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Fault Compensation Controller for Markovian Jump Linear Systems

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Abstract: In this paper, we tackle the fault-compensation controller in the context of Markovian Jump Linear Systems (MJLS). More specifically, we propose the design of $\mathcal{H}_\infty$ Fault-Compensation Controllers under the MJLS formulation, which is provided in terms of linear matrices inequalities optimization problems. These particular controllers have as the main motivation the network communication loss which is inherent to any automation process. We present a numerical example of a coupled tank system, where a Monte Carlo simulation illustrates the feasibility of the proposed solution. The results show that the proposed approach is indeed a valuable alternative to compensate for the fault occurrence.

Keywords: Fault-Tolerant Control, Stochastic control and game theory, Robust linear matrix inequalities

1. INTRODUCTION

The main purpose of any automation process is to provide an optimal solution to minimize any kind of loss and as a by-product increase its performance. An important aspect that should always be considered is the occurrence of faults. The fault occurrence is unavoidable in any automation process, therefore, it is of utmost importance to provide approaches to deal with this problem. Regarding this issue, a particular way to deal with this is the design of a Fault-Tolerant Controller (FTC) (Noura et al., 2009; Blanke et al., 2006).

In the design of an FTC, apart from the regular information of the system (for instance, measured output, system states and exogenous signals), the fault occurrence is also considered. A particular approach for an FTC scheme is the Fault-Compensation Controller (FCC), where there is a primary controller responsible for performance and stability requirements and a secondary one which is actuated only when a fault occurs. For this specific approach, the target type of faults are the ones that do not require the process to stop, that is, it is possible to deal with the fault until it is convenient to stop the process and fix the problem properly.

Although the main purpose of fault-tolerant control is to consider all possible faults in the process, we consider in this paper faults with two main characteristics. The first one is the faults that do not require the process to stop, that is, it is possible to deal with the fault until it is convenient to stop the process and fix the problem properly. The second one, which is inherent to any automation process, is the communication loss between components. These faults can occur in any type of communication, such as the wireless communication which is prone to such problems (Al-Karaki and Kamal, 2004). A particular way to model the network communication loss is the Markovian Jump Linear Systems (MJLS) framework (Gonçalves et al., 2011, 2012).

In the recent literature, several works deal with FTC where the main problem is formulated for a multi-agent system (Khalili et al., 2018). In Han et al. (2018), an approach using the stochastic fuzzy system FTC is provided. An FTC for wind turbine pitch control using adaptive sliding mode estimation is presented in Lan et al. (2018). In Zhu et al. (2019), it is presented an active FTC which considers specific frequency range. Regarding the FTC under the MJLS framework, a recent work can be mentioned (Li et al., 2018), where the problem of a robust fault estimation and fault tolerant control with uncertainty in the transition rates is tackled.

Based on the aforementioned discussion, the novelty of our paper is the design of an $\mathcal{H}_\infty$ FCC under the MJLS framework, where to the best of authors' knownledge, apart from the unique way to approach the problem of dealing with faults and unlike what is found in the literature, we also incorporate the information about the regular control signals in the FCC design. We obtain our controllers using Linear Matrix Inequalities (LMIs) constraints. Additionally, this particular approach aims to design an FCC where the controller will only be actuated when the fault occurs. Another essential aspect considered in the FCC design is that the performance in nominal conditions, without fault, should not be depleted.

The present work is organized as follows. In Section 2, some preliminary information is introduced. In Section 3, the problem description and the proposed approach are presented. Section 4 describes the example used to illustrate the main results whereas Section 5 presents the simulation results. Section 6 concludes the paper with some final comments.

2. PRELIMINARIES

In this section, we present the notation and theoretical background that are necessary to implement the proposed solution.

2.1 Notation

The symbol ($'$) represents the transpose of a matrix or a vector, the symbol ($\bullet$) denotes a block of a symmetric matrix. The Markov chain state set is represented by $\mathbb{K} = \{1, 2, \dots, N\}$. The mathematical expectation is represented by $\mathcal{E}$. The convex combination of matrices of vectors X_j with $j = 1, \dots, N$ is denoted by $\mathbb{E}_i(X) = \sum_{j=1}^{N} \rho_{ij} X_j$ where $\sum_{j=1}^{N} \rho_{ij} = 1$, $\rho_{ij} > 0$. For a discrete-time stochastic signal w, its norm is obtained via $\|w\|_2^2 = \sum_{k=0}^{\infty} \mathbb{E}(w(k)'w(k))$. On the probabilistic space $(\Omega, \mathcal{F}, \mathcal{F}_k, P)$, the set of signals $w(k) \in \mathbb{R}^n$, such that $w(k)$ is $\mathcal{F}_k$ measurable, for all $k \in \mathbb{N}$ and $\|z\|_2 < \infty$, is indicated by $\mathcal{L}^2$. We denote $\mathrm{He}(X) := X + X'$.

2.2 Markovian Jump Linear System

Consider the generic discrete-time Markovian Jump linear system written as

$$\begin{cases} x(k+1) = A_{\theta(k)}x(k) + B_{\theta(k)}u(k) + J_{\theta(k)}d(k), \\ y(k) = C_{\theta(k)}x(k) + D_{\theta(k)}d(k), \\ x(0) = x_0, \ \theta(0) = \theta_0, \end{cases} \tag{1}$$

where the system states, measured output, exogenous signal, and control signal are, respectively, denoted by $x(k) \in \mathbb{R}^n$, $y(k) \in \mathbb{R}^s$, $d(k) \in \mathbb{R}^p$, and $u(k) \in \mathbb{R}^m$. The index $\theta(k) \in \mathbb{K}$ represents the Markov chain mode. The transitions between modes are presented by a transition probability matrix $\mathbb{P} = [p_{ij}]$.

2.3 Mean Square Stability

In Costa and Fragoso (1993), it is presented a definition of the Mean Square Stability (MSS). Considering the initial conditions $x(0) = x_0 \in \mathbb{R}^n$ and the initial distribution $\theta(0) = \theta_0 \in \mathbb{K}$, the MSS is defined as

$$\lim_{k \to \infty} \mathcal{E}(x(k)'x(k)|x_0, \theta_0) = 0,$$

for more details, refer to Costa and Fragoso (1993).

2.4 $\mathcal{H}_\infty$ norm

As presented in Seiler and Sengupta (2003), $x = \{x(k) \in \mathbb{R}^n, k = 1, 2, \dots\}$ represents the states of system (1) with $u(k) = 0$, and $w = \{w(k) \in \mathbb{R}^r, k = 1, 2, \dots\}$ is the exogenous input. The $\mathcal{H}_\infty$ norm can be defined as

$$\|G\|_\infty = \sup_{0 \neq w \in \mathcal{L}_2, \theta_0 \in \mathbb{K}} \frac{\|y\|_2^2}{\|w\|_2^2},$$

Considering that system (1) is MSS, the following LMI can be used to compute the $\mathcal{H}_\infty$ norm

$$\begin{bmatrix} A_i & J_i \\ C_i & D_i \end{bmatrix}' \begin{bmatrix} \mathbb{E}_i(P) & 0 \\ 0 & \gamma I \end{bmatrix} \begin{bmatrix} A_i & J_i \\ C_i & D_i \end{bmatrix} > 0, \tag{2}$$

where γ is the $\mathcal{H}_\infty$ guaranteed cost and $i \in \mathbb{K}$ denotes the Markov chain modes $\theta(k)$ [1].

Proof: The proof is presented in Seiler and Sengupta (2003).

The result shown in the LMI constraints (2) was first presented in Seiler and Sengupta (2003), and is well known as the Bounded Real Lemma (BRL).

2.5 State-feedback Controller

Consider the mode-dependent control law

$$u(k) = K_{\theta(k)}x(k), \tag{3}$$

where $x(k) \in \mathbb{R}^n$ represents the states of system (1). The closed loop system may be represents as $A_{cl_i} = A_i + B_iK_i$, $J_{cl_i} = J_i$, $C_{cl_i} = C_i + G_iK_i$, $D_{cl_i} = D_i$, $\forall i \in \mathbb{R}$. The following result can be used to design the controller (Gonçalves et al., 2012).

Lemma 1. There is a controller K_i, $i \in \mathbb{K}$ which renders system (1) in closed-loop internally stochastically stable, with γ being an upper bound for the $\mathcal{H}_\infty$ norm of system (1), if

$$\begin{bmatrix} \mathrm{He}(G_i) - X_i & \bullet & \bullet & \bullet \\ 0 & \gamma I & \bullet & \bullet \\ A_iG_i + B_iY_i & J_i & \mathrm{He}(H_i) - \mathbb{E}_i(Z) & \bullet \\ C_iG_i + D_iY_i & E_i & 0 & I \end{bmatrix} > 0,$$

$$\begin{bmatrix} Z_{ij} & \bullet \\ H_i & X_j \end{bmatrix} > 0$$

holds for all $i, j \in \mathbb{K}$. If a feasible solution is found, the controller gain is defined as $K_i = Y_iG_i^{-1}, i \in \mathbb{K}$.

Proof: The proof can be found in Gonçalves et al. (2012).

3. PROBLEM FORMULATION

In this section, we formulate the problem and present the main theoretical results.

3.1 MJLS for Fault-compensation problem

The MJLS for the fault-compensation problem is described as

$$\mathcal{G} : \begin{cases} x(k+1) = A_{\theta(k)}x(k) + B_{\theta(k)}u_{total}(k) \\ \qquad\qquad + J_{\theta(k)}d(k) + F_{\theta(k)}f(k), \\ y(k) = C_{\theta(k)}x(k) + D_{\theta(k)}d(k), \\ x(0) = x_0, \ \theta(0) = \theta_0. \end{cases} \tag{4}$$

where the system states are denoted by $x(k) \in \mathbb{R}^n$, the control input is represented by $u(k) \in \mathbb{R}^m$, the exogenous input is $d(k) \in \mathbb{R}^m$, the fault signal is denoted by $f(k) \in \mathbb{R}^m$ and the measured output is represented by $y(k) \in \mathbb{R}^m$.

3.2 Fault compensation Controller

The Fault Compensation Controller scheme is presented in Fig. 1. We see from this scheme that our main goal is to provide an FCC ($\mathcal{K}_{c_i}$) that generates the control signal $h(k)$ with the sole purpose of compensating the fault

[1] Hereafter, the index i denotes the Markov chain modes $\theta(k)$

signal $f(k)$. The control signal $h(k)$ should be close to zero when the system is working properly.

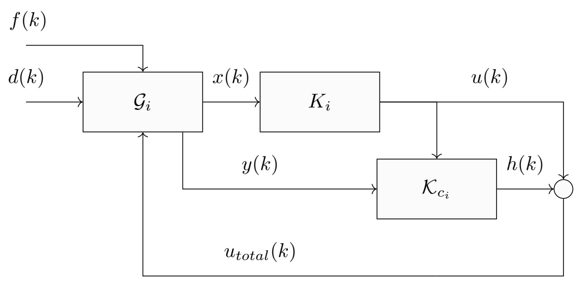

Figure 1. Fault compensation control scheme diagram.

The fault compensation controller can be described as

$$\mathcal{K}_c : \begin{cases} \eta(k+1) = \mathfrak{A}_{\theta(k)}\eta(k) + \mathfrak{M}_{\theta(k)}u(k) + \mathfrak{B}_{\theta(k)}y(k), \\ h(k) = \mathfrak{C}_{\theta(k)}\eta(k), \\ \eta(0) = \eta_0, \ \theta(0) = \theta_0. \end{cases} \tag{5}$$

where $\eta \in \mathbb{K}^q$ represents the FCC, $u(k)$ and $y(k)$, are respectively, the control signal from the regular controller and the measured signal from the system.

Considering system (1), the state feedback control law (3), and the FCC (5), as presented in Fig.1, the augmented system is given by

$$\mathcal{G}_{\text{aug}} : \begin{cases} \bar{x}(k+1) = \bar{A}_{\theta(k)}\bar{x} + \bar{B}_{\theta(k)}\bar{w}(k), \\ \bar{z}(k) = \bar{C}_{\theta(k)}\bar{x} + \bar{D}_{\theta(k)}\bar{w}(k) \\ \bar{x}(0) = \eta_0, \end{cases}$$

where $\bar{x}(k) = [x(k) \ \eta(k)]$ and $\bar{w}(k) = [d(k) \ f(k)]$, with the following augmented matrices are

$$\bar{A}_i = \begin{bmatrix} A_i - B_iK_i & B_i\mathfrak{C}_i \\ \mathfrak{B}_iC_i - \mathfrak{M}_iK_i & \mathfrak{A}_i \end{bmatrix}, \ \bar{B}_i = \begin{bmatrix} J_i & F_i \\ \mathfrak{B}_iD_i & 0 \end{bmatrix}, \\ \bar{C}_i = [0 \ -B_i\mathfrak{C}_i], \ \bar{D}_i = [0 \ F_i]. \tag{6}$$

The main goal of this paper is to design a FCC as presented in (5) where the difference $o(k) = F_if(k) - B_ih(k)$ is close to zero. Therefore, the optimization problem is described as

$$\|\mathcal{G}_{\text{aug}}\|_\infty = \sup_{\|\bar{w}\|_2 \neq 0, \bar{w} \in \mathcal{L}_2} \frac{\|o\|_2}{\|\bar{w}\|_2} < \gamma_c, \quad \gamma_c > 0, \tag{7}$$

Bearing the aforementioned information, and considering K_i the controller obtained beforehand using Lemma (1), it is possible to write the following theorem.

Theorem 1. There exist a mode-dependent FCC as described in (5) satisfying the constraint (7) for some $\gamma_c > 0$ if there exist symmetric matrices Z_i, X_i, and the matrices Δ_i, ∇_i, Ω_i, and Θ_i with compatible dimensions such that inequality (8) with

$$\Pi_i^{6,1} = \mathbb{E}_i(X)A_i - \mathbb{E}_i(X)B_iK_i + \Theta_iC_i + \nabla_iK_i + \Delta_i + \Omega_i,$$
$$\Pi_i^{6,2} = \mathbb{E}_i(X)A_i - \mathbb{E}_i(X)B_iK_i + \Theta_iC_i + \nabla_iK_i,$$
$$\Pi_i^{5,2} = \mathbb{E}_i(X)A_i - \mathbb{E}_i(X)B_iK_i,$$

holds for all $\mathbb{K}$. If a feasible solution is obtained, a suitable fault-compensation controller is given by

$$\mathfrak{A}_i = (\mathbb{E}_i(Z) - \mathbb{E}_i(X))^{-1}\Omega_i,$$
$$\mathfrak{M}_i = (\mathbb{E}_i(Z) - \mathbb{E}_i(X))^{-1}\nabla_i$$
$$\mathfrak{B}_i = (\mathbb{E}_i(Z) - \mathbb{E}_i(X))^{-1}\Theta_i$$
$$\mathfrak{C}_i = (\mathbb{E}_i(Z) - \mathbb{E}_i(X))^{-1}B_i^{-1}\Omega_i.$$

The proof of Theorem 1 is presented in the Appendix.

Remark: Note that, from (8), matrix B_i in (1) should be invertible. However, by requiring it only to be square, we can obtain the matrix $\mathfrak{C}_i$ using a Penrose inverse.

4. NUMERICAL EXAMPLE

In this section, the description of the plant and the control set up are described.

4.1 Plant Description

The numerical example used consists of a coupled tank system as described in Feedback Instruments Ltd. (2013). This system is composed of two identical tanks, which are connected by a pipe. The flow between the tanks are controlled by two pumps, one supplying the first tank, and another supplying the second tank. A scheme representing this system is given in Fig.2. In the following, we present the linear system description of the coupled tank system which is interconnected with a nominal feedback controller that gives result in the MJLS form as in (1) for the closed-loop nominal system.

Consider $x(k) = [H_1(k) \ H_2(k)]'$ the state vector and $\nabla H_1(k)$, $\nabla H_2(k)$ the height variation near the linearization point. The linearization point used is $H_1 = 25$ cm and $H_2 = 10$ cm, selected arbitrarily. The sampling time used is $T_s = 1$[s].

An important part in the FCC is the design of a nominal controller. In the proposed example we design a controller using the following matrices

$$A_{1,2} = \begin{bmatrix} -0.024 & -0.013 \\ 0.013 & -0.029 \end{bmatrix}, B_{1,2} = \begin{bmatrix} 0.71 & 0 \\ 0 & 0.71 \end{bmatrix},$$
$$B_{d_{1,2}} = 0.1B_{1,2}, \quad F_{1,2} = \text{diag}(I_1, 0_1)$$
$$C_1 = I_2, \quad C_2 = 0_2, \quad D_{1,2} = 0.1I_2.$$

Additionally, considering the transition matrix and the detector matrix are given by

$$\mathbb{P} = \begin{bmatrix} 0.8 & 0.2 \\ 0.8 & 0.2 \end{bmatrix}, \tag{9}$$

The nominal controller obtained using Lemma (1) is

$$K_1 = \begin{bmatrix} -1.3456 & 0.0154 \\ -0.0154 & -1.3398 \end{bmatrix}, \ K_2 = \begin{bmatrix} -1.3453 & 0.0154 \\ -0.0154 & -1.3398 \end{bmatrix}$$

and the $\mathcal{H}_\infty$ norm value is $\gamma = 0.1276$. The fault-compensation controller obtained designed using Theorem 1 is

$$
\begin{bmatrix}
Z_i & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
Z_i & X_i & \bullet & \bullet & \bullet & \bullet & \bullet \\
0 & 0 & \gamma_c I & \bullet & \bullet & \bullet & \bullet \\
0 & 0 & 0 & \gamma_c I & \bullet & \bullet & \bullet \\
\mathbb{E}_i(X)A_i - \mathbb{E}_i(X)B_iK_i + \Delta_i & \Pi_i^{5,2} & \mathbb{E}_i(X)J_i & \mathbb{E}_i(X)F_i & \mathrm{He}(\mathbb{E}_i(X)) - \mathbb{E}_i(Z) & \bullet & \bullet \\
\Pi_i^{6,1} & \Pi_i^{6,2} & \mathbb{E}_i(X)J_i + \Theta_i D_i & \mathbb{E}_i(X)F_i & \mathbb{E}_i(X) & \mathbb{E}_i(X) & \bullet \\
-\Delta_i & 0 & 0 & \mathbb{E}_i(X) & 0 & 0 & \mathrm{He}(\mathbb{E}_i(X)) - I
\end{bmatrix} < 0, \quad (8)
$$

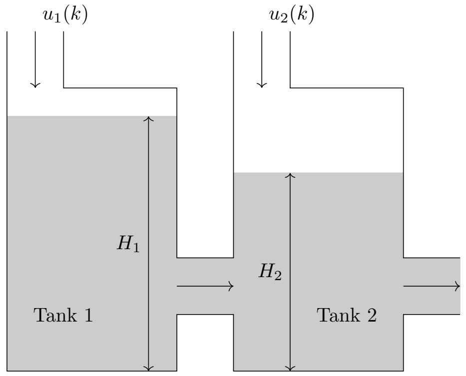

Figure 2. Plant scheme.

$$
A_{c1} = \begin{bmatrix} 0.2233 & -0.0080 \\ -0.0059 & 0.2731 \end{bmatrix}, \; A_{c2} = \begin{bmatrix} 0.0488 & -0.003 \\ -0.0013 & 0.0651 \end{bmatrix},
$$

$$
B_{c1} = \begin{bmatrix} -0.1745 & 0.0041 \\ 0.0045 & -0.2079 \end{bmatrix}, \; B_{c2} = \begin{bmatrix} -0.1745 & 0.0041 \\ 0.0045 & -0.2079 \end{bmatrix},
$$

$$
M_{c1} = \begin{bmatrix} -0.1701 & 0.0063 \\ 0.0016 & -0.2018 \end{bmatrix}, M_{c2} = \begin{bmatrix} -0.1701 & 0.0063 \\ 0.0016 & -0.2018 \end{bmatrix},
$$

$$
C_{c1} = \begin{bmatrix} -0.4597 & 0.0239 \\ -0.0006 & -0.5075 \end{bmatrix}, \; C_{c2} = \begin{bmatrix} -0.4596 & 0.0239 \\ -0.0006 & -0.5075 \end{bmatrix}.
$$

and the $\mathcal{H}_\infty$ norm value is $\gamma_c = 1.9002$.

Remark: It is important to consider that the control law is computed using the estimated state variables obtained, for example, by an observer or a Kalman filter.

5. RESULTS

In this section the simulation results are presented in two parts. The first consists in the results achieved for a fault signal and the second the ones obtained without fault.

5.1 Simulations with fault signal

In this example, the fault signal implemented is a sinusoidal wave as presented in Fig.3 The transition matrix is the same as (9). The noise signal is a white noise with zero mean and deviation equal to 0.01. The results presented herein were obtained via Monte Carlo simulations with 300 rounds. In all the simulation we made a comparison between the proposed approach (Comp), and a regular solution using only the controller designed using Lemma (1) (Not comp). The simulation results are organized in three sets of graphics, where the first and second ones shows, respectively, the mean and the standard deviation for both tank levels h_1 and h_2. The latter is the control signal for each actuator.

In Fig. 4 it is possible to observe that the fault is compensated for both levels, which can be seen by comparing

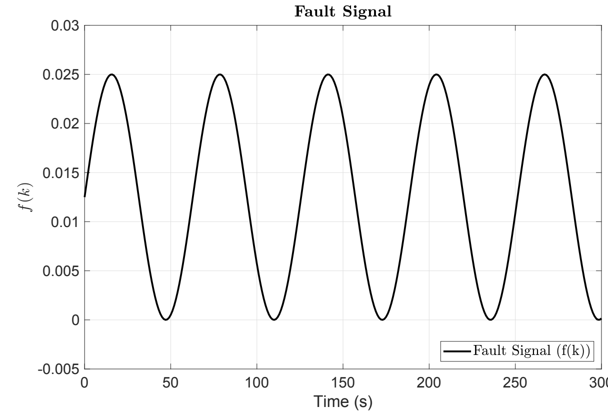

Figure 3. Fault signal.

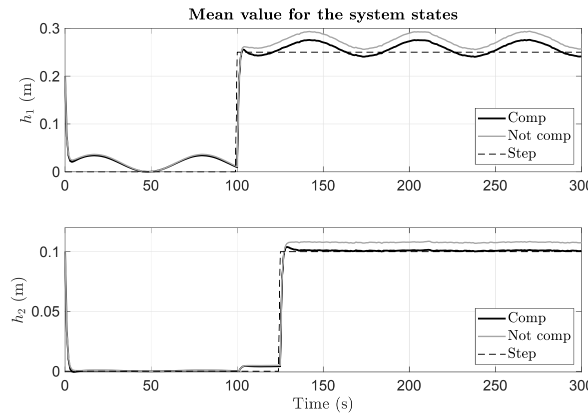

Figure 4. Mean for the tank levels with fault signal.

the mean value of the system states using the compensator and without using it. In both graphics the compensation is noticeable, the sinusoidal behavior is mitigated in both levels. Observing Fig. 5 allow us to state that the standard deviation for both the plant states are slightly higher, approximately 0.05 meter. Additionally, note that the control signals for both actuators, which are shown in Fig. 6, minimize the fault behavior while keeping the level near the linearization points, that is, 0.25m and 0.1m for the first and second tanks, respectively.

5.2 Simulations without fault signal

This subsection shows the results for the Monte Carlo simulations when there is no fault signal. This test is important since it is necessary to observe the FCC in the nominal situation. The simulation parameters are as same described in the previous subsection.

In Fig. 7, the mean value for both levels are presented, and comparing these results to the ones in Fig. 4, that is,

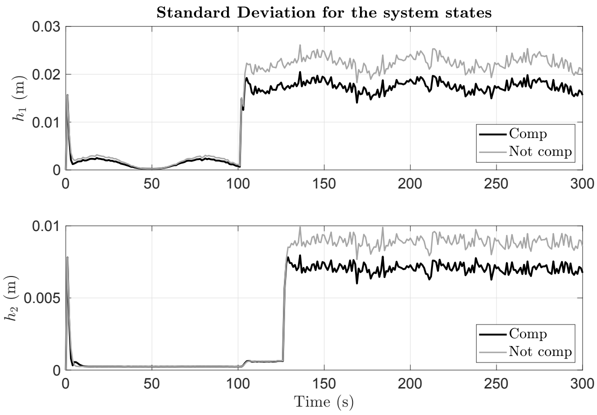

Figure 5. Standard deviation for the tank levels with fault signal.

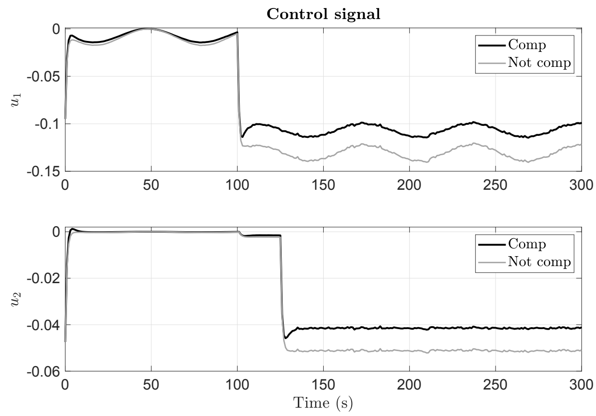

Figure 6. Comparison between control signal with and without compensation for the case with fault signal.

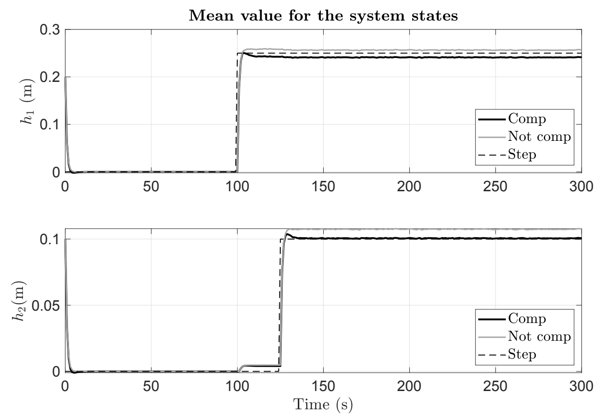

Figure 7. Mean for the tank levels without fault signal.

comparing the mean values for the cases with and without fault signals, we see that there is a noteworthy difference between them. The step response for the compensated approach is closer to the step signal. As seen in Fig. 5, Fig. 8 also shows a distinct difference between the graphics, however, this difference is around 0.001, which is acceptable. For the control signal presented in Fig. 9, there is a difference between the control signals for both actuators.

Based on the aforementioned results, we see that the FCC approach proposed in this paper indeed mitigate the fault

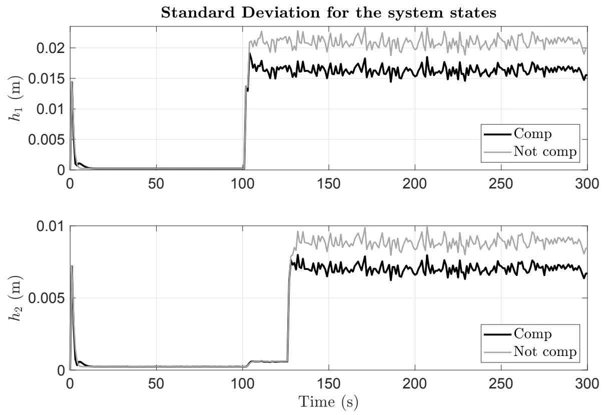

Figure 8. Standard deviation for the tank levels without fault signal.

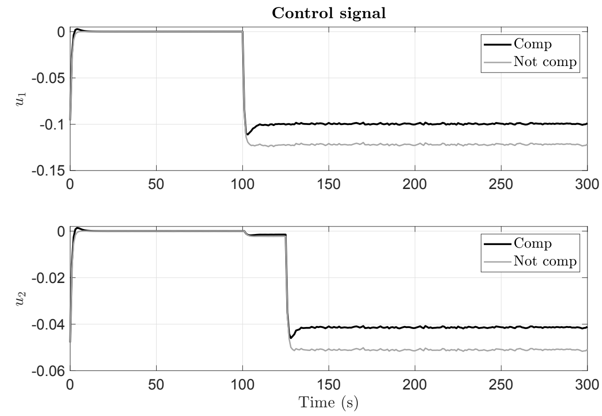

Figure 9. Comparison between control signal with and without compensation for the case without fault signal.

signal as intended. However, there is a slight difference between the FCC and the nominal controller, which was not optimal. This phenomenon can be explained due to the abrupt behavior step input, as the FCC detects this abrupt change as a fault.

6. CONCLUSION

In this paper, we focus on the Fault Compensation Controller. The main contribution is the use of linear matrices inequalities constraints to design the $\mathcal{H}_\infty$ FCC under the Markovian Jump Linear Systems framework, as described in Section 3. To illustrate the viability of the proposed solution for the FCC, and as presented in Section 5 the solution fulfill its purpose of minimizing the fault signal, and does not disturbs the nominal control when there is no fault occurrence.

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APPENDIX

Proof: The goal of the proof is to show that if the inequality (8) holds, then (2) is also satisfied. First, consider the following structures for the matrices

$$\begin{aligned} P_i &= \begin{bmatrix} X_i & U_i \\ U_i' & \hat{X}_i \end{bmatrix};\ P_i^{-1} = \begin{bmatrix} Y_i & V_i \\ V_i' & \hat{Y}_i \end{bmatrix}, \\ \mathbb{E}_i(P) &= \begin{bmatrix} \mathbb{E}_i(X) & \mathbb{E}_i(U) \\ \mathbb{E}_i(U)' & \mathbb{E}_i(\hat{X}) \end{bmatrix}, \mathbb{E}_i(P)^{-1} = \begin{bmatrix} R_{1i} & R_{2i} \\ R_{2i}' & R_{3i} \end{bmatrix}, \end{aligned} \tag{10}$$

and define the matrices Q_i and T_i as

$$T_i = \begin{bmatrix} I & I \\ V_i'Y_i^{-1} & 0 \end{bmatrix},\ Q_i = \begin{bmatrix} \mathbb{E}_i(X) & \mathbb{E}_i(X) \\ 0 & \mathbb{E}_i(U)' \end{bmatrix}.$$

As demonstrated in Gonçalves et al. (2010), by imposing that $U_i = Z_i - X_i$, it follows from (10) that $V_i = V_i'$, $V_i = Z_i^{-1}$. Setting the following matrices

$$T_i'P_iT_i = \begin{bmatrix} Y_i^{-1} & Y_i^{-1} \\ Y_i^{-1} & X_i \end{bmatrix},$$

$$Q_i'\bar{A}_iT_i = \begin{bmatrix} \nu_i^{11} & \mathbb{E}_i(X)A_i - \mathbb{E}_i(X)B_iK_i \\ \nu_i^{21} & \nu_i^{22} \end{bmatrix},$$

$$\nu_i^{11} = \mathbb{E}_i(X)A_i - \mathbb{E}_i(X)B_iK_i + \mathbb{E}_i(X)B_i\mathfrak{C}_i,$$

$$\begin{aligned} \nu_i^{21} = \mathbb{E}_i(X)A_i - \mathbb{E}_i(X)B_iK_i + \mathbb{E}_i(U)\mathfrak{B}_iC_i \\ -\mathbb{E}_i(U)\mathfrak{M}_iK_i - \mathbb{E}_i(X)B_i\mathfrak{C}_i, \end{aligned}$$

$$\nu_i^{22} = \mathbb{E}_i(X)A_i - \mathbb{E}_i(X)B_iK_i + \mathbb{E}_i(U)\mathfrak{B}_iC_i - \mathbb{E}_i(U)\mathfrak{M}_iK_i$$

$$Q_i'\bar{B}_i = \begin{bmatrix} \mathbb{E}_i(X)J_i & \mathbb{E}_i(X)F_i \\ \mathbb{E}_i(X)J_i + \mathbb{E}_i(U)\mathfrak{B}_iD_i & \mathbb{E}_i(X)F_i \end{bmatrix},$$

$$\bar{C}_iT_i = [-B_i\mathfrak{C}_i\ \ 0],\ \bar{D}_i = [0\ \ F_i].$$

as presented in de Oliveira et al. (1999), it is possible to write the $\mathrm{He}(\mathbb{E}_i(X)) - \mathbb{E}_i(Z) \leq \mathbb{E}_i(X)'\mathbb{E}_i(Z)^{-1}\mathbb{E}_i(X)$. This step allow us to write

$$Q_i'\mathbb{E}_i(P)^{-1}Q_i = \begin{bmatrix} \mathrm{He}(\mathbb{E}_i(X)) - \mathbb{E}_i(Z) & \mathbb{E}_i(X) \\ \mathbb{E}_i(X) & \mathbb{E}_i(X) \end{bmatrix}.$$

Therefore the inequality given in (8) can be written as

$$\begin{bmatrix} T_i'P_iT_i & \bullet & \bullet & \bullet \\ 0 & \gamma I & \bullet & \bullet \\ Q_i'\bar{A}_iT_i & Q_i'\bar{B}_i & Q_i'\mathbb{E}_i(P)^{-1}Q_i & \bullet \\ \mathbb{E}_i(X)\bar{C}_iT_i & \mathbb{E}_i(X)\bar{D}_i & 0 & \mathrm{He}(\mathbb{E}_i(X)) - I \end{bmatrix} > 0$$

Applying the congruence transform

$$\mathrm{diag}(T_i^{-1}, I, Q_i^{-1}, \mathbb{E}_i(X)^{-1})$$

in this last inequality,the following constraint is obtained

$$\begin{bmatrix} P_i & \bullet & \bullet & \bullet \\ 0 & \gamma I & \bullet & \bullet \\ \bar{A}_i & \bar{B}_i & \mathbb{E}_i(P)^{-1} & \bullet \\ \bar{C}_i & \bar{D}_i & 0 & I \end{bmatrix} > 0$$

which, by applying a Schur complement, can be recognized as the BRL (2), concluding the proof.