the case when the levels of use in the customer population are beta(2,2) distributed.
4.5. Analysis for variable in-house development cost

Figure 4.5: Fixed-fee versus pay-per-use licensing for the case when the levels of use in the customer population are beta(3,6) distributed.

Figure 4.6: Fixed-fee versus pay-per-use licensing for the case when the levels of use in the customer population are beta(4,2) distributed.
4.6 Conclusion

This chapter compares the profitability of pay-per-use and fixed-fee licensing for a monopoly software vendor that is selling packaged software to customers who are homogeneous in marginal value of software use but heterogeneous in level of use. Next to obtaining a copy from the software vendor, customers can develop the required software in-house. The situation when in-house development is not accounted for is equivalent to the case when the cost of in-house development is infinitely large for all customers. Our results show that the software vendor then prefers pay-per-use licensing over fixed-fee licensing. The same results show, however, that fixed-fee licensing is optimal if the cost of in-house development drops below a certain threshold value. When the assumption of a constant in-house development cost is relaxed by letting it vary among customers, it still holds that pay-per-use licensing is optimal if its average is relatively large. For low and medium values of the average cost of in-house development, however, fixed-fee licensing may no longer be optimal as the relative attractiveness of the two licensing strategies now depends on how dispersed the in-house development costs of individual customers are.

Future research can extend our model in various ways. First, it has been assumed that the software vendor charges a constant unit price when pay-per-use licensing is applied. In reality, however, unit prices often depend on a customer’s level of use, allowing the software vendor to price discriminate between different types of customer. It may therefore be interesting to explore how nonlinear pricing affects the relative attractiveness of fixed-fee and pay-per-use licensing. Second, we have assumed that customers are homogeneous in marginal value of software use. Future research can generalize our results by letting this parameter vary among customers. Finally, it has been assumed that the distribution of the cost of in-house development is independent of the distribution of the level of use. A natural way to extend our model is therefore to relax the assumption of a zero covariance.