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Output regulation of Euler-Lagrange systems based on error and velocity feedback

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Abstract: Based on a certainty equivalence property, we propose an adaptive internal model control law that solves global robust output regulation of uncertain Euler-Lagrange (EL) systems based only on error (or relative position) and velocity feedback. The proposed controller does not require *apriori* knowledge of reference signal and its derivatives, which are commonly assumed in literature. It enables a self-learning mechanism of the closed-loop EL systems where the adaptive internal model-based controller is able to learn the desired trajectories and adapt itself to the uncertain plant parameters. Furthermore, the analysis offers insights to the design of internal model-based output regulation for multivariable nonlinear systems with uniform vector relative degree two.

Key Words: Euler-Lagrange systems; trajectory tracking; output regulation; internal model principle; certainty equivalence principle

1 Introduction

For the past decades, major progresses have been achieved in the trajectory tracking control problem of EL systems with a broad of applications in electro-mechanical systems including high-precision mechatronics systems and advanced robotic systems. We refer to the monographs [1, 2, 3] for a general overview of progresses in this field. In early studies, one may refer to [4, 5, 6] for a variety of adaptive inverse dynamics control methods, and refer to [7, 8, 9, 10, 11, 12] for passivity-based adaptive control methods. Recent works that are relevant for the present paper and based on advances in nonlinear control theory are [13, 14, 15, 16, 17, 18] with relevant references thereof.

In all the aforementioned results, the output regulator relies on the *apriori* knowledge of the reference signal and its derivatives, which become essentially the feedforward part of the tracking controller. Consequently the high-level controller, which pre-computes the reference signals to solve and optimize higher-level tasks, is not independent/separated from the low-level tracking controller [19, 20]. In other words, the current output regulator design does not admit a self-learning mechanism of the references that enables a separation principle between the high-level and low-level controller.

For enabling such self-learning capability, we embed the classical internal model principle (see [21, pp. 216]) in the design of tracking controller. Generally speaking, the internal model-part of our controller is responsible in predicting the reference signals that can subsequently be used in the output regulator. This allows us to realize plug-and-play mechanism between the high-level and low-level controllers, as long as, they agree on the exosystems. In other words, a class of exosystems can firstly be defined as common kernels for both controllers, based on which, the high-level controller can use them for task and trajectory planning while the low-level controller employs them in the internal model-based controller.

As an illustrative example, let us consider a basic tracking control of single-link manipulator equipped with camera and encoder sensors to provide displacement and velocity measurements as depicted in Fig. 1. In this figure, the relative position or the displacement between the end-effector and moving target effector can be measured by a camera. Based only on these measurements, our proposed controller will then be able to generate the desired trajectory and to track it robustly with respect to parameter uncertainties. In this perspective, the use of teaching pendant, which records all the motions of the target robotic behaviour, is no longer needed for training robotic systems as commonly used nowadays in industry.

Specifically, we investigate global asymptotic tracking of EL systems based only on the use of error and velocity feedback in order to track any reference signals generated by known exosystems and be adaptive to system parameter uncertainties. For a class of fully actuated uncertain EL systems, we reformulate this problem as a global output regulation problem following the approach in [22, Chapter 7] for strict-feedback nonlinear systems. Our main contribution is to propose an adaptive internal model approach to achieve a constructive and smooth control law for solving the aforementioned problem. Our study also provides additional practical insights to the nonlinear output regulation problems, especially for multivariable strict-feedback nonlinear systems, such as, relevant studies on global output regulation in [22] and the semiglobal scenarios in [23, 24].

Outline: The rest of this paper is organized as follows. Section 2 formulates the concerned output regulation problem and lists some standing assumptions. Section 3 presents the main result of this paper. Section 4 illustrates the effectiveness of the proposed controllers on a two-link manipulator. Section 5 ends the paper with some conclusions.

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Notation: $\|\cdot\|$ is the Euclidean norm of a vector in $\mathbb{R}^n$ or the induced matrix norm in $\mathbb{R}^{n\times m}$. I is identity matrix of appropriate dimension. For matrices $A_i \in \mathbb{R}^{n\times m}$, $i = 1,\ldots,k$, block $\mathrm{diag}(A_1,\ldots,A_k)$ denotes a block matrix with its i-th diagonal entry to be A_i and all other entries zero.

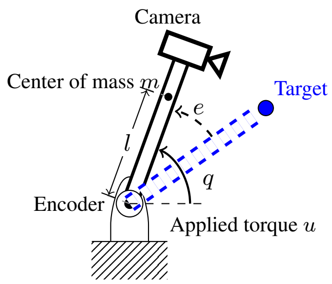

Fig. 1: Illustrative example of output regulation problem of single-link manipulator using error and velocity feedback.

2 Formulation and Background

Consider n-dimensional EL systems described by

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = u \tag{1}$$

where $q(t) \in \mathbb{R}^n$ is the generalized position; $\dot{q}(t) \in \mathbb{R}^n$ is the generalized velocity; $u(t) \in \mathbb{R}^n$ is the vector of control input; $H : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is the inertia matrix; $C : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is the Coriolis and centrifugal force matrix-valued function; $g : \mathbb{R}^n \to \mathbb{R}^n$ is the gradient of the potential energy that typically represents the gravitational forces/torques or spring torsion/forces, with $g(0) = 0$.

For the system in (1), we define the tracking error by

$$e = q - q_{\text{ref}}, \tag{2}$$

where the reference output $q_{\text{ref}}(t) \in \mathbb{R}^n$ is assumed to be generated by the following linear exosystem with a nonlinear output map

$$\dot{v} = Sv, \quad q_{\text{ref}} = Q(v) \tag{3}$$

with the exosystem state $v(t) \in \mathbb{R}^{n_v}$. For technical convenience, we assume that all the eigenvalues of exosystem matrix S are distinct and lie on the imaginary axis, and $v(0) \in \mathbb{V} \subset \mathbb{R}^{n_v}$ for a specified compact invariant set $\mathbb{V}$.

2.1 Standing Assumptions

The following assumptions are standing in literature, see, e.g., [2, pp. 22-24] for **H1** and **H2**, and see [22] for **H3**.

H1 The inertia matrix $H(q)$ is uncertain and positive definite, i.e., there exist constants $c_{\min}, c_{\max} > 0$ such that

$$c_{\min} I \le H(q) \le c_{\max} I, \quad \forall q \in \mathbb{R}^n.$$

Moreover, the matrix $\frac{\mathrm{d}}{\mathrm{d}t}H(q(t),\dot{q}(t)) - 2C(q(t),\dot{q}(t))$ is skew-symmetric, where $\frac{\mathrm{d}}{\mathrm{d}t}H(q,\dot{q}) = \sum_{j=1}^{n}\frac{\partial H}{\partial q_j}\dot{q}_j$.

H2 There are smooth functions $a(\cdot) \in \mathbb{R}^p$ and $Y(\cdot) \in \mathbb{R}^{n\times p}$ such that for any reference $q_{\text{ref}}(t) \in \mathbb{R}^n$ with a bias $q_{\text{ref},0}$ and continuous derivatives $\dot{q}_{\text{ref}}(t)$ and $\ddot{q}_{\text{ref}}(t)$,[1]

$$\begin{aligned} &H(q_{\text{ref}})\ddot{q}_{\text{ref}} + C(q_{\text{ref}},\dot{q}_{\text{ref}})\dot{q}_{\text{ref}} + g(q_{\text{ref}}) \\ &= Y(q_{\text{ref}} - q_{\text{ref},0}, \dot{q}_{\text{ref}}, \ddot{q}_{\text{ref}})a(w, q_{\text{ref},0}), \end{aligned} \tag{4}$$

where Y is the so-called dynamic regressor matrix and $a(\cdot)$ is defined varying in a given compact set $\mathbb{A} \subset \mathbb{R}^p$ that contains uncertain physical parameters $w \in \mathbb{R}^{n_w}$ and the unknown reference bias $q_{\text{ref},0} \in \mathbb{R}^n$.

H3 Each entry of $Q(v)$ is a nonlinear polynomial in v.

[1] In this paper, for any reference $q_{\text{ref}}(t) = q_{\text{ref},0} + \bar{q}(t)$ to be written as sum of a constant part $q_{\text{ref},0}$ and a time-varying part $\bar{q}(t)$, $q_{\text{ref},0}$ is said to be a bias of $q_{\text{ref}}(t)$.

Remark 2.1 If constant vector $q_{\text{ref},0} = 0$, condition **H2** is the standard parameter linearization property for EL systems, see [2, Chapter 2] and [1, Chapter 9] for instance.

2.2 Problem Definition

Problem 2.1 *[Global Adaptive Output Regulation for Fully-Actuated EL Systems]* Design a *smooth* dynamic controller of the form

$$\begin{aligned} \dot{x}_c(t) &= f(x_c(t), e(t), \dot{q}(t)), \\ u(t) &= h(x_c(t), e(t), \dot{q}(t)) \end{aligned} \tag{5}$$

such that, for every initial condition $v(0) \in \mathbb{V}$, $q(0), \dot{q}(0) \in \mathbb{R}^n$ and for every $x_c(0)$, the closed-loop system (1) and (5) satisfies,

- the trajectory exists for all $t \ge 0$ and is bounded over $[0,\infty)$; and
- the tracking error $e(t)$ satisfies $\lim_{t\to\infty} e(t) = 0$.

In literature, there are mainly two methodologies for tracking control of EL systems. One is the adaptive inverse dynamics control as developed in [4, 5, 6] and many others. The other is the passivity-based adaptive control such as those developed in [7, 8, 10, 12]. All the aforementioned studies are based on "feedforward" control method. That is, the availability of information on $q(t)$, $q_{\text{ref}}(t)$ and their derivatives is prerequisite, and instead of (5), the control law has generically the following form

$$\begin{aligned} \dot{x}_c &= f(x_c, q, \dot{q}, q_{\text{ref}}, \dot{q}_{\text{ref}}, \ddot{q}_{\text{ref}}), \\ u &= h(x_c, q, \dot{q}, q_{\text{ref}}, \dot{q}_{\text{ref}}, \ddot{q}_{\text{ref}}). \end{aligned} \tag{6}$$

On one hand, the real-time information on $\dot{q}, q_{\text{ref}}, \dot{q}_{\text{ref}}$ and $\ddot{q}_{\text{ref}}$ may not readily available in order to implement (6). Firstly, the velocity $\dot{q}$ may not be accurately obtained through standard encoder systems that provide q. Secondly, the computation of $q_{\text{ref}}, \dot{q}_{\text{ref}}$ and $\ddot{q}_{\text{ref}}$ by the high-level controller requires accurate knowledge of the kinematics of the EL systems whose parameters may be uncertain. Thirdly, we require a common frame of coordinates for defining q and q_{ref} which may not be accessible for industrial robots. These limitations have restricted the wide adoption of (6) beyond bespoke robotic solutions as developed and deployed for space or advanced industrial sectors.

On the other hand, the well-known internal model principle has played a crucial role in the solvability of the tracking problem using output error feedback [21, pp. 216]. The use of internal-model based controller has enabled the controller to recreate the reference trajectories internally within its dynamics [25]. It is able to self-learn the target's dynamical behavior based only on the output error feedback. In combination with adaptive control technique, the controller is able to learn both the target's behavior and the plant dynamics. Correspondingly, we will adopt these two approaches to solve the global adaptive output regulation problem for fully-actuated EL systems.

One may consider to directly apply the internal model-based control framework as presented in [26] for output regulation of lower triangular nonlinear systems. However it proves to be a non-trivial task as shown in the following example using a single-link manipulator.

Example 2.1 Consider a single-link manipulator as shown in Fig. 1 and modeled by

$$J\ddot{q}(t) + mgl\cos(q(t)) = u(t) \tag{7}$$

where m is the mass, J is the moment of inertia about the joint axis, l is the distance from its axis of rotation to the center of mass. Suppose that it should track a reference signal generated by (3). Following [22, pp. 83], the so-called zero-error constrained input can be written as

$$u^\star(v,w) = J\frac{\partial^2 Q(v)}{\partial v^2}Sv + mgl\cos(Q(v)) \tag{8}$$

which is obtained from (2), (3) and (7), where $w = [J \ \ mgl]^\intercal$ collects all physical parametric uncertainties. From equation (8), we observe that the zero-error constrained input function $u^\star(v,w)$ is not a polynomial in v. Moreover, the popular internal model design conditions of [26, 27, 28, 29] may not be verifiable. Thus standard internal model based controller as in [26] is not directly applicable to EL systems for global output regulation design. This is the main motivation of the present study for a new internal model approach towards output regulation of EL systems.

3 Main Result

This section is devoted to a constructive solution for the output regulation problem. We begin with addressing a useful certainty equivalence property relating to the designed internal model. Subsequently, we reformulate Problem 2.1 to a global robust adaptive stabilization problem for the relevant augmented system, leading to the whole control law design.

3.1 Internal Model and Certainty Equivalence Property

Firstly, we solve the *regulator equations* (see [30] or [22, Chapter 3]) associated to the systems (1) and exosystem (3). It has a globally defined solution

$$\{q^\star(v), \xi^\star(v), u^\star(v,w)\}$$

with

$$\begin{aligned}
q^\star(v) &= Q(v),\\
\xi^\star(v) &= \frac{\partial Q(v)}{\partial v}Sv,\\
u^\star(v,w) &= H(q^\star(v))z^\star(v) + C(q^\star(v),\xi^\star(v))\xi^\star(v)\\
&\quad + g(q^\star(v)), \ \ z^\star(v) := \frac{\partial \xi^\star(v)}{\partial v}Sv.
\end{aligned} \tag{9}$$

Secondly, we consider the intermediate composite system

$$\begin{aligned}
\dot{v} &= Sv,\\
\dot{q} &= \xi,\\
e &= q - Q(v)
\end{aligned} \tag{10}$$

with ξ as the virtual control input and e as the regulated output.

Lemma 3.1 *Consider system* (10) *under condition **H3** and functions $q^\star(v)$ and $z^\star(v)$ as given in* (9)*. Then the following two properties hold.*

P1 *(Internal Model Design) There exists a canonical internal model of the form*

$$\dot{\eta}(t) = M\eta(t) + N\xi(t) \tag{11}$$

with output ξ in the sense of [22, Definition 6.6], where (M,N) is a controllable matrix pair with M being Hurwitz.
Particularly, it satisfies, for a smooth function $\theta(v) \in \mathbb{R}^\ell$ and a linear output mapping $\Gamma\theta \in \mathbb{R}^n$,

$$\begin{aligned}
\frac{\partial \theta(v)}{\partial v}Sv &= M\theta(v) + N\xi^\star(v),\\
\xi^\star(v) &= \Gamma\theta(v), \ \ \forall v \in \mathbb{V}.
\end{aligned} \tag{12}$$

P2 *(Certainty Equivalence Property) There are linear mappings L_1 and L_2 such that*

$$\begin{aligned}
q^\star(v) &= q_{\text{ref},0} + L_1\theta(v),\\
z^\star(v) &= L_2\theta(v), \quad \forall v \in \mathbb{V}
\end{aligned} \tag{13}$$

where $q_{\text{ref},0}$ is the reference bias. Particularly, if $q^\star(v)$ is unbiased, then $q_{\text{ref},0} = 0$ can be set in (13)*.*

Proof of Lemma 3.1: The proof of **P1** can be found from [31] and [22]. To be self contained, it is given as follows. Consider $\xi^\star(v) = \begin{bmatrix}\xi_1^\star(v) & \cdots & \xi_n^\star(v)\end{bmatrix}^\intercal$. For each $i = 1,\dots,n$, under condition **H3**, $\xi_i^\star(v)$ is polynomial in v. By denoting $\Xi_i(v) = \begin{bmatrix}\xi_i^\star(v) & \frac{d\xi_i^\star(v)}{dt} & \cdots & \frac{d^{(\ell_i-1)}\xi_i^\star(v)}{dt^{(\ell_i-1)}}\end{bmatrix}$, and

$$\Phi_i = \begin{bmatrix} 0 & 1 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & 1\\ c_{i1} & c_{i2} & \cdots & c_{i\ell_i}\end{bmatrix}, \ \ \Psi_i = \begin{bmatrix}1\\0\\ \vdots\\ 0\end{bmatrix}^\intercal, \tag{14}$$

the triple $\{\Xi_i, \Phi_i, \Psi_i\}$ yields a steady-state generator

$$\frac{\partial \Xi_i(v)}{\partial v}Sv = \Phi_i\Xi_i(v), \ \ \xi_i^\star(v) = \Psi_i\Xi_i(v), \ \ \forall v \in \mathbb{V} \tag{15}$$

with output ξ_i in the sense of [22, Definition 6.1].

Based on (15), we can choose any controllable pair (M_i, N_i) with M_i being Hurwitz, and solve the Sylvester equation $T_i\Phi_i = M_iT_i + N_i\Psi_i$ for a unique nonsingular solution T_i (see [32, Theorem 2]). Denote $\ell = \ell_1 + \cdots + \ell_n$,

$$X = \text{block diag}(X_1,\dots,X_n), \ \ Y = \begin{bmatrix}Y_1^\intercal & \cdots & Y_n^\intercal\end{bmatrix}^\intercal \tag{16}$$

where X stands for M, N, T, Φ, Ψ and Γ, respectively, and Y for θ and Ξ, respectively. This leads to generator (12) with similarity transformation $\theta(v) = T\Xi(v)$ and $\Gamma = \Psi T^{-1}$. Finally, it shapes internal model (11).

Second, to show **P2**, consider $q^\star(v) - q_{\text{ref},0} = \bar{q}^\star(v) = \begin{bmatrix}\bar{q}_1^\star(v) & \cdots & \bar{q}_n^\star(v)\end{bmatrix}^\intercal$. For each $i = 1,\dots,n$, denote $\Pi_i(v) = \begin{bmatrix}\bar{q}_i^\star(v) & \frac{d\bar{q}_i^\star(v)}{dt} & \cdots & \frac{d^{(\ell_i-1)}\bar{q}_i^\star(v)}{dt^{(\ell_i-1)}}\end{bmatrix}$ and column stacking vector $\Pi(v) = \begin{bmatrix}\Pi_1(v) & \cdots & \Pi_n(v)\end{bmatrix}^\top$. Then, recalling the definition of Φ and Ψ in (14) and (16), we have the triple $\{\Pi, \Phi, \Psi\}$ to be a generator of $q^\star(v) - q_{\text{ref},0}$. It follows $\Xi(v) = \frac{\partial \Pi(v)}{\partial v} Sv = \Phi\Pi(v)$. Thus, we have

$$\begin{aligned} q^\star(v) - q_{\text{ref},0} &= \Psi\Pi(v) = \Psi\Phi^{-1}\Xi(v) \\ &= \Psi\Phi^{-1}T^{-1}\theta(v) \triangleq L_1\theta(v). \end{aligned}$$

Moreover, in view of the definition of $z^\star(v)$ in (9), and using (15) and (16), we obtain

$$z^\star(v) = \frac{\partial \xi^\star(v)}{\partial v} Sv = \Psi\Phi\Xi(v) = \Psi\Phi T^{-1}\theta(v) \triangleq L_2\theta(v).$$

The proof is complete.

Remark 3.1 The internal model design of **P1** in Lemma 3.1 is due to [31] for a canonical internal model with output ξ. It is used to reproduce the desired steady-state information of ξ for dynamic compensation control of system (10). One has the following detectability question: is it possible to reproduce the state information of q through the internal model state information η? In other words, can we solve $q^\star(v)$ of (9) from $\theta(v)$ satisfying (12)? This question is of interest in the present study for achieving displacement and velocity feedback, i.e., q is not available and only the displacement information e and velocity ξ are available. The property of **P2** in Lemma 3.1 is exploited as a certainty equivalence property for an answer to this question.

Remark 3.2 Lemma 3.1 is crucial for us to achieve output regulation of EL systems as explained using the zero-error constaint input (8) in Example 2.1. Using (9) and (13), it is not difficult to show that

$$\begin{aligned} u^\star(v, w) &= J\frac{\partial^2 Q(v)}{\partial v^2} Sv + mgl\cos(Q(v)) \\ &= Jz^\star(v) + mgl\cos(q^\star(v)) \\ &= JL_2\theta(v) + mgl\cos(q_{\text{ref},0} + L_1\theta(v)) \\ &= \underbrace{\begin{bmatrix} L_2\theta & \cos(L_1\theta) & -\sin(L_1\theta)\end{bmatrix}}_{Y(L_1\theta, \Gamma\theta, L_2\theta)} a(w, q_{\text{ref},0}) \end{aligned}$$

where

$$a(w, q_{\text{ref},0}) = \begin{bmatrix} J \\ mgl\cos(q_{\text{ref},0}) \\ mgl\sin(q_{\text{ref},0}) \end{bmatrix}, \text{ with } w = \begin{bmatrix} J \\ mgl \end{bmatrix},$$

collects all the parametric uncertainties, due to system parameters and reference bias. Hence, in virtue of Lemma 3.1, $u^\star(v, w)$ can be parameterized as

$$u^\star(v, w) = Y(L_1\theta, \Gamma\theta, L_2\theta)a(w, q_{\text{ref},0}).$$

The above equation is of importance for us to achieve the subsequent global adaptive stabilizing control for the augmented system pursued in the rest of this section.

3.2 Global Adaptive Stabilization Design

In what follows, we address a byproduct adaptive stabilization control problem to complete the output regulation design. Toward this end, substituting (11) to (1) gives us the following augmented system

$$\begin{aligned} \dot\eta &= M\eta + N\xi, \\ \dot q &= \xi, \\ H(q)\dot\xi &= u - C(q, \xi)\xi - g(q). \end{aligned}$$

Using the following specific coordinate transformations

$$\begin{aligned} \tilde\eta &= \eta - \theta - Ne, \\ e &= q - Q(v), \\ \bar\xi &= \xi - \Gamma\eta, \end{aligned} \tag{17}$$

we can obtain a translated augmented system of the form

$$\begin{aligned} \dot{\tilde\eta} &= M\tilde\eta + MNe, \\ \dot e &= \bar\xi + \Delta_1(\tilde\eta, e), \\ H(q)\dot{\bar\xi} &= u + \Delta_2(\tilde\eta, e, \bar\xi, v, a) - \rho(\eta)a, \end{aligned} \tag{18}$$

where

$$\begin{aligned} \rho(\eta) &= Y(L_1\eta, \Gamma\eta, L_2\eta), \\ \Delta_1 &= \Delta_1(\tilde\eta, e) = \Gamma\tilde\eta + \Gamma Ne, \\ \Delta_2 &= \Delta_2(\tilde\eta, e, \bar\xi, v, a) \\ &= \rho(\tilde\eta + \theta + Ne)a - HL_2(\tilde\eta + \theta + Ne) \\ &\quad + C(e + Q(v), \bar\xi + \Gamma(\tilde\eta + \theta + Ne)) \\ &\quad \cdot (\bar\xi + \Gamma(\tilde\eta + \theta + Ne)) + g(e + Q(v)), \end{aligned} \tag{19}$$

and $\Delta_1(\tilde\eta, e), \Delta_2(\tilde\eta, e, \bar\xi, v, a)$ satisfy

$$\Delta_1(0,0) = 0, \;\; \Delta_2(0,0,0,v,a) = 0, \;\; \forall v \in \mathbb{V}, a \in \mathbb{A}.$$

System (19) is in a block lower-triangular form with dynamic uncertainties. At this moment, we can use a recursive approach to synthesize a adaptive stabilizer as stated in the following lemma.

Lemma 3.2 *(Global Adaptive Stabilization) For system* (18)*, there is a positive definite matrix $\Lambda \in \mathbb{R}^{n\times n}$ and a smooth matrix-valued function $k : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ such that, under the control law*

$$\begin{aligned} \dot{\hat a} &= -\lambda\rho^\top(\eta)\tilde\xi, \\ u &= -k(\tilde\xi)\tilde\xi + \rho(\eta)\hat a, \;\; \tilde\xi = \Lambda e + \bar\xi \end{aligned} \tag{20}$$

where $\lambda \in \mathbb{R}^{n\times n}$ is an arbitrary positive definite constant matrix, the closed-loop system (18) *and* (20) *is globally asymptotically stable at $(\tilde\eta, e, \tilde\xi) = (0, 0, 0)$.*

Summarized from the above developments, we are ready to state the main theorem of the present study as follows.

Theorem 3.1 *Under assumptions **H1**, **H2** and **H3**, Problem 2.1 is solvable by a smooth control law of the form*

$$\begin{aligned} \dot\eta &= M\eta + N\xi, \\ \dot{\hat a} &= -\rho^\top(\eta)\tilde\xi, \;\; \tilde\xi = \Lambda e + \bar\xi, \;\; \bar\xi = \xi - \Gamma\eta, \\ u &= -k(\tilde\xi)\tilde\xi + \rho(\eta)\hat a \end{aligned}$$

where Λ and k are as in (20) *and $\rho(\eta)$ as in* (19).

Remark 3.3 Note that we only use a single internal model in the controller. Both zero-error constrained input and state are reconstructed by the same generator due to their common frequencies. This method is distinguished from that of [22, Chapter 7.2] for system (1) with a pair of internal models. The design conditions required in [22, Chapter 7.2] relating to EL systems are not verifiable.

Also note that from the proof of Lemma 3.2, the skew symmetric property in **H1** can be replaced by a growth condition on the time derivative of inertia matrix. Thus, the design of Theorem 3.1 is not limited to EL nonlinear systems and it is applicable to a strictly larger class of nonlinear systems than system (1) of the present study. For example, consider the following multivariable system

$$
\begin{aligned}
\dot{v} &= Sv,\\
\dot{x}_1 &= f_1(x_1, v) + b_1(x_1, v)x_2,\\
\dot{x}_2 &= f_2(x_1, x_2, v) + b_2(x_1, x_2, v)u,\\
y &= x_1,\\
e &= y - y_{\mathrm{ref}}(v)
\end{aligned}
$$

in a block lower-triangular form with state $(x_1, x_2) \in \mathbb{R}^n \times \mathbb{R}^n$, input $u \in \mathbb{R}^n$, and external signal $v \in \mathbb{R}^{n_v}$ (as a specific references/disturbances source), having uniform vector relative degree two [33, pp. 220]. It contains the EL system as a special case. Under certain conditions, this output regulation problem can be approached in the framework of [26] with (e, x_2) as the displacement and velocity measurement. Therefore, it is of interest to investigate the same problem by applying the adaptive internal model principle approach proposed in the present study based on strictly relaxed conditions than before.

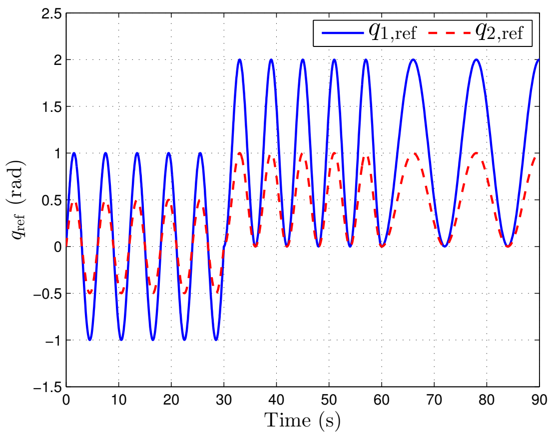

Fig. 2: Reference signal $q_{\mathrm{ref}}(t)$.

4 Illustration

Consider a two-link robot manipulator without gravity described by the EL equation as follows [1, Example 6.2]

$$
\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -\hbar\dot{q}_2 & -\hbar\dot{q}_1 - \hbar\dot{q}_2 \\ \hbar\dot{q}_1 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}
$$

with $q = [q_1 \;\; q_2]^\intercal$ being the two joint angles, $u = [u_1 \;\; u_2]^\intercal$ being the joint inputs, $w = [w_1 \;\; w_2 \;\; w_3]^\intercal$ collects system

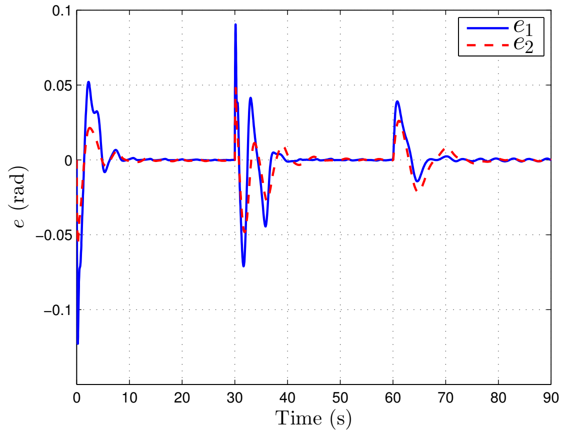

Fig. 3: Position tracking error $e(t)$.

physical parameters, and

$$
\begin{aligned}
H_{11} &= w_1 + 2w_3\cos(q_2),\\
H_{12} &= H_{21} = w_2 + w_3\cos(q_2),\\
H_{22} &= w_2, \quad \hbar = w_3\sin(q_2).
\end{aligned}
$$

Let $q(t) = q_0 + \bar{q}(t)$ with a bias q_0 where $q_0 = \left[\begin{smallmatrix} q_{01} \\ q_{02} \end{smallmatrix}\right]$ and $\bar{q} = \left[\begin{smallmatrix} \bar{q}_1 \\ \bar{q}_2 \end{smallmatrix}\right]$. Then, according to **H2**, we have

$$
\begin{aligned}
Y(q - q_0, \dot{q}, \ddot{q}) &= \begin{bmatrix} \ddot{q}_1 & \ddot{q}_2 & Y_{13} & Y_{14} \\ 0 & \ddot{q}_1 + \ddot{q}_2 & Y_{23} & Y_{24} \end{bmatrix},\\
a &= \begin{bmatrix} w_1 & w_2 & w_3\cos(q_{02}) & -w_3\sin(q_{02}) \end{bmatrix}^\intercal
\end{aligned}
$$

with

$$
\begin{aligned}
Y_{13} &= 2\ddot{q}_1\cos(\bar{q}_2) + \ddot{q}_2\cos(\bar{q}_2) - (2\dot{q}_1\dot{q}_2 + \dot{q}_2^2)\sin(\bar{q}_2),\\
Y_{14} &= 2\ddot{q}_1\sin(\bar{q}_2) + \ddot{q}_2\sin(\bar{q}_2) + (2\dot{q}_1\dot{q}_2 + \dot{q}_2^2)\cos(\bar{q}_2),\\
Y_{23} &= \ddot{q}_1\cos(\bar{q}_2) + \dot{q}_1^2\sin(\bar{q}_2),\\
Y_{24} &= \ddot{q}_1\sin(\bar{q}_2) - \dot{q}_1^2\cos(\bar{q}_2).
\end{aligned}
$$

In this numerical setup, the simulated reference signal $q_{\mathrm{ref}} = \left[\begin{smallmatrix} q_{1,\mathrm{ref}} \\ q_{2,\mathrm{ref}} \end{smallmatrix}\right]$ (rad) as shown in Fig. 2 is

$$
q_{\mathrm{ref}} = \begin{cases} \left[\sin(\frac{\pi}{3}t) \quad \frac{1}{2}\sin(\frac{\pi}{3}t)\right]^\intercal & 0 \le t < 30 \\ \left[1 - \cos(\frac{\pi}{3}t) \quad \frac{1}{2} - \frac{1}{2}\cos(\frac{\pi}{3}t)\right]^\intercal & 30 \le t \le 60 \\ \left[1 - \cos(\frac{\pi}{6}t) \quad \frac{1}{2} - \frac{1}{2}\cos(\frac{\pi}{6}t)\right]^\intercal & 60 \le t \le 90. \end{cases}
$$

The matrices in (14) are specified as follows:

$$
\begin{aligned}
\Psi &= \text{block diag}(\Psi_1, \Psi_2),\\
\Phi &= \begin{cases} \text{block diag}(\Phi_1, \Phi_1), & 0 \le t < 60, \\ \text{block diag}(\Phi_2, \Phi_2), & 60 \le t \le 100, \end{cases}\\
\Phi_1 &= \begin{bmatrix} 0 & 1 \\ -\frac{\pi^2}{9} & 0 \end{bmatrix}, \Phi_2 = \begin{bmatrix} 0 & 1 \\ -\frac{\pi^2}{36} & 0 \end{bmatrix}, \Psi_1 = \Psi_2 = [1 \;\; 0].
\end{aligned}
$$

As a result, the control law in Theorem 3.1 can be designed with parameters:

$$
\begin{aligned}
M_i &= \begin{bmatrix} 0 & 1 \\ -1 & -1.4142 \end{bmatrix}, \quad N_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad i = 1, 2,\\
M &= \text{block diag}(M_1, M_2), \quad N = \text{block diag}(N_1, N_2),\\
\lambda &= 100I, \quad \Lambda = 2I, \quad k(\tilde{\xi}) = \text{diag}(8(4 + \tilde{\xi}_1^2), 8(4 + \tilde{\xi}_2^2)).
\end{aligned}
$$

The simulation is performed with system parameters $w = [3.9\ \ 0.75\ \ 1.125]^{\mathsf{T}}$. All the initial conditions of system state and controller are set to be zero. The position tracking error is shown in Fig. 3.

5 Conclusion

We have studied global robust output regulation of uncertain EL systems and developed an adaptive internal model approach for the problem. A certainty equivalence principle is used to achieve the constructive internal model based smooth controller design. The future direction is two-folds. One is to further investigate the problem as noted in Remark 3.3 for more general nonlinear systems with input disturbances (see [34] for an interesting study) and unknown exosystems. The other is hoped to further approach the coordination problem for multiple EL systems setting as those in [25, 35] based on local displacements and velocity measurements.

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