10.1 Conclusion

Part I: Rationality and social influence in network games

Based on theoretical and behavioral economics in Chapter 3, we proposed novel dynamics for finite and convex network games that result from an intuitive mix of rational best responses and social learning. We have shown that for a general class of games these dynamics converge to a generalized Nash equilibrium and that the corresponding decision-making process is “compatible” with rational best response dynamics. That is, a mix of best responders and h-relative best responders will eventually reach an equilibrium action profile. These results make it possible to rigorously study how relative performance considerations of “irrational” or conforming decision makers affect the behavior and equilibrium profiles of complex socio-technical
Conclusion and Future Research

and socio-economic processes. Considering these effects is important because many technological challenges require increasingly complex models of large social systems that, in reality, are often affected by social learning effects that are not captured by best response dynamics.

In Chapter 4, we have shown that rational imitation dynamics in a general class of asynchronous public goods games on networks converge to an imitation equilibrium in finite time. By means of a counter-example, we have shown that this general case of convergence is not guaranteed when imitation is unconditional. For regular spatial structures and linear production functions, we have proven convergence either directly from the payoff functions or by using an algorithmic proof technique that takes advantage of the regularity of the network. We have shown that in the case of rational imitation, convergence is also guaranteed when the group structures are determined by a bipartite graph. Such a representation of a spatially structured social dilemma can, for instance, be used when the group structures are obtained from data that does not contain information about the entire social network. Next to convergence, we have provided evidence that in contrast to best response dynamics, rational imitation can effectively facilitate the evolution of cooperation via network reciprocity. Our results indicate that through the combination of rationality and imitation, beneficial dynamic features can arise that are able to sustain the availability of a publicly available good, providing new insights in the design of solutions to the tragedy of the commons.

In Chapter 5, we have shown how network games can be extended to include a subset of players that can employ different actions against different opponents. When the local games in the network admit a weighted potential function, convergence of the strategically differentiated version with myopic best response dynamics is guaranteed. For both imitation and best response dynamics the topology of the network, the existence and location of differentiators in the networks can crucially alter the action profile at an equilibrium of groupwise public goods games. When differentiators are plentiful, the equilibrium action profile becomes less sensitive to changes in the values of the payoff parameters and cooperation can exist for very low values benefit-to-cost ratios.

Part II: Strategic play and control in repeated games

In Chapter 6, we have extended the existing results for ZD strategies in repeated two-player two-action games to $n$-player two-action games. We focused on $n$-player social dilemma games because of their importance to the current literature. However, the fundamental relation between the memory-one strategy and the limit distribution is independent of the structure of the game and thus the results in this chapter can
be extended by considering \( n \)-player games that are not social dilemmas. Our theory supports the finding that due to the finite number of expected rounds or discounting of the payoffs, the initial probability to cooperate of the key player remains important, and we have shown that for the existence of generous strategies the ZD strategist must start to cooperate with probability one. Likewise, for extortionate strategies, this initial probability must be zero. These results indicate that even in large groups of players, a single player can unilaterally enforce the mutual cooperation payoff independent of the strategies of the other players. Especially the latter is an important feature of our results that distinguishes them from classical folk theorems in which it is assumed all players are rational. If one however assumes that other players are rational, the positive payoff relations that generous and extortionate ZD strategies enforce ensure that the collective best response of the selfish co-players is to maximize the ZD strategists payoff by cooperating every round.

In Chapter 7, a theory is developed that characterizes the efficiency of exerting control in terms of the minimum required number of expected interactions in social dilemmas. Based on the necessary conditions on the initial probability to cooperate, we derived expressions for the minimum discount factors above which a ZD strategist can enforce some desired generous or extortionate payoff relation. Because equalizer strategies do not impose such conditions on the initial probability to cooperate, one can identify a multitude of \( p_0 \) regions in the unit interval for which there exist different threshold discount factors. Consequently, we have derived an expression that ensures the desired equalizer strategy to be enforceable for any initial probability to cooperate in the open unit interval. The derived necessary and sufficient conditions for existence and the thresholds discount factors presented in this chapter may also be helpful in designing novel control techniques for repeated decision making processes in which the objective is to achieve a desired relative performance within a given number of rounds.

In Chapter 8 we have studied the evolutionary stability of ZD strategies in a finite population in which players interact in randomly formed \( n \)-player repeated contests. Necessary and sufficient conditions are provided for a resident ZD strategy to be stable with respect to a single mutant ZD strategy. These conditions can characterize when ZD strategies can enforce cooperation to evolve in a finite population. In particular, they suggest that under the classic Maynard-Smith conditions \( N = \infty, n = 2 \) extortionate strategies cannot be evolutionarily stable. In this case, only generous strategies and equalizers with generous slopes are favored by evolution. In sharp contrast, when the population size is equal to the group size \( n = N \), only extortionate strategies can be evolutionarily stable. In a finite population in which the group size of the contests is smaller than the population size \( n < N \) both generosity and extortion can be stable, however, this highly depends on the benefit-to-cost ratio, the
population size $N$, and group size of the contests $n$.

In Chapter 9 we have proposed a novel discounting method for repeated games that takes into account the added psychological complexity of uncertainty about the discount factor or continuation probability of the repeated game. With this generalized discounting framework it is possible to recover deterministic discounting methods that exist in the current literature, as well as hyperbolic discounting that has time-inconsistent discount rates. We have shown how ZD strategies, that are normally fixed memory-one strategies, can be adapted to time-varying memory-one strategies that take into account the changing discount rates that result from uncertainty in the continuation probability. In deterministic limits these novel risk-adapted ZD strategies recover the formulations of ZD strategies under deterministic discounting methods and ZD strategies for repeated games with an infinite number of expected rounds. Characterization of the enforceable slopes shows that in this uncertain setting, generous strategies cannot be enforced. This result highlights that certain continuation probabilities are necessary for mutual cooperation to be enforceable by a strategic player.

10.2 Recommendations for future research

Part I: Rationality and social influence in network games

For the $h$-RBR dynamics proposed in Part I many challenging open problems and future research directions can be identified. In this work, we have focused on deterministic decision-makers that do not deviate from their decision rule. In reality, trembling hands [154] or random explorations are inevitable. For myopic best response dynamics, these effects have been studied under a variety of noise models such as constant noise as in adaptive play [45], or a noise that is proportional to one’s expected payoff as in the log-linear response model [45]. These ideas can also be applied to relative best responses, and it is interesting to characterize how stochastically stable equilibria may change under the influence of social learning. For rational imitation dynamics, proportional noise models from imitation processes may be incorporated [37,155] as well. However, even for deterministic dynamics, the effects of social influence and network structure on the equilibria of network games are not yet fully characterized. In particular, it could be interesting to identify network structures that enhance the opportunities for rational cooperation to evolve in social dilemmas on networks. The same holds for the mechanism strategic differentiation.

From a more technical perspective, it would be interesting to study the convergence properties of synchronous $h$-RBR and rational imitation dynamics. In this case, the existence of a potential function is not immediately helpful in the convergence
10.2. Recommendations for future research

Analysis and because the constrained sets imposed by the $h$-RBR dynamics are not jointly convex, new analysis techniques must be developed to characterize the convergence properties of this class of synchronous dynamics. Nevertheless, it can also be interesting to study the effect that synchronous revisions have on the effectiveness of network reciprocity under rational imitation and $h$-RBR dynamics. Finally, it would be interesting to apply the idea of relative performance considerations in rational imitations and relative best responses to opinion dynamics. In this case, the relative performance of the players can, for instance, be associated with the differences in opinions (3). In such a model, players will only take into account the opinions of neighbors that are relatively close to their own opinion. In particular, it could be of interest to investigate under which conditions polarization or clustering of opinions will occur in such “relative” opinion dynamics.

Part II: Strategic play and control in repeated games

The theory developed in Part II can be extended in several ways. We will begin with the most immediate extensions. We have seen how uncertainty in the continuation probability or discount rate prevents the opportunities of an individual to enforce a generous payoff relation, but it remains an open problem how the parameters of the probability distribution affect the equilibrium payoffs of repeated games. In particular, the uncertain discounting framework from Chapter 9 can be used to investigate how classical folk theorems hold up under uncertain discounting.

In Chapter 6, we have focused on characterizing the enforceable payoff relations in social dilemmas, but as mentioned before, the fundamental relation between memory-one strategies and mean distributions of the repeated game do not require the social dilemma assumptions. In particular, the results in [63] indicate that ZD strategies possibly exist in the class of symmetric potential games. It would be interesting to extend the theory in this thesis by characterizing the enforceable payoff relations in this widely studied class of games.

Perhaps a more difficult research direction is to extend the theory of ZD strategies in repeated games with individual discount factors. Indeed, the folk theorem has been studied under these rather complex settings [53].

It would be also interesting to include other sources of psychological complexity and uncertainty in $n$-player games. For instance, psychologists and game theorists have recently studied the effect of uncertainty in the group size $n$ [156, 157]. It will be interesting to study how this will affect the strategic behavior of individuals in repeated games. Lastly, an interesting and challenging direction for future research is to study ZD strategies in continuous-time repeated games [158]. For this, a new analysis tool must be developed first. In particular, it is not yet clear how mean
distributions of continuous time stochastic processes can be related to a strategy of a strategic player in continuous time repeated games. With such a relation, one could apply continuous-time discounting methods [150] to repeated games. This, in turn, would allow us to formalize how the time “spent” in a certain action profile can affect the strategic decisions of individuals.