Modeling and analysis of a hysteretic deformable mirror with electrically coupled actuators

A.E.M. Schmerbauch¹,*, A.I. Vakis², R. Huisman³ and B. Jayawardhana¹

Abstract—In this paper, we present our novel concept of deformable mirror which enables a high density 2D actuation array through multiplexing. We investigate the resulting electrical coupling effect in the neighboring actuators through finite element analysis. These insights are used to consider the behavior within the least-square fitting performed to achieve wavefront error corrections.

I. INTRODUCTION

Deformable mirrors (DMs) are instruments used for the correction of light wavefront errors in telescope images and are applied in adaptive optical systems. In the search for planets outside our solar system (so called exoplanets), a critical technology development requirement is to advance DM technology. Astronomers have been able to find a few thousand exoplanets in orbit of distant stars, typically using indirect methods, where variations in the light coming from the star reveal the presence of an accompanying planet. To be able to characterize the exoplanet atmosphere, it is necessary to directly observe the light coming from the planet. In this case, the extreme contrast between the brightness of the star and the planet requires unprecedented wavefront control. Future large space telescopes, dedicated to direct imaging of exoplanets like LUVOIR [1], will rely on DMs to achieve clean and stable wavefronts over long periods of time, while reducing the negative effect of star light speckles originating from small surface shape errors of the optics. By increasing the number of actuators used to deform the mirror surface, the higher the quality of the obtained images. Current developments can reach 100 to 6000 actuators but rarely higher [2]. Nevertheless, scaling up the number of actuators will lead to design challenges because one of the major limitations for employing DMs on a space mission is the associated cable harness, electronics and related reliability issues. If every actuator has to be driven continuously to hold a particular position, an assigned channel with high voltage amplifier and converter is required which leads to a massive hardware of electronical components. To reduce the number of wires needed to connect and actuate, a new concept of DM called Hysteretic Deformable Mirror (HDM) is presented in [3] which relies on multilayered piezoelectric actuators with high hysteresis that are electronically addressed by time-division multiplexing. A schematic representation of the HDM can be found in Figure 1. The actuators are realized with a purposely designed piezoelectric material to exhibit asymmetric butterfly loops with remnant deformation, and intersecting electrodes to address the particular areas. Figure 2 shows an example of such a butterfly loop collected from previous material tests.

Fig. 1. A schematical representation of the hysteretic deformable mirror which shows the concept in 3D (left) and exploded view (right). The integral components are the mirror surface, an isolation layer, parallel electrodes, piezoelectric layers and perpendicular electrodes.

Fig. 2. Asymmetric butterfly hysteresis loop with remnant deformation. The axial displacement was measured while a certain voltage was applied.

Based on this concept, we can strive for arrays with high numbers of actuators because reducing the cable harness consequently increases the reliability of the DM. Relying on the interconnection electrode design of the HDM decreases the numbers of cables needed to wire the electrodes of the mirror which will enable high density arrays in the long term. However, relying on such a concept poses further challenges. Based on the specific electrode layout, actuators become electrically coupled and will complicate the control process.

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for creating the desired wavefront correction. The modeling and control for the associated butterfly hysteresis behavior is presented in [4] and [5]. In the following, we will present finite element method (FEM) simulations that show the electrical coupling between the actuators, and propose one approach how to consider this behavior in the control system and in the surface fitting process to achieve a wavefront error correction.

This paper is organized as follows. In Section II we explain the electrical coupling between certain actuators while actuating the mirror, and validate the behavior with FEM simulations. We present a plate model in Section III for modeling the reflective mirror surface by consideration of the particular interconnection layout of the electrodes. In Section IV we use this plate model to finally fit the mirror to Zernike polynomials, which are commonly used to represent wavefronts for aberration correction in optical systems, and consider the electrical coupling behavior between the actuators. All theoretic aspects are elucidated by means of a 3×3 actuator array. Conclusions are given in Section V.

II. ELECTRICAL COUPLING BETWEEN ACTUATORS

![3×3 actuator array visualization as electrical circuit.](image)

Fig. 3. Hysteretic deformable mirror for a 3×3 actuator array visualized as electrical circuit.

A. The origin of the electrical coupling

The HDM benefits from time-division multiplexing and subsequently, provides a very simple electrode layout. The top and bottom electrodes lie perpendicular to each other to form intersecting areas of the electrodes presenting an actuator. The mirror addressing is bundled by sharing the same electrodes for actuators along a line. The voltage is transmitted over a shared top electrode while the corresponding bottom electrode for the desired actuator is grounded. Surrounding electrodes are floating, meaning free of potential and able to let a potential appear on a single electrode when all other potentials on the other electrodes are held constant. Considering this configuration, the neighboring actuators which share the same electrode with the actuated one will be affected due to the spillover of the electrical charges and also experience mechanical deformation that diminishes with increasing distance due to interaction effects. We refer to this effect as the electrical coupling of the actuators. To exemplify the electrical coupling, we can consider a simple 3×3 array of our HDM and its associated electrical circuit diagram which is shown in Figure 3. Each actuator is represented as a parallel-plate capacitor having a capacitance C. When one actuator is addressed by application of an electrical potential V, the quantity of charge density on the plates Q is given by

\[ Q = CV. \]

Due to constant thickness and size ratios, we assume that every actuator possesses the same C and without loss of generality, let us consider the case when \( C = 1 \). Following from this, Equation (1) simplifies to \( Q = V \). Due to the fact that certain actuators share either top or bottom electrodes, it is implied that neighboring electrodes along a line share the introduced charges on the plates. An equilibrium of charges will build up and at this equilibrium, the plates of neighboring actuators become charged with the charges \(-Q/2\) and \(+Q/2\); this results in an additional deformation on the intersecting areas. Electrical coupling can be further illustrated by considering the 3×3 actuator array. Addressing the central actuator affects the direct neighbors (midline actuators) as well as the remaining ones (diagonal actuators). Nevertheless, with diagonal actuators the transferred charges are much smaller due to the high number of intersections between electrodes, and consequently the electrical coupling here is negligible.

B. Simulation results

FEM simulations have been performed to simulate the electric field in the piezoelectric array and the actuator response. When an electric potential is placed on one of the top electrodes, while one of the bottom electrodes is connected to the ground, there is a strong electric field at the interconnection (Figure 4), which will yield a piezoelectric response and deformation of the central actuator (Figure 5 and Figure 6).

The FEM simulation and the model are set up with the dimensions, material characteristics, and boundary conditions summarized in Table I. Analysis of the results shows that the electrical coupling is present. The midline electrodes which surround the central actuator reveal a smaller deformation as well. Diagonal actuators are minimally affected and exhibit a very low deformation.

The behavior of electrically coupled actuators has an impact on accurate shape control and needs to be considered in the fitting process. Although it can be expected that due to the limited field strength observed at the neighboring actuators, together with the strongly nonlinear nature of the remnant deformation, unacceptable high shape changes at other actuators are avoided, but nevertheless we resolve this and continue with modeling the facesheet and describe how to consider the electrical coupling in the determination of required pressures to fit the mirror surface to Zernike polynomials.

III. PLATE MODEL WITH INTERCONNECTION LAYOUT

An influence function defines the deformed mirror shape resulting by the actuation of each actuator. Besides their char-
Fig. 4. FEM simulation showing the distribution of the electric field in the xy-midplane of a 3×3 actuator array. An electrical potential of 100 V was applied to the mid-top electrode while the mid-bottom electrode was grounded.

Fig. 5. FEM simulation showing the deformation of the central actuator after actuation and the electrical coupling to the neighboring actuators in an isometric view. The maximum displacement of the center amounts to 31 nm. This deformation was achieved by applying an electrical potential of 100 V to the mid-top electrode while the mid-bottom electrode was grounded.

Fig. 6. FEM simulation showing the deformation of the central actuator and its two neighboring actuators in a cross-section view of the xz-plane. The maximum displacement of the center amounts to 31 nm. This deformation was achieved by applying an electrical potential of 100 V to the mid-top electrode while the mid-bottom electrode was grounded.

Table I

<table>
<thead>
<tr>
<th>Electrodes</th>
<th>Width</th>
<th>1 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spacing (center to center)</td>
<td>2 mm</td>
<td></td>
</tr>
<tr>
<td>Piezoelectric layer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Material</td>
<td>PZT-5H</td>
<td></td>
</tr>
<tr>
<td>Thickness</td>
<td>0.5 mm</td>
<td></td>
</tr>
<tr>
<td>Diameter</td>
<td>10 mm</td>
<td></td>
</tr>
<tr>
<td>Boundary conditions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electrodes</td>
<td>Thin layer boundary condition</td>
<td></td>
</tr>
<tr>
<td>Center top electrode</td>
<td>Grounded</td>
<td></td>
</tr>
<tr>
<td>Center bottom electrode</td>
<td>Electrical potential of 100 V</td>
<td></td>
</tr>
<tr>
<td>Remaining electrodes</td>
<td>Floating potential</td>
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<tr>
<td>Mesh</td>
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<td>Domain elements</td>
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<tr>
<td>Boundary elements</td>
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<tr>
<td>Edge elements</td>
<td>3456</td>
<td></td>
</tr>
<tr>
<td>Number of DOFs solved for</td>
<td>1247337</td>
<td></td>
</tr>
</tbody>
</table>

In order to increase the modeling accuracy for the HDM and give an exact definition of the electrode shape that exerts a pressure on the mirror surface, we briefly explain our model for calculating the influence functions whose complete description is out of the scope of this work and refer the interested reader to [14].

Given the concept of the HDM, the electrodes have an interconnection layout creating square areas lying under a thin circular facesheet. The actuators are separated by a specified distance. To model this, we consider the Poisson equation and incorporate the square shape of the electrodes during integration. Therefore, each actuator is separated into several areas which can be described by a coordinate transformation using the Cartesian coordinates as well as the radial and angular limits. It is assumed that the thickness of electrodes can be neglected and the actuators modeled as springs in parallel to a force source which creates pressure on the facesheet.

A. Determination of influence matrix

We obtain a mechanical model to describe the relation between the facesheet deflection z and the pressures q applied by the actuators. We consider the Poisson equation [15]

\[ \nabla^2 z = -\frac{q}{T} \]  \hspace{1cm} (2)

for small surface displacements for a membrane under tension T. Incorporating the HDM’s particular arrangement of the electrodes into the solution to Poisson’s equation in polar coordinates \( (r, \phi) \) leads to

\[ z(r, \phi, \bar{r}, \bar{\phi}) = D \int_A \mathcal{F}(r, \phi, \bar{r}, \bar{\phi}) q(\bar{r}, \bar{\phi}) d\bar{\phi} d\bar{r} \]  \hspace{1cm} (3)
with

\[ A = \{(\bar{r}, \bar{\phi}) | \phi_1(\bar{r}) \leq \bar{\phi} \leq \phi_2(\bar{r}), 0 \leq \bar{r} \leq 1\} \]  

where

\[ \mathcal{F}(r, \phi, \bar{r}, \bar{\phi}(\bar{r})) = \begin{cases} f_1(r, \phi, \bar{r}, \bar{\phi}(\bar{r})) & \text{if } 0 < \bar{r} < r \\ f_2(r, \phi, \bar{r}, \bar{\phi}(\bar{r})) & \text{if } r < \bar{r} < 1 \end{cases} \]  

and \( z(r, \phi) \) is the out-of-plane displacement of the thin facesheet, \((\bar{r}, \bar{\phi})\) are the integration variables, \(q(\bar{r}, \bar{\phi})\) are the distributed forces over the particular electrode area, and constant \(D = a^2/T\) contains the relation between the facesheet radius \(a\) and the surface tension. Edge deflection and slopes are both equal to zero.

The solutions of Equation (3) can be written in matrix form [12] and the surface deflection on a specific point on the facesheet calculated according to

\[ z(r, \phi) = \sum_{j=1}^{N_e} \mathcal{M}(r, \phi, j)q(r, \phi, j) \]  

where \( \mathcal{M} \) represents the coefficients derived from the solutions of the Poisson equation, \(q(r, \phi, j)\) are piecewise constant pressures exerted on the respective \(j\)-th electrode, and \(N_e\) is the total number of electrodes.

**B. Actuator model**

The stiffness of the facesheet is responsible that actuators become coupled. Generally, a mechanical coupling between neighboring actuators, i.e., inter-actuator coupling, can improve the surface accuracy and smooth the deformation on the facesheet [16]. We consider the inter-actuator coupling by modeling each actuator simplified as a spring in parallel with a force source that interact with the facesheet over a specified area (Figure 7).

![Fig. 7. Simplified actuator model, modeled by a stiffness \(k\) in parallel to a force source over an area \(\Phi\).](image)

It is assumed that all the actuators are identical. The pressure term \(q(r, \phi)\) can be split so that it captures both components in terms of stiffness and force source over that area. Consequently, the relation from Equation (6) may be described by

\[ z(r, \phi) = \sum_{j=1}^{N_e} \mathcal{M}(r, \phi, j) \left( \Phi_P j(V) - k_j \bar{z}_j \right) \]  

with

\[ \Phi_P j(V) = Y_j \Phi_T j(V) \]  

and

\[ \bar{z}_j := \frac{\sum_{i \in E_j} z_i / n_e}{A_e} \]  

where \(\Phi_P j(V)\) denotes the Preisach operator capturing the highly nonlinear hysteresis of the actuators in regard to the total deformation in relation of the initial thickness dimension, \(Y_j\) the diagonal matrix containing the Young’s modulus, \(\Phi_T j(V)\) the longitudinal elongations of the actuators, \(k_j\) the diagonal stiffness matrix containing the actuators’ stiffness, and \(\bar{z}_j\) the mean surface deflection above the respective electrode with area \(A_e\) calculated by means of \(n_e\) surface displacement points \(z_i\) on a specific position within the electrode area. A model for describing the electric-field dependence on the strain in piezoelectric materials purposely designed to exhibit loops with remnant deformation was presented in [4] based on the use of the Preisach operator. The complete definition of this operator can be found in [5].

**IV. Results**

**A. Least-square fitting with consideration of electrical coupling**

To represent the optical imperfections which can occur in adaptive optical systems, we use a common representation of wavefront aberrations which is a series of Zernike polynomials. Zernike polynomials are defined on a unit circle of wavefront aberrations which is a series of Zernike polynomials. The set of polynomials [17] can be given by

\[ Z^m_n(r, \theta) = R^m_n(r) \sin(m \theta) \quad \text{for } m < 0 \]  

\[ Z^m_n(r, \theta) = R^m_n(r) \cos(m \theta) \quad \text{for } m \geq 0 \]  

where

\[ R^m_n(r) = \frac{(n-m)!}{2^{(n-m)/2} S! [(n+m)/2-S]! [(n-m)/2+S]!} \]  

Each displacement of a respective point on the facesheet which is defined by \((r, \phi)\) is fit to the corresponding point on the Zernike polynomial so that we can calculate the required pressure terms which can generate the desired shape. An over-determined set of equations is solved in the least-square sense resulting in

\[ \Phi = (\mathcal{M}^T \mathcal{M})^{-1} \mathcal{M}(z_d + \mathcal{M} k \bar{z}_d) \]  

which aims at minimizing the root-mean-square deviation (RMSD) between the two quantities.

The obtained results are generalized measures to the desired pressures for each actuator of the HDM. Taking into account the existing electrical coupling, a separation between direct and indirect introduced pressures has to be performed. The direct pressure is exerted by the addressed actuator, while the indirect pressure is caused by the neighboring actuators which are electrically coupled. Therefore, following
of equations in matrix form can be given by

\[
\begin{bmatrix}
\Phi_{p_1} \\
\Phi_{p_2} \\
\vdots \\
\Phi_{p_{N_e}}
\end{bmatrix} = \begin{bmatrix}
\lambda_{p_1} & \lambda_{p_2} & \ldots & \lambda_{p_{N_e}} \\
\lambda_{p_2} & \lambda_{p_3} & \ldots & \\
\vdots & \vdots & \ddots & \\
\lambda_{p_{N_e-1}} & \lambda_{p_{N_e-2}} & \ldots & \lambda_{p_{N_e}}
\end{bmatrix}
\begin{bmatrix}
\Phi^*_{\lambda_1} \\
\Phi^*_{\lambda_2} \\
\vdots \\
\Phi^*_{\lambda_{N_e}}
\end{bmatrix}
\]  

(14)

where \( \Phi \) is a vector containing the constant terms from the least-square fitting, \( \Lambda \) a \( N_e \times N_e \) matrix including the coupling factors (\( \lambda_{p_j} \) is 1 if the actuator is actuated directly) and \( \Phi^* \) a vector with the unknown pressures. If the matrix \( \Lambda \) is square and has full rank, then the system has a unique solution given by

\[
\Phi = \Lambda^{-1} \Phi^*
\]

(15)

where \( \Lambda^{-1} \) is the inverse of \( \Lambda \).

As an example according to the \( 3 \times 3 \) actuator array, with numbering beginning at the center and continuing with the right upper corner clockwise from 1 to 9, we achieve the following matrix

\[
\begin{bmatrix}
\Phi_{p_1} \\
\Phi_{p_2} \\
\Phi_{p_3} \\
\Phi_{p_4} \\
\Phi_{p_5} \\
\Phi_{p_6} \\
\Phi_{p_7} \\
\Phi_{p_8} \\
\Phi_{p_9}
\end{bmatrix} = \begin{bmatrix}
\lambda_{p_1} & 0 & \hat{\lambda}_n & 0 & \hat{\lambda}_n & 0 & \hat{\lambda}_n & 0 & \hat{\lambda}_n \\
0 & \lambda_{p_2} & \hat{\lambda}_n & \hat{\lambda}_n & 0 & 0 & \hat{\lambda}_n & \hat{\lambda}_n & \hat{\lambda}_n \\
\hat{\lambda}_n & \hat{\lambda}_n & \lambda_{p_3} & \hat{\lambda}_n & 0 & 0 & \hat{\lambda}_n & 0 & 0 \\
0 & \hat{\lambda}_n & \hat{\lambda}_n & \lambda_{p_4} & \hat{\lambda}_n & \hat{\lambda}_n & 0 & 0 & 0 \\
\hat{\lambda}_n & 0 & 0 & \hat{\lambda}_n & \lambda_{p_5} & \hat{\lambda}_n & \hat{\lambda}_n & \hat{\lambda}_n & \hat{\lambda}_n \\
0 & 0 & 0 & \hat{\lambda}_n & \hat{\lambda}_n & \lambda_{p_6} & \hat{\lambda}_n & \hat{\lambda}_n & 0 \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots
\end{bmatrix}
\begin{bmatrix}
\Phi^*_{\lambda_1} \\
\Phi^*_{\lambda_2} \\
\Phi^*_{\lambda_3} \\
\Phi^*_{\lambda_4} \\
\Phi^*_{\lambda_5} \\
\Phi^*_{\lambda_6} \\
\Phi^*_{\lambda_7} \\
\Phi^*_{\lambda_8} \\
\Phi^*_{\lambda_9}
\end{bmatrix}
\]

(16)

The coupling behavior is known from the presented FEM simulations in Section II-B and the coupling factors \( \hat{\lambda}_n \) can be extracted as estimation for solving this system. The resulting pressures denoted with \( \Phi^* \) are needed to control the mirror because they assume the initial state for the input. Together with the electrical coupling behavior the actuators will create the Zernike polynomial. As an example, Figure 8 shows the fitted mirror surface to the Zernike polynomial \( Z_{1}^{1} \) with a RMSD of 7.17%. The calculation of coefficients and the simulations were performed using MATLAB R2019a. The active area was restricted to 70% of the mirror diameter. The stiffness of the actuators were estimated based on general available information of piezoelectric actuators. Low order Zernike polynomials can still be fitted with a low density array but the higher the degree of the polynomial the higher the resulting RMSDs. This can be demonstrated by Figure 9 which shows the fitted mirror surface to the Zernike polynomial \( Z_{2}^{1} \) with a RMSD of 28.63%. By means of high density arrays a higher fitting accuracy and reduction of the RMSD can be reached.
V. CONCLUSION

We discussed the electrical coupling between the actuators in DMs which underly the concept of the hysteretic deformable mirror and have validated this behavior with FEM simulations. An approach was presented to model the mirror facesheet including the particular interconnection layout of the electrodes. The electrical coupling was analysed and integrated based on the known information. With this, it was possible to simulate the fitting of the mirror surface to Zernike polynomials. It serves as the basis method to calculate the initial pressure of each actuator for controlling and creating desired wavefront error corrections with this mirror.

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