CHAPTER 3

Cross-shareholding in the Japanese banking sector*

3.1 Introduction

There is ample evidence that nowadays firms often acquire shares in their rivals, and mostly these shareholdings do not give control power. For example, Hansen and Lott (1996, Table 1) give evidence for substantial cross-ownership relations in the American computer and automobile industries for 1994-1995, and state that “slightly over 77 percent of Intel and 71 percent of Compaq are owned by institutions that have holdings in at least one of the other five computer industry companies listed [Apple, Compaq, IBM, Intel, Microsoft, Motorola]. Fully 56 percent of Chrysler is held by institutions that simultaneously hold shares in Ford and/or General Motors” (p. 49). In 2002, the leader of the wireless communications businesses in Korea – SK Telekom – acquired 11.3% of Korea Telecom, the leader in the wireline communications business, which in its turn already owned 9.3% of equity of the first company (see Choi et al., 2003, p.498). Firms’ acquisitions of stocks largely cross the national borders as well. For instance, in 2001, General Motors increased its equity holding in Suzuki Motor from 10.0% to 20.0%, and acquired also a 21.1% stake in Fuji Heavy Industries.1 Since shareholding interlocks of firms is a widespread phenomenon,2 it is essential to analyze the implication of the presence

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1 Section 3.2 is partly based on a paper published in the Journal of Economic Studies, vol. 36, no. 3, pp. 296-306, 2009a, while the rest of this chapter is based on joint work with Stanislav Stakhovych.
2 See Gilo (2000) for more cases of equity acquisitions in various industries.
of ownership links on the behavior of firms.

Cross-shareholding is, in particular, an important characteristic of Japanese, German and Swedish business groups (see e.g., Kester, 1992). However, due to antitrust concerns most cross ownership is *silent* (or partial) by its nature. Financial interests are silent when firms do not control the policies (e.g., outputs, prices) of their competitors.³ That is, firms take the choices of these competitors as given, although in the presence of cross ownership decisions of one firm affect also the profits of its rivals. It has been shown that partial cross ownership (PCO) of firms, when compared to the case without PCO, leads to higher prices,⁴ lower industry outputs, and thus lower welfare (see e.g., Reynolds and Snapp, 1986; Flath, 1992a; Reitman, 1994; Dietzenbacher et al., 2000). Nonetheless, Farrell and Shapiro (1990) show that welfare may still rise even if prices increase, which occurs when a small firm acquires shares in a rival in which it previously had no financial interest.

Given the fact that passive investments in rivals were largely neglected by antitrust agencies (see e.g., Gilo, 2000), much attention in the literature was given to the study on explicit links between PCO and tacit collusion. Reitman (1994) shows that for any number of firms an individually rational PCO equilibrium exists if the market is more rivalrous than Cournot oligopoly and is close to price competition. Malueg (1992) concludes that passive investments have an ambiguous effect on the likelihood of collusion. In a repeated Cournot game, he shows that the effect of an increase in cross ownership on tacit collusion depends critically on the form of the market demand. However, Gilo et al. (2006) find that in a Bertrand supergame an increase in PCO never hinders tacit collusion and surely facilitates it under certain conditions. They show that an increase of firm \( r \)'s stake in firm \( s \) strictly facilitates collusion if (i) firm \( s \) is not an industry maverick (a firm with the strongest incentive to deviate from a collusive agreement), and (ii) each industry maverick has a direct and/or an indirect stake in firm \( r \) (firm \( i \) has an *indirect* stake in firm \( r \) if it has a share in a firm that has a stake in firm \( r \), or has a stake in a firm that has a stake in a firm that holds a stake in firm \( r \), and so on).⁵

The results of empirical research on the effect of PCO on market structure mostly support the collusion hypothesis, which states that a complex web of PCO is an

³ The term “silent financial interests” was introduced by Bresnahan and Salop (1986). Equivalently, such equity interests in the literature are also termed passive investments, partial ownership arrangements, and partial cross ownership links. We will also use all these terms interchangeably throughout this chapter.

⁴ Interestingly, Weinstein and Yafeh (1995) find that keiretsu firms had price-cost margins lower by as much as 2.5 percentage points than those of non-keiretsu firms.

⁵ An extension of Gilo et al. (2006) to the case where firms have asymmetric costs will be presented in Chapter 4.
important factor for the existence of collusive prices. The focus of such studies are specific industries, such as the US mobile telephone industry (Parker and Röller, 1997), the Dutch financial sector (Dietzenbacher et al., 2000), and the Norwegian-Swedish electricity market (Amundsen and Bergman, 2002). Alley (1997) finds that tacit collusion does occur in both the Japanese and the US domestic automobile industries, but its degree is lower in Japan.

In this chapter we take into full account both direct and indirect interests of firms in each other due to PCO, which is ignored, to the best of our knowledge, in all empirical estimations of the level of tacit collusion. As mentioned above, for example, if firm $i$ owns a share in firm $k$ that has a share in $j$ then firm $i$ is said to have an indirect share in firm $j$ (via firm $k$). In general, the number of intermediate firms in the indirect links can be infinity when there are cycles present in the ownership paths (for instance, when firm $i$ holds shares in firm $j$ and, vice versa, $j$ has a stake in $i$). PCO is incorporated in the analysis of Alley (1997), but he considers only direct shareholdings. It has been shown that indirect interests might be significant in size, thus should not be neglected in the analysis of industries (economies) with the presence of PCO (see e.g., Flath, 1992b; Dietzenbacher and Temurshoev, 2008).

We first discuss different profit formulations of firms with cross-shareholdings that have been used in the literature, where the differences are due to the distinct ways of considering direct and/or indirect PCO links. Then using the conjectural variation model we find that (unlike in the case without PCO) the link between firms’ price-cost margins and the degree of collusion is nonlinear in the presence of PCO. Hence, if shareholding links among firms are present, ignoring PCO would most likely give biased parameters’ estimates due to model misspecification. It is shown that given market shares, number of firms, price elasticity of demand, and collusion degree, firms with shareholdings exert strictly higher market power than those without PCO, provided that the market conduct is consistent with Cournot or a more collusive environment. This is because shareholding interlocks among firms cause commonality of interests of firms, implying greater monopoly power for firms with PCO holdings.

The model is applied to the Japanese banking sector for the fiscal year 2003. The results of our estimations show that Japanese banks are competing in a modest collusive environment. However, disregarding banks’ PCO gives biased result.

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6 Dietzenbacher et al. (2000) fully consider PCO links in a Cournot and a Bertrand setting, and find that such links reduce Dutch banks’ price-cost margins, hence reduce competition. We, however, focus directly on the indicator of market performance that ranges from perfect competition to monopoly (perfect cartel).
indicating a Cournot oligopoly. It is further shown that banks with passive investments in rivals exert a strictly larger market power than those without any PCO, which confirms the hypothesis that acquiring shares in rivals is one of the crucial means for a firm to enhance its market power. In particular, city banks with many shareholdings are found to exercise a much higher market power than regional banks with none or few stockholdings.

The model presented here belongs to the conjectural variations (CV) literature. CV models are often used in empirical research in order to infer the degree of market power from real data (see e.g., Brander and Zhang, 1990; Haskel and Martin, 1994; Richards et al., 2001; Fischer and Kamerschen, 2003; Brissimis et al., 2008). It is well known that these models are subject to some criticism from a theoretical point of view because they describe the dynamics of firms’ interaction using a static setting (see e.g., Tirole, 1988, pp. 244-45). However, Cabral (1995) shows that CV models can be interpreted as a reduced form of the equilibrium in a quantity-setting supergame with linear demand and marginal cost functions, justifying their use in estimating the competition level among oligopolists. In the same fashion, for his infinite horizon adjustment cost model, Dockner (1992) shows that any steady state closed-loop (subgame-perfect) equilibrium coincides with the CV equilibrium. In addition, Pfaffermayr (1999) proves that CV models represent the joint profit maximizing reduced form of a price-setting supergame with product differentiation, which “...provides a comprehensive theoretical foundation of the widely criticized static CV models” (p. 323).

The rest of this chapter is organized as follows. Section 3.2 discusses different profit specifications of firms in the presence of PCO used in the literature. Section 3.3 describes the CV model with cross-shareholdings and examines the effect of PCO linkages on firms’ market power. Section 3.4 focuses on the empirical estimation of the degree of tacit collusion in the Japanese commercial banking sector for 2003, and diagnoses market power of the banks. Section 3.5 concludes. All proofs are relegated to the Appendix at the end of the chapter.

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7 Some authors therefore believe that CV parameters have nothing to do with real conjectures or expectations of firms. To avoid this confusion Krouse (1998, p. 688), for example, refers to them as “equilibrium solution parameters”. 
3.2 Profits of horizontally interrelated firms

In this section we briefly present profit formulations of firms in the presence of partial cross ownership (PCO) that have been used in the literature. The differences in these profit specifications are the result of the different ways of taking account of a complex web of interfirm ownership links. Consider an industry with $n$ firms that are interdependent through PCO ties. Reynolds and Snapp (1982) was one of the first studies that brought attention to the analysis of firms’ PCO holdings and formulated the profit of firm $i$ as follows

$$
\pi_i = z_i + \sum_{k \neq i} w_{ik} z_k,
$$

where $\pi_i$ and $z_i$ denote, respectively, the profits and the operating earnings of firm $i$, and $w_{ik}$ $(i, k = 1, \ldots, n)$ represents the share in firm $k$ that is held by firm $i$.\(^8\) That is, equation (3.1) states that firm $i$’s profits consists of its own operating earnings (profits from ordinary production) plus its direct shareholdings in operating earnings of all other firms. This formulation is also used in Bresnahan and Salop (1986), who study a competitive joint venture, in which parent firms own non-controlling ownership rights.

Reynolds and Snapp (1986) consider the case of joint ventures, whose profits are divided according to each partner’s share of equity, and they define profits of firm $i$ as\(^9\)

$$
\pi_i = \left(1 - \sum_{k \neq i} w_{ki}\right) z_i + \sum_{k \neq i} w_{ik} z_k,
$$

which differs from (3.1) in that firm $i$ also considers competitors’ financial interests in its operating earnings. This specification of the firms’ objective was used in Alley (1997) in analyzing the effect of non-controlling (partial) shareholdings on the degree of competition in the US and Japanese automobile industries.

The above specifications totally disregard indirect financial interests, when, for example, firm $i$ has an indirect stake in firm $j$ via intermediate firms. In many

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\(^8\) First and second subscripts in $w_{ik}$ denote, respectively, the owner and the owned firm. Throughout this chapter it is assumed that a firm cannot own equity interest in itself, i.e., $w_{ii} = 0$ for all $i$. However, one can also allow for $w_{ii} > 0$, which would reflect, for example, the share repurchases by firms due to the tax advantage of capital gains. Note that while in Chapter 2 the cross-shareholding matrix was denoted by the matrix $S$, in this chapter its transpose is denoted by $W$.

\(^9\) For other profit specifications depending on the kind of behavior imputed on the joint ventures see e.g., Bresnahan and Salop (1986) and Martin (2002, Chapter 12.10).
cases indirect shareholdings are significant in size and thus call for a proper consideration. Hence, equations (3.1) and (3.2) are not adequate when an industry is characterized by extensive shareholding interlocks. These shareholding links are fully taken into account in Flath (1991), who defines firm $i$’s profit as the sum of its operating earnings and the revenue from shareholding in rivals’ profits:

$$\pi_i = z_i + \sum_{k \neq i} w_{ik} \pi_k.$$  \hspace{1cm} (3.3)

Equivalently, in matrix form, (3.3) can be rewritten as $\pi = z + W\pi$, where $W$ is the $n$-square PCO matrix with its typical element $w_{ij}$, and $\pi$ and $z$ are, respectively, the column vectors of profits and operating earnings. Solving the last equation with respect to profits gives

$$\pi = (I - W)^{-1}z,$$  \hspace{1cm} (3.4)

where $I$ is the $n$-square identity matrix.

Assuming that each firm has external shareholders (i.e., private owners and firms outside the industry) implies that the column sum of the matrix $W$ is smaller than one, which guarantees non-singularity of the matrix $(I - W)$ (see e.g., Solow, 1952).\footnote{Although, the existence of external shareholders perfectly corresponds with the real life observations, it is - mathematically speaking - not necessary that all column sums of $W$ are smaller than one. For the existence of $(I - W)^{-1}$ it suffices that no column sum of $W$ is larger than one and, at least, one column sum is strictly less than one, provided that $W$ is an indecomposable matrix. (A square matrix $A$ is called\textit{decomposable} if there exists a permutation matrix $P$ such that $P^{-1}AP = \begin{pmatrix} A_{11} & A_{12} \\ O & A_{22} \end{pmatrix}$, where $A_{11}$ and $A_{22}$ are square submatrices, and $O$ is a null matrix of appropriate dimension. If this is impossible, $A$ is called\textit{indecomposable}.) Hence, (if $W$ is an indecomposable matrix) for all but one firm it may even be the case that no external shareholders exist.} Define $L \equiv (I - W)^{-1}$ that, similar to the Leontief inverse in input-output economics, can be written as the matrix power series expansion $L = I + W + W^2 + \ldots$ (see e.g., Miller and Blair, 2009). The last expression together with (3.4) allow us to separate direct and indirect effects of PCO. Namely, profits of firm $i$ consist of three components (Dietzenbacher et al., 2000, p. 1226). First, its own operating earnings reflected by the $i$-th element of the vector $z$. Second, firm $i$’s direct shareholdings in rivals, reflected by the $i$-th element of the vector $Wz$. Finally, the third term gives the indirect equity returns of firm $i$ in other firms and is equal to the $i$-th element of the vector $(W^2 + W^3 + \ldots)z$. So even if $w_{ij} = 0$, the entry $(i, j)$ of the matrix $W^3$ is positive if firm $i$ partially owns firm $k$ that has a share in firm $l$ that in its turn holds a stake in firm $j$.

The profit specification in (3.4) is widely used in the literature (see e.g., Flath,
These profits “overestimate” industry-wide operating earnings. To see this, let \( \mathbf{i} \) be the summation vector of ones. Then \( \mathbf{i}' \mathbf{\pi} = \mathbf{i}'(\mathbf{I} - \mathbf{W})^{-1} \mathbf{z} = \mathbf{i}'(\mathbf{I} + \mathbf{W} + \mathbf{W}^2 + \ldots) \mathbf{z} > \mathbf{i}' \mathbf{z} \) in the presence of PCO. However, this “overestimation” does not cause any problem since these profits indicate the value of the firms, and should increase when firms become interlinked. Say, in a two firms setting, PCO creates a multiplier effect in the sense that firm A gets a share in firm B’s profit, which includes firm B’s share in firm A’s profit, which includes firm A’s share in firm B’s profit, and so on. However, what should concern us is whether there is a problem of overestimation of profits accruing to “real” (i.e., external) shareholders. The last is equal to \( \mathbf{i}'(\mathbf{I} - \mathbf{W}) \mathbf{\pi} = \mathbf{i}'(\mathbf{I} - \mathbf{W})(\mathbf{I} - \mathbf{W})^{-1} \mathbf{z} = \mathbf{i}' \mathbf{z} \), hence although the aggregate profits “…overstate the firms’ cash flows … the aggregate payoffs of ‘real’ equityholders are not overstated and do sum up to [industry operating earnings]” (Gilo et al., 2006, p. 86). This approach is very similar to the input-output technique, where multiplication of, say, the direct employment coefficients vector by the Leontief inverse gives total (direct and indirect) labor requirements per unit of final demand (see e.g., Miller and Blair, 2009). Here, similarly, multiplication of external shareholders’ direct shares in firms, \( \mathbf{i}'(\mathbf{I} - \mathbf{W}) \), by the “Leontief inverse” of the form \( (\mathbf{I} - \mathbf{W})^{-1} \) results in the total (direct and indirect) equity interests of owners in firms per unit of operating earnings, or, equivalently, in Gilo et al. (2006) terminology, in the total effective stake of the “real” equityholders in firms’ profits.

The issue of profits overestimation in Flath’s approach is considered in Merlone (2007). In terms of our notations, his proposed new formulation of net profits is \( \mathbf{\pi}^{net} = (\mathbf{I} - \mathbf{i}' \mathbf{W})(\mathbf{I} - \mathbf{W})^{-1} \mathbf{z} \), where \( \mathbf{i}' \mathbf{W} \) is the diagonal matrix with the column sums of \( \mathbf{W} \) on its main diagonal and zero elsewhere. The last, unlike the profits in (3.4), sum up to the overall operating earnings, i.e., \( \mathbf{i}' \mathbf{\pi}^{net} = \mathbf{i}' \mathbf{z} \) since \( \mathbf{i}'(\mathbf{I} - \mathbf{i}' \mathbf{W}) = \mathbf{i}'(\mathbf{I} - \mathbf{W}) \). However, as we just showed above, \( \mathbf{\pi}^{net} \) is nothing else than the profits accruing to “real” equityholders of firms.\(^{11}\)

A few studies focused only on the real cash flows due to firms’ PCO links, hence effectively neglected the notion of a firm value considered in (3.4). Futatsugi (1978, 1986, 1987) writes firm \( i \)'s profits as

\(^{11}\) We should note that Merlone’s (2007) view that his profit specification results in different cartelizing effects of shareholding interlocks than those based on equation (3.4) is entirely wrong. In fact, the Lerner indices for homogeneous and product-differentiated oligopolies proposed by Merlone (2007) are nothing else than the corresponding indicators in Merlone (2001). This is because Merlone’s profit specification is a netted version of firms’ objective in (3.4). Thus both profit formulations have exactly identical optimality conditions (from which Lerner indices are derived), since in the maximization process the structure of PCO is taken as given.
\[
\pi_i = z_i + \sum_{k \neq i} w_{ik} r_k \pi_k,
\]

where \(r_k \in (0, 1)\) is the payout ratio (dividend propensity) of firm \(k\). Note that if \(r_k = 1\) for all \(k\), then (3.5) boils down to (3.3). Hence, unlike (3.3), the last equation considers only dividend returns of firms due to PCO. Its netted version, where dividend outflows due to PCO are also taken into account, is given in Temurshoev (2009a) as follows

\[
\pi^\text{net}_i = (1 - r_i) \left( z_i + \sum_{k \neq i} w_{ik} r_k \frac{\pi^\text{net}_k}{1 - r_k} \right),
\]

where \(\pi^\text{net}_i\) denotes firm \(i\)'s profits after dividend payments, hence \(\pi^\text{net}_i / (1 - r_i) = \pi_i\) is the gross profit including dividend payments.\(^{12}\) Equations (3.5) and (3.6) in matrix form can be rewritten, respectively, as \(\pi = (I - W\hat{r})^{-1} z\) and \(\pi^\text{net} = (I - \hat{r})(I - W\hat{r})^{-1} z\), where \(\hat{r}\) is the diagonal matrix with payout ratios on its main diagonal and zero otherwise. Since in the analysis \(\hat{r}\) and \(W\) are given, the first-order conditions for profit maximization are exactly the same for (3.5) and (3.6).

However, equations (3.5) and (3.6) are not suitable for the economic analysis of cross-shareholdings. The main focus in economic analysis is the value of the firm, and not its total cash flows due to PCO. For instance, if no firm announces dividend payments (i.e., \(r_i = 0\) for all \(i\)), then both (3.5) and (3.6) reduce to \(\pi_i = \pi^\text{net}_i = z_i\). Although from a pure accounting view this is the correct amount of (current) earnings, it is a wrong representation of the PCO presence as far as economic analysis is concerned. This is because – in that case – (3.5) and (3.6) do not reflect the PCO links which give firms shares in the profits of rival firms (which in this case are held as retained earnings). Essentially, an investor’s income from equity consists of dividends and retained earnings. The difference between the two is only the timing at which they are received: dividends are received whenever the firm distributes them, whereas retained earnings are realized either when the equityholder sells his shares or when the firm is liquidated. Equations (3.5) and (3.6) represent a one period model, where there should not be any difference between equity sales and firm liquidation, because the firm is effectively liquidated at the end of the period (after its profits are realized), and its profits are fully distributed. Therefore, divi-

\(^{12}\)To see this, let \(r_i = d_i / \pi_i\), where \(d_i\) denotes the dividend obligations of firm \(i\). By definition \(\pi^\text{net}_i = \pi_i - d_i\), which implies \(\pi^\text{net}_i / (1 - r_i) = \pi_i\).
dends do not matter in a static one period model.\textsuperscript{13} Hence, the only correct profit specification for economic analysis of PCO is Flath’s formulation given in (3.3) or (3.4).

### 3.3 Theoretical framework

In order to diagnose market power of firms and analyze market performance in the presence of cross ownership links, we modify the well-known conjectural variation model of Clarke and Davies (1982) by taking into account both direct and indirect PCO linkages among firms. Assume there are \(n\) firms in an industry that are interdependent through PCO ties. The profit of firm \(i\) consists of its operating earnings plus the revenue from shareholding in other firms and is given in equation (3.3) in the previous section.

Consider a homogeneous product industry. Firm \(i\)’s total cost \(c_i(x_i)\) is a function of its own output level \(x_i\). Further, the inverse demand function is \(p(X)\), where \(X = \sum_{i=1}^{n} x_i\). Let \(l_{ij}\) be the generic element of the matrix \(L = (I - W)^{-1}\). Since the operating earnings of firm \(i\) is \(z_i = p(X)x_i - c_i(x_i)\), using (3.4) firm \(i\)’s profit can be written as

\[
\pi_i = \sum_{j=1}^{n} l_{ij} \left[ p(X)x_j - c_j(x_j) \right].
\]

We consider only passive financial interests of firms, thus in maximizing profits firms take the choices of their rivals as given. Following Clarke and Davies (1982) we further assume that in choosing its output, firm \(i\) forms a conjectural variation about the output response of all other firms to a unit change in its own output level. Denote the constant conjectural elasticity parameter of firm \(i\) by \(\alpha\), which is defined as

\[
\frac{\partial x_j}{\partial x_i} = \alpha \frac{x_j}{x_i} \quad \text{for all } j \neq i. \tag{3.7}
\]

The conjectural elasticity \(\alpha\) is interpreted simply as the percentage change in firm

\textsuperscript{13} In fact, Miller and Modigliani (1961) show that for a given investment policy, a firm’s dividend policy is irrelevant to its current market valuation. In particular, they state: “[L]ike many other propositions in economics, the irrelevance of dividend policy, given investment policy, is ‘obvious, once you think of it.’ It is, after all, merely one more instance of the general principle that there are no ‘financial illusions’ in a rational and perfect economic environment. Values there are determined solely by ‘real’ considerations—in this case the earning power of the firm’s assets and its investment policy—and not by how the fruits of the earning power are ‘packaged’ for distribution” (p. 414).
j’s output that firm i expects in response to a one percent change in its own output. Note that this parameter is assumed to be the same for all firms and measures the degree of (tacit) collusion inherent in an industry. Positive values of \( \alpha \) indicate the presence of collusion, and its degree is larger if \( \alpha \) is larger. This is more obvious if we rewrite (3.7) as \( \partial x_j / x_j = \alpha (\partial x_i / x_i) \). If \( 0 < \alpha < 1 \), lower values of \( \alpha \) imply that firm i’s rivals will react with a smaller (percentage) change to the change in output, so that firm i believes that there is some scope for improving its market share.\(^{14}\) Let \( c_i' \) be the marginal cost of firm i, then the first-order condition (FOC) \( \partial \pi_i / \partial x_i = 0 \) is \( \sum_j l_{ij} [(p - c_i') \partial x_j / \partial x_i + x_j \sum_k (d p / d X) (\partial x_k / \partial x_i)] = 0. \)

Define firm i’s price-cost margin by \( m_i \equiv (p - c_i') / p \), its market share by \( s_i \equiv x_i / X \), and the price elasticity of demand by \( \varepsilon \equiv (p / X) (\partial X / \partial p) \). Using \( \partial x_j / \partial x_i = \alpha (s_j / s_i) \) as an equivalent expression for (3.7), firm i’s FOC after some rearrangements yields\(^{15}\)

\[
m_i = \frac{1}{\varepsilon} \left[ 1 + \frac{\sum_{j \neq i} l_{ij} s_j}{l_{ii} s_i} \right] \left[ \alpha + (1 - \alpha) s_i \right] - \frac{\alpha \sum_{j \neq i} l_{ij} s_j m_j}{l_{ii} s_i}.
\] (3.8)

To represent (3.8) succinctly in matrix form, let \( \hat{L} \) be the diagonal matrix with \( l_{ii} \) along its main diagonal and zero otherwise, \( m \) and \( s \), respectively, be the vectors of firms’ markups and market shares. Then (3.8) can be rewritten as\(^ {16}\) (see Appendix 3.A)

\[
m = \alpha Q m + \frac{\alpha}{\varepsilon} x_1 + \frac{1 - \alpha}{\varepsilon} x_2,
\] (3.9)

where \( Q \equiv \hat{s}^{-1} (I - \hat{L}^{-1} L) \hat{s} \), \( x_1 \equiv \hat{s}^{-1} \hat{L}^{-1} L s \), and \( x_2 \equiv \hat{L}^{-1} L s \).

In empirical work equation (3.9) can be used for the estimation of the effect of PCO on the degree of market power of firms, and on the overall level of tacit collusion in an industry. For the first task it is obvious that a firm exercises market power if its markup is positive. In the context of this model, firm i exercises market power if \( m_i \) in (3.9) is significantly (in a statistical sense) positive. Without PCO,

\(^{14}\) Throughout the paper the notions of market conduct, degree of tacit collusion, market performance, and market competitive intensity are used interchangeably for \( \alpha \).

\(^{15}\) Equation (3) in Alley (1997) is \( m_i = \frac{1}{\varepsilon} \left[ 1 - \frac{\sum_{j \neq i} w_{ij} s_j}{(1 - \sum_{j \neq i} w_{ij}) s_i} \right] \left[ \alpha + (1 - \alpha) s_i \right] - \frac{\alpha \sum_{j \neq i} w_{ij} s_j m_j}{(1 - \sum_{j \neq i} w_{ij}) s_i} \). He disregards indirect shareholdings and since in the PCO presence \( l_{ii} \geq 1 \) and \( l_{ij} \geq w_{ij} (i \neq j) \), in general, these two equations will give different estimates of \( \alpha \) and \( \varepsilon \).

\(^{16}\) Theoretically, we can allow for different conjectural elasticities, in which case the scalar \( \alpha \) in (3.9) is replaced by the diagonal matrix \( \hat{\alpha} \) with \( \alpha_{ii} \) on its ii-th entry and zeros elsewhere. However, for empirical estimation we need to make an identical conjectural elasticity assumption, hence \( \alpha \) instead of \( \alpha_i \) or \( \hat{\alpha} \) is entered in all equations. Alley’s model can be also written in the form of (3.9) with the redefinition of \( \hat{L}^{-1} L = I + (I - \hat{J} W)^{-1} W \).
$L = I$, and the market power diagnosis of firm $i$ reduces to the condition $m_i = \left[ \alpha + (1 - \alpha)s_i \right] / \epsilon > 0$ (see Martin, 1988). In order to identify the market competitiveness, one needs to estimate the value of $\alpha$ empirically.\(^{17}\)

Without PCO, $L = I$, hence (recalling that $t$ is the summation vector of ones) (3.9) boils down to (see e.g., Martin, 2002)

$$m = \frac{\alpha}{\epsilon} t + \frac{1 - \alpha}{\epsilon} s.$$  \hspace{1cm} (3.10)

The important difference between (3.9) and (3.10) is that without PCO price-cost margins are linearly related to the conjectural elasticity, while with PCO this relation is nonlinear. This is because the solution of (3.9) is $m = (I - \alpha Q)^{-1} (\frac{\alpha}{\epsilon} x_1 + \frac{1 - \alpha}{\epsilon} x_2)$ and $(I - \alpha Q)^{-1}$ is nonlinear in $\alpha$. Hence, it follows that the failure of taking firms’ direct and indirect cross-shareholdings in the presence of PCO is likely to give biased parameter estimates due to model misspecification.\(^{18}\)

Using (3.9) the range of the market competitive intensity $\alpha$ consistent with the economic interpretations is given in the following result, which helps to infer the industry market performance.

**Theorem 3.1.** Irrespective of whether PCO is present or absent, the reasonable range of the market competitive intensity is $\alpha \in \left[ -\frac{1}{(n - 1)}; 1 \right]$. \(^{19}\)

In Cournot competition we have $\partial x_j / \partial x_i = 0$ for all $j \neq i$, which corresponds to zero conjectural elasticity, i.e., $\alpha = 0$. In this case markups in (3.9) become $m = (1/\epsilon) \tilde{L}^{-1} Ls$ (Merlone, 2001, p. 335). The value of $\alpha$ equal to the lower bound of $-1/(n - 1)$ characterizes the perfect competition outcome, because then price-cost margins equal zero. The case $\alpha = 1$ reflects the perfect cartel since then markups equal the inverse of the price elasticity of demand.\(^{19}\)

Given the expressions for price-cost margins with and without PCO, respectively, in (3.9) and (3.10), the obvious question is how the two are interrelated. Clearly, it is impossible to compare two different real-world environments with and without PCO as all the endogenous variables (i.e., price-cost margins and mark-

\(^{17}\)It is not possible to directly run an OLS regression of (3.9), since the inverse matrix $(I - \alpha Q)^{-1}$ (which would solve (3.9) for the vector of markups) contains the unknown market conduct parameter $\alpha$. This problem is similar to the so-called spatial autoregressive models in Spatial Econometrics, where $Q$ and $\alpha$ can be reinterpreted as a spatial weight matrix, and a spatial autoregressive parameter, respectively (see Anselin, 1988). The only difference is that $\alpha$ is also included in the regression coefficient vector.

\(^{18}\)Similarly, one may get biased estimates if only direct PCO holdings are taken into account, which in the model is equivalent to the case when $L = I + W$ and $\tilde{L} = I$.

\(^{19}\)Note also that if $\alpha$ is close to its lower bound, we say that the market competitive intensity is high, and, similarly, an increase in $\alpha$ is referred to as the decrease in the market competitive intensity. For the conjecture’s range without PCO see e.g., Kwoka and Ravenscraft (1986).
ket shares) are different within the two frameworks. Hence, let us focus on the difference between the markups assuming that $\alpha$, $\varepsilon$, $n$, and $s$ are identical in both the PCO and the no PCO case.\footnote{Note that the assumption $s_i = s_i^0$ for each $i$, where the superscript ‘0’ refers to the no PCO case, does not necessarily imply that all firms have equal market shares of $1/n$.}

**Theorem 3.2.** Let $m_i < 1/\varepsilon$ for all $i = 1, \ldots, n$. For given $\alpha$, $\varepsilon$, $n$, and $s$, price-cost margins of firms with PCO are higher than those of firms without PCO provided that $\alpha \in [0, 1)$.

The intuition behind Theorem 3.2 is simple. In this setting, shareholding interlocks among firms cause a common interest of firms that in turn leads to greater monopoly power of firms with PCO holdings. Recall that the requirement $m_i < 1/\varepsilon$ means that firm $i$ is not a monopolist (hence the above result excludes the perfect cartel case).

### 3.4 Empirical estimation and results

In practice, simple direct use of accounting price-cost margins is insufficient as marginal costs defined by economists are unobservable, i.e., firms’ costs should also include opportunity costs. One way to deal with this problem in the literature is assuming constant returns to scale (CRS), which means that marginal costs equal average costs. Average costs of firm $i$, $ac_i$, besides costs of variable inputs, include also the normal rate of return on investments, i.e., $ac_i = (v'^l_i + \mu K_i)/x_i$, where $l_i$ and $v$ are, respectively, the vectors of variable inputs of firm $i$ and input prices, $\mu$ and $K_i$ are, respectively, the rental cost of capital services and the value of capital assets of firm $i$. Plugging the last expression in the definition of the price-average cost margin, one gets firm $i$’s economic earnings per unit of sales, or, equivalently, price-cost margins under the CRS assumption as (see e.g., Martin, 2002, p. 137)

$$m_i = \frac{px_i - v'^l_i - \mu K_i}{px_i} = \frac{px_i - v'^l_i}{px_i} - \frac{\mu K_i}{px_i} = PCM_i - \frac{\mu K_i}{px_i}, \quad (3.11)$$

which is equal to accounting price-cost margins $(PCM_i)$ minus the normal rate of return on investments. Solving (3.9) for the vector of markups and combining it with (3.11) yields the final model for empirical estimation as\footnote{Evidently (3.12) is a nonlinear function of the unknown parameters $\alpha$ and $\varepsilon$. Therefore, we numerically estimate parameters in (3.12) using a nonlinear least-squares approach. In MATLAB this is implemented by the function *lsqnonlin*, which finds the minimum of the objective function on the basis of the Levenberg-Marquardt method.}

\begin{align*}
\text{Note that the assumption } s_i &= s_i^0 \text{ for each } i, \text{ where the superscript ‘0’ refers to the no PCO case, does not necessarily imply that all firms have equal market shares of } 1/n. \\
\text{Evidently (3.12) is a nonlinear function of the unknown parameters } \alpha \text{ and } \varepsilon. \text{ Therefore, we numerically estimate parameters in (3.12) using a nonlinear least-squares approach. In MATLAB this is implemented by the function *lsqnonlin*, which finds the minimum of the objective function on the basis of the Levenberg-Marquardt method.} \end{align*}
PCM_i = \frac{\alpha}{\epsilon} [(I - \alpha Q)^{-1} x_1]_i + \frac{1 - \alpha}{\epsilon} [(I - \alpha Q)^{-1} x_2]_i + \mu KS_i + \upsilon_i, \quad (3.12)

where \([(I - \alpha Q)^{-1} x_1]_i\) is the i-th element of the vector \((I - \alpha Q)^{-1} x_1\), \(KS_i = K_i / (px_i)\) is firm i’s capital-sales ratio, and \(\upsilon_i\) is a random error term. Without PCO, \(L = I\), thus \(Q\) is a null matrix, \(x_1 = i\) and \(x_2 = s\), and as a consequence (3.12) reduces to (Martin, 2002, eq. (6.11))

\[
PCM_i = a_0 + a_1 s_i + \mu KS_i + \upsilon_i, \quad (3.13)
\]

where \(a_0 = \alpha / \epsilon\) and \(a_1 = (1 - \alpha) / \epsilon\). Hence, estimates for \(a_0\) and \(a_1\) provide the estimates of \(\alpha\) and \(\epsilon\).

3.4.1 Data

As an empirical application, we study the banking sector in Japan. Conventional wisdom is that the Japanese economy is collusive due to the existence of keiretsu groups that are historically interlinked through strong shareholding interlocks. We select city and regional banks from the Bankscope database published by Bureau van Dijk Electronic Publishing. Trust banks, long-term credit banks, security firms, and other smaller cooperative institutions (such as Shinkin banks) are excluded from the analysis because the sample should be consistent with the homogeneity assumption of the theoretical model described in Section 3.3 in the sense that all banks face the same inverse market demand function. Trust banks (next to having banking business) are also engaged in trust business (i.e., asset management services). Security firms apparently have different lines of business than commercial banks, hence do not compete with each other in the same market either. Similarly, long-term credit banks are mainly specialized in the provision of long-term loans and debentures. Hence, the city banks and the regional banks constitute the “ordinary banks”. Legally, the two are not distinguished from each other and it is basically the size and area of business that distinguishes them. Regional banks are much smaller and operate in restricted areas, whereas city banks have nation-wide branch networks and operation. Uchida and Tsutsui (2005) reports that in 1996 the shares of city and regional banks in the Japanese loan market were, respectively, 49.6% and 33.1%, thus by analyzing these two groups one is able to cover 82.7% of the total outstanding loans in Japan. (The total outstanding loan in Japan is de-
### Table 3.1: Descriptive statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>St. deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Overall sample (63 obs.)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Markups (PCM)</td>
<td>0.2274</td>
<td>0.1274</td>
<td>0.0288</td>
<td>0.8574</td>
</tr>
<tr>
<td>Market share (s)</td>
<td>0.0159</td>
<td>0.0341</td>
<td>0.0008</td>
<td>0.2058</td>
</tr>
<tr>
<td>Capital-sales ratio (KS)</td>
<td>2.6753</td>
<td>0.8051</td>
<td>1.0616</td>
<td>4.6928</td>
</tr>
<tr>
<td>Growth rate (GR)</td>
<td>0.3808</td>
<td>1.9184</td>
<td>-0.3210</td>
<td>15.3398</td>
</tr>
<tr>
<td><strong>City banks (4 obs.)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Markups (PCM)</td>
<td>0.4763</td>
<td>0.3181</td>
<td>0.1983</td>
<td>0.8574</td>
</tr>
<tr>
<td>Market share (s)</td>
<td>0.1377</td>
<td>0.0496</td>
<td>0.0878</td>
<td>0.2058</td>
</tr>
<tr>
<td>Capital-sales ratio (KS)</td>
<td>2.3848</td>
<td>0.8442</td>
<td>1.6474</td>
<td>3.1439</td>
</tr>
<tr>
<td>Growth rate (GR)</td>
<td>3.8812</td>
<td>7.6394</td>
<td>-0.0100</td>
<td>15.3398</td>
</tr>
<tr>
<td><strong>Regional banks (59 obs.)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Markups (PCM)</td>
<td>0.2105</td>
<td>0.0869</td>
<td>0.0288</td>
<td>0.5069</td>
</tr>
<tr>
<td>Market share (s)</td>
<td>0.0076</td>
<td>0.0049</td>
<td>0.0008</td>
<td>0.0265</td>
</tr>
<tr>
<td>Capital-sales ratio (KS)</td>
<td>2.6950</td>
<td>0.8061</td>
<td>1.0616</td>
<td>4.6928</td>
</tr>
<tr>
<td>Growth rate (GR)</td>
<td>0.1435</td>
<td>0.1146</td>
<td>-0.3210</td>
<td>0.6172</td>
</tr>
</tbody>
</table>

*Note: Computations are based on the data given in thousands of US dollars. GR is the growth rate of banks’ total revenue in 2003 relative to 2002.*

fined as the sum of loans of city banks, long-term credit banks, trust banks, regional banks, Shinkin banks, and credit cooperations.)

After deleting unprofitable banks and those without necessary data information, we end up with a sample of 63 commercial banks for the fiscal year 2003, which includes 4 city banks and 59 regional banks. Data on accounting price-cost margins \((PCM_i)\) and market shares \((s_i)\) are derived from the banks’ unconsolidated statements. The accounting price-cost margin is defined as the ratio of profit before tax over total revenue, where total revenue is the sum of net interest revenue and other operating income, and profit before tax is equal to the total revenue minus overheads, loan loss provisions, and other net expenses. The capital-sales ratio \((KS_i)\) is proxied by the ratio of total equity over total revenue. Descriptive statistics are reported in Table 3.1. It shows that city banks have both economically and statistically significant larger means of accounting price-cost margins and market shares than regional banks. In particular, on average, city banks have higher accounting markups and market shares by factors of 2.3 and 18.1, respectively. The averages of the capital-sales ratios of these banks are roughly identical (i.e., with an insignificant difference).

Data on ownership are available only for the last year of the bank’s reports, which varies from 2002 to 2005. Thus in constructing the cross-shareholding matrix for Japan, we assume that these direct shareholdings were also valid for 2003.
However, we should note that the ownership data, though crucial for this analysis, represent an incomplete picture of the shareholding ties due to its partial (and in some cases total) unavailability in the Bankscope dataset. In general, the city banks are the most intensive shareholders in the Japanese commercial banking sector.\textsuperscript{22} For illustration purposes, a few banks from the sample are chosen and their partial ownership links are graphed in Figure 3.1. For the sake of simplicity, we disregard outside shareholding links of these banks, which do exist. As an example, Figure 3.1 shows that Bank of Fukuoka owns 2.36\% of the shares in Higo Bank. Two remarks are in place. First, there are cases of mutual shareholding ties in the Japanese banking sector. In the figure this is the case for Kagoshima Bank and Miyazaki Bank. Second, given this mutual relationship, one might expect that indirect shareholdings could matter for the Japanese banks. It is easily seen that Mizuho Corporate Bank has an indirect share in Miyazaki Bank, for example, Kagoshima Bank. However, in fact, because of the mutual shareholding described above there is an infinite number of paths of different length through which Mizuho Corporate Bank indirectly owns Miyazaki Bank (see Dietzenbacher and Temurshoev, 2008).

### 3.4.2 Estimation results

The results of the numerical nonlinear least-squares estimation are reported in Table 3.2. Since in the presence of local optima, finding the global optimal point de-
Table 3.2: Empirical results (year 2003, obs. = 63)

<table>
<thead>
<tr>
<th></th>
<th>Full PCO</th>
<th>Direct PCO</th>
<th>Alley</th>
<th>No PCO</th>
<th>Full PCO</th>
<th>No PCO</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\alpha} )</td>
<td>0.0435</td>
<td>0.0435</td>
<td>0.0435</td>
<td>0.0390</td>
<td>0.0281*</td>
<td>0.0255</td>
</tr>
<tr>
<td></td>
<td>(0.0404)</td>
<td>(0.0404)</td>
<td>(0.0404)</td>
<td>(0.0401)</td>
<td>(0.0163)</td>
<td>(0.0160)</td>
</tr>
<tr>
<td>( \hat{\varepsilon} )</td>
<td>0.6189*</td>
<td>0.6189*</td>
<td>0.6188*</td>
<td>0.6167*</td>
<td>0.3329**</td>
<td>0.3241**</td>
</tr>
<tr>
<td></td>
<td>(0.3631)</td>
<td>(0.3631)</td>
<td>(0.3630)</td>
<td>(0.3674)</td>
<td>(0.1466)</td>
<td>(0.1501)</td>
</tr>
<tr>
<td>( \hat{\mu} )</td>
<td>0.0495***</td>
<td>0.0495***</td>
<td>0.0495***</td>
<td>0.0521***</td>
<td>0.0410***</td>
<td>0.0430***</td>
</tr>
<tr>
<td></td>
<td>(0.0139)</td>
<td>(0.0139)</td>
<td>(0.0139)</td>
<td>(0.0147)</td>
<td>(0.0146)</td>
<td>(0.0152)</td>
</tr>
<tr>
<td>( \hat{\mu}_{GR} )</td>
<td>-0.0365**</td>
<td>-0.0371*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0177)</td>
<td>(0.0190)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SSR</td>
<td>0.7382</td>
<td>0.7383</td>
<td>0.7383</td>
<td>0.7542</td>
<td>0.5850</td>
<td>0.6008</td>
</tr>
</tbody>
</table>

Note: The superscripts (*), (**), and (***) denote statistical significance of the coefficients at 10%, 5%, and 1% levels, respectively. SSR denotes the sum of squared residuals. The robust standard errors are given in parentheses.

Table 3.2 depends on the initial parameters’ values, in the estimation we first constructed grids for all parameters (i.e., we created a grid structure for \( \alpha \), \( \varepsilon \) and \( \mu \)), and used all possible combinations of these grids as starting points. Then the minimum value of the sum of squared residuals (SSR) is chosen, and its corresponding estimates are given in Table 3.2. Column 2 gives the estimates of the parameters in (3.12) when both direct and indirect (full) PCO links are taken into account (i.e., when \( L = (I - W)^{-1} \)). Positive values of \( \hat{\alpha} \) are indicative of cooperative behavior of banks. The full PCO model gives the market conduct estimate of \( \hat{\alpha} = 0.0435 \), which is not statistically different from zero.\(^{23}\) Hence, at this point one may conclude that commercial banks in Japan in 2003 behaved as Cournot competitors. Table 3.2 also shows that the Japanese banking sector is characterized by inelastic demand (i.e., \( \hat{\varepsilon} = 0.6189 \) which is significant at 10% level). So, theoretically, banks in 2003 would have increased their revenues if they had raised the price. Finally, the sign of the capital-sales ratio coefficient is positive as expected, and is statistically significant for all estimated models. This is an estimate of the marginal rental cost of capital to the firm. One can also interpret the capital-sales ratio as a barrier to entry, and from this point of view its coefficient should also be positive, meaning that the higher the capital-sales ratio, the more difficult it is for a new firm to enter the industry.

The third column of Table 3.2 gives the results of the direct PCO model (i.e.,

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\(^{23}\) Standard errors are heteroscedastic-consistent. The error vector is \( \nu(\theta) \), where \( \theta \) is a vector of \( k \) parameters. Denote the \( n \times k \) Jacobian matrix by \( J(\theta) \) with \( (J(\theta))_{ij} = \partial \nu_i(\theta)/\partial \theta_j \). Then the heteroscedastic-consistent estimate of the variance-covariance matrix of the estimate \( \hat{\theta} \) is

\[
\hat{\Phi} \equiv \frac{1}{n-k} \hat{J}' \hat{J}^{-1} \hat{J}' \Omega \hat{J}^{-1},
\]

where \( \hat{J} = \partial \nu/\partial \theta |_{\hat{\theta}} \) and \( \Omega \) is a diagonal matrix with \( \hat{\nu}_i^2 \) on its main diagonal (see e.g., Cameron and Trivedi, 2005, Chapter 5.8).
when \( L = I + W \). The estimates of the parameters are identical to those of the full PCO model, implying that for our sample indirect ownership links are insignificant and do not have any impact on the results. The fourth column of Table 3.2 gives the results for Alley’s model (i.e., when \( \hat{L}^{-1}L = I + (I - \hat{r}W)^{-1}W \), see footnotes 15 and 16), which also gives estimates of the parameters that are very close to those of the full PCO and direct PCO specifications. Alley’s model is based on the profit specification given by (3.2), 
\[
\pi_i = (1 - \sum_{k \neq i} w_{ki})z_i + \sum_{k \neq i} w_{ik}z_k,
\]
which is different from (3.3) for the full PCO model. However, the closeness of the outcomes of all these three models is due to the fact that there is a small number of PCO links in our sample (to be discussed later) and direct shareholdings are small in size (on average 3.2%), both of which imply that indirect PCO links are negligible.

Column 5 in Table 3.2 reports the estimates of the parameters in (3.12) when all the elements of the PCO matrix are set to zero (i.e, \( L = I \), hence effectively (3.13) is estimated), which gives an estimate of the tacit collusion degree of \( \hat{\alpha} = 0.0390 \) that is not statistically different from zero either. Hence, without considering any other additional explanatory variable(s) in (3.12), neglecting PCO links does not give an economic bias in the results. That is, both the full PCO and the no PCO models predict that Japanese commercial banks compete in a Cournot oligopoly (although note that the point estimates are different).

Following Alley (1993, 1997) we re-estimate the full PCO and the no PCO models in Table 3.2 by adding the growth variable \( GR_i \) - the growth rate of a bank’s operating income relative to the year 2002, which allows for changes in demand and thus in accounting price-cost margins to be taken into account.\(^{24}\) Theoretically, the sign of the effect of the growth rate variable can be either positive, or negative. On the one hand, an increase in market demand may raise demand on inputs, thereby increasing their factor prices, hence may lead to lower accounting markups. On the other hand, the growth rate of demand may increase accounting price-cost margins by increasing output prices and/or expanding production volume. The results are given in the last two columns of Table 3.2, where the estimate \( \hat{\mu}_{GR} \) for the growth variable is negative, and statistically significant.

Note that including \( GR_i \) in the full PCO model gives a market conduct estimate of \( \hat{\alpha} = 0.0281 \) that is statistically significant (at 10% level), while in the no PCO case \( \hat{\alpha} = 0.0255 \), which does not differ statistically significantly from zero. Hence, neglecting cross-shareholding links in this case yields different economic outcomes:

\(^{24}\) We also estimated the models with other bank-specific factors, such as net loans and total fixed assets to account for risk and capacity differences. However, these coefficients were insignificant and did not change the results, hence are not reported.
the no PCO case predicts Cournot oligopoly in the 2003 Japanese banking sector, while the full PCO model predicts modest collusive environment in the industry. Although $\hat{\alpha}$ in the full PCO case is not highly statistically different from zero, this result suggests that ownership links should be taken into account in empirical studies of the Japanese banking sector. In addition, we think that the main reasons for the almost identical results of all the four models given in columns 2-5 of Table 3.2 are the following. First, as we already noted, the ownership data are incomplete, and it is quite difficult to obtain the true picture of these linkages. This yields underestimation of the PCO effects. Second, some banks with partial ownership data were excluded from the sample for their unprofitability and/or unavailability of other required data. Third, in our $63 \times 63$ PCO matrix there are 67 cases of shareholding links, which comprises only 1.7% of the total number of possible ownership ties of $n(n - 1) = 3906$. Fourth, in general, in the Japanese financial system city banks, long-term credit banks and trust banks are the main shareholders of other financial (and nonfinancial) institutions (see footnote 22). Hence, we expect that studies that concentrate also on the last two types of banks should consider PCO links, otherwise the (economic) bias of the results might be significant. In this chapter, however, we do not consider trust banks and long-term credit banks, which would require using a different model of a differentiated-product nature.

### 3.4.3 Comparison with related studies

There are few studies that estimate the degree of competition in the Japanese banking sector. Before comparing our results with these studies, we first briefly discuss different approaches in estimating the competition level (see for details e.g., Bresnahan, 1989). CV models are frequently used for this purpose, starting with the early important paper of Iwata (1974). The Clarke and Davies (1982) model, adopted in this chapter, also belongs to this strand of literature. Since CV models provide theoretical foundations for firms’ structure-conduct-performance reduced-form relationships (which explains the term “structure-conduct-performance paradigm”), they are widely used to infer the degree of market competition. The disadvantage of using such models is that cost data are required, which in many cases are difficult to obtain. The attempt of avoiding cost data resulted in the so-called “new empirical industrial organization” (NEIO) literature pioneered by Bresnahan (1982) and

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25 Dietzenbacher et al. (2000) analyzed the sensitivity in their analysis of the Dutch banking sector, because banks were only required to report if shares were larger than 5%. They showed that direct interests below 5% are relevant and have a substantial effect on the estimates of banks’ price-cost margins.
Lau (1982). Its econometric approach is structural because both demand and supply sides are explicitly considered. However, modeling the competition level does not differ from the CV literature, which is stated by Bresnahan (1989, p. 1027) as follows: “As a matter of fact, there is absolutely no difference between [CV and NEIO approaches to modeling collusion and] ... the two specifications can nest the same models”.

Another widely used approach is that of Panzar and Rosse (1987). The Panzar-Rosse $H$ statistic is the sum of the elasticities of the reduced-form revenues with respect to all factor prices. Its advantage is that few data are required on endogenous variables (revenue is always observable even when price and quantity are not), though it will require information on all the variables that shift demand or cost. However, using $H$ statistics in empirical work relies on the assumption that markets are in the long-run equilibrium in each point of time. In general, speaking about above methods and others including time-series data analysis, event studies, studies of the determinants of the price, and fully dynamic models, Martin (2002, p. 225) concludes: “No one of these are immune from criticism. Broadly speaking, these diverse methodologies yield consistent results, tending to support the hypotheses advanced by the structure-conduct-performance school”.

The paper closest to our work in terms of the methodology used is Alley (1993), who uses exactly the same theoretical model, but without considering PCO linkages. The author finds that the degree of competitive intensity for 1986-87 Japanese regional and Sogo banks is $\hat{\alpha} = 0.6013$, indicating a high degree of collusion. This estimate is much larger than our estimate of $\hat{\alpha} = 0.0281$ for the Japanese commercial banking sector (column 6 in Table 3.2). Two remarks are in place in this regard. First, it might very well happen that the estimate of $\alpha$ is biased (upward), given the fact that back in the 1980s-1990s shareholding interlocks were quite extensive in the Japanese banking system compared to the current situation (see e.g., Miyajima and Kuroki, 2007). Second, if the result would not change with PCO consideration, then comparison of the two would suggest that competition has significantly improved between 1986-87 and 2003.

Molyneux et al. (1996) employing Panzar-Rosse $H$ statistics, conclude that Japanese commercial (city and regional) banks behaved as if under monopoly in 1986, but the market conduct improved in 1988 whereby it becomes consistent with mo-

\[26\text{If } H \text{ is negative then firms' policies are consistent with the monopoly conduct, } 0 < H < 1 \text{ represents monopolistic competition and } H = 1 \text{ under perfect competition. These interpretations can be deduced from the effect of an upward shift in firms' marginal, average and total cost curves on the firms' equilibrium revenues.}\]
nopolistic competition. Using the NEIO approach and long-term panel data from 1974 to 2000, Uchida and Tsutsui (2005) conclude that market competition largely improved from the 1970s to the 1980s, but deteriorated after 1997. They also find that the degree of competition is higher for city banks than for regional banks. Finally, Lee and Nagano (2008) compare the pre-merger period of 1986-1997 to the post-merger wave period of 1998-2005 in a set of Japanese regional banks that is divided into seven regions. Essentially their results in terms of $H$ statistics suggest that in six regions the monopolistic competition environment holds for both periods, while in only one region there is a tendency towards a more competitive environment.\textsuperscript{27} In relation to this chapter, we think that similar to the first point made with regard to Alley’s (1993) study, there is a possibility of bias in the estimates of the market conduct or $H$ statistics due to ignorance of PCO linkages, which is again much more probable for the results on earlier periods in these studies. The market performance indicator for Japanese regional banks in 2000 in Uchida and Tsutsui (2005) shows a collusive environment, which is consistent with our result for 2003. However, their study does not reject Cournot competition for city banks in 2000. All in all, we think that taking into account PCO links in all these studies is crucial, which might even change the results, especially, for the period before the mid 1990s when cross-shareholding was believed to be one of the main distinguishing features of the Japanese business groups.

### 3.4.4 Market power test

In this section the market power test of each individual bank is carried out. Having the estimates of the market competitive intensity and the price elasticity of demand, one can estimate firms’ markups using equation (3.9). Then in the context of our model, firm $i$ exercises market power if its estimated price-cost margin is in a statistical sense significantly positive. As mentioned in Section 3.3, in an industry without PCO, the market power diagnosis of firm $i$ reduces to the condition 

$$[\hat{\alpha} + (1 - \hat{\alpha})s_i]/\hat{\epsilon} > 0$$

(see Martin, 1988).

The \textit{delta method} is used in order to compute t-statistics of the markups in (3.9).\textsuperscript{28} The estimated markups and their t-statistics based on the estimates of the full PCO

\textsuperscript{27} We should note that the authors’ own conclusion is, however, different. Lee and Nagano (2008) state that “... the banking sector in Japan’s metropolitan area is very competitive, becoming more competitive than that of 1986-1997” (p. 614). This conclusion is not consistent with the values of the $H$ statistics with their appropriate 95\% confidence intervals given in their Table 1 on pp. 612-613.

\textsuperscript{28} Let price-cost margins depend on $k$ parameters given by the vector $\theta$ and let $C(\theta) = \partial m(\theta)/\partial \theta'$. Then according to the delta method, the estimated (asymptotic) variance-covariance matrix of the markups is given by $\hat{C}\Phi\hat{C}'$, where $\Phi$ is defined in footnote 23 (see e.g., Greene, 2003).
model from Table 3.2 (i.e., column 6) are reported in Table 3.3. Note that estimating markups for the actual no PCO case does not make sense, since we do not know anything about the real environment without cross-shareholdings between banks. That is, markups and market shares would be different in that case, implying that using our data for this purpose would be totally misleading.

The t-statistics of all these markups are computed on the null hypothesis that the true value of the statistics are zero, which is a market power test for each bank. Several conclusions can be drawn from Table 3.3. First, given that the smallest t-statistic in the entire sample is 2.183, we conclude that each bank exercises some degree of market power (at a 5% significance level). Second, on average, banks that hold shares in other banks have higher markups than banks without any stockholdings in rivals (i.e., 0.271 vs. 0.106). This difference is statistically significant (the one-sided two-sample t significance test of means gives \( p = 0.0184 \) with 9 degrees of freedom), implying that PCO increases the market power of banks owning shares in rivals. In our sample there are in total 67 cases of shareholdings (the sum of the column “Sub” in Table 3.3, which denotes the number of subsidiaries, or, equivalently, the sum of the column “Share” for the number of shareholders) that are made by the 10 banks that hold shares in other banks, consisting of all four city banks and six regional banks. Note also that the regional banks, and not the city banks, are owned by others. Moreover, the correlation coefficient between the estimated markups and the number of banks’ subsidiaries for the entire sample is 0.68, while that between the estimated price-cost margins and the number of banks’ shareholders is equal to \(-0.26\). All in all, this confirms the conjecture that owning shares in rivals increases (resp. decreases) market power of firms-owners (resp. owned firms). Third, city banks, on average, have significantly higher price-cost margins than regional banks (i.e., 0.501 vs. 0.107, and the difference is highly statistically significant with \( p = 0.0054 \)). One of the explanations for this (in light of the second point made above) is that city banks own many more banks with larger shareholding size than regional banks. Table 3.3 shows that the four city banks, on average, own 14.5 banks with an average direct stake of 4.98%, while they are not owned themselves. On the other hand, on average, a regional bank owns only 0.2 banks with 0.21% as the average share, but 3.10% of its shares are owned by 1.1 banks. The six regional banks with shareholdings, on average, hold 2.01% shares in 1.5 banks, whereas 2.75% of their shares are owned by 2.5 banks (not shown in Table 3.3). Hence, among other factors, owning larger shares in many regional banks allows city banks to exercise a larger market power.
<table>
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<tr>
<th>No</th>
<th>Bank Name</th>
<th>Type</th>
<th>( \hat{m} )</th>
<th>t-stat.</th>
<th>Sub.</th>
<th>Share.</th>
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</tbody>
</table>

**Overall sample average:**

- 0.132
- 1.1
- 0.51
- 1.1
- 2.91

**City banks average:**

- 0.501
- 14.5
- 4.98
- 0.0

**Regional banks average:**

- 0.107
- 0.2
- 0.21
- 1.1
- 3.10

**All shareholders average:**

- 0.271
- 6.7
- 3.23
- 1.5
- 1.37

**All non-shareholders average:**

- 0.106
- 0.0
- 1.0
- 3.19
3.5 Concluding remarks

Nowadays there is ample evidence of the presence of partial cross ownership (PCO) links among firms. This study examines empirically the influence of PCO on the degree of competitive intensity of an industry and on firms’ market power. The model of Clarke and Davies (1982) is adopted and modified by taking into full account both direct and indirect interests of firms in each other via PCO ties. To the best of our knowledge, in all empirical estimations of the degree of tacit collusion, PCO is totally neglected, except for Alley (1997), who, however, disregards indirect shareholdings.

It has been shown that, unlike in the no PCO case, with cross-shareholding the link between firms’ price-cost margins and the market competitive intensity is non-linear. Hence, in the presence of extensive shareholding links among firms, ignoring PCO leads to biased parameter estimates due to model misspecification. It has been shown that when market shares, number of firms, price elasticity of demand, and collusion degree are given, firms with shareholdings exert a strictly larger market power than those without PCO, provided that the market conduct is consistent with Cournot or a more collusive environment. This is because shareholding interlocks among firms cause a common interest of firms, implying greater monopoly power for firms with PCO holdings.

As an empirical application we have studied the Japanese banking sector in 2003. We found that the Japanese banks are competing in a modest collusive environment, while neglecting PCO yields a different economic outcome that indicates a Cournot oligopoly. (By modest we mean that the degree of collusion is relatively small being closer to the Cournot outcome rather than a monopoly.) Secondly, banks with passive investments in rivals exert a strictly larger market power than those without any PCO, which confirms the hypothesis that acquiring shares in rivals for a firm is one of the crucial means of enhancing its market power. Also, city banks with many shareholdings were found to exercise a much larger market power than regional banks with none or few stockholdings.

A few simplifying assumptions have been made throughout the chapter and need some clarification. First, we did not consider product differentiation, and focused only on homogeneous market environment, which, in general, does not hold in the real world. Analyzing a differentiated-product industry is rather complex, since one has to compute all the own- and cross-price elasticities, for instance.29

29 See, for example, the “menu approach” for identifying market conduct proposed by Nevo (1998).
Nonetheless, the empirical results of the homogeneous model used here are still useful in discovering the collusion degree within an industry, as the estimates of the market competitive intensity indicate “the similarity of margins between firms of different size” (Clarke et al., 1984, p. 447). As a matter of fact this has been confirmed in our study, as the low degree of collusion implies rather different levels of firms’ market power. Second, the PCO structure has been assumed to be exogenous, which might not reflect the optimal decisions of firms. However, similar to the Gilo et al. (2006) study, our analysis was done from the perspective of antitrust agencies facing a given pattern of PCO. Third, in the empirical part we have disregarded the PCO of banks with other financial and non-financial institutions. This allowed us to focus on the commercial banking sector only, while neglecting the potential effect of banks’ shareholding interlocks with firms in other industries. However, for that one needs to use a different theoretical model for an industry with differentiated products, which is beyond the scope of the current study.

30 Ideally, one would like to consider all possible shareholding links, but this would be unfeasible or, at least, a complicated task in light of unavailability of (access to all) ownership data of firms in the all involved industries.
3.A Proofs

Derivation of equation (3.9). Equation (3.8) can be expressed in matrix form as

\[ m = \frac{\alpha}{\epsilon} [I + \hat{L}^{-1} s^{-1} (L - \hat{L}) \hat{s}] i + \frac{1 - \alpha}{\epsilon} [I + \hat{L}^{-1} (L - \hat{L})] s - \alpha [\hat{L}^{-1} s^{-1} (L - \hat{L}) \hat{s}] m, \]

(3.A.1)

where \( i \) is the summation vector of ones.

All the three terms in square brackets can be further simplified as

\[ I + \hat{L}^{-1} s^{-1} (L - \hat{L}) \hat{s} = \hat{s}^{-1} \hat{L}^{-1} L \hat{s}, \quad I + \hat{L}^{-1} (L - \hat{L}) = \hat{L}^{-1} L, \]

\[ \hat{L}^{-1} s^{-1} (L - \hat{L}) \hat{s} = \hat{s}^{-1} (\hat{L}^{-1} L - I) \hat{s}. \]

(3.A.2)

Plugging results from (3.A.2) in (3.A.1) we obtain

\[ m = (1/\epsilon) \left[ \alpha \hat{s}^{-1} \hat{L}^{-1} L s + (1 - \alpha) \hat{L}^{-1} L s \right] - \alpha \hat{s}^{-1} (\hat{L}^{-1} L - I) \hat{s} m. \]

(3.A.3)

With definitions \( Q \equiv \hat{s}^{-1} (I - \hat{L}^{-1} L) \hat{s}, x_1 \equiv \hat{s}^{-1} \hat{L}^{-1} L s, \) and \( x_2 \equiv \hat{L}^{-1} L s \) the equation (3.A.3) yields (3.9).

Proof of Theorem 3.1. From economic point of view, the lower limit of \( \alpha \) corresponds to zero price-cost margins for all firms \( i = 1, \ldots, n \). Using \( x_2 = \hat{s} x_1 \), (3.9) can be rewritten as \( m = (1/\epsilon) (I - \alpha Q)^{-1} [\alpha I + (1 - \alpha) \hat{s}] x_1 \), which together with (3.10) imply that markups are zero both with and without PCO when \( \alpha I = -(1 - \alpha) \hat{s} \), or, equivalently, when \( \alpha = -(1 - \alpha) s_i \). This implies that market shares should be equal, thus plugging \( s_i = s = 1/n \) in the last condition gives \( \alpha = -1/(n - 1) \). The highest possible price-cost margins are those of the monopolist (perfect cartel) that equal the inverse of the price elasticity of demand, which is the case when \( \alpha = 1 \) in (3.9), since then \( m = (1/\epsilon) (I - Q)^{-1} x_1 = (1/\epsilon) (I - I + \hat{s}^{-1} \hat{L}^{-1} L s)^{-1} x_1 = (1/\epsilon) i \). The same holds for the no PCO case in (3.10), hence the (economic) upper bound of the conjectural elasticity both with and without PCO is \( \alpha = 1 \).

Proof of Theorem 3.2. For simplicity denote \( A \equiv \hat{L}^{-1} L \) and \( B \equiv A - I \). Premultiplication of (3.9) by \( \epsilon \hat{s} \) yields \( \epsilon s m = \alpha A s + (1 - \alpha) \hat{s} A s - \epsilon A B s m \). Add to and subtract from the right-hand side (rhs) of the last equation \( \alpha s + (1 - \alpha) \hat{s} s \), which in turn is equal to \( \epsilon s m^0 \) as follows from (3.10), where \( m^0 \) is the vector of markups in the no PCO case provided that \( a^0 = \alpha, \epsilon^0 = \epsilon, n^0 = n \), and \( s^0 = s \). This yields
\[ \varepsilon \hat{s}m = \alpha Bs + (1 - \alpha)\hat{s}Bs - \varepsilon \alpha B\hat{s}m + \varepsilon \hat{s}m^0. \] Hence,

\[ \varepsilon \hat{s}(m - m^0) = \alpha Bs + (1 - \alpha)\hat{s}Bs - \alpha \varepsilon B\hat{s}m = (1 - \alpha)\hat{s}Bs + \alpha B\hat{s}(1 - \varepsilon m). \quad (3.A.4) \]

For \( \alpha = 0 \), the rhs in (3.A.4) is \( \hat{s}Bs \) and its \( i \)-th element is strictly positive if and only if firm \( i \) owns shares in rival(s). Hence, with \( \alpha = 0 \) we have \( m_i > m_i^0 \) for all \( i \) with PCO holdings, otherwise \( m_i = m_i^0 \). Equally, for \( \alpha \in (0, 1) \) and \( m_i < 1/\varepsilon \), the \( i \)-th element in the rhs of (3.A.4) is positive if \( i \) has shareholdings, implying again that \( m_i > m_i^0 \) for all firms \( i \) with PCO holdings. Note that if \( \alpha = 1 \) (or equivalently \( \varepsilon = \varepsilon m \)), the rhs in (3.A.4) is a zero vector, hence we get an expectable outcome of \( m = m^0 \). \[ \blacksquare \]