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Galactic dynamics in the era of Gaia

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Galactic dynamics in the era of Gaia

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Chapter 1

Introduction

1.1 Prologue

19 December 2013, 10:12 CET: “10..9..8..7..6..5..4..3..2..1..top, décollage!” Via a live stream connected to Europe’s spaceport in French Guiana, I watched the launch of Soyuz flight VS06. At that time I was a student enrolled in the master’s program ‘Astronomy’ of the University of Groningen. It was a coincidence that the launch happened to be during a lecture of the course ‘Space Mission and Technology’. What a piece of technology had just been launched into space: on board of the rocket was the *Gaia* space observatory (see Fig. 1.1)! Although I could not have realised this by then, *Gaia* turned out to be the basis for my next step in academia: a PhD position which led to this thesis.

It took *Gaia* a month to get to its final destination. *Gaia* is currently in a large Lissajous orbit around a virtual point in space which is about 1.5 million kilometres away from Earth, almost four times as far away as the Moon, and positioned behind the Earth as seen from the Sun (second Lagrangian point, L2). The initial conditions of *Gaia*’s orbit around L2 were chosen such that the spacecraft would not enter the Earth’s shadow for at least six years after the start of the mission. While being out there, *Gaia*’s major task is to survey about a billion stars of our Galaxy. The primary goal of the *Gaia* space mission namely is “to investigate the origin and subsequent evolution of our Galaxy, the Milky Way”.

Figure 1.1 – Left: The lift-off of Soyuz flight VS06, with on board the *Gaia* space observatory. Right: The *Gaia* space observatory. Note the persons next to the satellite to get an idea on how big it is. The disk-like structure is a deployable sunshield with a diameter of 10.2 metres. Photos taken from: <https://sci.esa.int/web/gaia/multimedia-gallery>.

1.2 Galaxy Formation and Evolution

To understand how observing stars in our Galaxy at the current epoch can give information about the history of our Galaxy, we have to place our Galaxy in the context of the Universe as a whole. In current cosmological models, the Universe started around 14 billion years ago with an event called the Big Bang. From an initially extremely high-density and high-temperature state the Universe rapidly expanded and cooled down. Light elements were created by fusion of protons and neutrons in a process called Big Bang Nucleosynthesis. The Universe continued to cool down and roughly 377.000 years after the Big Bang, without the destroying power of ionising temperatures, it became possible for electrons to bind to the charged elements in order to form the lightest neutral atoms, predominantly hydrogen and helium. In the next hundreds of millions of years the first stars and galaxies formed under the influence of gravity. As time proceeded gravity continued to play an important role in the growth of structure in the Universe. Overdense regions became denser, underdense regions became more diffuse (e.g. Peebles 1980; Dodelson 2003).

The process of ‘hierarchical growth’ indicates that small structures successively aggregate into larger structures and it are the signatures of such building blocks of galaxies that we can try to uncover in order to learn about the formation history of galaxies. Our own Milky Way (MW), an average galaxy, is an ideal test bed for this purpose.

One of these signatures is the chemical composition of the stars of a building block (Freeman & Bland-Hawthorn 2002). Stars fuse their elements into heavier elements and do not have eternal life. When no more elements can be efficiently fused, stars lose their outer layers whose elements subsequently mix with the gas in the interstellar medium (ISM). In general high-mass stars will lose a different mixture of elements than low-mass stars, resulting in distinct enrichments of the ISM, from which subsequently new generations of stars are born. By studying the chemical composition of stars we thus gain information about the environment in which they are born.

Another signature of a common origin is that the stars of a building block typically have very similar orbits in the galaxy they accreted onto. In fact, there is convincing evidence that minor galaxies are currently being shredded apart as part of their merging process with the Milky Way. These stellar streams are perfect examples of a merging process in action, see the Field of Streams (Fig. 1.2). Thus, by studying the kinematics of stars we can try to identify groups of stars that come from the same progenitor.

1.3 Dark matter

Besides ‘normal’ baryonic matter, such as atoms, the Universe also contains dark energy and dark matter (DM), currently with energy budgets of $\sim 5\%$, $\sim 68\%$, and $\sim 27\%$ of the total matter-energy budget available (Planck Collaboration et al. 2018). Galaxies are thought to be predominantly composed of baryonic and dark matter. The latter does barely seem to interact with radiation or baryonic matter, beyond through gravity, and therefore has not yet been observed directly. When small galaxies merge onto a large galaxy they thus contribute to the dark matter content of the combined system. On the scales of individual galaxies, their large fraction of dark matter is of great importance for the dynamics of stars. The nature of dark matter itself is, however, still unknown. One possibility favoured a new elementary particle which is non-relativistic at very early times, and which is generically referred to as cold dark matter (CDM).

Figure 1.2 – Field of Streams (Belokurov et al. 2006) in the Sloan Digital Sky Survey (SDSS) data for the northern (upper panel) and southern (lower panel) hemisphere. Figure by Bonaca et al. (2012).

Although the existence of dark matter halos has been inferred from for example kinematics in clusters of galaxies (e.g. Zwicky 1933), galaxy rotation curves (e.g. Freeman 1970), and gravitational lensing (Walsh et al. 1979), the existence of the expected large number of subhalos for CDM models has not been demonstrated. Klypin et al. (1999) and Moore et al. (1999) introduced the ‘missing satellites problem’, stating that cold dark matter (Λ CDM) cosmological models overpredict the number of satellites compared to the number of luminous satellite galaxies observed around galaxies like the Milky Way. The number and size of the satellite galaxies around our Milky Way, which are believed to be embedded in the largest subhalos of our galaxy, might thus give us a hint on the mass of the Galaxy.

Related to this, Boylan-Kolchin et al. (2011) stated that the predicted subhalos from Λ CDM simulations are too big to host the largest (but still relatively small) Milky Way dwarf satellites, although Vera-Ciro et al. (2013) showed that this ‘too big to fail problem’ is dependent on the mass of the host halo. The latter authors found no discrepancy for a Milky Way mass around $8 \times 10^{11} M_{\odot}$. On the other hand, if a relatively high Milky Way mass will be determined, then a different explanation for the missing satellites is needed, maybe requiring changes in the Λ CDM model.

Another prediction was made about the shape of the dark matter halos: dark matter only cosmological simulations showed that they should be triaxial (Jing & Suto 2002). Although effort has been put into investigating the shape of the Milky Way dark halo, the results of different researchers/methods are not all in agreement with each other (e.g. Pearson et al. 2015). By analysing the dynamically young tidal debris streams of the Sagittarius dwarf galaxy, Law & Majewski (2010) found evidence for a triaxial halo with the minor axis in a direction close to the Galactic plane. Bovy et al. (2016) argued for a near spherical inner halo by modelling the nearby Pal-5 and GD-1 streams. Also Wegg et al. (2019) argued for a spherical halo by studying the kinematics of RR Lyrae stars. On the other hand, oblate models were favoured by studying SDSS halo stars with new data from *Gaia*, LAMOST and SEQUE (Loebman et al. 2014), and by studying the GD-1 stream solely (Malhan & Ibata 2019). To complete the tension, prolate models have been inferred by studying the leading stream of Sagittarius dwarf galaxy (Helmi 2004), the angular velocity dispersions and density of tracers of halo stars (Bowden et al. 2016), and the Galactic globular cluster population (Posti & Helmi 2019). Finally, Vera-Ciro & Helmi (e.g. 2013) have argued for a halo shape changing as function of Galactocentric distance. The variety of contradicting results in the literature are largely a consequence of limitations in the models. More general models that fit all constraints simultaneously are therefore needed to pin down the actual shape of our Galaxy.

The past years significant progress has been made in improving cosmological simulations by including the effects of baryonic physics, such as magneto-hydrodynamical simulations in which, e.g., gas dynamics, chemical processes, magnetic fields, stellar feedback, radiative cooling and heating, and star formation are incorporated. It seems that most problems related to predictions from dark matter only models can be alleviated or solved by including baryonic physics in the simulations (Zolotov et al. 2012; Brooks et al. 2013; Wetzel et al. 2016; Kim et al. 2018). Hydrodynamical simulations have also indicated that baryons tend to make dark matter halos rounder at all radii compared to DM-only simulations (Debattista et al. 2008; Abadi et al. 2010; Prada et al. 2019).

1.4 The Milky Way

The Milky Way is just one of the hundreds of billions of galaxies that exist in the Universe. It is only special in the sense that it is *our* home galaxy, with the Sun positioned very close to the Galactic midplane, at a distance of around 8.2 kpc from the Galactic centre. It is a spiral galaxy and its main components are the central super massive black hole (SMBH), central bulge and bar, stellar disk, a stellar halo, a halo with hot gas and a dark halo (see Fig. 1.3 and Bland-Hawthorn & Gerhard 2016, and references therein). The SMBH mass has now been determined by accurate measurements of the innermost stellar orbits and is given by $M_{\bullet} \sim 4 \times 10^6 M_{\odot}$.

In the past years, evidence was obtained that the inner part of our galaxy has a box/peanut shaped bulge, which in its turn is the central part of a three-dimensional galactic bar extending up to roughly 5 kpc from the Galactic centre (Wegg et al. 2015). The Galactic disk has a diameter of roughly 50 kpc and its stellar component is dominated by the thin and thick disk. The thick disk has, at fixed $[\text{Fe}/\text{H}]$, an enhanced $[\alpha/\text{Fe}]$ ratio with respect to the thin disk, is generally more metal-poor and has an older age. The radial scale length of the thin disk seems to be slightly larger (2.6 ± 0.5 kpc) than that of the thick disk (2.0 ± 0.2 kpc). The thin disk's scale height is roughly 300 ± 50 pc at the solar position, whereas the thick disk's vertical scale length is 900 ± 180 pc at the same radial distance from the Galactic centre.

The stellar halo contains roughly 1% of the total stellar mass of the Milky Way and the most metal-poor stars of the Galaxy can be found here. The stellar halo also contains roughly 150 compact stellar clusters, globular clusters. Because of its long dynamical time scales, tidal features of accreted objects remain observable for over Gyrs of time. The stellar halo therefore provides a good opportunity to study the accretion history of the Milky Way (e.g. Helmi 2008).

A diffuse hot plasma (or corona) with temperatures around $T \sim 2 \times 10^6$ K surrounds the MW disk, but it is not clear yet how far it extends into the halo. The dark matter halo completes our picture of the Milky Way. Its virial mass (M_{200}) is estimated to lie in the range from 0.5 to $2 \times 10^{12} M_{\odot}$ (e.g. Callingham et al. 2019).

Figure 1.3 – A broad schematic overview of the most prominent components of the Milky Way galaxy. Credits: Freedman & Kaufmann (2008), *Universe*, 8th edition (W.H. Freeman and Company).

1.5 Gaia - Data Release 1 and 2

The first data release (DR) of *Gaia* took place on 14 September 2016 (Gaia Collaboration et al. 2016). In this data release the magnitudes and sky positions of over a billion stars with *G*-band magnitudes up to 21 mag were determined. By comparing the positions of the stars that were in common with those of the Tycho-2 catalog, parallaxes and proper motions could be determined for around two million stars, the Tycho-Gaia Astrometric Solution (TGAS, Lindegren 2018). Radial velocities could be included by cross-matching the catalog to external spectroscopic surveys such as APOGEE or RAVE.

The big leap was made with the second data release of *Gaia* on 25 April 2018 (Gaia Collaboration et al. 2018b). This data release no longer needed to rely on the previous less accurate astrometric catalogs determined by the Hipparcos satellite (1989-1993) and much more accurate positions, proper motions, and parallaxes were provided for almost 1.7 billion sources. In addition, *Gaia*'s radial velocity spectrograph (RVS) had successfully determined line-of-sight velocities for around seven million stars with G_{RVS} -band magnitudes brighter than 13 mag (Gaia Collaboration et al. 2018d). Besides the *G*-band magnitudes, also magnitudes in the blue and red photometer (BP and RP) passbands were determined, enabling to make a coloured view of our Galaxy (see Fig. 1.4).

Figure 1.4 – Top: Count map of *Gaia* DR1. In such a map the Small and Large Magellanic clouds are very prominent (bottom right part of the panel). The striped patterns are related to *Gaia*'s scanning law. Bottom: Flux map of *Gaia* DR2, which provided a much better sampling of the sky. In addition, colours of stars were measured and for a large subset of stars line-of-sight velocities have become available. Credits: ESA/*Gaia*/DPAC.

Figure 1.5 – Left: The snail-shell shaped phase-space spiral for a local sample of stars as determined by Antoja et al. (2018), indicating a satellite interaction with the Galactic disk. Right: In-plane velocity map of stars with distances smaller than 200 pc, showing the huge amount of asymmetric kinematic substructure (Gaia Collaboration et al. 2018d).

Gaia DR2 has substantially increased our knowledge about the Milky Way and its history. Antoja et al. (2018) noticed a snail-shell pattern when plotting the Galactic heights *vs.* the vertical velocities for a local sample of stars (see left panel of Fig. 1.5). They argued that such a pattern is likely the relic from an interaction with a satellite between 300 and 900 Myr ago, and could possibly be due to the last pericentre passage of the Sagittarius dwarf galaxy. In addition, Gaia Collaboration et al. (2018d) showed that the in-plane velocity maps contain much more substructure than observed before (see right panel of Fig. 1.5). Several works have put these signatures in context of the Galactic spiral arms or resonances with the Galactic bar (e.g. Hunt et al. 2019; Monari et al. 2019; Monari et al. 2019; Trick et al. 2019). Gaia Collaboration et al. (2018d) also used the full RVS subset of *Gaia* DR2 to study the velocity field of the Galactic disk and showed that streaming motions exist in all velocity components. Furthermore, Gaia Collaboration et al. (2018a) noticed that the HR diagram for stars belonging to the Galactic halo show a double sequence (see Fig. 1.6): a blue and a red sequence, which were subsequently related to stars from a major merger event around 10 Gyr ago and the dynamically perturbed stars of a pre-existing Galactic disk, respectively (Helmi et al. 2018, see also Belokurov et al. (2018); Gallart et al. (2019)).

Figure 1.6 – The double sequence HR diagram for halo stars with tangential velocities greater than 200 km/s (Gaia Collaboration et al. 2018a). Further analysis has related this to a major merger event that took place around 10 Gyr ago.

Gaia DR2 has provided new constraints on the mass and shape of the Milky Way. Using the proper motions obtained by Gaia Collaboration et al. (2018c) and Sohn et al. (2018), Posti & Helmi (2019) modelled the Milky Way globular cluster system as a two-component distribution function, a disk-like component and a halo-like component. They simultaneously fitted the mass and shape of the Milky Way and inferred a virial mass of $(1.3 \pm 0.3) \times 10^{12} M_{\odot}$ and for the density axis ratio of the dark matter halo they found $q = 1.30 \pm 0.25$. Although the error on the shape estimate is non-negligible, oblate models with $q < 0.8$ and very prolate halos with $q > 1.9$ were ruled-out. Callingham et al. (2019) applied a similar approach to the MW satellite galaxies and found a consistent virial mass of $1.17^{+0.21}_{-0.15} \times 10^{12} M_{\odot}$. Monari et al. (2018) selected around 2850 counter-rotating halo stars for which full phase-space information was available. They inferred the escape velocity over Galactic radii ranging from 5.0 kpc out to 10.5 kpc. At the position of the Sun they inferred an escape speed of (580 ± 63) km/s. Assuming a Navarro-Frenk-White (NFW, Navarro et al. 1996) profile and considering the escape velocity as the minimum speed to be able to reach a distance of three virial radii, they determined

a dark matter mass of $M_{200} = 1.28^{+0.68}_{-0.50} \times 10^{12} M_{\odot}$. Deason et al. (2019), also using counter-rotating halo stars in *Gaia* DR2, found $528^{+0.24}_{-0.25}$ km/s by using a different prior on the slope k of the high-velocity tail of the distribution than assumed by Monari et al. (2018). The authors then inferred a total (baryonic + dark) mass of $M_{200} = 1.00^{+0.31}_{-0.24} \times 10^{12} M_{\odot}$ for their Galactic model. They also found an escape velocity consistent with that of Monari et al. (2018) when using the same priors on k . On the other hand, Eilers et al. (2019) determined the circular velocity curve for a radial range between 5 and 25 kpc from the Galactic centre and inferred a lower virial mass of $(7.25 \pm 0.26) \times 10^{11} M_{\odot}$.

One should, however, keep in mind that the complexities seen in the data, such as the asymmetries and substructures shown in Fig. 1.5, or the significant imprints of merger events shown in Fig. 1.6, hint towards an intricate evolutionary history of the Milky Way. This may have consequences for most dynamical modelling studies of the Galaxy, which often assume that the system is symmetric and in dynamical equilibrium.

1.6 This thesis

The goal of this thesis is to learn more about the shape and dark matter content of the Milky Way and its satellite galaxies. In the different chapters we investigate the Solar Neighbourhood (SN) in order to infer the local dark matter density (Chapter 2), the Milky Way disk in order to put constraints on the shape of the Galactic potential (Chapter 3), the Galactic thick disk and halo to understand the effects of realistic Galactic models on the orbital properties of stars (Chapter 4), and test the ability to retrieve reliable mass and shape measurements for dwarf spheroidal galaxies, dSphs (Chapter 5).

1.6.1 Thesis outline

The structure of this PhD thesis is as follows:

- In Chapter 2 we use *Gaia* DR1 to study a local sample of stars. The availability of high-precision kinematics allows us to estimate the density of dark matter at our location in the Milky Way. We also test the impact of the choices made on the determination of this local dark matter density. We find that the determined local dark matter density is strongly dependent on the scale height assumed for our

tracer stars. The systematic uncertainties on the local dark matter estimate are significantly larger than the statistical ones, indicating that more sophisticated models are needed to improve on the estimate being made.

- In Chapter 3 we use *Gaia* DR2 to study a sample of disk and halo stars. We investigate the kinematics of the stars and focus on the orientation of the velocity ellipsoid as a function of position in the Milky Way in the hope to put constraints on the shape of the potential. We find that the *Gaia* DR2 distances determinations are not accurate enough to assess whether or not the ellipsoid is spherically aligned everywhere. We analyse the influence of these systematic parallax errors on the measurement of the orientation of the velocity ellipsoid and reflect our findings to Galactic models. Future *Gaia* releases are expected to give a more robust measurement and will likely make it possible to pin down whether or not the alignment changes towards cylindrical alignment for the innermost Galactic radii.
- In Chapter 4 we apply *Gaia* DR2 to study the orbits of stars in different Galactic models. We characterise the orbits by performing a frequency analysis on their trajectories. This enables us to explore a whole new parameter space, besides the well-studied velocity space and the more standard orbital parameter spaces. We find that substructures in orbital parameter space, especially apocentre *vs.* maximum distance from the Galactic mid-plane, are not necessarily always due to merging events. Instead, we find that resonances in the orbital frequencies, due to the nature of the Galactic potential, can be the cause of the over-densities and depleted regions found.
- In Chapter 5 we move on to the satellites of our Galaxy. The dSphs are one of the most dark matter dominated systems in the Universe, which make them very interesting to study their dark matter content, thus potentially yielding complementary results on dynamical models of the Milky Way. We test how accurately the flattening and mass of dwarf spheroidal galaxies can be retrieved by setting up a realistic mock dwarf spheroidal galaxy. We release the commonly used assumption of sphericity and consider axisymmetry instead. The mass of the mock galaxy is always recovered well, independently of

the flattening assumed for the potential of the system. It is, however, not possible to determine the flattening itself. We argue that this is mostly likely due to limitations (such as an isothermal distribution function) in the mock galaxy model used.

1.7 Future Outlook

In the coming years, future *Gaia* data releases will continue to improve the quality of the data, reducing the astrometric biases currently known to exist. Line-of-sight velocities will be determined for stars up to $G_{\text{RVS}} \sim 15$ mag, thus increasing the number of stars with full phase-space information. Future data releases will also provide stellar parameters such as effective temperatures, metallicities and gravity measurements, making it easier to study different populations of stars. Astrometric measurements will be improved for all stars up to $G \sim 20.5$ mag. Wide-field spectroscopic surveys such as the ‘William Herschel Telescope Enhanced Area Velocity Explorer’ (WEAVE) in the Northern Hemisphere and the ‘4-metre Multi-Object Spectrograph Telescope’ (4MOST) in the Southern Hemisphere will soon start to determine the line-of-sight velocities and stellar parameters for stars in this magnitude regime. Moreover, for a large subset of stars chemical abundances will be determined.

With the increasing quality of the datasets, the models of our Galaxy themselves also need to be improved. Ideally assumptions on axisymmetry and dynamical equilibrium should be released to generate as realistic as possible models of the Galaxy. It is already known that such effects play an important role in the determination of the local dark matter density (Chapter 2), and thus better data will only decrease the statistical error on the measurements, but will not reduce the systematic biases - such as those caused by assuming time-independence and thus e.g. neglecting ongoing mergers -, which already seem to dominate such estimates.

Once these problems have been tackled mass and shape determinations of the Milky Way and therefore estimates on the dark matter content of the Galaxy can be significantly improved. We can further hope to fully pin down the evolution history of the Milky Way, which - when compared to predictions from current cosmological models - can lead to further improvements in the models and therefore to a better understanding of our Universe.

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Chapter 2

The vertical force in the solar neighbourhood using red clump stars in TGAS and RAVE

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Abstract

We investigate the kinematics of red clump (RC) stars in the solar neighbourhood by combining data from Tycho-Gaia Astrometric Solution (TGAS) and Radial Velocity Experiment (RAVE) to constrain the local dark matter density. After calibrating the absolute magnitude of RC stars, we characterized their velocity distribution over a radial distance range of 6-10 kpc and up to 1.5 kpc away from the Galactic plane. We then applied the axisymmetric Jeans equations on subsets representing the thin and thick disks to determine the (local) distribution of mass near the disk of our Galaxy. Our kinematic maps are well behaved, permitting a straightforward local determination of the vertical force, which we find to be $K_z^{\text{thin}} = -2454 \pm 619 \text{ (km/s)}^2/\text{kpc}$ and $K_z^{\text{thick}} = -2141 \pm 774 \text{ (km/s)}^2/\text{kpc}$ at 1.5 kpc away from the Galactic plane for the thin and thick disk samples and for thin and thick disk scale heights of 0.28 kpc and 1.12 kpc respectively. These measurements can be translated into a local dark matter density $\rho_{\text{DM}} \sim 0.018 \pm 0.002 M_{\odot}/\text{pc}^3$. The systematic error on this estimate is much larger than the quoted statistical error, since even a 10% difference in the scale height of the thin disk leads to a 30% change in the value of ρ_{DM} and a nearly equally good fit to the data.

2.1 Introduction

With the launch of the *Gaia* satellite a wealth of new data is becoming available on the motions and positions of stars in the Milky Way and its satellite galaxies (Gaia Collaboration et al. 2016a,b). For example, its first data release (*Gaia* DR1) containing the Tycho-Gaia Astrometric Solution (TGAS) has provided parallaxes that are roughly a factor two more precise (and proper motions of similar quality) than those in the Hipparcos catalogue (ESA 1997), but for a sample that is unbiased and nearly 20 times larger. Furthermore, and specifically for the Hipparcos stars themselves, the improvement in the proper motions is more than a factor 10 (Lindgren et al. 2016). The power of these data increases even further when combined with spectroscopic surveys such as the Radial Velocity Experiment (RAVE, Kunder et al. 2017), the Apache Point Observatory Galactic Evolution Experiment (APOGEE, Majewski et al. 2017) and the Large Sky Area Multi-Object Fiber Spectroscopic Telescope (LAMOST, Cui et al. 2012), as this provides knowledge of the full phase-space distribution of stars near the Sun, as shown in, for example, Allende Prieto et al. (2016); Helmi et al. (2017); Liang et al. (2017); Monari et al. (2017); Yu & Liu (2018).

New kinematic maps of the solar neighbourhood can be used, for example, to obtain more precise estimates of the local dark matter density ρ_{DM} . Most modern measurements of ρ_{DM} using the vertical kinematics of stars seem to be consistent with a value just below $\sim 0.01 M_{\odot}/\text{pc}^3$ when assuming a total baryonic surface mass density Σ_{baryon} of $55 M_{\odot}/\text{pc}^2$ (Read 2014). McKee et al. (2015) argued for a value of $0.013 M_{\odot}/\text{pc}^3$ for $\Sigma_{\text{baryon}} = 47.1 M_{\odot}/\text{pc}^2$, and Bienaymé et al. (2014) found $\rho_{\text{DM}} = 0.0143 \pm 0.0011 M_{\odot}/\text{pc}^3$ for $\Sigma_{\text{baryon}} = 44.4 \pm 4.1 M_{\odot}/\text{pc}^2$ using red clump (RC) stars in RAVE DR4. Recently Sivertsson et al. (2017) determined a dark matter density of $0.012 M_{\odot}/\text{pc}^3$ using SDSS-SEGUE G-dwarf stars for $\Sigma_{\text{baryon}} = 46.85 M_{\odot}/\text{pc}^2$. These estimates are consistent with those inferred from studying the local equation of centrifugal equilibrium (Salucci et al. 2010) or from studies that model the mass of the Milky Way globally (e.g. Piffl et al. 2014).

Despite the apparent good agreement between the various values reported in the literature, local dark matter density estimates are affected by different systematic uncertainties (see e.g. Silverwood et al. 2016). These include of course, uncertainties on Σ_{baryon} , which depend on the gas mass density, which typically has an error of 50%, and the local stellar densities

Figure 2.1 – Selection and calibration of RC stars. Left: Distribution of giant stars in our sample in the extinction-corrected colour vs. surface gravity space. The blue lines enclose our RC sample. Right: Extinction-corrected HR diagram of stars in our sample. The solid and dashed light blue lines indicate the adopted mean and standard deviation of the RC absolute K_S -band magnitude, respectively.

although these are typically determined with 10% accuracy or better (e.g. Holmberg & Flynn 2000; Bovy 2017). Furthermore, the presence of multiple populations and assumptions made regarding their distributions (e.g. Moni Bidin et al. 2012; Büdenbender et al. 2015; Hessman 2015) or deviations from equilibrium, such as bending or breathing of the disk (Widrow et al. 2012; Williams et al. 2013), may also affect the conclusions reached. For example, Banik et al. (2017) showed that breathing modes alone may lead to systematic errors of order 25% in ρ_{DM} .

In this chapter we use TGAS and RAVE data to derive an estimate of the local dark matter density and explore the impact of uncertainties in (some of) the characteristic parameters of the Galactic thin and thick disks. In Sect. 2.2 we present the data and selection criteria used in this chapter as well as the new kinematic maps of the solar neighbourhood. In Sect. 2.3 we present our local dark matter measurement and the impact of the disk parameters on the estimate. We conclude in Sect. 2.4.

2.2 Data

The dataset we use stems from the cross-match of TGAS and RAVE DR5. The TGAS dataset contains ~ 2 million stars that are in common between the *Gaia* DR1 catalogue and the Hipparcos and Tycho-2 catalogues and

provides accurate positions on the sky, mean proper motions and parallaxes. The RAVE survey is a magnitude-limited survey ($9 < I < 12$) of stars in the southern hemisphere (~ 450000 stars), that for low Galactic latitudes ($b < 25^\circ$) uses a colour criterion, $J - K_s \geq 0.5$, which preferentially selects giant stars. RAVE provides radial velocities, astrophysical parameters, as well as a spectro-photometric parallaxes. The overlap between RAVE DR5 and TGAS contains ~ 250000 stars with full phase space information. McMillan et al. (2017, PJM2017 hereafter) used the TGAS astrometric parallax measurements as priors to derive *improved* spectro-photometric parallaxes and astrophysical parameters. This is the dataset that we use here, after applying a few extra quality cuts. We keep only those stars that have $\text{SNR}_K > 20$, an `ALGO_CONV` parameter equal to either 0 or 4, `eHRV` < 8 km/s, and `flag_any` = 0. This leaves us with a ‘reliable’ sample containing 108679 stars. The median distance of stars in this sample is of ~ 0.5 kpc, and their median relative distance error is $\sim 13\%$.

2.2.1 Selecting a red clump stars’ sample

To gain more in distance accuracy we use RC stars, as they act as standard candles (e.g. Paczyński & Stanek 1998; Groenewegen 2008; Girardi 2016). We select RC stars on the basis of the surface gravity $\log(g)_{\text{PJM}}$ and the extinction-corrected 2MASS bands J_{corr} and $K_{S,\text{corr}}$, where $J_{\text{corr}} = J - A_J$ and $K_{S,\text{corr}} = K_S - A_{K_S}$, and $A_J = 0.282 A_V$ and $A_{K_S} = 0.112 A_V$, where A_V is taken from PJM2017. Finally we define our ‘RC sample’ by:

$$0.52 \leq (J - K_S)_{\text{corr}} \leq 0.72 \quad \text{and} \quad 2.35 \leq \log(g)_{\text{PJM}} \leq 2.60, \quad (2.1)$$

and which contains 26653 stars. We calibrate the RC absolute K_s -band magnitude by considering a subsample of 3211 RC stars with a maximum relative parallax error of 10%. We find these stars to have a mean absolute K_s -band magnitude of $M_{K_S}^{\text{RC}} = -1.604$ mag and a dispersion of 0.064 mag, which we round-off to $M_{K_S}^{\text{RC}} = -1.60$ mag and a dispersion of 0.1 mag (which translates into a $\sim 5\%$ error in distance) in the remainder of this chapter. These values depend slightly on the maximum parallax error imposed for the calibration, and they are consistent with Hawkins et al. (2017), who find $M_{K_S}^{\text{RC}} = -1.61 \pm 0.01$ mag and a dispersion of 0.17 ± 0.02 , and Ruiz-Dern et al. (2017) who find $M_{K_S}^{\text{RC}} = -1.606 \pm 0.009$ mag, both works mainly using TGAS parallaxes and APOGEE spectroscopy.

In the left panel of Fig. 2.1 we show the distribution of giant branch stars in our sample in the extinction-corrected $(J - K_s)_{\text{corr}}$ colour versus surface gravity $\log(g)_{\text{PJM}}$ space. Our selection of RC stars is given by the light blue box. In this region there will be some contamination of red giant branch (RGB) stars since RAVE does not provide asteroseismic information that can be used to discriminate between RC and RGB stars (e.g. Bedding et al. 2011; Chaplin & Miglio 2013). According to Girardi (2016) the typical contamination fraction is of order 30% of the $\log(g)$ box width in dex used to define the RC sample, thus resulting in a contamination estimate of 7.5% for our RC selection criteria. In the right panel of Fig. 2.1 we show a similar region of an extinction-corrected HR diagram using the PJM2017 improved parallaxes for the stars from the reliable sample. The mean and standard deviation of the adopted RC absolute K_S -band magnitude are plotted as the horizontal solid and dashed lines, respectively.

2.2.2 General properties of the RC sample

Now that we calibrated the mean absolute magnitude of RC stars and its spread, we proceed to compute distances to our stars. We define a default realization in which all RC stars are assumed to have $M_{K_S}^{\text{RC}} = -1.60$ mag. The resulting spatial distribution of stars is shown in Fig. 2.2 and is computed in bins with widths 0.5 kpc in R and 0.1 kpc in z after setting $R_{\odot} = 8.3$ kpc (Schönrich 2012) and $z_{\odot} = 0.014$ kpc (Binney et al. 1997) for the position of the Sun with respect to the Galactic centre. This figure shows that our sample covers $R \sim 6 - 10$ kpc and $|z| \lesssim 1.5$ kpc. Since the RAVE survey is more orientated to the southern and inner part of the Galaxy, there are more RC stars observed in these regions.

When converting the observables to Galactocentric cylindrical coordinates and velocities, we additionally use

- $(U, V, W)_{\odot} = (11.1, 12.24, 7.25)$ km/s (Schönrich et al. 2010) for the peculiar motion of the Sun with respect to the local standard of rest (LSR), and where U is radially inward, V in the direction of Galactic rotation, and W perpendicular to the Galactic plane and positive towards the north Galactic pole;
- $v_c(R_{\odot}) = 240$ km/s (Piffl et al. 2014) for the circular velocity at $R = R_{\odot}$.

Figure 2.2 – Meridional plane RC star counts in bins of 0.5 kpc in R and 0.1 kpc in z (as indicated by the box in the upper right corner). The white contours indicate where the number of RC stars in the bins have dropped to 200, 100, and 50 from inner to outer contours respectively. The black symbol marks the position of the Sun. This figure is based on the default realization in which the absolute magnitudes of all RC stars are set to $M_{K_S}^{\text{RC}} = -1.60$ mag.

Figs. 2.3 and 2.4 show kinematic maps, i.e. mean velocities and their standard deviations, respectively, in the meridional plane using the same default realization and bin widths as before. These figures show the well-known decrease of the average rotational velocity $\langle v_\phi \rangle$ with height above the plane and the increase of the velocity dispersions with increasing $|z|$. We also see a variation (of order 5 km/s/kpc) in $\langle v_R \rangle$ with respect to R and some asymmetry in the vertical direction, as found in other works (e.g. Casetti-Dinescu et al. 2011; Siebert et al. 2011; Williams et al. 2013). On the other hand we do not find strong evidence for a bending or breathing mode in $\langle v_z \rangle$, at least up to 0.7 kpc (see also Carrillo et al. 2017).

2.3 Local dark matter density estimate

Now that we have constructed a good quality kinematic dataset, we proceed to estimate the local dark matter density in the steady-state axisymmetric limit. Although our kinematic maps reveal some deviations, these are of

Figure 2.3 – The mean velocities of RC stars in the meridional plane based on the default realization in which the absolute magnitudes of all RC stars are set to $M_{K_S}^{\text{RC}} = -1.60$ mag. The white contours and black symbol have the same meaning as in Fig. 2.2.

Figure 2.4 – Similar to Fig. 2.3, but now showing the standard deviation of the velocities.

sufficiently small amplitude¹ that we neglect them in our analysis. In this section, we first discuss the basic equations that relate the mass density to the kinematic moments, then describe how we measure these moments, and finally present our new determination and discuss the influence of the uncertainties on the main parameters of our mass model.

2.3.1 Surface mass density and the vertical Jeans equation

The (integrated) Poisson equation in cylindrical coordinates links the total surface mass density $\Sigma(R, z)$ to the components of the gravitational force

¹In the analysis carried out below we find $\langle v_R \rangle = 5.8 \pm 3.3$ km/s and $\langle v_z \rangle = -0.6 \pm 1.9$ km/s, for the thick disk, and 1.4 ± 1.1 and 0.5 ± 0.6 km/s, respectively, for the thin disk RC stars in our sample (at large heights).

per unit mass in the radial, F_R , and vertical direction, K_z , via

$$-2\pi G\Sigma(R, z) = K_z(R, z) + \int_0^{+z} \frac{1}{R} \frac{\partial(RF_R)}{\partial R} dz', \quad (2.2)$$

where $\Sigma(R, z) = \int_{-z}^{+z} \rho_{\text{tot}}(R, z') dz'$.

Under the assumption of equilibrium, we can use the Jeans equations to relate the moments of the distribution function of a population, such as its density and velocity moments, to the gravitational potential in which it moves $\Phi(R, z)$. In this case,

$$\begin{aligned} K_z &\equiv -\frac{\partial\Phi}{\partial z} \\ &= -\frac{\langle v_z^2 \rangle}{z} [\gamma_{*,z} + \gamma_{\langle v_z^2 \rangle, z}] + \frac{\langle v_R v_z \rangle}{R} [1 - \gamma_{*,R} - \gamma_{\langle v_R v_z \rangle, R}] \end{aligned} \quad (2.3)$$

and

$$\begin{aligned} F_R &\equiv -\frac{\partial\Phi}{\partial R} \\ &= -\frac{\langle v_\phi^2 \rangle}{R} + \frac{\langle v_R^2 \rangle}{R} [1 - \gamma_{*,R} - \gamma_{\langle v_R^2 \rangle, R}] - \frac{\langle v_R v_z \rangle}{z} [\gamma_{*,z} + \gamma_{\langle v_R v_z \rangle, z}]. \end{aligned} \quad (2.4)$$

In these equations we have defined

$$\gamma_{Q,x} \equiv -\frac{\partial \ln [Q(x)]}{\partial \ln [x]},$$

where $\gamma_{*,x}$ is the log-slope of the stellar density profile of the population with respect to coordinate x ($=R$ or z), and otherwise $\gamma_{Q,x}$ denotes the log-slope of the velocity moments. The steady-state assumption implies that $\langle v_R \rangle = \langle v_z \rangle = 0$, and hence $\text{cov}(v_R, v_z) = \langle v_R v_z \rangle$, and $\sigma^2(v_R) = \langle v_R^2 \rangle$, and analogously for $\sigma^2(v_z)$.

From Eq. 2.3 and 2.4 and using the kinematic moments, we can derive the force field, which when inserted in Eq. 2.2, allows us to derive the total surface mass density. This includes the contributions of all baryonic components and a putative dark matter component, whose contribution we can establish with the data. We note, therefore, that not only accurate measurements of the velocity moments and their variation with R and z are needed, but also knowledge of the radial and vertical slopes of the density distribution, and the surface densities of the various baryonic components;

i.e. interstellar medium (ISM) and stars. Since our dataset does not allow us to derive these quantities reliably, we have to make additional assumptions.

To reduce the complexity of the problem, one may note that the last term of Eq. 2.2 is approximately zero near $R = R_\odot$ and $z = 0$, since the circular velocity curve $v_c^2(R) = -RF_R(R, z = 0)$ is approximately flat at the solar Galactocentric radius (e.g. Reid et al. 2014). Kuijken & Gilmore (1989) showed that in this case the term can be neglected up to a few kpc in z . Furthermore, Bovy & Tremaine (2012) showed that $\frac{\partial(RF_R)}{\partial R}$ decreases as one moves away from the midplane for reasonable models of the Milky Way (although this has been challenged by Moni Bidin et al. 2015). Therefore dropping the term leads to an underestimate of the surface mass density $\Sigma(R, z)$, which Bovy & Tremaine (2012) found in their models to be a few percent at $z = 1.5$ kpc up to roughly 15% at $z = 4$ kpc.

Our dataset does not extend to heights larger than 1.5 kpc, which implies that we can solve the integrated Poisson equation by only evaluating the vertical Jeans equation. We assume an exponential disk and that $\sigma^2(v_R)$ and $\sigma^2(v_z)$ follow an exponential profile in R with the same scale length as the density (e.g. van der Kruit & Searle 1982; Lewis & Freeman 1989). For the tilt angle, which relates to the last term in Eq. 2.3, we do not assume spherical alignment² but that it is constant with R . We then combine Eq. 2.2 and 2.3 to yield

$$-2\pi G\Sigma(R, z) \simeq -\frac{\sigma(v_z)^2}{h_z} + \frac{\partial\sigma(v_z)^2}{\partial z} + \text{cov}(v_R, v_z) \left[\frac{1}{R} - \frac{2}{h_R} \right], \quad (2.5)$$

where h_z and h_R are the vertical and radial scale heights of the population traced by the stars explored. Eq. 2.5 can be applied to multiple populations that satisfy the assumptions described.

We emphasize that as significantly more data with much better quality will be available soon (e.g. *Gaia* DR2), one should aim to solve the full set of equations (i.e. including all terms in Eq. 2.2), especially when going towards larger Galactic heights. Getting a better handle on both the vertical and radial slopes of the velocity moments and on the density profiles of the samples traced would reduce the number of assumptions needed to estimate the dark matter density.

²We find that the difference in the amplitude of K_z for a spherically aligned ellipsoid or one where $\text{cov}(v_R, v_z)$ varies as $\sigma^2(v_R)$ and $\sigma^2(v_z)$, is less than 10% of the uncertainty on K_z due to measurement errors.

2.3.2 Analysis of the data

To mimic the uncertainties of the distances, we draw 1000 realizations of the absolute K_S -band magnitude for each star in our RC sample, assuming a Gaussian distribution characterized by our calibrated mean and dispersion (see Sect. 2.2.1). This effectively leads to 1000 distance realizations for the RC stars selected from the PJM2017 sample. In each realization we transform the observables to a Galactocentric cylindrical coordinate frame and propagate the errors. For this procedure, we assume no error on the positional coordinates and use the TGAS values for the proper motion errors and their correlations. Since radial velocities are observed independently by RAVE there are no correlations with the proper motion measurements from TGAS.

Given that we are only interested in the vertical trend of the gravitational force K_z , in each realization we consider only those stars that are within 0.5 kpc in R from $R_\odot$. We fold the data below the plane towards positive z by flipping the signs of z and v_z and select the subset of stars that trace the population under consideration (see Sect. 2.3.2). We then take bins in z such that they contain at least 100 stars from the population and from each bin we remove the outlier stars, by iteratively eliminating stars outside the tilted velocity ellipsoid that would contain 99.994% of the stars in case of a perfect multi-variate Gaussian distribution. This clipping is performed in (v_R, v_z, v_ϕ) -space.

Since measurement errors in general inflates the observed velocity dispersion, we subsequently attempt to solve for the intrinsic velocity dispersions of the population by maximizing the bivariate Gaussian likelihood function L of the (v_R, v_z) -data. The error terms are set by the true intrinsic dispersions σ_{intr} of the population and the measurement errors ϵ_i of star i , summed in quadrature,

$$\begin{aligned} L_i &= L_i[\langle v_R \rangle, \sigma(v_R)_{\text{intr}}, \langle v_z \rangle, \sigma(v_z)_{\text{intr}}, \text{cov}(v_R, v_z)_{\text{intr}}] \\ &= \frac{1}{\sqrt{\det(2\pi \Sigma_i)}} \exp \left[-\frac{1}{2} (\mathbf{x}_i - \boldsymbol{\mu})^\top \Sigma_i^{-1} (\mathbf{x}_i - \boldsymbol{\mu}) \right], \end{aligned} \quad (2.6)$$

where $\mathbf{x}_i = [v_{R,i}, v_{z,i}]$, $\boldsymbol{\mu} = [\langle v_R \rangle, \langle v_z \rangle]$, and

$$\Sigma_i = \begin{bmatrix} \sigma^2(v_R)_{\text{intr}} + \epsilon^2(v_{R,i}) & \text{cov}(v_R, v_z)_{\text{intr}} + \text{cov}(v_{R,i}, v_{z,i}) \\ \text{cov}(v_R, v_z)_{\text{intr}} + \text{cov}(v_{R,i}, v_{z,i}) & \sigma^2(v_z)_{\text{intr}} + \epsilon^2(v_{z,i}) \end{bmatrix},$$

Figure 2.5 – Influence of the adopted metallicity ranges for the thin (left) and thick (right) disk RC samples on the intrinsic vertical velocity dispersions (for the default realization). For the thick disk velocity dispersion profile only there is a dependence on the adopted upper metallicity boundary, partly due to contamination by the low metallicity tail of the thin disk. The bins plotted are fully independent (non-overlapping).

with $\text{cov}(v_{R,i}, v_{z,i})$ the covariance of the measurement errors in the (v_R, v_z) components for star i . Thus

$$L = \prod_{i=1}^N L_i, \quad (2.7)$$

given the N stars of the population in the bin under consideration.

We employ Markov chain Monte Carlo (MCMC) modelling (Foreman-Mackey et al. 2013) to solve for the intrinsic velocity dispersions $\sigma(v_R)_{\text{intr}}$ and $\sigma(v_z)_{\text{intr}}$, the mean velocities $\langle v_R \rangle$ and $\langle v_z \rangle$, and the covariance term $\text{cov}(v_R, v_z)_{\text{intr}}$ in each bin in z . Each MCMC run also returns an error estimate of these moments. Priors to the MCMC model are added such that the dispersions in v_R and v_z should be positive and such that the absolute value of the correlation of v_R and v_z is always smaller or equal to one.

The procedure of taking bins in z , removing outlier stars, and solving for the intrinsic moments is repeated for every realization of the RC sample and for each population of stars.

Figure 2.6 – Mean intrinsic vertical velocity dispersion profile over all realizations for both the thin metal-rich (red) and thick metal-poor (blue) disk RC stars. The error bars show the spread over all realizations due to the spread in the absolute magnitude of the RC stars and as a result of the error deconvolution.

Vertical velocity dispersion profiles: Two components

The Milky Way contains a thin and a thick disk that in the solar neighbourhood are known to have different spatial, kinematical, and metallicity distributions. In particular, the thick disk is hotter, has a larger scale height, and is typically more metal poor, becoming more dominant for $[\text{Fe}/\text{H}] \lesssim -0.5$ dex. These two populations should thus be treated separately when attempting to solve Eq. 2.5.

Since the distributions of thin and thick disk stars show a certain amount of overlap in most observables, we first investigate how the choice of different metallicity ranges for the thin and thick disk tracer RC samples affect the intrinsic z -velocity dispersion profiles. For the metallicity we use the calibrated values, `Met_N_K`, as provided by RAVE. In Fig. 2.5 we show the results using the default realization (i.e. for the distance) after deconvolution of the measurement errors (see Eqs. 2.6 and 2.7). For this figure we require a minimum of 50 stars per bin.

This figure shows that a change in the adopted metallicity boundary for the thin disk has no significant effect on the vertical velocity dispersion profile (left panel). Therefore, for the thin disk we simply choose

$\text{Met_N_K} \geq -0.25$ dex. For the thick disk on the other hand, changing the upper metallicity limit can have a significant influence on the recovered dispersion profile (see the right panel of the figure), most likely driven by contamination by thin disk stars. To get a clean sample (and to avoid the inclusion of halo stars) we adopt the following range in what follows: $-1.00 \leq \text{Met_N_K} \leq -0.50$ dex. Although we could have chosen an even lower value for the upper limit, this comes at the cost of an important decrease in the number of stars. For example in the default realization there are 586, 427, or 386 stars in the thick disk sample in between $z = 0.55$ and $z = 1.7$ kpc when using upper metallicity boundaries of -0.5 , -0.55 , and -0.6 dex, respectively.

After finding the intrinsic velocity moments as functions of z for each of the 1000 realizations for the samples representing the thin and thick disk, we compute their mean and dispersion over all realizations, also taking into account the errors on the moments in each single realization that were determined in the MCMC modelling procedure. These dispersions thus account for the error on the moments due to the errors in proper motion and radial velocity and to the unknown distances to the RC stars. In Fig. 2.6 we show the mean vertical velocity dispersion profiles for both the thick (blue) and thin (red) disk sample, overplotted on the dispersion profiles from all individual realizations (for which the error bars are omitted in this plot). From this figure we see that the velocity dispersion profiles increase quickly with z and that above $z \sim 0.5$ kpc their variation is much shallower, particularly for the thick disk sample.

The K_z force

Because our goal is to measure the contribution of dark matter more reliably, we focus on the bins at large Galactic heights, since for small z the baryons (are expected to) dominate the gravitational force. We thus choose to explore only those bins for which the central z -coordinate satisfies $|z| \geq 0.6$ kpc.

We are now ready to insert the moments (and their errors) in Eq. 2.5. This equation also requires knowledge of the variation of $\sigma(v_z)$ with z . Rather than differentiating the data directly, we fit a linear function to $\sigma(v_z)$ as function of z , whose slope and uncertainty are then used in Eq. 2.5.

In order to compute K_z , we set $R = R_\odot$ in the last term of Eq. 2.5, and we fix the scale lengths of both disks to an intermediate value of 2.5 kpc

Figure 2.7 – Derived vertical force K_z (black lines) for the thin (solid lines) and the thick disk samples (dashed lines). The coloured lines show the decomposition of K_z into the terms concerning the vertical velocity dispersion (blue), the slope of the vertical variance as function of z (green), and the mixed velocity moment (red). The curves shown were computed for the combination $h_z^{\text{thin}} = 0.28$ kpc and $h_z^{\text{thick}} = 1.12$ kpc.

(e.g. Siegel et al. 2002; Jurić et al. 2008; Bensby et al. 2011; McMillan 2011; Bovy et al. 2012; Robin et al. 2012, 2014; Bland-Hawthorn & Gerhard 2016). Finally we explore how K_z varies for a set of scale heights for the thin and thick disk populations.

Fig. 2.7 shows the derived vertical forces in black for $h_z^{\text{thin}} = 0.28$ kpc and $h_z^{\text{thick}} = 1.12$ kpc. The solid line corresponds to the thin disk sample, the dashed line to the thick disk sample. Since we folded our data towards positive z , the forces derived are negative. We find $K_z^{\text{thin}} = -2454 \pm 619$ (km/s)²/kpc and $K_z^{\text{thick}} = -2141 \pm 774$ (km/s)²/kpc at 1.5 kpc away from the Galactic plane. We also show the decomposition of K_z into the three terms of Eq. 2.5: the first associated with vertical velocity dispersion (blue), the second with the slope of the vertical variance as function of z (green), and the third with the mixed velocity moment (red). The first term dominates K_z for both samples. In addition, we find that the error on K_z is dominated by that on the vertical velocity dispersion for the thin disk sample, while for the thick disk it is the error on the slope.

2.3.3 Mass model

Now that we determined K_z , and hence the total surface mass density $\Sigma(z)$ according to Eq. 2.2 (which is strictly equivalent only if we assume $\partial(RF_R)/\partial R = 0$ for all z), we proceed to compare it to the surface mass density derived from the contribution of all baryonic components and explore the need for dark matter.

We characterize the baryonic components with a double exponential stellar thin and thick disk, and an infinitely thin ISM disk with a surface mass density equal to $13M_\odot/\text{pc}^2$ (Holmberg & Flynn 2000), and consider a constant dark matter density³. We further fix the midplane stellar density at the solar radius to $0.043M_\odot/\text{pc}^3$ (McKee et al. 2015), which is very similar to the value of $(0.040 \pm 0.002)M_\odot/\text{pc}^3$ found by Bovy (2017). We fix the midplane thick-to-thin disk density ratio to 12% (Jurić et al. 2008)⁴.

To explore the effect of the assumed thin and thick disk scale heights on the local dark matter density estimates, we sample the thin disk scale height from 200 to 400 pc and the thick disk scale height from 700 to 1300 pc, such that typical estimates reported in the literature are covered (e.g. Bland-Hawthorn & Gerhard 2016). For each combination we determine the value of ρ_{DM} by minimizing the total χ^2 -value defined as

$$\chi^2 = \sum_{j=1}^{N_{\text{thin}}} \left(\frac{\text{model}(j) - \text{data}(j)}{\text{error}(j)} \right)_{\text{thin}}^2 + \sum_{j=1}^{N_{\text{thick}}} \left(\frac{\text{model}(j) - \text{data}(j)}{\text{error}(j)} \right)_{\text{thick}}^2 \quad (2.8)$$

where j runs over all data points for which surface mass densities have been derived for both the thin and thick disk data. The best-fit model thus minimizes jointly $\chi_{\text{thin}}^2 + \chi_{\text{thick}}^2$, but to understand the quality of the models, we inspect separately the reduced $\chi_{\text{thin,red}}^2$ and $\chi_{\text{thick,red}}^2$.

The results for various combinations of scale height parameters are shown in Fig. 2.8. In the left panel we show the χ^2 values as functions of the local dark matter density parameter for a number of models. The thick black curve shows the case of a model with scale heights $h_z^{\text{thin}} = 0.28$

³The assumption of a constant dark matter density is a reasonable approximation for the volume probed by our data, since for example the difference in surface mass density with an NFW halo with a scale radius of 14.4 kpc and a flattening 0.9 as in Piffi et al. (2014), is only $\sim 1\%$ at $z = 1.5$ kpc.

⁴Varying this ratio has a minor impact on the results described in this section, as factor two decrease leads to an increase in ρ_{DM} of only 4.5%.

Figure 2.8 – Left: χ^2 values as functions of the local dark matter density for various combinations of the scale heights of the thin and thick disks. The (light to dark) blue curves have $h_z^{\text{thin}} = 0.28$ kpc and $h_z^{\text{thick}} = [0.85, 0.94, 1.03, 1.21, 1.30]$ kpc, while those from orange to dark red have $h_z^{\text{thin}} = [0.22, 0.24, 0.26, 0.27, 0.29, 0.30, 0.32, 0.34]$ kpc and $h_z^{\text{thick}} = 1.12$ kpc. The thick black curve corresponds to the global minimum of the χ^2 landscape, which is found for the model with $h_z^{\text{thin}} = 0.28$ kpc and $h_z^{\text{thick}} = 1.12$ kpc. The horizontal dashed line indicates a $\Delta\chi^2 = 1$ with respect to this minimum χ^2 -value. Right: Map showing the dependence of the best-fit values of the local dark matter density on the adopted scale height for the thin (x-axis) and thick (y-axis) disk. The total χ^2 is minimum at the position of the star symbol. The yellow lines indicate where the total baryonic surface mass density in the models equals 35, 40, 45, 50, and $55M_\odot/\text{pc}^2$.

and $h_z^{\text{thick}} = 1.12$ kpc. This is the model with the lowest χ^2 among all the different combinations explored. The dashed horizontal line corresponds to $\Delta\chi^2 = 1.0$ with respect to its minimum value, which is the criterion we use to quantify the error on the dark matter density determination. In this panel we also show (from orange to dark red) the χ^2 curves for models with varying thin disk scale height for fixed $h_z^{\text{thick}} = 1.12$ kpc. Models with fixed $h_z^{\text{thin}} = 0.28$ kpc, but with varying thick disk scale height are given with colours varying from light to dark blue. We thus see that a relatively large change in thick disk scale height does not alter the inferred dark matter density by much, although $\Delta\chi^2 > 1$ for, for example $h_z^{\text{thick}} \leq 0.95$ kpc. On the other hand, a small change in the thin disk scale height has a large influence on the inferred dark matter density.

This is also depicted in the right panel of Fig. 2.8, where we show the best-fit dark matter densities for all combinations of scale heights explored. The green star indicates the location in $(h_z^{\text{thin}}, h_z^{\text{thick}})$ -space where the model can fit the data the best. The larger impact of the thin disk scale height

Figure 2.9 – Maps of the reduced χ^2 values computed for the thin (left) and thick (right) disk samples separately. The thin disk sample is generally fitted well with the minimization of the joint χ^2 , and this is because it contributes with a larger number of data points with smaller error bars. This is not the case for the thick disk, where for certain combinations of the scale heights the fit is poor for this sample ($\chi^2_{\text{thick,red}} > 1$).

on the fitted dark matter density is probably driven by the smaller error bars on the gravitational force profile implied from the thin disk sample in comparison to that from the thick disk sample (see e.g. Fig. 2.6). Yellow lines indicate where the total baryonic surface mass density in the models equals $35, 40, 45, 50$, and $55 M_\odot/\text{pc}^2$. Most previous works seem to agree on baryonic surface mass densities in the range $40 - 55 M_\odot/\text{pc}^2$ (e.g. McKee et al. 2015, and references therein), thus the models with combinations of scale heights that result in smaller or larger baryonic contributions are less plausible.

In Fig. 2.9 we explore further the quality of the fits obtained by comparing the reduced χ^2 values as computed from the thin (left) and thick disk data (right). The panels show that the thin disk data is always fitted well, but that models with a very low (or large) thin and large (or low) thick disk scale height do not lead to a high quality fit for the thick disk data.

In summary, the model and data are thus most consistent for $h_z^{\text{thin}} = 0.28$ kpc and $h_z^{\text{thick}} = 1.12$ kpc, which results in $\Sigma_{\text{baryon}} = 44.8 M_\odot/\text{pc}^2$. In this case, the inferred local dark matter density is $0.018 \pm 0.002 M_\odot/\text{pc}^3$. This model is shown in Fig. 2.10 and has $\chi^2 = 2.6$. With 11 degrees of freedom (12 data points and 1 model parameter), this value implies that the quality of the fit is good. We note that the uncertainty on the scale heights of the populations used constitute a much larger source of error on

Figure 2.10 – Surface mass density as implied from the axisymmetric Jeans equation, derived for the thin (red) and thick (blue) disk samples. In this example the scale height for the thin disk tracer stars (red) is set to 0.28 kpc, the scale height for the thick disk tracer stars (blue) to 1.12 kpc. This combination of parameters gives the lowest χ^2 value (see Fig. 2.8) when fitting our mass model (black), which is the sum of the baryonic (solid yellow) components (thin, ISM, and thick disks from top to bottom in dashed yellow) and the dark matter component (purple).

the estimate of the local dark matter density than the measurement errors alone, which lead to a relative error of order 10% only.

In the analysis presented thus far we considered all data within $|R - R_\odot| \leq 0.5$ kpc. When we restrict ourselves to a smaller volume, within 0.25 kpc from $R_\odot$, the dataset is smaller and this leads to less constraining power, especially for the thick disk set, on the value of the local dark matter density. For this more local sample, we find that the model does not fit the thin disk data well ($\chi^2_{\text{thin,red}} > 1$) for $h_z^{\text{thin}} > 0.35$ kpc and for the thick disk if $h_z^{\text{thick}} \lesssim 3.3h_z^{\text{thin}}$.

2.4 Conclusions

We have studied the kinematics of RC stars from the RAVeXTGAS dataset. The kinematic maps obtained are well behaved, do not show evidence for strong bending or breathing modes up to 0.7 kpc from the midplane,

and only show a small hint of radial motions towards the outer disk. We therefore applied the axisymmetric Jeans equation relating kinematic moments to the vertical force, ultimately yielding a new measurement of the dark matter density in the solar neighbourhood. To account for the presence of multiple populations, we divided the RC sample into thin and thick disk tracer samples according to the metallicity of the stars, as estimated from the RAVE dataset, and fitted both populations simultaneously. This allowed us to determine a local value of the dark matter density with a relative internal error (due to measurement errors on the observables) of only 13.5%, $\rho_{\text{DM}}(R_{\odot}, 0) = 0.018 \pm 0.002 M_{\odot}/\text{pc}^3$, which is in reasonable agreement with previous work.

It is however misleading to consider only the internal errors on the dark matter density, as they do not account for the large systematic uncertainties in the stellar disk parameters (especially the scale heights), the thick-to-thin disk density ratio, the scale lengths of the disks, and the ISM mass. These (external) sources of uncertainty can lead to a large systematic error on the dark matter density near the Sun. For example a 10% difference in the scale height of the thin disk leads to a 30% change in the value of ρ_{DM} , and a nearly equally good fit to the data. The change due to uncertainties on the scale height of the thick disk is slightly weaker. It is therefore extremely important to get accurate constraints on the stellar disk parameters of the tracer stars used. Future *Gaia* data releases will characterize better and more precisely the various stellar components in the Galaxy, and thus allow a more global and accurate approach to determining the contribution of the dark matter, not just locally, but also as a function of position in the Galactic disk.

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Chapter 3

The tilt of the velocity ellipsoid in the Milky Way with *Gaia* DR2

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Abstract

The velocity distribution of stars is a sensitive probe of the gravitational potential of the Galaxy, and hence of its dark matter distribution. In particular, the shape of the dark halo (e.g. spherical, oblate, or prolate) determines velocity correlations, and different halo geometries are expected to result in measurable differences. Here we explore and interpret the correlations in the (v_R, v_z) -velocity distribution as a function of position in the Milky Way. We selected a high-quality sample of stars from the *Gaia* DR2 catalogue and characterised the orientation of the velocity distribution or tilt angle over a radial distance range of $[4 - 13]$ kpc and up to 3.5 kpc away from the Galactic plane while taking into account the effects of the measurement errors. We find that the tilt angles change from spherical alignment in the inner Galaxy ($R \sim 4$ kpc) towards more cylindrical alignments in the outer Galaxy ($R \sim 11$ kpc) when using distances that take a global zero-point offset in the parallax of $-29\mu\text{as}$. However, if the amplitude of this offset is underestimated, then the inferred tilt angles in the outer Galaxy only appear shallower and are intrinsically more consistent with spherical alignment for an offset as large as $-54\mu\text{as}$. We further find that the tilt angles do not seem to strongly vary with Galactic azimuth and that different stellar populations depict similar tilt angles. Therefore we introduce a simple analytic function that describes the trends found over the full radial range. Since the systematic parallax errors in *Gaia* DR2 depend on celestial position, magnitude, and colour in complex ways, it is not possible to fully correct for them. Therefore it will be particularly important for dynamical modelling of the Milky Way to thoroughly characterise the systematics in astrometry in future *Gaia* data releases.

3.1 Introduction

The second data release of the *Gaia* space mission (Gaia Collaboration et al. 2018a) contains more than 1.3 billion stars with measured proper motions and positions and a subset of over 7 million stars with full six-dimensional (6D) phase-space information. The availability of the motions and positions of stars in the Milky Way and its satellite galaxies has already led to new insights about the Galaxy (e.g. Antoja et al. 2018; Belokurov et al. 2018; Helmi et al. 2018; Poggio et al. 2018; Price-Whelan & Bonaca 2018), and many more discoveries will likely follow before *Gaia*’s next data release.

Studies of the Galaxy provide insight about the formation and evolution of galaxies in general, and hence about elements of the cosmological paradigm. For example, detailed dynamical modelling of the Milky Way and its satellites, and in particular their mass distribution, provide critical constraints on the nature of dark matter (e.g. Bonaca et al. 2018). Mass models of the Galaxy, such as those by McMillan (2011), Piffl et al. (2014), and McMillan (2017), have been developed to fit many different observational constraints simultaneously, although this is very challenging. Therefore many works often focus on a specific aspect such as the characterisation of the velocity distribution across the Galaxy.

The in-plane velocity distribution $f(v_R, v_\phi)$ in the Solar vicinity has long been known to be complex, and many moving groups are known to exist (e.g. Proctor 1869; Eggen 1965; Dehnen 1998; Antoja et al. 2008). With *Gaia* DR2 the level of detail visible in the velocity distribution of stars has increased immensely (see e.g. Gaia Collaboration et al. 2018c; Antoja et al. 2018), and a plethora of substructures have become apparent. On the other hand, the 2D velocity distribution describing the radial and vertical velocity components, $f(v_R, v_z)$, shows significantly less substructure and the traditional velocity moments can still describe the data well to first order.

Such velocity moments and thus the axial ratios of the velocity ellipsoid, however, depend on the stellar distribution function and are different for different populations of stars. In contrast, its orientation (or better known as alignment or tilt) is directly related to (the shape of) the underlying gravitational potential in which the stars move (e.g. van de Ven et al. 2003; Binney & Tremaine 2008; Binney & McMillan 2011; An & Evans 2016) and is the focus of this chapter.

Nearly spherically aligned velocity ellipsoids were found for the halo (Smith et al. 2009; Bond et al. 2010; King et al. 2015; Evans et al. 2016) by mainly using data from the Sloan Digital Sky Survey (York et al. 2000). Similar findings were obtained by Posti et al. (2018) for dynamically selected nearby halo stars. These authors obtained full 6D phase-space information by combining radial velocity measurements from the RAdial Velocity Experiment (RAVE DR5, Kunder et al. 2017) to the 5D subset of the *Gaia* DR1 catalogue (Gaia Collaboration et al. 2016). Most recently, Wegg et al. (2018) used 15,651 RR Lyrae halo stars with accurate proper motions from *Gaia* DR2 and also inferred a nearly spherically aligned velocity ellipsoid over a large range of distances between 1.5 kpc and 20 kpc from the Galactic centre. When fed into the Jeans equations, this result seems to imply a spherical dark matter distribution.

Studies focusing on the orientation of the velocity ellipsoid in local samples of the Milky Way disk have also been consistently reporting (close to) spherical alignment. Siebert et al. (2008) have used RAVE DR2 and found a tilt angle γ equal to $7.3^\circ \pm 1.8^\circ$ for red clump stars at $R = R_\odot$ and $z = 1$ kpc, where $\gamma_{\text{sph}} = 7.1^\circ$ would be expected for spherical alignment at this location. Casetti-Dinescu et al. (2011) found $8.6^\circ \pm 1.8^\circ$ for a sample of stars with heights between 0.7 kpc and 2.0 kpc and representative of the metal-rich thick disk, which can be compared to $\gamma_{\text{sph}} = 8.0^\circ$ given the mean location of the sample. Subsequently, Smith et al. (2012) reinforced these findings using data from the Sloan Digital Sky Survey DR7 (SDSS, Abazajian et al. 2009). Binney et al. (2014) using RAVE data, and Büdenbender et al. (2015), using Sloan Extension for Galactic Understanding and Exploration (SEGUE, Yanny et al. 2009), characterised the tilt angle around the Galactic radius of the Sun up to $z \sim 2.0$ kpc by $\gamma(z) \approx a_0 \arctan(z/R_\odot)$. They found $a_0 \sim 0.8$ and $a_0 = 0.9 \pm 0.04$ respectively, values close to, but significantly different from, spherical alignment for which $a_0 = 1.0$. Recently, Mackereth et al. (2019) have analysed the kinematics of mono-age, mono-[Fe/H] populations for both low and high $[\alpha/\text{Fe}]$ samples. They have cross matched the Apache Point Observatory Galactic Evolution Experiment (APOGEE DR14, Majewski et al. 2017) with *Gaia* DR2 to obtain a sample of 65,719 red giant stars located between 4 kpc and 13 kpc in Galactic radius and up to 2 kpc from the Galactic plane. Mackereth et al. (2019) report that the tilt angles found are consistent with spherical alignment for all populations, although they note that the uncertainties are very large.

In this chapter we characterise the orientation of the velocity ellipsoid over a larger section of the Milky Way by using a dataset of more than 5 million stars from *Gaia* DR2. The chapter is organised as follows. In Sect. 3.2 the dataset is introduced as well as the selection criteria applied. In Sect. 3.3 we characterise the velocity distribution and the measurement errors. The results are presented in Sect. 3.4. In that section we also explore differences with azimuth, investigate trends with stellar populations, and put forward a fit that reproduces the variation of the tilt angle with position in the Galaxy. In Sect. 3.5 we explore the effect of systematic errors on our measurements and show that the systematic parallax errors present in *Gaia* DR2 have a significant impact on the tilt angles found. In that Section we therefore also discuss our findings in the context of Galactic models. We summarise in Sect. 3.6.

3.2 Data

We used the subset of *Gaia* DR2 with full 6D information (Gaia Collaboration et al. 2018c). We use the Bayesian distance estimates $\hat{d}$ provided by McMillan (2018) who uses the *Gaia* DR2 parallaxes ϖ and G_{RVS} magnitudes as input. McMillan (2018) takes into account *Gaia* DR2's overall parallax offset of $-29\mu\text{as}$ with a RMS error of $43\mu\text{as}$ (Lindgren et al. 2018).

To construct a high-quality sample we select stars with at most 20% relative distance errors, that is $\hat{d}/\hat{\epsilon}(\hat{d}) > 5$, and $\hat{d} < 5$ kpc. The sample contains 5,796,226 stars. Stars with $\hat{d} < 1$ kpc, typically have distances better than 5% (median 2.8%) and for stars at $4 < \hat{d} < 5$ kpc the relative distance errors are in between 12% and 20% (median 17.1%).

In Fig. 3.1 we show the extent of our sample in a number density map. To compute the Galactocentric cylindrical coordinates (R, z, ϕ) , we assume¹ $R_{\odot} = 8.3$ kpc (Schönrich 2012) and $z_{\odot} = 0.014$ kpc (Binney et al. 1997, and $\phi_{\odot} = 180^{\circ}$) for the position of the Sun. Because of the imposed maximum distances to the stars, the sample extends from $R \sim 4$ kpc up to $R \sim 13$ kpc and reaches up to $z = \pm 4$ kpc. The white contours in Fig.3.1 indicate the location of bins containing 2,000 and 100 stars respectively. This shows

¹Use of the value of $R_{\odot} = 8178 \pm 13_{\text{stat.}} \pm 22_{\text{sys.}}$ pc, as determined by Gravity Collaboration et al. (2019), does not affect the main conclusions of this chapter.

Figure 3.1 – Star counts from our high-quality *Gaia* DR2 6D sample in bins of width 1.0 kpc in R and z , as indicated by the box in the upper right corner. The central coordinates of the bins are separated by 0.5 kpc in R and z , thus the bins are not fully independent. The white contours indicate the location of bins with 2,000 (inner contour) or 100 (outer contour) stars. The position of the Sun is indicated by the white symbol. Only stars with $\hat{d}/\hat{\epsilon}(\hat{d}) > 5$ are considered in our sample.

that Galactic heights up to ~ 3.5 kpc are still covered with a statistically significant number of stars.

We derive the velocities of the stars in our sample in a Galactocentric cylindrical coordinate system (v_R, v_z, v_ϕ) . For the motion of the Local Standard of Rest (LSR), that is the velocity of a circular orbit at $R = R_\odot$, we assume $v_c(R_\odot) = 240$ km/s (Piffl et al. 2014; Reid et al. 2014). The peculiar motion of the Sun with respect to the LSR is taken to be $(U, V, W)_\odot = (11.1, 12.24, 7.25)$ km/s (Schönrich et al. 2010), where U denotes motion radially inwards and V in the direction of Galactic rotation (both in the Galactic plane), and W perpendicular to the Galactic plane and in the direction of the Galactic north pole. We propagate the errors and correlations in the observables to determine the errors on the velocities (and their correlations). Here we assume that the Bayesian distances are not correlated with the remaining astrometric parameters. The velocity errors for the stars in our sample at $\hat{d} < 1$ kpc are typically smaller than 2 km/s

with a median value of ~ 1 km/s for the v_R -, v_z -, and v_ϕ -components. At $4 < \hat{d} < 5$ kpc the median errors are in the range from ~ 3 km/s to ~ 8 km/s and generally smaller than 15 km/s.

The characterisation of the kinematics, in terms of the mean motions and velocity dispersions, of a large part of the Milky Way disk have been presented in Gaia Collaboration et al. (2018c) using the 6D dataset from *Gaia* DR2. This characterisation has put on firm ground the evidence of the presence of streaming motions in all velocity components (Siebert et al. 2008; Williams et al. 2013; Tian et al. 2017; Carrillo et al. 2018) and revealed a large amount of substructure in the velocity distributions. In this chapter we proceed to focus on the correlation between the radial and vertical velocity components across a large fraction of the Milky Way galaxy.

3.3 Methods

The 3D velocity distribution of stars $f(v_\phi, v_R, v_z)$ at a given point in the Galaxy may be characterised by its various moments. As described in the Introduction, the tilt of the velocity ellipsoid refers to the orientation of the 2D velocity distribution $f(v_R, v_z)$, which would be obtained by integrating over v_ϕ . As shown in Smith et al. (2009) and Büdenbender et al. (2015), this is equivalent to taking the moments of the 3D velocity distribution and neglecting the cross terms with v_ϕ . These cross-terms are interesting in their own right, as they reveal also other physical mechanisms at work, such as for example the presence of substructures associated to resonances induced by the rotating Galactic bar (Dehnen 1998), but are not the focus of this chapter.

3.3.1 The tilt angle: the orientation of the velocity ellipse

In the Galactocentric cylindrical coordinate system we define the tilt angle γ , following for instance Smith et al. (2009), as:

$$\tan(2\gamma) = \frac{2\text{cov}(v_R, v_z)}{\text{var}(v_R) - \text{var}(v_z)}, \quad (3.1)$$

which therefore takes values from -45 degrees to $+45$ degrees, and is measured counterclockwise (i.e. from the v_R -axis towards the positive v_z -

axis). For exact cylindrical alignment $\gamma_{\text{cyl}} = 0^\circ$ and the major and minor axis align with the Galactocentric cylindrical coordinates.

It is also possible to define a tilt angle α with respect to the spherical coordinate system (r, θ, ϕ) , where $\tan(\theta) \equiv R/z$, that is:

$$\tan(2\alpha) \equiv \frac{2\text{cov}(v_r, v_\theta)}{\text{var}(v_r) - \text{var}(v_\theta)}. \quad (3.2)$$

The tilt angle α thus measures directly the deviation from spherical alignment, which corresponds to $\alpha = 0^\circ$. In such a case one of the principal axes of the ellipse points to the Galactic centre. The relation between α and γ at every (R, z) is

$$\tan(2\gamma) = -\tan(2\theta + 2\alpha). \quad (3.3)$$

From now on, we always refer to the tilt angle γ , thus as defined in the cylindrical coordinate system, unless stated otherwise. To explore the spatial variation of the tilt angle we measure the intrinsic moments of Eq. 3.1 after projecting all stars onto the (R, z) -plane, thus ignoring in the first stage the Galactic azimuthal angle of the stars (although this is considered in Sect. 3.4.2). We bin the meridional plane as in Fig. 3.1 and always require at least 100 stars per bin.

3.3.2 Accounting for measurement errors

Measurement errors affect the observed velocity moments and can therefore have a significant effect on the inferred tilt angles (Siebert et al. 2008). To establish their effect we here explore two ‘methods’ to account for the errors and for recovering the (intrinsic) velocity moments.

Method 1. We assume that the stars in a given spatial bin have similar measurement errors. This assumption is reasonable because the measurement errors in a particular bin are usually much smaller than the intrinsic velocity dispersion. If the measurement errors were exactly the same for all stars in a bin, the intrinsic velocity covariance matrix can be recovered by subtracting the error covariance matrix from the observed velocity covariance matrix. This follows from the fact that convolving a Gaussian distribution with Gaussian distributed measurement errors again results in a Gaussian with covariance matrix $\Sigma_{\text{obs}} = \Sigma_{\text{intr}} + \Sigma_{\text{error}}$, where Σ_{obs} and Σ_{intr} are the observed and intrinsic covariance matrix

of the velocity distribution respectively. In our approximation $\Sigma_{\text{error}} \approx \text{median}(\Sigma_{\text{error},i})$ for

$$\Sigma_{\text{error},i} = \begin{bmatrix} \text{var}(v_{R,i}) & \text{cov}(v_{R,i}, v_{z,i}) \\ \text{cov}(v_{R,i}, v_{z,i}) & \text{var}(v_{z,i}) \end{bmatrix}, \quad (3.4)$$

in which the diagonal terms denote the variance error of the corresponding velocity component of star i . Similarly $\text{cov}(v_{R,i}, v_{z,i})$ denotes the error covariance for the (v_R, v_z) measurements of star i . For the required typical errors we take the relevant median errors of the stars in the bin. The recovered intrinsic velocity moments are then used to characterise the velocity distribution. The errors on these moments are analytically estimated and then propagated into uncertainties on the recovered tilt angles. More details can be found in Appendix 3.A.

Method 2. We perform Markov Chain Monte Carlo (MCMC) modelling (Foreman-Mackey et al. 2013) for bins with a smaller number of stars (with $100 < N < 2,000$). This aims to solve for the intrinsic velocity dispersions $\sigma(v_R)_{\text{intr}}$ and $\sigma(v_z)_{\text{intr}}$, the mean velocities $\langle v_R \rangle$ and $\langle v_z \rangle$, and the covariance term $\text{cov}(v_R, v_z)_{\text{intr}}$ in each bin. This is done by maximizing the bivariate Gaussian likelihood function $L = \prod_{i=1}^N L_i$, where

$$\begin{aligned} L_i &= L_i[\langle v_R \rangle, \sigma(v_R)_{\text{intr}}, \langle v_z \rangle, \sigma(v_z)_{\text{intr}}, \text{cov}(v_R, v_z)_{\text{intr}}] \\ &= \frac{1}{\sqrt{\det(2\pi\Sigma_i)}} \exp \left[-\frac{1}{2}(\mathbf{x}_i - \boldsymbol{\mu})^\top \Sigma_i^{-1}(\mathbf{x}_i - \boldsymbol{\mu}) \right], \end{aligned} \quad (3.5)$$

in which $\mathbf{x}_i = [v_{R,i}, v_{z,i}]$, $\boldsymbol{\mu} = [\langle v_R \rangle, \langle v_z \rangle]$ and $\Sigma_i = \Sigma_{\text{intr}} + \Sigma_{\text{error},i}$. Whereas in Method 1 $\Sigma_{\text{error},i}$ was assumed to be the same for each star, we here use $\Sigma_{\text{error},i}$ for each star separately. We add priors to the model that only allow for positive velocity dispersions in v_R and v_z and that restrict the correlation coefficient between v_R and v_z always to be within $[-1, 1]$. For a given bin, the samples drawn by the MCMC run translate into a distribution of tilt angles. We take the median as the best estimate of the tilt angle. For its error we take half the difference between the tilt angles corresponding to the 16th and 84th percentile.

In general we find that the effect of the measurement errors on the recovered moments is small. Moreover, for most bins the velocity measurement errors are sufficiently similar and small that we may use the computationally much faster Method 1 instead of the MCMC-based

Figure 3.2 – Velocity ellipses in the meridional plane. The ellipses are colour-coded by their misalignment with respect to spherical alignment. The orientation that corresponds to spherical alignment is indicated by the dotted grey line through each ellipse. The inset in the top right of the figure shows the velocity ellipse for a non-tilted distribution with dispersions $\sigma(v_R) = 100$ km/s and $\sigma(v_z) = 50$ km/s (see Sect. 3.4 for more information). The contours show the (relatively small) formal statistical errors on the recovered tilt angles and are drawn for error levels of [0.5, 1.0, 2.0, 4.0] degrees. See Sect. 3.5 for a discussion on the effect of systematic errors.

deconvolution. We have also compared the results to the case in which we simply compute the variances of the observed stellar velocities in the bins of interest, and take these at face value, meaning that we do not take into account the measurement errors. The results are again rather similar, see for example, Fig. 3.12 of Appendix 3.A which shows the distributions of the measurement errors for the bin located at $R = 11.5$ kpc and $z = 1.5$ kpc. In what follows, we use the results from Method 1 unless stated otherwise.

3.4 Results

We present our measurement of the tilt angles by showing velocity ellipses in the meridional plane. At each position (R, z) , we define a set of axes with v_R into the R -direction and v_z in the z -direction. The centre of each velocity ellipse is always placed at its position (R, z) . The size of the major and minor axis of each ellipse scale with the intrinsic velocity dispersions along

these directions. The R - and z -axis are both scaled by the same constant c_x . Similarly, all v_R - and v_z -axes are scaled by a constant c_v , thus both sets of axes have an aspect ratio of 1. As a consequence, the velocity ellipses drawn will actually point to the Galactic centre when there is spherical alignment. As a reference, the inset in the figures shows a velocity distribution aligned in cylindrical coordinates and with $\sigma(v_R) = 100$ km/s and $\sigma(v_z) = 50$ km/s (unless stated otherwise).

3.4.1 Tilt angles projected onto the (R, z) -plane

Fig. 3.2 shows the velocity ellipses colour-coded by their angular misalignment with respect to spherical alignment. For $z \geq 0$ kpc we define this misalignment as $\gamma - \gamma_{\text{sph}}$, whereas for $z < 0$ kpc the misalignment is $\gamma_{\text{sph}} - \gamma$. Steeper tilt angles result in positive misalignment (from light to dark red), shallower tilt angles in negative misalignment (from light to dark blue). Ellipses that are consistent with spherical alignment are greyish. At the midplane it is however not possible to distinguish between spherical and cylindrical alignment, since both $\gamma_{\text{sph}} = \gamma_{\text{cyl}} = 0^\circ$ at $z = 0$ kpc, thus here consistency with spherical alignment also implies consistency with cylindrical alignment. Only away from the midplane it is possible to differentiate between these types of alignment.

We further add contours of constant formal statistical error values on the recovered tilt angles in Fig. 3.2. We have drawn contours for errors reaching 0.5, 1.0, 2.0, and 4.0 degrees. These contours show the great quality of our dataset over the distance range explored.

From this figure it is evident that there are just a few bins that have tilt angles much steeper than spherical alignment (i.e. there are just two dark red ellipses). These are however located in the inner regions of the Galaxy and at those positions where the error on the tilt angle is also large.

In general, however, the following trend is apparent: for Galactocentric spherical radius $r \sim 4$ kpc, the orientations of the velocity ellipses seem to be slightly steeper than spherical alignment. For $r \sim 7$ kpc they seem fully consistent with spherical alignment. For larger radii, that is $R > 8$ kpc and $|z| \gtrsim 1$ kpc, the ellipses have a negative misalignment, meaning that the orientations of the ellipses become shallower compared to prediction for spherical alignment. Here the orientation thus changes into the direction of cylindrical alignment and is no longer consistent with spherical alignment.

Figure 3.3 – Tilt angles as a function of Galactic height for different positions across the Galaxy. We show the trends with z for $R = [5, 7, 9, 11]$ kpc. The red squares, green diamonds, and blue crosses are based on the methods described in Sect. 3.3.2 (see text). The solid black line shows the trend that would correspond to spherical alignment. The tilt angle is changing from spherical alignment in the inner Galaxy ($R \sim 5$ kpc) towards shallower tilt angles at $R \sim 11$ kpc. The cyan line shows the analytic description of the data as proposed in Sect. 3.4.4.

To be able to assess whether the tilt angles found are more consistent with spherical or cylindrical alignment we show them with error bars in Fig. 3.3 as a function of height for four Galactic radii, namely $R = [5, 7, 9, 11]$ kpc. The red squares (without error bars; labelled ‘Raw data’) follow from computing the moments directly from the data, and the green diamonds (‘Analytic’) and blue crosses (‘MCMC’) are derived using Method 1 and Method 2 respectively, thus accounting for the measurement errors (see Sect. 3.3.2). Except for a few points, they give consistent results given the error bars, although the MCMC-method seems to result in slightly steeper tilt angles.

The black curve in Fig. 3.3 shows the expectation in the case of spherical alignment. At $R = 5$ kpc (left panel) the recovered tilt angles are in agreement with spherical alignment for the heights explored. At $R = 7$ kpc (left centre panel) the data is consistent with spherical alignment up to $|z| \sim 2$ kpc. For larger heights the tilt angles are only mildly shallower. For $R = 9$ kpc and $R = 11$ kpc, however, the tilt angles are becoming increasingly shallower with respect to spherical alignment. In fact, for $R = 12$ kpc (see Fig. 3.2) the orientation of the ellipses become more consistent with cylindrical alignment for the heights probed.

3.4.2 Tilt angles for different azimuthal angles

Since the Galaxy is not axisymmetric we now investigate whether the tilt angles vary with azimuth by taking into account the 3D location of the individual stars in our dataset. We bin the data into Cartesian bins (x, y, z) whose volume is fixed to $1 \times 1 \times 1$ kpc³, which implies that the different azimuthal cones we explore contain independent data for $R > 4$ kpc. These cones are centred on three different angles $\phi = [165^\circ, 180^\circ, 195^\circ]$.

The resulting maps are shown in Fig. 3.4. Since the data is effectively sliced in ϕ , the number of stars at a given (R, z) is lower and as a consequence the spatial bins cover a smaller spatial extent in comparison to Sect. 3.4.1. A coarse comparison of the different panels in this figure suggests that the variations with azimuth are relatively small compared to the global trend that is still apparent in each panel: the misalignment changes from positive to negative when moving outwards in Galactic radius.

The most prominent differences are seen for the bins at $R \sim 4$ kpc and $z \sim 1$ kpc. The $\phi = 180^\circ$ -slice indicates much steeper tilt angles than the $\phi = 195^\circ$ -slice. The statistical errors on these tilt angles are however large. In fact, most of these bins have consistent tilt angles given their error bars.

For a more direct comparison we show in Fig. 3.5, for specific radii $R = [6, 8, 10]$ kpc, the tilt angles for the different Galactic azimuths as a function of Galactic height. Here the different symbols, namely red squares, green diamonds, and blue crosses correspond to the measurements for $\phi = [165^\circ, 180^\circ, 195^\circ]$, respectively. The black starred symbols show the measurements from all stars at the given R and z and irrespective of azimuth (as in Sect. 3.4.1). At $R = 10$ kpc the tilt angles for the different azimuths are less consistent with spherical alignment than those at $R = 6$ kpc, especially at positive Galactic heights.

Figure 3.4 – Velocity ellipses in the meridional plane, now for different positions in azimuth ($\phi = [165^\circ, 180^\circ, 195^\circ]$ from top to bottom, respectively). The spatial bins are cubes in (x, y, z) , of 1 kpc on a side. The colours of the ellipses represent the misalignment with respect to spherical alignment (as in Fig. 3.2).

Figure 3.5 – Tilt angles as a function of Galactic height for different radial and azimuthal positions across the Galaxy. The red squares, green diamonds, and blue crosses show the measurements for $\phi = [165^\circ, 180^\circ, 195^\circ]$, respectively. The black starred symbols show the measurements irrespective of azimuth (as in Sect. 3.4.1). Given the error bars, there are only small differences in the tilt angles for the different azimuths explored. The solid black line denotes the trend expected for spherical alignment.

Even though some bins reveal slight differences in the tilt angles when varying Galactic azimuth, the overall qualitative trends are similar to the case in which we projected all stars onto the (R, z) -plane, thus justifying the approach used in Sect. 3.4.1. These results also suggest that the degree of non-axisymmetry, in terms of the tilt angles, is modest over the azimuthal range explored.

3.4.3 Variations with stellar populations

In this section we explore whether different populations of stars follow similar trends in tilt angle. To this end we have cross matched the full *Gaia* DR2 catalogue with three spectroscopic datasets: the Large Sky Area Multi-Object Fiber Spectroscopic Telescope (LAMOST DR4, Cui et al. 2012), RAVE DR5, and APOGEE DR14. We correct the LAMOST DR4 radial velocities given their offset of +4.5 km/s (Anguiano et al. 2018). If a star has radial velocity measurements from more than one survey, we take the measurement with the smallest quoted error. As for the spectroscopic sample delivered as part of *Gaia* DR2 (Arenou et al. 2018), we only consider stars whose radial velocity errors have been estimated to be smaller than 20 km/s. By adding radial velocities from these other surveys the number of stars with full phase-space information is increased by over 30%.

To explore dependences on populations, we only use metallicities from LAMOST DR4 since this survey probes a much larger region than either RAVE or APOGEE. We refrain from merging the metallicity information from the different surveys to avoid possible offsets between metallicity scales. Finally, only stars with metallicity uncertainties up to 0.2 dex are considered in our analysis.

A downside of extending our sample is that Bayesian distances are missing for the newly added stars to our sample. Since the purpose of this section is to inspect variations between different populations, we here approximate the distances to the stars by $\hat{d} = 1/\hat{\varpi}$, where

$$\hat{\varpi} = \varpi + 0.029 \text{ mas}, \text{ and } \hat{\epsilon}_{\varpi} = \sqrt{\epsilon_{\varpi}^2 + 0.043^2}. \quad (3.6)$$

Here ϖ is the observed parallax and ϵ_{ϖ} the formal random parallax error. For the following analysis, we select those stars with at most 20% relative distance errors, that is $\hat{\varpi}/\hat{\epsilon}_{\varpi} > 5$, and $\hat{d} < 5$ kpc. We proceed to classify the stars according to a halo population as those with $[M/H] < -1.0$ dex, a thick disk population for $-1.0 < [M/H] < -0.5$ dex, and a thin disk

Figure 3.6 – Velocity ellipses in the meridional plane, as in Fig. 3.2, but now for the subsamples representing halo (top), thick disk (middle) and thin disk (bottom) populations. We note that the scaling of the velocity ellipses, indicated by the insets in the bottom right of each panel, are different. The colour coding of the ellipses represents the misalignment with respect to spherical alignment and is the same as in Fig. 3.2. There is no strong evidence that the tilt angles of the different populations behave differently.

population for $[M/H] > -0.4$ dex. With these criteria, our sample contains $\sim 23,000$ halo stars, $\sim 260,000$ thick disk stars, and ~ 2 million thin disk stars.

Fig. 3.6 shows the velocity ellipsoids and tilt angles as a function of position in the meridional plane for the halo (top), thick disk (middle), and thin disk (bottom) subsamples. The different spatial coverage of the subsets reflect differences in the number of stars (recall that to reliably measure a tilt angle we require at least 100 stars in a spatial bin). In addition the ellipses for the halo population are much larger compared to those of the thick and thin disks. In fact, we have had to use different scales for the panels: the insets in the bottom right of each panel show ellipses whose semi-major and semi-minor axes correspond to dispersions of $\sigma(v_R) = 200$ km/s and $\sigma(v_z) = 100$ km/s for the halo and thick disk populations, and to $\sigma(v_R) = 100$ km/s and $\sigma(v_z) = 50$ km/s for the thin disk.

As in previous sections, the colours in Fig. 3.6 represent the misalignment of the tilt angles with respect to spherical alignment. The same trends as found earlier are visible for the populations independently: at $R \lesssim 7$ kpc the alignment is closer to spherical, while outwards from $R \sim 9$ kpc the misalignment becomes negative, which means that the tilt angles become shallower. This can be seen more easily when comparing the tilt angles derived for each population at specific radii, as shown in Fig. 3.7.

There are also some differences seen. For example, at $R = 8.5$ kpc, the halo sample seems to be more consistent with spherical alignment than both disk samples. For $R = 9.5$ kpc, however, the differences between the populations are minor, except for the flatter thin disk tilt angles at $z \sim 2.5$ kpc. Therefore we may conclude that the results shown in Sect. 3.4.1 are not strongly dependent on the different populations present throughout the volume probed by our dataset.

3.4.4 Quantifying the degree of spherical alignment

Because the trends seen in the tilt angles are not strongly dependent on Galactic azimuth nor on stellar population, we here aim to provide a simple description of their variation with radius R and height z as found in Sect. 3.4.1. Since we infer near spherical alignment for $R \sim 6$ kpc, we

Figure 3.7 – Tilt angles as a function of Galactic height for different populations of stars. We show the trends with z for $R = 7.5$ kpc (top), $R = 8.5$ kpc (middle) and $R = 9.5$ kpc (bottom). The red squares, green diamonds, and blue crosses show the results for the halo, thick, and thin disk population described in Sect. 3.4.3, respectively. The light blue triangles correspond to all LAMOST stars with metallicity information with uncertainties smaller than 0.2 dex, while the black stars are for all stars in the extended sample regardless of whether or not they have metallicity information. The solid black line shows the trend that would correspond to spherical alignment.

consider expanding α around a point (R_0, z_0) :

$$\begin{aligned}\alpha(R, z) = & \alpha(R_0, z_0) + a_1 (R - R_0) + a_2 (z - z_0) \\ & + a_3 (R - R_0)(z - z_0) \\ & + a_4 (R - R_0)^2 + a_5 (z - z_0)^2 + \dots,\end{aligned}\tag{3.7}$$

where a_i are constants and both R and z in kpc^2 . By definition $\alpha(R_0, z_0) = 0^\circ$. We further set $z_0 = 0$ kpc (i.e. the symmetry plane of α is set to be the Galactic midplane). Moreover, $a_1 = a_4 = 0$, since for most realistic models the tilt angle does not vary at the midplane. By symmetry arguments the coefficients of all even powers of z (including a_5) must be zero, since α is expected to be either antisymmetric with respect to the midplane or zero. Since we have found that at $R \sim 6$ kpc the tilt angles are consistent with spherical alignment for all z probed (see left panels of Fig. 3.3), we additionally set $a_2 = 0$ such that at $R = R_0$: $\alpha(R_0, z) = 0^\circ$. With these choices:

$$\alpha(R, z) \approx a_3 (R - R_0)z.\tag{3.8}$$

We thus fit this functional form to the data to derive values for R_0 and a_3 such that the χ^2 -statistic defined as:

$$\chi^2 = \sum_{j=1}^{N_{\text{bins}}} \left(\frac{\alpha(R_j, z_j)_{\text{model}} - \alpha(R_j, z_j)_{\text{obs}}}{\epsilon[\alpha(R_j, z_j)_{\text{obs}}]} \right)^2\tag{3.9}$$

is minimised. Here j runs over the number of bins N_{bins} where a measurement is made, in other words where $N > 100$ stars.

For most bins at $|z| \leq 2.0$ kpc and $5 \leq R \leq 12$ kpc the inferred statistical errors on the tilt angles are very small (e.g. see the dashed contours in Fig. 3.2). In that case systematic errors need to be considered. One such source of systematic errors are substructures. We performed tests to estimate the effect of substructures in velocity space on the tilt angle. To this end we inserted $N_{\text{sub}} = 1, 4, 9, 16, 25$, or 36 substructures on smooth

²We prefer to quantify the deviation from spherical symmetry directly on the spherical tilt angle α than to use the purely geometric parametrisation by Binney et al. (2014) of the cylindrical tilt angle $\gamma' = a_0 \arctan(z/R) = a_0(\pi/2 - \theta)$ where θ indicates the spatial location of the bin (see also Eq. 3.3). Although $a_0 = 1$ implies spherical alignment and $a_0 = 0$ cylindrical alignment, it is not intuitively clear what the quantitative meaning of other a_0 values is.

Figure 3.8 – Left: Differences in tilt angles, $\Delta\gamma_{\text{GUMS}}$, between error convolved realisations (taking into account random and systematic parallax errors) and the error-free GUMS catalogue. Centre: Standard deviation of the tilt angles over all realisations. Right: Division of the differences by the corresponding standard deviation. At distances at around 2 kpc the changes are significant with respect to the scatter present between realisations.

non-tilted velocity distributions with velocity dispersions of 20 km/s and 35 km/s in v_z and v_R (i.e. values representative of the thin disk near $R \sim R_\odot$), respectively. Each substructure was assigned a random number of stars such that the total fraction of stars in substructures is $f_{\text{sub}} = 5\%$, 10%, 15%, or 20%. We randomly assigned velocity dispersions to the substructures, drawn uniformly from 1 km/s to 5 km/s in both directions. For each combination of $(N_{\text{sub}}, f_{\text{sub}})$ we considered 100 realisations. The median (absolute) tilt angle found from these experiments is ~ 1 degree, implying that this value is representative of the error introduced by neglecting the presence of substructures in a velocity distribution. This result is independent of the total number of stars N for $N \gtrsim 10,000$ (a value that is representative of the number of stars in the bins with $\epsilon[\alpha(R_j, z_j)] < 1^\circ$). Thus, when minimising the χ^2 we consider a floor for the statistical error $\epsilon[\alpha(R_j, z_j)]$ in each bin of 1° .

We fit to find $R_0 = (6.16 \pm 0.16)$ kpc and $a_3 = (0.72 \pm 0.04)^\circ/\text{kpc}^2$ resulting in a reduced χ^2 of 1.65. The cyan line in Fig. 3.3 shows the tilt angles predicted by this fit, which reproduces relatively well the trends observed in the data. The model goes through the 1σ -error bars for approximately 60% of all spatial bins, while for 98% of bins the model matches the data within $3\times$ the estimated uncertainty. This indicates that our simple model provides a fair description of the behaviour of the tilt of the velocity ellipsoid across the Galactic volume probed by our dataset.

The fact that the total reduced χ^2 -value is greater than unity indicates that the tilt angles for some bins are not fitted very well by the model.

For example at $R \sim 10$ kpc the tilt angles as inferred from the data are asymmetric with respect to the $z = 0$ plane: at $z > 0$ kpc they more or less attain a constant value of $\sim 2.0^\circ$, whereas below the midplane the tilt angles become steeper with z (e.g. -15° at $z = -3.0$ kpc). The fits at such radii are therefore relatively poor. For the bins between $R = 11$ kpc and $R = 12$ kpc, we notice that the observed tilt angles seem to have a small positive offset from zero near $z = 0$. These offsets are small (of order 2 degrees), although they do affect the goodness of fit measure.

3.5 Discussion

3.5.1 The impact of (parallax) measurement errors on the recovered tilt angles

Gaia Collaboration et al. (2018a) have reported the presence of a systematic error on the *Gaia* DR2 parallaxes in the form of a zero-point offset of a few 10 of μas (in the sense that *Gaia* parallaxes are too small) and whose exact amplitude depends on location on the sky. Such systematic zero-point offset affects the tangential velocities of the stars, which are determined from both distances and proper motions. The overall systematic parallax offset in *Gaia* DR2 was determined using distant quasars by Lindegren et al. (2018) to be approximately $-29\mu\text{as}$, with a large RMS of $\sim 43\mu\text{as}$. Arenou et al. (2018) using different samples of objects (RR Lyrae stars, Magellanic Clouds, open clusters, dwarf spheroidal galaxies, etc.) report important variations in the zero-point offsets, highlighting the complexity of the offset. Nonetheless all values are consistent given the large estimated RMS.

Around the time the paper corresponding to this chapter was submitted in its original form, Schönrich et al. (2019) reported a new estimate of the parallax zero-point offset based on the distance estimation method used in Schönrich & Aumer (2017) (also see Schönrich et al. 2012). These authors argue for a much larger zero-point for the parallaxes in the RVS subset of *Gaia* DR2, namely of magnitude $-54 \pm 6\mu\text{as}$. Zinn et al. (2018) and Khan et al. (2019) applied asteroseismology to determine distances to Red Giant Branch (RGB) and Red Clump (RC) stars with *Gaia* *G*-band magnitudes similar to those present in the RVS subset of *Gaia* DR2 and determined an offset close to $-50\mu\text{as}$, while Sahlholdt & Silva Aguirre (2018), using asteroseismology information on dwarfs, report that the offset could be

$\sim -35 \pm 16 \mu\text{as}$. More recently Hall et al. (2019), using RC stars with asteroseismology, estimate the mean offset to be $-41 \pm 10 \mu\text{as}$. These comparisons suggest that the offset could well be larger for the brighter stars of the *Gaia* RVS sample but that its amplitude is quite uncertain.

Quantification of the impact of a zero-point offset

We first quantify how the tilt angles are affected if parallaxes are underestimated. For illustration purposes, we estimate the impact on the recovered tilt angles induced by a systematic error (with mean $-29 \mu\text{as}$) while also including the effects of random errors³. Their effect is examined by using the *Gaia* Universe Model Snapshot (GUMS, Robin et al. 2012), which is based on the Besançon Galaxy Model (Robin et al. 2003).

We mimic the *Gaia* DR2 subsample with full phase-space information, by selecting stars in GUMS that have $G < 13$ mag, as this is roughly the magnitude limit for radial velocities in *Gaia*'s current data release. We generate 100 data realisations by convolving the (error-free) GUMS sample with a Gaussian with *Gaia* DR2-like random and systematic errors for the parallaxes (Lindgren et al. 2018). The systematic parallax offsets for the stars are drawn from a Gaussian with mean $-29 \mu\text{as}$ and standard deviation of $30 \mu\text{as}$ ⁴. To obtain a distance estimate we invert the parallaxes and consider only those stars that satisfy $\varpi/\epsilon(\varpi) > 5$ and $\varpi \gtrsim 200 \mu\text{as}$. Again ϖ is the observed parallax and $\epsilon(\varpi)$ the random parallax error and thus the same quality criteria are applied as to the real data (see Sect. 3.2).

For each spatial bin the median (over all realisations) of the distribution of tilt angles is compared to the tilt angles from the error-free model, on the meridional plane. The error-free GUMS model has close to cylindrically aligned velocity ellipses ($\gamma_{\text{GUMS}} \sim 0^\circ$). The impact of the random and systematic parallax uncertainties on the tilt angles depends on location as can be seen in the left panel of Fig. 3.8. At $R \lesssim 7$ kpc the orientations of the velocity ellipses change towards the direction of spherical alignment ($\Delta\gamma_{\text{GUMS}} > 0$ for $z > 0$ and $\Delta\gamma_{\text{GUMS}} < 0$ for $z < 0$), while for $R \gtrsim 9$ kpc the change is in the opposite sense.

The middle panel of Fig. 3.8 shows the spread in tilt angles over all realisations, and reveals that the errors result in a spread with a typical

³In Appendix 3.B we analytically compute how the v_R - and v_Z -velocities (and thus their moments and tilt angles) are affected by the parallax zero-point offset alone.

⁴as estimated in Gaia Collaboration et al. (2018b).

Figure 3.9 – Tilt angles as a function of Galactic height for different positions across the Galaxy. We show the trends with z for $R = [5, 7, 9, 11]$ kpc for different distance estimates for the stars. The blue squares and orange diamonds use distances based on inverting the parallaxes after correcting the parallaxes for an offset of $-29\mu\text{as}$ and $-54\mu\text{as}$, respectively. The green crosses and red starred symbols use Bayesian distances from McMillan (2018) and Schönrich et al. (2019), respectively. The solid black line shows the trend that would correspond to spherical alignment.

amplitude of $\lesssim 4^\circ$, except for the outermost bins, where it can be twice as large, and hence comparable to $\Delta\gamma_{\text{GUMS}}$. The right panel shows at which locations the median change in tilt angles, caused by parallax errors, is larger than the RMS from realisation to realisation. For bins located at distances of ~ 2 kpc a change in tilt angle due to parallax errors is thus likely to occur in a preferential direction, with the amplitude of this change varying from realisation to realisation.

These findings imply that, if parallaxes are underestimated, the tilt angles inferred may appear steeper than they really are in the inner Galaxy, while the opposite happens in the outer Galaxy, thus the tilt angles become shallower there. If we take the results from GUMS at

face value, $|\Delta\gamma_{\text{GUMS}}| \approx 6^\circ$ at $(R, |z|) \sim (5, 3)$ kpc, which means that an unaccounted for zero-point offset of magnitude $29\mu\text{as}$ in the parallaxes affects the inferred tilt angles such that they appear steeper by $\sim 6^\circ$. This does not radically change the type of alignment at this location (where spherical alignment would imply $\gamma \sim 30^\circ$). For $(R, |z|) \sim (11, 2)$ kpc we find that $|\Delta\gamma_{\text{GUMS}}|$ can attain values close to 5° , which is of similar amplitude as the misalignment seen in Fig. 3.2. Although the GUMS tilt angles intrinsically have $\gamma_{\text{GUMS}} \sim 0^\circ$, we find similar amplitudes for the cases explored in Appendix 3.B, where we start from both intrinsically spherically and cylindrically aligned ellipsoids.

In the analysis presented in previous sections, we have effectively corrected for the parallax offset by using the McMillan (2018) distances. If the assumed parallax zero-point is too small, the results presented in this section indicate that, especially towards the outer Galaxy, the zero-point offset could produce tilts that are less steep than what they are intrinsically. We explore such a larger offset next.

A zero-point offset as large as $-54\mu\text{as}$

Everall et al. (2019) have derived tilt angles using the Schönrich et al. (2019) Bayesian distance estimates (with parallax zero-point of $-54\mu\text{as}$). These authors showed that the tilt angles appear to be much more consistent with spherical alignment when using those distances.

Since the method used in Schönrich et al. (2019) assumes spherical alignment, we preferred not to directly use their distances while testing for the effect of a large $-54\mu\text{as}$ offset. Therefore we here also explore how the tilt angles change if the parallax offset would be as large as $-54\mu\text{as}$, by comparing them to the case in which the offset is $-29\mu\text{as}$. For both cases we take the extended sample and invert the parallaxes after correcting for the zero-point offset (as in Sect. 3.4.3), such that the changes due to the differences in parallax offset can be easily compared.

In Fig. 3.9 we show the results. The blue squares have been calculated after correcting for a parallax zero-point offset of $-29\mu\text{as}$, whereas for the orange diamonds a value of $-54\mu\text{as}$ is assumed. For the outer Galaxy ($R = 9$ kpc and $R = 11$ kpc) such a larger parallax zero-point can modify the tilt angles such that they are more consistent with spherical alignment, in agreement with our analysis of the previous section.

A direct comparison of the tilt angles obtained using McMillan (2018) Bayesian distances (who assumes a zero-point of $-29\mu\text{as}$, green crosses) with the results obtained from inverting the parallaxes after correcting for a zero-point of $-29\mu\text{as}$ (blue squares), shows good agreement except for $R = 5$ kpc. At this location, it would seem as if the choice of the distance estimator would play a role in the determination of the tilt angle. The Bayesian distances result in tilt angles that are just slightly steeper than expected for spherical alignment, while inverting the parallaxes results in much shallower tilt angles (the larger the offset assumed the shallower the tilt angles). On the other hand, comparing the tilt angles obtained using Schönrich et al. (2019) Bayesian distances (who find a zero-point of $-54\mu\text{as}$, red starred symbols), with the results obtained from inverting the parallaxes after correcting for a zero-point of $-54\mu\text{as}$ (orange diamonds), shows rather similar trends at $R = 5$ kpc. At the other radii shown, these Bayesian distances also result in tilt angles that are in good agreement with inverting the parallaxes.

The analysis presented in the last two sections shows that the amplitude of the systematic error in the parallax, in the form of a zero-point offset, plays a role in the determination of the tilt angles for the outer Galaxy ($R > 9$ kpc). Since the offset is known to vary with celestial position, magnitude and colour, it is difficult at this point to properly correct for it, and this impairs a very accurate determination of the tilt angle throughout the range of distances probed. However, recall that the range of zero-point offsets is bracketed by the values explored (i.e. from $-54\mu\text{as}$ to $-29\mu\text{as}$), so the analysis presented here gives us a handle on the possible outcomes.

3.5.2 Constraints to models of the Milky Way

Several models of the Milky Way have been proposed by matching a variety of constraints (e.g. McMillan 2011; Piffl et al. 2014; McMillan 2017). Particularly useful for the interpretation of the findings reported in this chapter are Stäckel models (e.g. de Zeeuw 1985; Dejonghe & de Zeeuw 1988). Axisymmetric models with a potential of Stäckel form have the property that the equations of motion are separable in their spheroidal coordinates. Therefore the principal axes of the velocity ellipsoids are always aligned with these coordinates (also see: Eddington 1915). The foci of such a coordinate system then determine the alignment at each position.

For a composite model to be of a Stäckel form, the locations of the foci must be identical for all components.

Famaey & Dejonghe (2003), for example, have extended the two-Stäckel component work of Batsleer & Dejonghe (1994) by adding a third component, such that the model could allow for a thin and thick disk, in addition to a halo component. The authors use constraints such as the (flat) rotation curve, circular velocity at the position of the Sun, the Oort constants, and the local total mass density in the disk to search for a set of consistent parameters for their Stäckel models. Here we take the set of prolate spheroidal coordinates, (λ, ϕ, ν) , from Famaey & Dejonghe (2003, mass model II). The foci of this oblate mass model are located at $(R, z) = (0, \pm 0.88)$ kpc. At $R \sim 0$ and $|z| \lesssim 0.88$ kpc such spheroidal coordinates align with the cylindrical coordinate system (see Fig. 3.10). Outside of these foci and with increasing distance from the Galactic centre the spheroidal coordinates approach the spherical coordinate system. In general, any (composite) Stäckel model predicts a change in the tilt of the velocity ellipse from cylindrical to spherical alignment. The transition radius depends on the location of the foci.

Since the observed tilt angles at $R \sim 4$ kpc already show near spherical alignment, this implies foci at $|z| \lesssim 4$ kpc. Their exact position would depend on whether the innermost region of the Galaxy, not probed by our dataset, is cylindrically aligned or not, and if so at what distance the transition occurs. However, the tilt angles in the outer Galaxy ($9 \lesssim R \lesssim 12$ kpc) derived using the McMillan (2018) distances are not consistent with Stäckel models that have foci at $|z| \lesssim 4$ kpc, and would require a larger focal distance. We have numerically checked these statements by comparing the predicted tilt angles of both oblate and prolate Stäckel models (for a large range of different focal distances) to the observed tilt angles while taking into account their errors.

There are of course many more models with bulge, disk and halo components, for example spherical bulge, exponential disk, Navarro-Frenk-White (NFW, Navarro et al. 1996) halo, or Miyamoto & Nagai (1975) models. The separable models are in that sense a subset but have the advantage that for them the tilt of the velocity ellipsoid is dictated by the coordinate system in which the equations of motion (Hamilton-Jacobi equation to be more precise) separate.

Piffl et al. (2014) have applied a five component mass model (gas disk, thin and thick disk, flattened bulge and dark halo) to RAVE DR4

Figure 3.10 – Contours of constant prolate spheroidal coordinates, (λ, ν) , with foci at $R = 0$ and $z = \pm 0.88$ kpc (see text). Contours of constant λ are shown in blue, contours of constant ν in red. The green ellipses show some of our measured velocity ellipses (Method 1). Their orientation does not align with the coordinate contours at $R \gtrsim 10$ kpc and $|z| \gtrsim 2$ kpc.

stars. Using their best-fitting parameters we computed the relevant velocity moments from the distribution function for a similar range in R and z as probed in our dataset. The tilt angles for this model are spherically aligned for $R \gtrsim 7$ kpc and are, as in the separable models discussed above, changing towards cylindrical alignment with decreasing R .

In Fig. 3.11 we show the tilt angles for both the Stäckel model (purple line) of Famaey & Dejonghe (2003) and the Piffl et al. (2014) model (orange line), for radii $R = 6$ kpc and $R = 10$ kpc. The green diamonds indicate the tilt angles as found by Method 1. Since this Stäckel model has focii at $|z| \lesssim 0.88$, which is very close to the Galactic centre with respect to the innermost radius probed in our dataset, the Stäckel model is almost indistinguishable from spherical alignment for all positions probed. The Piffl et al. (2014) model has tilt angles that are shallower at $R = 6$ kpc, but also approaches spherical alignment with increasing Galactic radius. At $R = 10$ kpc, for example, the tilt angles from the Piffl et al. (2014) model are seen to nearly coincide with the expectation for spherical alignment.

Figure 3.11 – Tilt angles for both the Stäckel (purple line) and Piffl et al. (2014, orange line) model for radii at $R = 4$ kpc and $R = 8$ kpc (see text). For comparison we add our measurement as green diamonds (Method 1).

We note that if the parallax zero-point is larger than assumed here the tilt angles do become more consistent with spherical alignment for large radii (see Sect. 3.5.1). This is in line with predictions for both composite Stäckel models as well as for the Piffl et al. (2014) model. In addition, it would be interesting to know whether the tilt angles become shallower towards the central regions of the Galaxy (at $R \lesssim 4$ kpc). In principle it would then be possible to solve for the focal distance. However, the effects of both the type of distance estimator and the assumed parallax zero-point are too large to make firm statements in this region. Future data releases will for sure enable to probe regions closer to the Galactic centre more robustly.

3.6 Conclusions

We have studied the trends in the tilt angle of the velocity ellipsoids in the meridional plane for a high-quality sample of more than 5 million stars located across a large portion of the Galaxy, from $R \sim 4$ kpc to $R \sim 13$ kpc, and reaching a maximum distance from the plane of ~ 3.5 kpc.

We find that the tilt angles are somewhat dependent on the offset of the *Gaia* DR2 parallaxes, and that the effects are particularly important for the outer Galaxy. When using the McMillan (2018) Bayesian distances, derived assuming an offset of $-29\mu\text{as}$, we find that the tilt angles are consistent with (near) spherical alignment at $R \lesssim 7$ kpc for all heights probed ($|z| \lesssim 3$ kpc). Beyond $R \gtrsim 9$ kpc the tilt angles clearly become more shallower than expected for spherical alignment. These trends remain when the stars

are separated into ‘populations’ according to their metallicity (as given by LAMOST DR4). We provide a simple analytic function for the tilt angle in spherical coordinates $\alpha(R, z)/[\text{deg}] \approx 0.72(R - 6.16)z$, that fits well the trend observed as a function of Galactic radius and height, after projecting the stars onto the (R, z) -plane.

We find that if the amplitude of the zero-point offset in the parallax is underestimated, the angles tend to appear shallower than they intrinsically are in the outer Galaxy (i.e. changing into the direction of cylindrical alignment if the ellipsoid is intrinsically spherically aligned). We quantify the impact on the tilt angles when assuming a parallax zero-point as large as $-54\mu\text{as}$, as estimated in Schönrich et al. (2019) (also see Everall et al. 2019). Such a large offset (the upper limit of estimates reported in the literature by other authors) does indeed lead to tilt angles that are more consistent with spherical alignment than obtained when using the McMillan (2018) distances. Therefore it will be particularly important to pin-down, in future *Gaia* data releases, the amplitude of the parallax zero-point as well as its local variations as these affect our ability to constrain the mass distribution in our Galaxy.

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(<http://www.rave-survey.org>), whose funding has been provided by institutions of the RAVE participants and by their national funding agencies. (3) the LAMOST survey (www.lamost.org), funded by the National Development and Reform Commission. LAMOST is operated and managed by the National Astronomical Observatories, Chinese Academy of Sciences. For the analysis, the following software packages have been used: *vaex* (Breddels & Veljanoski 2018), *NumPy* (Oliphant 2015), *matplotlib* (Hunter 2007), *Jupyter Notebook* (Kluyver et al. 2016), *TOPCAT* and *STILTS* (Taylor 2005, 2006).

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Appendix 3.A: Standard errors of sample (co)variances

To estimate the error on the inferred tilt angles from Method 1 of Sect. 3.3.2 we propagate the errors of the relevant velocity moments from Eq. 3.1.

The error on a sample variance, s^2 , can be estimated (e.g. Rao 1973; Mood et al. 1974) by using

$$\text{var}(s^2) = \frac{1}{N} \left(\mu_4 - \frac{N-3}{N-1} \text{var}(v)^2 \right) \quad (3.10)$$

Figure 3.12 – Error distributions for the bin at $R = 11.5$ kpc and $z = 1.5$ kpc for the different velocity components: v_R (left), v_z (middle), and its covariance (right). The corresponding medians of the error distributions are shown by the vertical grey dotted lines. The vertical grey dashed lines indicate the values of the velocity moments taken from the data directly (i.e. not accounting for the errors). The black vertical solid lines show the recovered intrinsic velocity moment from Method 1 (see Sect. 3.3). Even at this bin, which still contains 2,016 stars, the impact of the measurement errors on the recovered velocity moments is relatively small.

for N stars. Here, μ_4 denotes the intrinsic 4th central moment and $s^2 = \frac{1}{N-1} \sum_{i=1}^N (v_i - \langle v \rangle)^2$, for which v_i is the relevant velocity component, either v_R or v_z , of star i and $\langle v \rangle$ its mean taken over all stars in the bin considered. The intrinsic velocity moments are estimated by their observed values, which is a good approximation given the relatively small errors in the data for the bins explored.

The error on a sample covariance S_{xy} of x and y can be estimated (see Stuart & Ord (1987) or Rose & Smith (2002) for using mathStatica) by

$$\text{var}(S_{xy}) = \frac{1}{N} \left[\mu_{22} - \frac{N-2}{N-1} \text{cov}(x, y)^2 + \frac{1}{N-1} \text{var}(x) \text{var}(y) \right], \quad (3.11)$$

where $\mu_{22} = E[\{x - E(x)\}^2 \{y - E(y)\}^2]$ for E denoting the expectation value. We have defined $S_{xy} = \frac{1}{N-1} \sum_{i=1}^N (x_i - \langle x \rangle)(y_i - \langle y \rangle)$. In our application x is replaced for v_R and y for v_z . The intrinsic moments are again estimated by taking the equivalent moments directly from the observed velocity distribution.

As an example for Sect. 3.3.2 we show in Fig. 3.12 the error distributions for the bin at $R = 11.5$ kpc and $z = 1.5$ kpc. This bin is near the edge of the volume investigated, but still contains 2,016 stars. The vertical grey dashed lines indicate the values of the velocity moments that would be derived by using the data directly (i.e. not accounting for the errors). The medians of the error distributions are indicated by the vertical grey dotted lines. The recovered intrinsic velocity moments from Method 1 are visualised by the

vertical black solid lines (as here, these usually coincide with the vertical grey dashed lines). Thus, even for this outer bin, the effects of measurement errors are relatively small.

Appendix 3.B: The impact of a systematic parallax offset on the recovered tilt angles

Here we explain how a systematic parallax offset can affect the inferred tilt angles. For this purpose, we now only consider the (x, z) -plane and we assume that all parallaxes are shifted by the same offset $\Delta\varpi = -0.029$ mas.

For Galactic longitude l and latitude b the (U, V, W) -velocities in km/s can be computed the usual way (Johnson & Soderblom 1987; Bovy 2011):

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} \cos(l) \cos(b) & -\sin(l) & -\cos(l) \sin(b) \\ \sin(l) \cos(b) & \cos(l) & -\sin(l) \sin(b) \\ \sin(b) & 0 & \cos(b) \end{pmatrix} \begin{pmatrix} v_{\text{los}} \\ \frac{k}{\varpi} \mu_{l^*} \\ \frac{k}{\varpi} \mu_b \end{pmatrix}. \quad (3.12)$$

Here, $\mu_{l^*} = \mu_l \cos(b)$ and μ_b denote the proper motions in mas/yr in the direction of l and b , respectively, ϖ is the parallax in mas, and $k = 4.74047 \frac{\text{km/s}}{\text{kpc mas/yr}}$ (assuming a Julian year).

When only considering an error in the parallaxes the ‘observed’ velocities are affected as:

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix}_1 = \begin{pmatrix} U \\ V \\ W \end{pmatrix}_0 + \frac{\partial}{\partial \varpi} \begin{pmatrix} U \\ V \\ W \end{pmatrix} \bigg|_0 \Delta\varpi + O(\Delta\varpi^2). \quad (3.13)$$

Subscript 0 denotes the true position and velocities, subscript 1 the ‘observed’ quantities. Furthermore:

$$\frac{\partial}{\partial \varpi} \begin{pmatrix} U \\ V \\ W \end{pmatrix} = -\frac{1}{\varpi} \begin{pmatrix} U - \cos(l) \cos(b) v_{\text{los}} \\ V - \sin(l) \cos(b) v_{\text{los}} \\ W - \sin(b) v_{\text{los}} \end{pmatrix}, \quad (3.14)$$

and:

$$v_{\text{los}} = \cos(b) \cos(l) U + \cos(b) \sin(l) V + \sin(b) W. \quad (3.15)$$

Let us now define the tilt angle δ as:

$$\tan(2\delta) = \frac{2\text{cov}(U, W)}{\text{var}(U) - \text{var}(W)}. \quad (3.16)$$

In a steady state axisymmetric system $\langle v_R \rangle = \langle v_z \rangle = 0$. Therefore, at the (x, z) -plane $\langle U \rangle = \langle W \rangle = 0$, and thus $\text{var}(U) = \langle U^2 \rangle$, $\text{var}(W) = \langle W^2 \rangle$, and $\text{cov}(U, W) = \langle UW \rangle$. For $l = 0^\circ$ and $l = 180^\circ$ we also notice that $U = -v_R$ and $W = v_z$, and therefore that $\delta = -\gamma$. In the remainder of this Appendix we refer to δ when we use ‘tilt angle’ (unless stated otherwise).

Plugging Eq. 3.13 up to first order in $\frac{\Delta\varpi}{\varpi_0}$ into Eq. 3.16 we get:

$$\tan(2\delta_1) \simeq \frac{2\langle U_0 W_0 \rangle + \epsilon_A}{\langle U_0^2 \rangle - \langle W_0^2 \rangle + \epsilon_B}, \quad (3.17)$$

in which:

$$\begin{aligned} \epsilon_A &= [\pm (\langle U_0^2 \rangle + \langle W_0^2 \rangle) \sin(2b) - 2\langle U_0 W_0 \rangle] \left(\frac{\Delta\varpi}{\varpi_0} \right) \\ \epsilon_B &= 2 [\langle W_0^2 \rangle \cos^2(b) - \langle U_0^2 \rangle \sin^2(b)] \left(\frac{\Delta\varpi}{\varpi_0} \right), \end{aligned} \quad (3.18)$$

where $\pm$ holds for $l \in \left\{ \begin{array}{c} 0^\circ \\ 180^\circ \end{array} \right\}$.

To further explore the effect of a shift in the parallaxes we now investigate what would happen to the tilt angles in two different cases of alignment: spherical alignment and cylindrical alignment.

We start by rewriting Eq. 3.17 into the form of

$$\begin{aligned} \delta_1 &= \frac{1}{2} \arctan [(1+x) \tan(2\delta_0)] \\ \delta_1 &= \delta_0 + \frac{1}{4} \sin(4\delta_0) x + O(x^2) \\ \Delta\delta &\simeq \frac{1}{4} \sin(4\delta_0) x. \end{aligned} \quad (3.19)$$

We then get:

$$\begin{aligned} \tan(2\delta_1) &\simeq \frac{2\langle U_0 W_0 \rangle}{\langle U_0^2 \rangle - \langle W_0^2 \rangle} \left[\frac{1 - \epsilon_C}{1 - \epsilon_D} \right] \\ \tan(2\delta_1) &\simeq \tan(2\delta_0) \left[\frac{1 - \epsilon_C}{1 - \epsilon_D} \right], \end{aligned} \quad (3.20)$$

in which:

$$\begin{aligned} \epsilon_C &= \left[1 \mp \left(\frac{\langle U_0^2 \rangle + \langle W_0^2 \rangle}{2\langle U_0 W_0 \rangle} \right) \sin(2b) \right] \left(\frac{\Delta\varpi}{\varpi_0} \right) \\ \epsilon_D &= 2 \left[\frac{\langle U_0^2 \rangle \sin^2(b) - \langle W_0^2 \rangle \cos^2(b)}{\langle U_0^2 \rangle - \langle W_0^2 \rangle} \right] \left(\frac{\Delta\varpi}{\varpi_0} \right). \end{aligned} \quad (3.21)$$

Then, under the assumptions that $|\epsilon_C| \ll 1$ and $|\epsilon_D| \ll 1$, we get:

$$x \simeq \epsilon_D - \epsilon_C. \quad (3.22)$$

We highlight the effects for four different latitudes:

$$\begin{aligned} b = 0^\circ : \quad & x = - \left(\frac{\Delta\varpi}{\varpi_0} \right) \left[\frac{\langle U_0^2 \rangle + \langle W_0^2 \rangle}{\langle U_0^2 \rangle - \langle W_0^2 \rangle} \right] \\ |b| = 90^\circ : \quad & x = + \left(\frac{\Delta\varpi}{\varpi_0} \right) \left[\frac{\langle U_0^2 \rangle + \langle W_0^2 \rangle}{\langle U_0^2 \rangle - \langle W_0^2 \rangle} \right] \\ b = +45^\circ : \quad & x = \pm \left(\frac{\Delta\varpi}{\varpi_0} \right) \left[\frac{\langle U_0^2 \rangle + \langle W_0^2 \rangle}{2\langle U_0 W_0 \rangle} \right] \\ b = -45^\circ : \quad & x = \mp \left(\frac{\Delta\varpi}{\varpi_0} \right) \left[\frac{\langle U_0^2 \rangle + \langle W_0^2 \rangle}{2\langle U_0 W_0 \rangle} \right]. \end{aligned} \quad (3.23)$$

Since the velocity ellipse is mostly non-tilted ($\delta_0 = 0^\circ$) at the Galactic midplane the inferred tilt angles at $b = 0^\circ$ are not affected by an error in the parallax. Geometrically this is not surprising since, at $b = 0^\circ$, the U -component of the velocities are not affected. The W -velocities are only inflated and do not change the tilt angle. However, if $\delta_0 \neq 0^\circ$, then the term between the square brackets becomes larger than one, since for typical values of the velocity moments at the midplane $\sigma(v_R) > \sigma(v_z)$ (see e.g. Gaia Collaboration et al. 2018c). The inferred tilt angle is therefore steeper (more positive if $\delta_0 > 0^\circ$ and more negative if $\delta_0 < 0^\circ$). At $|b| = 90^\circ$, the effect is reversed and the tilt angle becomes shallower (less positive if $\delta_0 > 0^\circ$ and less negative if $\delta_0 < 0^\circ$) due to the parallax offset. For the case of spherical alignment the relation $\tan(2\delta_0) = \tan(2\theta)$ can be applied.

The approximations used so far fail for $\langle U_0 W_0 \rangle \simeq 0$, since then $|\epsilon_C| \not\ll 1$, and for $\langle U_0^2 \rangle \simeq \langle W_0^2 \rangle$, since then $|\epsilon_D| \not\ll 1$, and thus $|x| \not\ll 1$. In the case of cylindrical alignment ($\delta_0 = \langle U_0 W_0 \rangle = 0$) and for $|\epsilon_D| \ll 1$ we get⁵:

$$\delta_1 \simeq \pm \frac{1}{2} \sin(2b) \left(\frac{\langle U_0^2 \rangle + \langle W_0^2 \rangle}{\langle U_0^2 \rangle - \langle W_0^2 \rangle} \right) \left(\frac{\Delta\varpi}{\varpi_0} \right), \quad (3.24)$$

where we used that $\tan(2\delta_1) \simeq 2\delta_1$ for small deviations around $\delta_1 = 0^\circ$. This means that at $l = 0$ ($l = 180^\circ$) and for $\sigma(v_R) > \sigma(v_z)$ the tilt angles appear to be negative (positive) for $b > 0^\circ$, and positive (negative) for $b < 0^\circ$.

⁵If, hypothetically, both $\langle U_0^2 \rangle = \langle W_0^2 \rangle$ and $\langle U_0 W_0 \rangle = 0$, then $\tan(2\delta_1) = \pm \tan(2b)$.

Figure 3.13 – The effect of a constant shift in the parallaxes of the stars ($\Delta\varpi = -0.029$ mas) on the tilt angle γ , as measured in Galactocentric cylindrical coordinates for different types of intrinsic alignment. The left columns show intrinsic tilt angles γ_0 as a function of R and z . The middle columns show the tilt angles γ_1 computed from the ‘observed’ velocity moments. The right column shows $\Delta\gamma = \gamma_1 - \gamma_0$. Be aware of the different colourbar ranges. In the top panels we set the velocity covariances such that the input alignment is spherical. In the bottom panels the input alignment is cylindrical. Black contours denote regions where the tilt angle is not affected, i.e. $\Delta\gamma = 0^\circ$. For spherical alignment this is expected to be the case on the line passing through the Galactic centre and the position of the Sun, thus along $z \approx 0$ kpc, and on the circle that goes through the Galactic centre and the position of the Sun. For cylindrical alignment this is expected to occur at both $z = z_\odot \approx 0$ kpc and $R = R_\odot$.

We have inserted the relevant Galactic velocity dispersions as a function of R and z and set the covariance term such that there is either spherical or cylindrical alignment throughout the extent of the dataset. We find that the tilt angles are affected very similarly. This is visualised in Fig. 3.13 (recall that $\gamma = -\delta$ since we here consider $l = 0^\circ$ and $l = 180^\circ$ only). We therefore think that our test performed in Sect. 3.5.1 is realistic, even though the intrinsic tilt angles of the GUMS catalogue are more or less cylindrically aligned.

Besides the fact that the orientation of the velocity ellipse changes due to the parallax offset, obviously the stars under consideration also move in position. Thus, in fact a sample of stars with tilt angle δ_0 at parallax ϖ_0 gets ‘observed’ at ϖ_1 with tilt angle δ_1 . We have not taken this effect into account in the analytic description from this Appendix.

Chapter 4

Characterization of the orbital parameters of nearby thick disk and halo stars with *Gaia* DR2

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Abstract

We study the distribution of nearby thick disk and halo stars in subspaces defined by their characteristic orbital parameters. Our aim is to establish the origin of the structure reported in particular in the $R_{\text{max}} - z_{\text{max}}$ space. To this end we compute the orbital parameters of stars for a generic and for a Stäckel Milky Way potential. We find that all populations (thick disk and halo) show very similar prominent structures for the generic potential, while no structure is seen for the Stäckel model. This indicates that their origin is not merger related, but due to non-integrability of a generic Milky Way potential. This conclusion is strengthened by our frequency analysis of the orbits of stars in such a potential, which reveals the presence of prominent resonances, with $\sim 30\%$ of the halo stars associated to resonance families. In fact, the stars in resonances define the structures seen in the spaces of characteristic orbital parameters, while the existence of irregular orbits leads to empty or depleted regions in these spaces. Furthermore, we find that some debris streams may be close to resonant orbits, and that some stars in our sample are on the same resonance as the Sagittarius dwarf, suggesting this system has also influenced the distribution of stars in the Galactic thick disk and halo components. Our study constitutes a step towards disentangling the imprint of merger debris from structures driven by internal dynamics. Given their prominence, these resonant-driven overdensities could potentially be useful to constrain the exact form of the Galactic potential.

4.1 Introduction

With the second data release of the *Gaia* space mission (Gaia Collaboration et al. 2018b), a catalogue comprising the full six-dimensional (6D) phase-space information for more than 7 million stars has become available. This subset, known as the RVS sample (Katz et al. 2019), has empowered holistic studies of dynamical processes taking place in the Milky Way. For example, substructures in integrals-of-motion spaces of nearby stars, have been related to the Galactic bar or spiral arms (e.g. Dehnen 2000; Monari et al. 2017; Khoperskov et al. 2019; Trick et al. 2019; Hunt et al. 2019; Monari et al. 2019), or to interactions with satellite galaxies (Antoja et al. 2018; Laporte et al. 2018, 2019).

Also the assembly history of the Milky Way can be studied by identifying and characterising substructures in integrals-of-motion space, as pioneered by Helmi & de Zeeuw (2000). Structures comprising tens to hundreds of stars have been linked to shredded dwarf galaxies whose debris can still be traced in the solar neighbourhood (Helmi et al. 1999; Chiba & Beers 2000; Klement et al. 2008, 2009; Williams et al. 2011; Helmi et al. 2017; Myeong et al. 2018; Koppelman et al. 2018a, 2019, see also the reviews of Newberg & Carlin 2016; Klement 2010). Besides these small groups, recently the relics of a massive dwarf galaxy (now known as Gaia-Enceladus) have been identified in the same kind of spaces (Belokurov et al. 2018; Helmi et al. 2018).

With the dawn of machine learning, it has become popular to deploy automated classification algorithms to search for substructures (Necib et al. 2019; Borsato et al. 2019; Du et al. 2019; Koppelman et al. 2019). Substructures do not necessarily always have an accreted origin (see, e.g. Gómez & Helmi 2010; Gómez et al. 2013; Jean-Baptiste et al. 2017) and therefore their interpretation is not always straightforward. In this work, we investigate structures reported in z_{max} -apocentre space (see e.g. Haywood et al. 2018) supplemented by information obtained from the orbital frequencies of the stars.

Substructure identified in the space characterised by z_{max} -apocentre is always amenable to intuitive interpretation. Schuster et al. (2012) noted that the low- $[\alpha/\text{Fe}]$ stars in the halo, identified in Nissen & Schuster (2010), have orbits that reach out to much larger apocentre and z_{max} than the high- $[\alpha/\text{Fe}]$ stars. A similar conclusion was reached by Haywood et al. (2018), who identified distinct ‘wedges’ in this space. These wedges are also present

when using the chemically defined halo sample of Chiba & Beers (2000), with updated astrometry from *Gaia* DR2, ruling out that they are due to a kinematic bias. Haywood et al. (2018) relate the wedges to the merger with *Gaia*-Enceladus and link them to impulsive heating of an ancient Milky Way disk. The authors suggest that the largest gap may hint at a phase transition in the assembly history of the Milky Way, transitioning from significant to quiescent accretion.

Because substructure in spaces such as $z_{\max}$ -apocentre may also be due to orbital resonances, we explore here a possible link to frequency space. The space of orbital frequencies is particularly interesting for two reasons. As shown in Gómez & Helmi (2010) and Gómez et al. (2010), individual streams associated to an accreted galaxy can be easily identified here. Moreover its time of accretion can in principle be determined from the separation between two adjacent streams in frequency space. On the other hand, Valluri et al. (2012) have argued that the strengths and locations of resonances for halo stars in frequency space are both dependent on the stellar distribution function as well as on the global shape of the halo. Moreover, they argue that the diffusion rates of the orbital frequencies can help in distinguishing between the true and an incorrect potential.

In this work we investigate the properties of stars in the Solar vicinity in spaces associated to different orbital parameters, -such as to pericentre, apocentre, eccentricity, $z_{\max}$ and orbital frequencies-, and also explore the kinematics of substructures identified. The chapter is organised as follows. In Sect. 4.2 we introduce the data and present the selection criteria applied. Then, in Sect. 4.3, we identify substructure in different orbital parameters spaces. In Sect. 4.4 we perform an orbital frequency analysis and show how the structures in the orbital parameters spaces, in particular $z_{\max}$ -apocentre are linked to resonances in frequency space. We reflect on these results in Sect. 4.5, and present our conclusions in Sect. 4.6.

4.2 Data

We use the subset of *Gaia* DR2 with line-of-sight velocities (Gaia Collaboration et al. 2018c). This subset, also referred to as the *Gaia* RVS subset, of more than 7 million stars thus contains accurate measurements of the 3D positions and 3D velocities of the stars. *Gaia* DR2 parallaxes are known to suffer from systematic parallax errors that vary with position, G-band magnitude, and stellar type. Therefore we use Bayesian distances $\hat{d}$

derived by McMillan (2018). The author has derived distances by assuming an overall parallax offset of $-29\mu\text{as}$ with a RMS error of $43\mu\text{as}$ for the *Gaia* RVS subset as found for the full *Gaia* DR2 release (Lindegren et al. 2018). We focus here on a local sample with $\hat{d} < 2.5$ kpc and select only those stars with at most 20% relative distance errors, thus $\hat{d}/\hat{e}(\hat{d}) > 5$. With such quality criteria the sample still contains 5,015,006 stars.

For these stars we compute their positions $\mathbf{x}$ and velocities $\mathbf{v}$ in a Galactocentric cylindrical coordinate system $(R, z, \phi, v_R, v_z, v_\phi)$. For the position of the Sun we assume $R_\odot = 8.2$ kpc (McMillan 2017)¹ and $z_\odot = 0.014$ kpc (Binney et al. 1997). The circular velocity for an orbit at $R = R_\odot$, the Local Standard of Rest (LSR), is for consistency reasons also taken from McMillan (2017). We thus assume $v_c(R_\odot) = 232.8$ km/s. We further set the motion of the Sun relative to the LSR to $(U, V, W)_\odot = (11.1, 12.24, 7.25)$ km/s (Schönrich et al. 2010). Here U expresses radially inward motion, V into the direction of Galactic rotation, and W perpendicular to the Galactic plane and towards the Galactic north pole. We further define v_ϕ such that it is positive in the sense of Galactic rotation.

In this work we focus on a thick disk (TD) and a halo sample. We “isolate” these based on the kinematics of the stars (also see Venn et al. 2004; Bensby et al. 2014; Posti et al. 2018). This is done by selecting stars that satisfy $v_{\min} < |\mathbf{v}(\mathbf{x}) - \mathbf{v}_c(R)| < v_{\max}$. We also assume that $v_c(R) = v_c(R_\odot)$ over the distance range probed. For the thick disk we choose $v_{\min}^{\text{thick}} = 100$ km/s and $v_{\max}^{\text{thick}} = 210$ km/s, while for the halo we only set a minimum speed ($v_{\min}^{\text{halo}} = 210$ km/s), as shown in Fig. 4.1. The thick disk sample now contains 216,672 stars, and the halo sample 17,704 stars. Since for such traditional definition the halo still consists of (at least) two main components (Gaia Collaboration et al. 2018a; Koppelman et al. 2018a; Haywood et al. 2018; Gallart et al. 2019), we further split the halo sample in a thick disk tail (‘TDtail’) sample and a pure halo sample (‘pureHalo’). The separation occurs at $v_{\min}^{\text{TDH}} = 260$ km/s, such that the ‘pureHalo’ sample contains 10,370 stars and the ‘TDtail’ sample 7,334 stars.

¹This value is consistent with Gravity Collaboration et al. (2019) who have recently determined $R_\odot = 8178 \pm 13_{\text{stat.}} \pm 22_{\text{sys.}}$ pc.

Figure 4.1 – Velocity distribution of the stars in our sample, highlighting our various kinematical selections. The background image shows a logarithmic star count map of the rotational velocity *vs.* the amplitude of the velocity in the perpendicular plane. Stars within the cyan semi-circles are considered to belong to the thick disk (TD), while those outside the outermost cyan contour to be halo stars. These stars are further subdivided into a pure halo sample (orange points, ‘pureHalo’) and a thick disk tail sample (blue points, ‘TDtail’). Only stars with $\hat{d}/\hat{\epsilon}(\hat{d}) > 5$ and $\hat{d} < 2.5$ kpc are considered in our analysis.

4.3 Analysis

In this section we review the distribution of stars in our sample in velocity space as well as in spaces defined by often used characteristic orbital parameters.

For most of the analysis presented in this chapter we adopt the Milky Way (MW) potential determined by McMillan (2017), unless stated otherwise. This axisymmetric potential consists of a stellar thin and thick disk, HI gas disk, molecular gas disk, a flattened bulge and a spherical halo component (NFW). This system has a virial mass of $(1.3 \pm 0.3) \times 10^{12} M_{\odot}$, with $(5.43 \pm 0.57) \times 10^{10} M_{\odot}$ in stars.

We compute the orbits of the stars in our sample in this potential and for a time span of ~ 80 Gyr for the halo subset and ~ 40 Gyr for the thick disk subset. We take advantage of the **AGAMA** software (Vasiliev 2019), which uses adaptive timesteps, to compute the orbits. The energies of our stars are always conserved to better than 0.01% with respect to their initial values. We output at fixed time intervals of roughly 1.25 Myr and 2.5 Myr

for the halo and thick disk samples respectively. This high time-sampling is necessary for the orbital frequency analysis presented later in the chapter.

4.3.1 Velocity space

For stars in a local sample, moving groups are apparent as clumps in velocity space (v_R, v_z, v_ϕ) . Such structures may be related to stars born together or may be of dynamical nature (e.g. see Antoja et al. 2018, on the Outer Lindblad Resonance (OLR) mode due to the Galactic bar).

In Fig. 4.2 we show the samples corresponding to the TD (top row), ‘TDtail’ (middle row) and ‘pureHalo’ (bottom row) in different projections of velocity space: (v_R, v_z) , (v_ϕ, v_R) , and (v_ϕ, v_z) . The central panels showing (v_ϕ, v_R) are predominantly shaped by our selection criteria for the different subsets or populations. The inner gap centred on $v_R \sim 0$ and $v_\phi \sim v_c(R_\odot)$ in the TD sample would be filled predominantly by thin disk stars that are not included in our analysis. The other arches visible in the remaining top panels are also due to this selection. On the other hand, the absence of stars in the outer regions of the panels reflects our cut on the traditional thick disk sample. The ‘TDtail’ component (middle row) consists of stars with “hotter” thick disk-like kinematics, and it is likely the result of the proto-disk being puffed up during the merger with Gaia-Enceladus (Helmi et al. 2018).

Gaia-Enceladus debris is part of the ‘pureHalo’ sample (bottom row). There is still a very thin arch visible in (v_ϕ, v_R) indicating that even this sample still contains a small fraction of heated thick disk stars. In the bottom left panel we also see a clear overdensity of stars in an arch near $v_z \sim \pm 250$ km/s, for a range of values of v_R . These stars define the two clumps in the bottom right panel seen at $(v_\phi \sim 120, v_z \sim \pm 250)$ km/s, and which correspond to the Helmi streams (Helmi et al. 1999; Koppelman et al. 2018b).

4.3.2 Characteristic orbital parameters

In an axisymmetric static potential, such as the one considered here, the integrals of motion energy (E) and angular momentum in the z -direction (L_z) are conserved. Stars from a common origin will be clustered in this space even if the debris is phase-mixed (Helmi & White 1999; Helmi & de Zeeuw 2000).

Figure 4.2 – Different projections of velocity space for the various subsets considered in this chapter: thick disk, thick disk tail, and pure halo from top to bottom. Note that the top panels have different axis ranges. The lower density of stars present in these top panels for small values of v_R and v_z is due to the exclusion of thin disk stars in our selection. Note that even our ‘pureHalo’ subset contains some stars with “hot” thick disk-like kinematics (as evidenced by the arch-like structure in the bottom central panel).

In the left panels of Fig. 4.3 we show the TD (top row), ‘TDtail’ (middle row) and ‘pureHalo’ (bottom row) samples in (L_z, E) -space respectively. The ‘pureHalo’ sample is dominated by stars near $L_z \sim 0$ kpc km/s, i.e. the blob identified as Gaia-Enceladus (Koppelman et al. 2018a; Helmi et al. 2018), though we also see small overdensities such as that tentatively linked to an accretion event termed Sequoia ($L_z \sim -2000$ kpc km/s, $E \sim -120000$ km²/s²), and Thamnos ($L_z \sim -1200$ kpc km/s, $E \sim -170000$ km²/s²) (see

(Myeong et al. 2019; Koppelman et al. 2019, for more details), as well as contamination from the hot thick disk.

Figure 4.3 – Orbital parameters spaces for the different subsets defined in Sec. 4.2; from top to bottom: thick disk, thick disk tail, and pure halo. The left panels correspond to the most often used space to identify merger debris, i.e. (L_z, E) . The substructure seen near $L_z = 250$ kpc km/s for highly bound energies is the globular cluster M4 (also known as NGC 6221), whose extent is largely the result of limitations in the *Gaia* DR2 astrometry for very dense stellar fields. The subspaces shown in the middle and right panels depict a large amount of structure, which appears to be independent of the subset or population considered.

In middle panels of Fig. 4.3 we plot $R_{\max}$ vs. $z_{\max}$, while in the right panels we show the eccentricity vs. $z_{\max}/R_{\max}$ (i.e. a proxy for orbital inclination). The eccentricity is defined as $e = [r_a - r_p]/[r_a + r_p]$, where

r_p and r_a denote the orbital pericentre and apocentre (measured as the minimum and maximum distances over the whole integration interval). For a perfect circular orbit $e = 0$, whereas purely radial orbits have $e = 1$.

Particularly for the latter orbital parameters spaces, we see a large amount of structure, similar to that reported by Haywood et al. (2018, but for a different Milky Way potential). Fig. 4.3 shows that the structures are present in all the subsets explored, although with different prominence partly because of the different selection criteria applied to each of the sets. The fact that they are present independently of the population indicates that the phenomenon that causes them must be of global nature, and not due to accretion events.

Figure 4.4 – $R_{\max}$ vs. $z_{\max}$ for the thick disk (left), thick disk tail (middle), and pure halo (right) populations, now computed for an integrable 2-component Galactic Stäckel potential. In all subsets the amount of structure has drastically decreased. The clear overdensity of stars in the thick disk tail sample and at $R_{\max} \sim 6.5$ kpc and $z_{\max} \sim 0.5$ kpc corresponds to the globular cluster M4 and is thus of no specific interest.

As a test of the dependence of our results on the adopted form of the Galactic potential, we have also integrated the orbits of stars in a Stäckel Milky Way-like potential (see Batsleer & Dejonghe 1994; Famaey & Dejonghe 2003). Fig. 4.4 shows the resulting distributions in the subspace $R_{\max}$ vs. $z_{\max}$ for the stars in our various subsets. This figure clearly reveals a much smoother distribution and very little substructure compared to that seen in Fig. 4.3. Possibly the biggest difference between a generic Milky Way-like potential and a Stäckel model is that the latter is fully integrable, i.e. all orbits are regular. This means that the structures seen in Fig. 4.3 are probably induced by resonances and chaotic diffusion.

4.3.3 Comparison to known substructures

To further strengthen this preliminary conclusion we now investigate the distribution of stars associated to known accreted substructures in the orbital parameters spaces just explored. Many new candidate substructures have been identified since *Gaia* DR2, such as Gaia-Enceladus (Helmi et al. 2018), Sequoia (Myeong et al. 2019), and Thamnos (Koppelman et al. 2019), and also confirmed such as the Helmi Streams (Helmi et al. 1999; Koppelman et al. 2018b).

Figure 4.5 – Distribution of accreted substructures in the ‘pureHalo’ subset following the identification by Koppelman et al. (2019), in velocity space (top) and in orbital parameters spaces (bottom).

We take the stars that belong to these substructures as identified by Koppelman et al. (2019) and take their intersection with our ‘pureHalo’ sample. In doing so we find 1186 Gaia-Enceladus stars, 54 members of the Helmi Streams stars, 92 in Sequoia, and 1098 Thamnos stars.

The velocity distribution of the stars in these various substructures is shown in the top panels of Fig. 4.5. The bottom left panel directly reflects the selection criteria used to identify them (where a lower limit in the energy E has been somewhat artificially imposed on the Gaia-Enceladus stars to reduce overlap with the bulge and the tail of the thick disk, and results in a

depletion in the number of stars with small V_R , and eccentricities $e < 0.8$). On the other hand, the middle and right hand side panels of the bottom row show that these accreted substructures are also themselves split into substructures or wedges, just like the stars in the various subsets considered in the previous section. For example, the nearly horizontally aligned gaps in $(R_{\max}, z_{\max})$ -space continue to be very prominent. Given the range of accretion times for the various objects (6 to 10 Gyr ago), and that their debris seems to be influenced in the same way, this analysis supports the idea that the origin must lie in a global phenomenon affecting the whole Galaxy.

4.4 Orbital Frequency Analysis

4.4.1 Methods

In this section we perform a frequency analysis to explore whether resonances might play a role in the substructures seen in the space of characteristic orbital parameters. To this end we perform a frequency analysis on the computed orbits by using the **SuperFreq** package² (Price-Whelan et al. 2016), which is a Python implementation very similar to the ‘Numerical Analysis of Fundamental Frequencies’ (NAFF) code written by Valluri & Merritt (1998); Valluri et al. (2010, 2012), which on itself is an implementation of the NAFF technique of Laskar (1990, 1993)³. The software finds the fundamental frequencies by computing the Fourier spectra for the phase space coordinates (more precise: of a complex time-series) used to describe the orbit. Since the orbits are computed in an axisymmetric potential we follow Valluri et al. (2012) by using a slightly different form of the cylindrical polar coordinates, namely the Poincaré’s symplectic polar coordinates. In this case, the complex time-series are chosen as: $f_R = R + iv_R$, $f_z = z + iv_z$, $f_\phi = \sqrt{2L_z} [\cos(\phi) + i \sin(\phi)]$. The frequencies in R and z are defined to be positive, while positive frequencies in the ϕ -coordinate correspond to motion in the direction of Galactic rotation, and negative frequencies to counter-rotating orbits.

By definition the fundamental frequencies are non-zero. However, a zero frequency line in the spectrum of a coordinate can indicate that the centre

²<https://superfreq.readthedocs.io/en/latest/>

³Wang et al. (2016) provides a comparison between the NAFF algorithm and another commonly used orbital frequency and classification code by Carpintero & Aguilar (1998).

of the orbit is not at the origin. Because we use symplectic coordinates and because the mean R -coordinate of an orbit is always positive the frequency spectrum in R will always have a zero-frequency component. On the other hand, a zero-frequency in z indicates an asymmetric orbit, such as a resonant banana orbit, whose resonance will be picked up (see below). In **SuperFreq** the first fundamental frequency will be the non-zero frequency with the highest amplitude. The software then continues by moving down along the remaining frequency lines in the Fourier spectrum corresponding to the other coordinates, in order of decreasing amplitude. The next fundamental frequency must be different from the first one found, and so on. These fundamental frequencies $\mathbf{\Omega} = (\Omega_R, \Omega_z, \Omega_\phi)$ are not necessarily equal to the dominant frequencies $\boldsymbol{\omega}$ in the Fourier spectra of each coordinate. The fundamental frequencies found are also not necessarily equal to the frequencies in which the angle variables in action-angle coordinates vary with time. For regular orbits the recovered fundamental frequencies will however be a linear combination of these ‘more fundamental’ (action-angle) frequencies (Valluri et al. 2012).

If, for a pair of coordinates, the fundamental frequencies are linearly related, then a resonance is said to be found⁴. A full resonance is said to exist at frequencies ratios denoted by $\Omega_z : \Omega_R : \Omega_\phi$. That is, a resonance is defined to exist if $\mathbf{n} \cdot \mathbf{\Omega} = 0$, in which $\mathbf{n} = (n_z, n_R, n_\phi)$ is a non-zero resonance integer vector. We here only consider resonance vectors with elements $|n_i| \leq 5$. In a spherical potential the true ‘fundamental’ frequencies corresponding to the nature of the potential satisfy: $\Omega_r/2 \leq |\Omega_\phi| \leq \Omega_r$ (Binney & Tremaine 2008). The lower limit of Ω_ϕ corresponds to the limiting case of a homogeneous sphere, and the upper limit corresponds to the case of a Kepler potential (point mass). In the epicyclic approximation $\Omega_\phi/\Omega_r = 1/\sqrt{2} \simeq 0.707$ for a flat rotation curve [$v_{\text{circ}}(r) = \text{cst}$]. In an axisymmetric disk potential vertical oscillations for most disk stars are expected to have the shortest periods (Ω_z is the largest in $\mathbf{\Omega}$).

The NAFF methodology provides reliable frequencies for regular orbits which have been traced for 20 – 30 periods, so we limit our analysis to stars with a minimum of 20 orbital periods. For each orbit we have computed the longest period as based on its smallest (absolute) frequency. Almost all

⁴Stars are defined to belong to a certain resonance if the orbital frequency ratio for the pair of coordinates deviates at most 0.001 from the rational number of the resonance under consideration.

of our halo and thick disk stars are integrated long enough to satisfy this criterion. For example, most halo stars are integrated for 100 – 600 orbital periods, while most thick disk stars for 150 – 300 orbital periods.

Figure 4.6 – Frequency maps for the different populations. From top to bottom: thick disk, thick disk tail, and pure halo population. From left to right: (Ω_R, Ω_z) , (Ω_ϕ, Ω_R) , and (Ω_ϕ, Ω_z) . Notice the presence of resonances (especially in the left panels) in the form of straightlines, and of depleted regions around them.

4.4.2 Results

We start by showing in Fig. 4.6 the different orbital fundamental frequencies for each of the populations/subsets considered. This figure clearly reveals the presence of resonances in Ω_R *vs.* Ω_z (left panels). Note as well the

regions around these resonances which are depleted of stars and likely reflect the presence of chaotic orbits. On the other hand, there are no very obvious resonances in Ω_ϕ *vs.* Ω_R (middle panels, these might be expected for thin disk stars, particularly in relation to the Galactic bar, see Dehnen 2000).

We find that roughly 10% of the stars in the thick disk sample are on resonant orbits. For the ‘TDtail’ and for the ‘pureHalo’ samples, these fractions are much higher, namely 25% and 28% respectively. To establish if there is a relation between the resonances and the structures we have seen in the previous section in orbital parameters space, we now study the most dominant resonances in each of the subsets.

Characterisation of the resonances in the different subsets

For the thick disk sample, the top panels of Fig. 4.7 show the selected resonances. These are the $\Omega_z : \Omega_R = 1:1$ (5.0%), 2:1 (0.3%), 1:2 (0.2%), 3:2 (3.6%), 3:4 (0.03%), and 5:4 (0.7%) resonances, where the values inside the brackets denote the fraction of stars in these resonances for the sample. The panels in the middle row of this figure show that the stars on these resonances occupy rather specific regions in the space of orbital parameters, revealing that there is a strong link between the frequencies and features seen in these spaces. In fact, the overdensities first shown in Fig. 4.3 correspond directly to some of the resonances highlighted in the top panels. On the other hand, depleted regions and gaps in orbital parameters’ space indicate regions of unstable orbits. Such orbits are likely trapped into the resonances explored here, leaving empty regions around the corresponding frequencies. This is for example clearly seen for the $\Omega_z : \Omega_R = 1:2$ (in green) and $\Omega_z : \Omega_R = 3:2$ (in red) resonances. The bottom panels of Fig. 4.7 show that stars on a given resonance have a broad range of velocities, but that in some cases (e.g. the 1:2) they may occupy rather specific regions of velocity space.

On the other hand, Fig. 4.7 reveals more complexity for the $\Omega_z : \Omega_R = 1:1$ or $\Omega_z : \Omega_R = 2:1$ resonances. These stars on these families surround the most prominent gap of Fig. 4.3, located near $R_{\max} \sim 8$ kpc and $z_{\max} \sim 2$ kpc. These resonances are thus not fully responsible for this depleted region. We return to this point in the next section.

We now follow the same procedure for the halo samples, plotting separately the ‘TDtail’ and the ‘pureHalo’ samples in Fig. 4.8 and 4.9 respectively. We select the $\Omega_z : \Omega_R = 1:1$ (10.9%, 10.1%), 2:1 (4.2%, 6.2%),

Figure 4.7 – Selected resonances in the thick disk subset and their mapping onto other informative subspaces. Top row: Frequency maps highlighting the main resonant families. Middle row: Characteristic orbital parameters colour-coded according to the different resonances. Bottom row: Their distribution in velocity space.

3:2 (4.8%, 3.9%), 3:4 (3.1%, 4.3%), 5:4 (1.9%, 1.8%), 1:2 (0.6%, 1.7%), and 5:2 (0%, 0.4%) resonances, where inside the brackets we give the relative fractions of stars for the ‘TDtail’ and ‘pureHalo’ samples, respectively. As for the thick disk sample, the stars associated to the resonances also occupy rather specific regions in orbital parameters’ space. Depleted or empty regions appear again to be associated with irregular orbits.

Figure 4.8 – Similar to Fig. 4.7 but now for the ‘TDtail’ sample.

4.4.3 Commonalities

In all three subsamples, we find similar families of resonances being dominant. The $\Omega_z : \Omega_R = 1:1$ is the most prominent resonance in all samples. In the thick disk sample the other dominant resonance is the 3:2 resonance. For the ‘TDtail’ sample, this resonance is also the second most dominant frequency, but the 2:1 and 3:4 resonances are also prominent. In the ‘pureHalo’ sample, the second most dominant resonance is the 2:1, then followed by the 3:4 and 3:2 resonances.

We now return to the most prominent gap seen in Fig. 4.3 and present in all subsamples, that is the gap that goes through $(R_{\max}, z_{\max}) \sim (8, 2)$ kpc. We focus for practical purposes on the thick disk subset. We have previously

Figure 4.9 – Similar to Figs. 4.7 and 4.8 but now for the ‘pureHalo’ sample.

noted from the middle row panels of Fig. 4.7 that the gap also breaks up the stars associated to the resonances $\Omega_z : \Omega_R = 1:1$, $2:1$, and $1:2$, implying that these resonances on their own can not fully explain it.

To gain further insight, we select stars around the gap for the $\Omega_z : \Omega_R = 1:1$ resonance, as shown in leftmost panel of Fig. 4.10. The rightmost panel of this figure shows that the stars in this region cluster around $\Omega_\phi : \Omega_z = 2:3$, but that those above the gap have $\Omega_\phi/\Omega_z < 2/3$, while those below have values greater than $2/3$. This suggests that a full resonance might be playing a role in producing this gap, as if leading to a bifurcation. Similar results are found for the other resonant families, with

Figure 4.10 – Selection of stars (left panel) around the gap in $R_{\max}$ *vs.* $z_{\max}$ for the $\Omega_z : \Omega_R = 1:1$ resonance for stars in the thick disk sample. Their mapping to eccentricity *vs.* $z_{\max}/R_{\max}$ and the distributions of their frequency ratios Ω_ϕ/Ω_z are respectively shown in the middle and right panels. This last panel shows that a splitting/bifurcation appears to occur for $\Omega_\phi/\Omega_z = 2:3$.

the gap seen in the characteristic orbital parameters of stars associated to the $\Omega_z : \Omega_R = 2:1$ resonance, being related to $\Omega_\phi : \Omega_z = 1:3$.

We also see that the velocity distributions of stars associated to resonances reveal the presence of overdensities, which could potentially be confused with debris from accretion events (i.e. such as the Arcturus stream, see Navarro et al. (2004) or Kushniruk & Bensby (2019)). A more recently example is the reported prograde stellar stream, Nyx, with azimuthal velocities around 140 km/s (Necib et al. 2019), which could also be due to the Galactic bar acting on thick disk-like stars, but which we have not considered here (e.g. see Monari et al. (2013) or Antoja et al. (2015)).

Known accreted substructures in frequency space

We now explore what regions of frequency space the accreted substructures introduced in Sect. 4.3.3 and plotted in Fig. 4.5 occupy. This is shown in Fig. 4.11.

Gaia-Enceladus and Thamnos are seen to span rather large regions in frequency space, and to be affected by resonances just like the whole halo subset. On the other hand, the Helmi Streams and Sequoia, occupy relatively narrow lines in Ω_ϕ *vs.* Ω_R , although the Helmi stream stars (red points) appear to be split further in two lines. This is also seen in Ω_R *vs.* Ω_z . One of the two resonant lines is the $\Omega_z/\Omega_R = 1 : 2$, and these stars have the lowest eccentricities, compared to the remaining stars that have

$\Omega_z/\Omega_R \sim 0.70 - 0.75$, but no other obvious distinction is found. The effect of measurement errors seems to be too small to have a role in this splitting.

Sequoia stars have rather retrograde orbits and the least bound energies. They appear to be located close to the $\Omega_z : \Omega_R = 3:4$ resonance, as comparison to Fig. 4.9 also shows.

Figure 4.11 – Frequency maps of the known accreted substructures for the stars present in our ‘pureHalo’ sample.

4.5 Discussion

The analysis carried out in previous sections shows that substructures present in spaces associated to the orbital parameters of stars can be driven by properties of the gravitational potential in which the stars move. In particular, we have seen that a realistic (but non-integrable) Milky Way potential, with a disk and halo components, leads to the presence of well-populated orbital resonances, which appear to trap orbits, which then also lead to the depletion of stars in the regions around these resonances.

These results do not strongly depend on the choice for the gravitational potential assumed for the Milky Way, provided this is generic enough (and not of Stäckel form). To test this we have also integrated our samples of stars in two different Galactic potentials, namely the Piffl et al. (2014) models and the *MWPotential2014* (Bovy 2015). We confirm that also in these potentials very similar substructures observed in $R_{\max}$ *vs.* $z_{\max}$, and that they are associated to resonant families, and the presence of non-integrability in the system.

Our analysis therefore directly shows that not all substructure is due to accretion as often considered in the literature, nor to the settling of the gravitational potential after major merger activity as had been suggested

by Haywood et al. (2018). Nonetheless the characteristics of substructures from merger events may sometimes be related to the presence of resonances. For example, we have seen that the Helmi streams appear to be close to a resonance, $\Omega_\phi/\Omega_R \sim 0.66$ and $\Omega_z/\Omega_\phi \sim 1$. This would perhaps explain why its stars are distributed asymmetrically in velocity space (the stream with $v_z < 0$ has more stars than that with $v_z > 0$). This asymmetry has been used to constrain its time of accretion to approximately 6 Gyr ago, which is a puzzling low value given that these stars are on relatively bound orbits. Since stars near a resonance take longer to spread out in space (Vogelsberger et al. 2008), this can lead to an underestimation of their time of accretion. All in all, this means that chaotic dynamics may have to be considered when modeling the evolution of tidal debris, as already hinted by Price-Whelan et al. (2015) in their modeling of cold thin streams further away in the halo.

Probably the best way to tell whether substructures are related to accretion events is via a chemical tagging analysis. Stars that originate from a satellite will follow characteristic paths in chemical abundance space. On the other hand, stars that group together because of dynamical resonances have no reason to be chemically distinct from other stars.

How stars populate different resonant families depends on their distribution function and the gravitational potential in which they move. In this work we only considered that of the Milky Way, but recent analysis of nearby thin disk stars has shown that also the Sagittarius dwarf plays a role in their dynamics (see Antoja et al. 2018). Interestingly, we have found that the orbit of Sagittarius falls on the $\Omega_z : \Omega_R = 1:2$ resonance. The stars in our sample on this resonance (green points in Figs. 4.7, 4.8, and 4.9) define a rather distinct branch in frequency space. They also have rather different velocities, with high v_z values, while remaining cold in (v_R, v_ϕ) . Although our integration did not include the gravitational potential due to the Sagittarius dwarf, this “coincidence” suggests that its effect on the hotter components of the Milky Way is non-negligible and would be even more reinforced if we had considered it in our orbital integrations.

4.6 Conclusions

We have studied the dynamical properties of nearby stars in the thick disk and stellar halo using data from the 2nd data release of the *Gaia* mission. We have explored spaces of characteristic orbital parameters for a generic

Galactic potential, and found that these host a large amount of structure. Further analysis using orbital frequencies shows that these structures are due to the presence of resonant families, and to the depletion of orbits around them due to non-integrability. Therefore these structures reflect intrinsic properties of the gravitational potential of the Milky Way.

These findings are interesting on their own, and highlight that a large number of stars in these components (nearly 30% for the halo sample) are on resonances. This gives us hope to use them to pin-down more precisely the gravitational potential of the Milky Way. Valluri et al. (2012) argue that most stars will not be launched on regular tori, and that therefore, chaotic diffusion should occur. The expectation would thus be that the fraction of stars on such irregular orbits will be the lowest for the true potential (as this would indicate self-consistency between the distribution function and the gravitational potential).

Frequency analysis is also interesting because it can reveal the individual streams originating in an accretion event (Gómez & Helmi 2010). Our analysis shows that their presence may be more subtle and disguised by the resonances. We have yet to find imprints of such events directly in frequency space.

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Chapter 5

Axisymmetric Schwarzschild models of an isothermal axisymmetric mock dwarf spheroidal galaxy

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Abstract

The goal of this chapter is to test the ability of Schwarzschild's orbit superposition method in measuring the mass content, scale radius and shape of a flattened dwarf spheroidal galaxy. Until now, most dynamical model efforts have assumed that dwarf spheroidal galaxies and their host halos are spherical, even though their observed light distribution is clearly not spherical. We use an Evans model (1993) to construct an isothermal mock galaxy whose properties somewhat resemble those of the Sculptor dwarf spheroidal galaxy. This mock galaxy contains flattened luminous and dark matter components, resulting in a logarithmic profile for the global potential. We have tested how well our Schwarzschild method could constrain the characteristic parameters of the system for different sample sizes, and also if the functional form of the system's potential was unknown. When assuming the true functional form of the system's potential, the Schwarzschild modelling technique is able to provide an accurate and precise measurement of the characteristic mass parameter of the system and reproduces well the light distribution and the stellar kinematics of our mock galaxy. When assuming a different functional form for the model's potential, such as a flattened NFW profile, we also constrain the mass and scale radius to their corresponding values. However in both cases, we find that the flattening parameter remains largely unconstrained. This is likely because the information content of the velocity dispersion on the geometric shape of the potential is too small, since σ is constant across our mock dSph. Our results using Schwarzschild's method indicate that the mass enclosed can be derived reliably, even if the flattening parameter is unknown, and already for samples containing 2000 line-of-sight radial velocities, such as those currently available. Further applications of the method to more general distribution functions of flattened systems are needed to establish how well the flattening of dSph dark halos can be determined.

5.1 Introduction

In the current cosmological Λ CDM model most of the mass is believed to be in the form of (cold) dark matter. While successful on large scales, on the scales of dwarf galaxies, the model suffers a number of challenges, including the missing satellites problem (Klypin et al. 1999; Moore et al. 1999), the cusp-core conundrum (Hui 2001), and the too big to fail problem (Boylan-Kolchin et al. 2011), although all may be solved one way or another by considering the effects of baryonic physics (e.g. Zolotov et al. 2012; Brooks et al. 2013; Wetzel et al. 2016; Kim et al. 2018). The dwarf spheroidal satellite galaxies (dSph's or dSph galaxies) of our Milky Way can provide particularly strong constraints on the nature of dark matter, since their high mass-to-light ratios suggest that they are fully dark matter dominated (Strigari et al. 2008; Walker et al. 2007; Wolf et al. 2010).

Various methods have been used to develop dynamical models of dSph galaxies using line-of-sight velocity measurements for large samples of individual stars in these systems (e.g. Battaglia et al. 2006, 2008a,b, 2011; Walker et al. 2009a, 2015). Modelling via the Jeans Equations, distribution functions, and orbit superposition methods like Schwarzschild modelling are amongst those most often used (Battaglia et al. 2013). All these methods have in common that they assume that the systems are in dynamical equilibrium.

The Jeans Equations are derived by taking moments of the Collisionless Boltzmann Equation, which itself describes the conservation of probability in phase-space (Binney & Tremaine 2008). Not every solution of the Jeans Equations has an associated distribution function that is physical (i.e. positive) everywhere. Furthermore finding a solution requires additional assumptions, for example on the functional form of the density profile and on the velocity anisotropy (because this is generally not known, although see the work by Massari et al. 2018, who determined directly the anisotropy of a sample of stars in the Sculptor dSph using proper motions derived from *Gaia* and HST). Because Jeans modelling is very flexible and fast it has become the most widely used tool to model dSph galaxies, particularly in the spherical limit. It has, for example, allowed a robust (independent of the velocity anisotropy) measurement of the mass enclosed within approximately the half light radii of the dSph galaxies (Walker et al. 2009b; Wolf et al. 2010), and the determination that the masses of the classical dSph's are in the range $\sim (10^8 - 10^9)M_{\odot}$ (e.g. Walker et al. 2007).

On the other hand, it has not been possible to rule out cusped or cored profiles on the basis of these types of models (e.g. Evans et al. 2009; Strigari et al. 2017).

The Schwarzschild (1979) modelling technique relies on the idea that a system can be seen as a superposition of stellar orbits. In Schwarzschild modelling one only needs to assume a specific gravitational potential form. The method does require a significant amount of computing power and therefore a smaller set of gravitational potentials can be explored in comparison to Jeans modelling. Breddels & Helmi (2013) have applied this method to 4 dwarf spheroidal galaxies and by modelling both the second and fourth line-of-sight velocity moments and assuming spherical symmetry they find that, independently of the particular form assumed for the potential, it is possible to constrain not only the mass at around the half-light radius (more precisely at r_{-3} where the logarithmic slope of the luminous density is -3) but also the logarithmic slope of the dark matter density at this radius.

Most work thus far has assumed that dwarf spheroidal galaxies and their host halos are spherical, despite the fact that their light distribution is typically not round (Irwin & Hatzidimitriou 1995; McConnachie 2012). Furthermore, dark matter halos are predicted to be triaxial (Jing & Suto 2002) when no baryonic effects are taken into account, although subhalos in cold dark matter simulations that could host dSph's are only mildly triaxial, and almost axisymmetric (Vera-Ciro et al. 2014). This implies that it is important to establish how many and which of the previously mentioned results still stand when taking into account deviations from spherical symmetry.

Kowalczyk et al. (2017, 2018) have in fact studied the ability of recovering the mass profile and anisotropy of the remnants of the mergers of dwarf disk galaxies (one postulated channel for the formation of dSph) when using spherical Schwarzschild models. These authors have shown that for spherical remnants the method can break the mass-anisotropy degeneracy, whereas for non-spherical (prolate) remnants the anisotropy will always be underestimated, although the total mass profile will be recovered well for data along the minor axis (however not if the data are along the major axis).

On the other hand, Hayashi & Chiba (2012, 2015) used axisymmetric Jeans modelling to infer the axis ratio of the dark matter density distribution (Q) in several dSph's assuming a constant velocity anisotropy

β_z . They report rather low axis ratios ($Q = [0.3 - 0.5]$) compared to the observed projected axial ratio in the light ($q'_* \sim 0.7$). These low values are somewhat counterintuitive, though the results may be affected by degeneracies between Q , the velocity anisotropy profile, the viewing angle of the dSph, and the inner slope of the dark matter density profile. In Hayashi et al. (2016), a very similar technique was applied to unbinned data, and for e.g. Scl dSph, the authors found that the flattening parameter is largely unconstrained.

In this chapter we explore the performance of the Schwarzschild modelling technique in the axisymmetric regime, to free ourselves from the assumptions inherent to Jeans models. We test the method on a mock Sculptor-like dSph and consider axisymmetric mass distributions for both the light and the dark matter component and establish how well the characteristic parameters of the potential can be recovered, for different sample sizes.

The chapter is organised as follows. In Sect. 5.2, we set up a mock galaxy and simulate a realistic dataset. In Sect. 5.3 we describe the Schwarzschild method and its implementation in this chapter. Then, in Sect. 5.4.1, we apply the Schwarzschild method and show that we can recover the characteristic mass parameter of the mock galaxy potential, irrespective of the potential flattening parameter assumed. In Sect. 5.4.2 we model our mock galaxy with an axisymmetric NFW potential form and show that, even in this case, the Schwarzschild method is able to constrain the mass and scale radius to the corresponding values for datasets containing a realistic number of stars. We present our conclusions in Sect. 5.5 where we also discuss our findings.

5.2 The mock galaxy

5.2.1 Potential, luminous density and characteristic parameters

We have built a mock galaxy inspired by the Sculptor dSph. We have assumed a flattened stellar density profile ($q_* = 0.8$), no net rotation and a line-of-sight velocity dispersion of ~ 10 km/s (Mateo 1998; Battaglia et al. 2008a; Walker et al. 2009b). For simplicity, we have set up the mock galaxy following Evans (1993), who uses an elementary distribution function to describe a composite axisymmetric system. This distribution

function is ergodic, i.e. it leads to a velocity ellipsoid that is isotropic and has a constant amplitude and thus is not generic¹. Even though the velocity anisotropies of most classical dSph's do not appear to be too far from isotropic (see e.g. Hayashi & Chiba (2015), Hayashi et al. (2016), Hayashi & Obata (2019), Massari et al. (2019), but also Massari et al. (2018)), our specific choice of an isotropic model might be a simplification of reality.

The gravitational potential of the composite system as a whole follows the form

$$\Phi_E(R, z) = \frac{1}{2}v_0^2 \ln \left(R_c^2 + R^2 + \frac{z^2}{q^2} \right) + \Phi_0, \quad (5.1)$$

where (R, ϕ, z) denote the cylindrical coordinates. Here v_0^2 relates to the mass of the system and R_c is the core radius. The parameter q is the axial ratio of the potential, and has to satisfy $1/\sqrt{2} = 0.707 \leq q \leq 1.08$ where the lower limit is set by the condition that the spatial density is positive everywhere (Binney & Tremaine 2008) and the upper limit yields a composite distribution function of the form used by Evans (1993) that is positive everywhere. The zero point of the potential is set by Φ_0 .

The density profile of the stellar component is described by

$$\rho_{\text{lum}}(R, z) = \frac{\rho_0 R_c^p}{(R_c^2 + R^2 + z^2/q_*^2)^{p/2}}, \quad (5.2)$$

where ρ_0 is the central density, p denotes a slope parameter, and q_* is the axial ratio of the stellar density. The associated stellar distribution function is given by

$$f_{\text{lum}}(E) \propto \exp[-pE/v_0^2] = \exp[-p\Phi_E/v_0^2] \exp[-pv^2/2v_0^2], \quad (5.3)$$

where E is the sum of the gravitational potential and kinetic energies of a star.

Throughout the chapter we define the flattening of a system as the axial ratio of the equipotential contours. In the Evans model $q_* = q$ and therefore the axial ratio of the stellar density component is the same as that of the potential (not the density) of the composite system. For $q < 1$ the system is said to be oblate, and prolate for $q > 1$. The surface brightness profile

¹nor is this distribution function ideal as we shall see later in the chapter, because it provides very little information on the symmetries of the system.

Figure 5.1 – The surface brightness profile of our mock galaxy in an edge-on view. The black horizontal and vertical lines show the boundaries of the kinematic-bins. We only show the positive quadrant of our FOV ($x' > 0$ kpc, $y' > 0$ kpc). The yellow contours correspond to the isophotes of the system ($q_* = q = 0.8$). In the top panel we have plotted the surface brightness normalised to its central value as function of x' , i.e. along the (projected) major axis of the galaxy.

of the mock galaxy can be found by integrating the luminous density along the line-of-sight.

The line-of-sight velocity profile is exactly Gaussian with a velocity dispersion that is isotropic and constant everywhere:

$$\sigma_E = \frac{v_0}{\sqrt{p}}, \quad (5.4)$$

and independent of the inclination, scale radius and flattening parameter.

We choose here $v_0 = 20$ km/s, $R_c = 1$ kpc, $q = 0.8$, and $p = 3.5$ for our mock galaxy. These values result in a velocity dispersion of roughly 10.7 km/s. For these values of p and q , the central total density should be at least 1.13 times the central stellar density to yield positive phase-space densities for both the stellar and dark components everywhere.

5.2.2 Observing the mock galaxy

We place the mock galaxy at a distance of 80 kpc, and “observe” it with a square field of view (FOV), centred on the mock galaxy, with a

size of $7832'' \times 7832''$ (which then corresponds to roughly 3×3 kpc)². The sky coordinates therefore range from roughly -1.5 kpc to $+1.5$ kpc. Throughout this chapter we assume an edge-on view.

In Fig. 5.1 we show the resulting 2D surface brightness profile of the mock galaxy for an edge-on view and as a function of the sky coordinates x' and y' . Since the galaxy is axisymmetric, we only show the positive quadrant. Contours of constant surface brightness follow ellipses with axial ratios q'_* , which because of the edge-on view are identical to those of the intrinsic density (i.e. $q'_* = q_* = 0.8$). The 1D surface brightness profile along the major axis is plotted in the top panel of this figure. The surface brightness decreases a factor two with respect to its central value at a projected ellipsoidal radius of 0.86 kpc, however, the projected half light radius is much larger (3.87 kpc).

We generate kinematic data for the mock galaxy by drawing positions of stars following the luminous density distribution (see Eq. 5.2) and velocities from the Gaussian distribution function (see Eq. 5.3). We thus assume that the dataset of the stars with measured line-of-sight velocities follows the same distribution as the light.

The typical line-of-sight velocity measurements of individual stars have errors of order $dv = 2$ km/s (Battaglia et al. 2008b; Walker et al. 2009a). Therefore, to simulate a realistic kinematic dataset we convolve the line-of-sight velocities with a Gaussian distribution having a standard deviation of 2 km/s. We compute velocity moments by combining the velocities of all available stars in a certain spatial bin on the sky (in what follows a *kinematic-bin*) in our FOV. This binning is thus performed in the projected sky coordinates x' and y' . The black horizontal and vertical lines visible in Fig. 5.1 show the borders of the kinematic-bins from the positive quadrant. The velocity moments are estimated by correcting for the measurement errors (see Appendix 5.A), similarly to Breddels et al. (2013). We assume that the surface brightness profile can be measured without error in much smaller spatial bins on the sky (which we refer to as *light-bins*). To produce a reasonable galaxy, we also assume that the three-dimensional light distribution is known to much larger radii, but for many fewer bins (more details can be found in Sect. 5.3.1).

²For an extended system like the one studied here, it would be better to use a larger FOV, or to have multiple pointings. The size of the FOV used here is however reasonable for dSph galaxies like Sculptor.

5.3 The Schwarzschild orbit superposition method

In Schwarzschild modelling, orbits are used as building blocks of a dynamical system. Given a potential Φ , a complete set of orbits are integrated numerically and for each orbit the predicted observables are stored in a so-called orbit library. Varying the parameters of the potential (or varying the potential form as a whole), will result in different libraries. The library which provides a combination of weighted orbits that matches the observations (light profile + kinematics) best, will be said to yield the best-fit parameters of the potential. The orbital weights themselves provide the corresponding distribution function. Since the orbital weights are positive by construction, the distribution function will be non-negative everywhere.

5.3.1 Generating orbit libraries

In this chapter we use a slightly modified version of the Schwarzschild code from van den Bosch et al. (2008), who modelled the elliptical galaxy NGC4365. In what follows, we briefly describe how we generated the orbit libraries, how the orbital integration has been done and how the libraries are stored. For more information we refer the reader to van den Bosch et al. (2008)³.

Given an energy E_i , initial positions x_0 and z_0 are sampled on an open polar grid, which is defined by N_{I_2} polar angles and N_{I_3} radii in between a thin orbit and the equipotential. The polar angles are sampled linearly, but to obtain a better sampling of orbits near the major axis of the system, 50% of the polar angles are sampled from the z -axis towards 10° above the midplane, and the remaining 50% from 10° down to the $z = 0$ plane. The initial y -coordinates and initial velocities in the x - and z -directions are set to zero. The initial velocities in the y -direction, $v_{y,0}$, are determined by $E_i - \Phi(x_0, 0, z_0) = 0.5v_{y,0}^2$. This is done for all N_{ener} energies, which are defined by $E_i = \Phi(x = x_i, y = 0, z = 0)$. The locations x_i that fix the energy grid are logarithmically sampled between 25 pc and 50 kpc from

³Note that the Schwarzschild code by van den Bosch et al. (2008) was developed to model triaxial systems, and therefore also generates initial conditions for box orbits, which have zero time-averaged angular momentum and which can cross the centre (Schwarzschild 1979, 1993). In an axisymmetric potential L_z is conserved and such box orbits will therefore never attain velocities in the azimuthal direction. As this could cause non-axisymmetries in our model we do not specifically generate box orbits.

the centre. Such sampling makes it also possible to model a reasonable light distribution at distances well outside our FOV (in what follows more detailed information about a 3D grid which is used for this purpose). This ‘orbit library’ thus consists of $N_{\text{orb}} = N_{\text{ener}} \times N_{I_2} \times N_{I_3}$ orbits (z -tubes in our axisymmetric potential).

To account for slowly precessing orbits in the library we also compute 17 copies of each orbit, where each copied orbit is subsequently rotated by 10° in the xy -plane. These 18 copies are summed into a single orbit and replace the non-rotated orbit, such that each orbit now follows the axisymmetric requirements. Besides ensuring axisymmetric behaviour of our models, adding rotations also increases the sampling of an orbit. Note as well that each orbit has a counter-rotating sibling, obtained by appropriately changing the sign of the velocity vector.

We further improve the accuracy of the model by ‘dithering’: every orbit is split into N_{dither}^3 suborbits by replacing each of its three nonzero initial coordinates by N_{dither} slightly different coordinates. In fact, the initial conditions of all suborbits are found by increasing N_{ener} , N_{I_2} , and N_{I_3} by a factor of N_{dither} . The observables of each set of adjacent N_{dither}^3 suborbits are combined and stored as being the observables of the (bundled) orbit. Choosing an odd number for N_{dither} ensures that the original orbit is the central suborbit of the bundle. In all our Schwarzschild models we use $N_{\text{dither}} = 5$. Every main orbit is thus made from a bundle of $5^3 = 125$ neighbouring suborbits.

We use a Runge Kutta integrator to compute the stellar trajectories over roughly 200 orbital time scales. We require that the energy of each suborbit is always conserved better than 1% by increasing the accuracy of the integrator if necessary. For each orbit the kinematic information is stored in a velocity grid, which consists of a line-of-sight velocity axis (N_v velocity bins) and an axis associated to the location on the sky (N_{kin} kinematic-bins). On equally spaced time intervals, a count is added to the element of the grid associated to the velocity and location at the given time. The sky projected path of the orbit is determined in a similar way and stored in the surface brightness grid containing $N_{2\text{D light}}$ light-bins. In an additional 3D grid containing $N_{3\text{D light}} = 800$ bins (40 radial, 5 azimuthal and 4 polar bins in the positive octant) the 3-dimensional path of an orbit is stored. This 3D grid contains the same radii as used to sample the energies for the initial conditions and thus reaches well beyond the FOV (in contrast

to the velocity and surface brightness grid). It is used to match the (3D) light distribution of the system at such radii.

In this chapter we set $N_{2\text{D light}}$ equal to $99 \times 99 = 9801$ and N_{kin} to $9 \times 9 = 81$, unless stated otherwise. The velocity axis of the velocity grid contains $N_v = 41$ bins and has a total velocity width of 80 km/s, such that we cover velocities up to $\pm 4\sigma_E$. The central velocity bin is centred on 0 km/s. To be able to track how long an orbit spends in a given kinematic-bin, we need to account for the time spent with velocities outside the velocity grid (at that position). Therefore counts will also be added to the first or last velocity bin if velocities are beyond the limits of the velocity grid⁴.

5.3.2 Fitting orbital weights

Once the orbit libraries are in place, we find the orbital weights such that the total luminous mass, the surface brightness profile and the kinematics within the FOV, and the 3D light profile of the system are reproduced.

The 2D light profile is fitted using the surface brightness grid, where we define:

$$m_j^{\text{light}} = \sum_{i=1}^{N_{\text{orb}}} w_i m_{ij}^{\text{light}}, \quad (5.5)$$

where we sum over all orbits i . Here, m_{ij}^{light} is the fraction of time orbit i spent in light-bin j and m_j^{light} is the fractional surface brightness in light-bin j . The orbital weights are denoted by w_i and add to unity by construction. The 3D light profile is fitted similarly using the 3D grid.

At the same time as we fit the light, we also fit the kinematics. In every kinematic-bin k we compute the first 4 mass-weighted velocity moments

⁴When taking too few (i.e. too wide) velocity bins for the velocity grid, the velocity moments might not be recovered correctly. We have also checked that if we bin the theoretical Gaussian line-of-sight velocity profile of our mock galaxy as described above, thus discarding the contribution of velocities that are outside the range of the grid, the velocity moments are recovered well, i.e. the first and third moments are not affected, while the second and fourth velocity moments might result in relative errors of order 0.1% and 2% given the choices made for binning the velocity data.

$\langle v_k^n \rangle$ by defining:

$$m_k^{\text{kin}} \langle v_k^n \rangle = \sum_{i=1}^{N_{\text{orb}}} w_i m_{ik}^{\text{kin}} \langle v_{ik}^n \rangle, \quad (5.6)$$

where again we sum over all orbits i . This time m_{ik}^{kin} is the fraction of time orbit i spent in kinematic-bin k and m_k^{kin} is the fractional surface brightness in kinematic-bin k . The n^{th} moment of orbit i in kinematic-bin k is given by $\langle v_{ik}^n \rangle$:

$$\langle v_{ik}^n \rangle = \frac{\sum_{l=2}^{N_v-1} h_{ikl} v_{\text{cen},l}^n \Delta v}{\sum_{l=2}^{N_v-1} h_{ikl} \Delta v}, \quad (5.7)$$

where Δv is the size of the velocity bin and h_{ikl} is the fraction of time that orbit i spent in kinematic-bin k and velocity bin l . Velocity bin l has velocity range $[v_{\text{cen},l} - \frac{1}{2}\Delta v, v_{\text{cen},l} + \frac{1}{2}\Delta v]$, where $v_{\text{cen},l}$ denotes its central velocity. We sum over the N_v velocity bins, although we discard the contributions of the first and last velocity bin. This is done since we did not set a stringent outer velocity boundary in these velocity bins: as described before, counts will be added here even if a star has a velocity outside the range of the grid and therefore the typical velocities of these bins are not known. Note that $m_{ik}^{\text{kin}} = \sum_{l=1}^{N_v} h_{ikl}$ with this choice.

Now that we have defined the relation between the observables and the quantities in our model, we can describe how we fit the orbital weights. The fit is based on minimising χ_{tot}^2 :

$$\chi_{\text{tot}}^2 = \sum_{u=1}^{N_{\text{obs}}} \left[\frac{\text{Model}[u] - \text{Data}[u]}{\text{Error}[u]} \right]^2, \quad (5.8)$$

where u runs over all N_{obs} observables. The number of observables is given by:

$$N_{\text{obs}} = 1 + N_{\text{2D light}} + N_{\text{3D light}} + 4N_{\text{kin}}, \quad (5.9)$$

which includes the contribution of the total light of the system, the fractional light for each 2D and 3D light-bin, and the four velocity moments for each kinematic-bin, respectively. We choose to use four velocity moments, since using higher order moments might reduce the degeneracy between the velocity anisotropy and the mass profile (e.g. Merrifield & Kent 1990; Richardson & Fairbairn 2013). We do not use higher moments since these are observationally harder to constrain. The odd velocity moments are used to ensure that the Schwarzschild model also matches the observed mean rotation and the skewness of velocity distribution.

We use a non-negative least-squares solver to ensure that all orbital weights are positive. The light is weighted by assigning an error of 2% to each of the 2D and 3D light-bins. We note that we can investigate the individual contribution to the total χ^2_{tot} by decomposing it, e.g:

$$\chi^2_{\text{tot}} = \chi^2_{\text{total light}} + \chi^2_{\text{2D light}} + \chi^2_{\text{3D light}} + \chi^2_{\text{kin}}. \quad (5.10)$$

We stress that χ^2_{tot} is being minimised. We do not minimise the terms on the right-hand side individually. The term associated to the total light of the system turns out to be negligible, since it is always recovered very well. The same holds for $\chi^2_{\text{3D light}}$. These terms are only added to Eq. 5.8 to ensure that the model returns a realistic galaxy (in the sense that the luminous component might resemble a galaxy). Most of the constraining power thus comes from the surface brightness profile and the kinematics.

5.4 Results

In this section we show that the Schwarzschild method can recover some of the characteristic parameters of the mock Sculptor-like dwarf spheroidal galaxy. We first show in Sect. 5.4.1 that if the true functional form of the system's potential is known, we can constrain the characteristic mass parameter of the mock galaxy. In reality however, the true functional form of the system's potential is not known. Therefore, in Sect. 5.4.2 we demonstrate how well we can constrain characteristic parameters when assuming an axisymmetric form of a Navarro-Frenk-White (NFW, Navarro et al. 1996) potential.

Figure 5.2 – $\Delta\chi^2$ -distribution of the characteristic parameters q and v_0 of the Evans models obtained after applying the Schwarzschild method. In this case our mock kinematic data consist of 10^5 stars inside the FOV (3×3 kpc). We use 9×9 kinematic-bins and assume the functional form of the system’s potential and inclination are known. The black circles show the locations where the Schwarzschild models were evaluated. The green circle indicates the input parameters of the mock system. The best-fit model is indicated by the white cross and recovers the mock galaxy mass parameter. In white, grey and black we show the $\Delta\chi^2 = [2.3, 6.18, 11.8]$ -contours respectively. The coloured landscape on the left shows interpolated $\Delta\chi^2$ -values, and goes up to a maximum of $\Delta\chi^2 = 10$. On the right we show the $\Delta\chi^2$ -landscapes when decomposing the landscape into $\chi_{2D \text{ light}}^2$ (top), χ_{kin}^2 (middle), or $\chi_{3D \text{ light}}^2$ (bottom). These right panels sum up to the large panel. We stress that only χ_{tot}^2 is being minimised in the fit.

5.4.1 Two parameter Evans models: recovering the mock galaxy parameters

Here we assume the true functional form of the system’s potential is known, i.e. we use it to build the orbit libraries for the Schwarzschild models. Our aim is to establish whether we can recover the correct values of the characteristic input parameters with this method. To this end we make a grid of models in which we vary the values of the characteristic parameters q and v_0 (see Eq. 5.1). We thus fix the core radius to $R_c = 1$ kpc, i.e. to its true value. We sample q from 0.72 to 0.96, and v_0 from 11 km/s to 29 km/s, with higher sampling (decided iteratively) with steps in q of

Figure 5.3 – The difference of the best-fit and the observed velocity dispersion in terms of the observed error, for all 9x9 kinematic-bins. The figure is obtained after fitting the q94v21 library to our mock data consisting of 10^5 stars with measured line-of-sight velocities in our FOV, assuming an edge-on view. The top and the right panels show the fit (red full line) obtained along the major and minor axis respectively. The data points with 68% error bars are shown in black. Black dashed lines indicate the true velocity dispersions from theory (Eq. 5.4).

0.02 and in v_0 of 1 km/s. We name the models by the values of their parameters: qXXvYY in which $XX = 100q$ and $YY = v_0$ in km/s. For the orbit sampling, we set $N_{\text{ener}} = 32$, $N_{I_2} = 32$ and $N_{I_3} = 16$ such that a total of $32 \times 32 \times 16 \times 5^3 = 2048000$ suborbits are integrated (see Sect. 5.3.1) and $2 \times 32 \times 32 \times 16 = 32768$ orbital weights are determined (see Sect. 5.3.2).

Results for a large sample

We start with an idealised case in which the kinematic data consist of 10^5 stars. For 9x9 kinematic-bins on the sky, the typical error of the velocity dispersion in a kinematic-bin is ~ 0.25 km/s.

The large panel of Fig. 5.2 shows the results obtained by fitting the Schwarzschild models to the data. The small black circles show the grid

of tested values for q and v_0 , the green circle the true input values, and the white cross indicates the values of the parameters corresponding to the maximum likelihood estimator (MLE). For the best-fit model q94v21 we find $\chi^2_{\text{tot}} = 207.7$. The contribution of the kinematics (see Eq. 5.8) to this value is 205.6. Using 81 kinematic-bins to fit 4 velocity moments, this corresponds to 0.64 per kinematic constraint.

We have computed $\Delta\chi^2(q, v_0) = \chi^2_{\text{tot}}(q, v_0) - \min[\chi^2_{\text{tot}}]$ for each of these models and define 68%, 95% and 99.7% -confidence intervals (white, grey and black contours, respectively) at $\Delta\chi^2 = [2.3, 6.18, 11.8]$ (Press et al. 1992)⁵. The coloured background shows the $\Delta\chi^2$ -landscape and is truncated at $\Delta\chi^2 = 10$. The smaller panels on the right show the $\Delta\chi^2$ -landscapes when only considering $\chi^2_{2\text{D light}}$ (top), χ^2_{kin} (middle), or $\chi^2_{3\text{D light}}$ (bottom). The $\Delta\chi^2$ -landscape based on χ^2_{tot} is thus slightly dominated by the differences in $\chi^2_{2\text{D light}}$, although the kinematics provide similar constraints.

To estimate the error on the mass parameter we first marginalise over the flattening parameter by selecting for each v_0 the minimum $\Delta\chi^2$ along q . We define the 68% error at those values where $\Delta\chi^2 = 1.0$ (Press et al. 1992). For this experiment we find $v_0 = 21^{+1.33}_{-2.11}$ km/s. We therefore conclude that we can recover the input mass parameter of our mock galaxy well, but as Figure 5.2 shows we do not constrain well the flattening parameter q .

In Fig. 5.3 we show how well the velocity dispersion is fitted in the best-fit q94v21 model. For each kinematic-bin, we show how much the model deviates from the data expressed in units of the error on the data. On top we show the fit along the major axis while the subpanel on the right shows the fit along the minor axis. In the panels the axisymmetric behaviour of our models is clearly seen. The observed velocity moments are subject to sampling effects and measurement errors and thus do not exactly obey such symmetry conditions. The figure shows that the fit is very good (and in fact, it is almost indistinguishable from the fit obtained for what would be the input parameters model, i.e. q80v20).

⁵We used the `scipy.interpolate.LinearNDInterpolator` to interpolate the $\Delta\chi^2$. Although using linear interpolation might not be optimal, it does not strongly affect the quantitative results of the chapter

Downsampling and folding data

We now consider the more realistic case of a sample of 10^4 stars with measured line-of-sight velocities. We do not change our assumption that the surface brightness profile can be measured without error in our light-bins, and hence we here only investigate the influence of the number of stars with line-of-sight velocities available. To reduce the observed uncertainties on the kinematics we decided to fold the kinematic data (but not the light). Since the system is axisymmetric, we fold our data into the kinematic-bins located in the first quadrant. We can simply move each star towards its corresponding kinematic-bin without changing its velocity, because our system has an identical Gaussian line-of-sight profile everywhere (see Sect. 5.2.1). In general, however, one should change the velocities following the assumed symmetry.

Since we fold the kinematic data from 9×9 bins of our FOV into the first quadrant, we effectively have 10^4 stars with measured line-of-sight velocities located in the resulting 5×5 kinematic-bins. A typical kinematic-bin now contains 400 stars with measured line-of-sight velocities on average, and the typical error on the velocity dispersion is ~ 0.45 km/s.

We fit the folded data with the Schwarzschild orbit superposition method and find the MLE for model q90v22 (see Fig. 5.4). As in the case of 10^5 stars, and thus as expected, the flattening parameter remains fairly unconstrained. We find a slightly larger mass parameter $v_0 = 22^{+1.02}_{-1.44}$ km/s, but $v_0 = 20$ km/s is still within the 95%-confidence region.

For the best-fit model q90v22 we find $\chi^2_{\text{tot}} = 16.5$. The contribution of the kinematics (see Eq. 5.8) to this value is 13.2. Both values are much lower than in the case of 10^5 stars, and this can be explained by the decrease in the number of kinematic constraints and the fact that the data have now been folded.

It is encouraging that a more realistic number of stars with measured line-of-sight velocities still gives such tight constraints. Comparing the 10^4 stars folded case to the case of 10^5 stars, the 2D 68%-probability contours are shifted towards just slightly larger masses. Note that the uncertainty on the mass parameter did not increase.

To further test how the results depend on the number of stars with measured line-of-sight velocities, we decreased even further this number of stars to a sample of 2000 stars. This is the typical size of currently available kinematic datasets used to put constraints on the mass of dSph galaxies (e.g.

Figure 5.4 – Similar to Fig. 5.2, but now using 10^4 stars for the kinematics and using the approach of folding the data from 9x9 into 5x5 kinematic-bins. The parameter inferences are similar, though slightly larger masses are preferred.

Walker et al. 2009b; Breddels & Helmi 2013; Hayashi & Chiba 2015). We again fold the data from 9x9 into 5x5 kinematic-bins. The resulting typical error on the velocity dispersion in a kinematic-bin is then ~ 0.9 km/s. In this case we find a best-fit model q92v23 ($\chi^2_{\text{tot}} = 38.9$, $\chi^2_{\text{kin}} = 32.6$). The $\Delta\chi^2$ -distribution is shown in Fig. 5.5. The best models are again reproduced by the most round models, although statistically the flattening parameter remains unconstrained. The region spanned by the contour drawn at $\Delta\chi^2 = 11.8$ is of similar size, but is shifted towards slightly higher masses ($\Delta v_0 \sim 1$ km/s) in comparison to the case of 10^4 stars. We now find that the mass parameter is constrained to $v_0 = 23^{+0.47}_{-2.00}$ km/s. The true q80v20 model is nevertheless still within the inferred 99.7%-confidence interval.

At first sight it may seem unrealistic that the case of 2000 results in slightly tighter constraints on the parameters of interest. We however note that the differences are minor and that these might be related to the fact that only one data realisation is studied for each case. Note as well that not only the parameters of the potential are being fitted in Schwarzschild models but also the orbital weights. This implies that the fitted distribution function can therefore be quite different from one model to the next

Figure 5.5 – Similar to Fig. 5.4, now using 2000 stars for the kinematic data. The parameter inferences are similar, though slightly larger masses are inferred ($\Delta v_0 \sim 1$ km/s).

(especially without the use of regularisation), and the marginalisation over the orbital weights is not being considered when computing the confidence contours on the potential parameters shown in Fig. 5.5 (also see Magorrian 2006). Nonetheless, when comparing the predicted line-of-sight velocity dispersions for the range of models within these contours (see Fig. 5.12 in Sect. 5.4.2) weaker constraints are in fact obtained for smaller datasets, as expected.

The weak trend found for smaller samples to prefer slightly higher values of v_0 may be due to the fact that, for small radii (compared to R_c), the potential (see Eq. 5.1) is proportional to $v_0^2[\ln R_c^2 + (R/R_c)^2] + (v_0/q)^2(z/R_c)^2$. Therefore, there is a weak degeneracy in the term v_0/q , that may manifest itself more when the sampling is sparse, and thus lead to a small shift in preferred values of v_0 for larger q .

From the tests performed in this Section we conclude that, with a kinematic sampling that follows the light, we cannot aim to constrain the flattening parameter of an *isothermal* dSph galaxy⁶, not even if the true functional

⁶Slightly better results can be obtained by sampling uniformly with distance, see Appendix 5.B.

form of the system's potential is known. This is likely because the information content in a velocity dispersion regarding the geometric shape of the potential is too small (since σ is constant across the whole system). We can however still reliably constrain the mass parameter of such a system, i.e. even though the true flattening parameter remains unknown. This can already be done for a realistic number of stars.

5.4.2 Axisymmetric NFW models

We have shown that the Schwarzschild method can correctly constrain the mass parameter when the true functional form of the system's potential is known. Now, we will tackle the problem more realistically by allowing a different functional form for the model's potential. We consider an axisymmetric NFW-profile, and follow the parametrisation of Vogelsberger et al. (2008):

$$\Phi_V(\tilde{r}) = -4\pi G \rho_0 R_s^3 \left[\frac{\ln(1 + \tilde{r}/R_s)}{\tilde{r}} \right], \quad (5.11)$$

where R_s is the scale radius and ρ_0 a characteristic density parameter. In comparison to the spherical NFW-profile, the radius $r = \sqrt{R^2 + z^2}$ is replaced by a newly defined radius:

$$\tilde{r} = \frac{(r_a + r)r_E}{r_a + r_E}, \quad (5.12)$$

where, for the axisymmetric case, $r_E = \sqrt{\left(\frac{R}{a}\right)^2 + \left(\frac{z}{c}\right)^2}$ is the ellipsoidal radius with a and c specifying the relative lengths of the major and minor axes, and where r_a is a transition radius. In addition, we require that $2a^2 + c^2 = 3$, such that when $a = c = 1$, this results in the spherical NFW profile. For $r \gg r_a$, $\tilde{r} \rightarrow r$, whereas for $r \ll r_a$, $\tilde{r} \rightarrow r_E$. Therefore, the gravitational potential is axisymmetric in the central regions and becomes spherical in the outer regions. We set the transition radius to $r_a = 10$ kpc. In all our Vogelsberger models we keep the transition radius r_a fixed.

To additionally guarantee that the total mass density is positive up to at least the orbits possessing the highest energies in our library (~ 50 kpc), the flattening parameter must satisfy $c/a \gtrsim 0.7$ for a case with $R_s = 1$ kpc. For smaller scale radii, larger lower limit values of c/a are needed to satisfy the positive density criterion.

For convenience, we define a characteristic mass parameter, $M_{1\text{kpc}}$ expressed in units of $M_\odot$, which corresponds to the total enclosed mass within 1 kpc from the centre for a spherical NFW profile with scale radius R_s , i.e.

$$M_{\text{NFW}}(r = 1\text{kpc} | R_s) = 4\pi\rho_0 R_s^3 \left[\ln\left(\frac{R_s + r}{R_s}\right) - \left(\frac{r}{R_s + r}\right) \right]_{1\text{kpc}}. \quad (5.13)$$

From this equation we determine the value of ρ_0 , and it is this value of ρ_0 that we use for the axisymmetric Vogelsberger potential in Eq. 5.11.

The “matched” Vogelsberger model

Before we can test the Schwarzschild orbit superposition method while assuming Vogelsberger mass models, we need to know when a result can be considered satisfactory. Since we could not constrain the flattening parameter for the case when the true functional form of the system’s potential is known, we will not aim to constrain the flattening parameter for the Vogelsberger models. Nevertheless, the corresponding scale radius R_s and mass $M_{1\text{kpc}}$ of our system will depend on the c/a -value assumed. In this section we therefore establish what are good parameters for the mass $M_{1\text{kpc}}$, scale radius R_s , and flattening parameter c/a , such that the properties of the Evans mock galaxy are reproduced the best.

Because most stars of our mock galaxy will have projected radii in between 0.5 and 2.0 kpc from the centre, we require that the flattening of the “matched” Vogelsberger model should be comparable to that of the mock galaxy over this region. At a given position we define the Vogelsberger potential flattening q_V as the axis ratio of the equipotential contour that goes through that point. For a position (R, z) , we thus define $q_V(R, z) = z_\Phi/R_\Phi$, where $\Phi(R = 0, z_\Phi) \equiv \Phi(R_\Phi, z = 0) \equiv \Phi(R, z)$. On such an equipotential, it must hold that $\tilde{r}(R = 0, z_\Phi) = \tilde{r}(R_\Phi, z = 0)$, and since $\tilde{r}$ only depends on c/a , $q_V(R, z)$ is independent of our mass parameter and scale radius⁷. We take values for z_Φ from 0.5 to 2.0 kpc in steps of 0.05 kpc along the minor axis and compute the corresponding R_Φ -values (i.e. the radii where the equipotential contours that belong to z_Φ cross the major axis). For a given c/a we then compute the mean of the absolute differences between the Evans mock galaxy potential flattening ($q = 0.8$)

⁷More precisely, $\tilde{r}$ depends on r_a and $r_E(c/a)$, but we have chosen to fix the value of r_a (and make it independent of R_s).

Figure 5.6 – Estimating the ‘true’ parameters of the Vogelsberger system by comparing the differences in potential flattening on the left and the differences in the gradients of the potentials on the right. The comparisons are based on the distance interval from 0.5 up to 2.0 kpc (with steps of 0.05 kpc) from the centre of the galaxy. Left: The mean absolute difference of the Vogelsberger potential flattening and the true potential flattening of the mock galaxy as a function of the flattening parameter c/a of the Vogelsberger potential (black line). The grey horizontal line marks the positions where this difference has doubled, with respect to the minimum 0.007 at $c/a \simeq 0.776$. Right: We minimise the mean of the absolute differences in the gradients of the potential along the major and minor axis (compared to the mock galaxy) by varying the Vogelsberger model parameters $M_{1\text{kpc}}$ and R_s . The figure is obtained after setting the flattening parameter to $c/a = 0.776$. The colour bar is truncated at 5.0 km/s. The green circle indicates the location at $\log_{10}(M_{1\text{kpc}}[M_\odot]) \simeq 7.69$ and $R_s = 4.9$ kpc where the differences are minimum ($\langle \Delta v \rangle_{\min} = 0.31$ km/s). Grey lines indicate the contours of constant mean absolute differences and are spaced by 1 km/s. As a proxy for the error on the Vogelsberger parameters, a green contour is drawn where the differences are doubled with respect to the minimum difference.

and the Vogelsberger potential flattening along the defined range for z_Φ , i.e. we compute: $\text{mean}(|q - q_V(R = 0, z_\Phi)|)$. We find that for $c/a \simeq 0.776$ this average difference is smallest (see left panel of Fig. 5.6). For our range of z_Φ and $c/a = 0.776$, the Vogelsberger potential flattening increases almost linearly with z_Φ , though the gradient is small (0.018 kpc^{-1}).

Given this value for the flattening parameter c/a , we proceed to obtain the corresponding values for the mass and scale radius of the mock galaxy, now described by the Vogelsberger profile. We do this by comparing $|RF_R| \equiv \sqrt{R \left| \frac{\partial}{\partial R} \Phi(R, z = 0) \right|}$ along the major axis and $|zF_z| \equiv \sqrt{z \left| \frac{\partial}{\partial z} \Phi(R = 0, z) \right|}$ along the minor axis with respect to their

Figure 5.7 – Left: The major (full lines) and minor (dashed lines) axis gradients of the potential as function of R and z respectively for the true Evans model (red) and for the “matched” Vogelsberger model with $c/a \simeq 0.776$, $R_s \simeq 4.9$ kpc and $\log_{10}(M_{1\text{kpc}}[M_\odot]) \simeq 7.69$ (blue). Right: Comparison of the isopotential contours for the true Evans model (red) and the “matched” Vogelsberger model (blue). For the purpose of this figure the zero-point of the potential is chosen here such that $\Phi_E = \Phi_V$ at $(R, z) = (1, 0)$ kpc. For each potential, contours are drawn at the positions where Φ has changed in steps of $50 \text{ km}^2/\text{s}^2$. In the region from 0.7 up to 2 kpc the “matched” Vogelsberger model follows well the true Evans potential. For more inner radii the (cusped) Vogelsberger models cannot reproduce the (less steep) cored behaviour of the Evans potential.

values for the mock galaxy. We investigate their trends for R - and z -values identical to those used for z_Φ previously.

We vary the scale radius and the mass parameter $\log_{10}(M_{1\text{kpc}}[M_\odot])$ and compute the mean of the absolute differences with respect to the mock galaxy obtained along the major and minor axis for $c/a = 0.776$. We denote this by $\langle \Delta v \rangle \equiv \text{mean}[0.5\{\text{abs}(\Delta|RF_R|) + \text{abs}(\Delta|zF_z|)\}]$. From the right panel of Fig. 5.6 we infer that $\langle \Delta v \rangle$ is minimum for mass parameter $\log_{10}(M_{1\text{kpc}}[M_\odot]) \simeq 7.69$ and scale radius $R_s = 4.9$ kpc (green circle), although any value with $R_s \geq 2$ kpc works well, as $\langle \Delta v \rangle$ does not vary strongly. To be able to compare these findings to the results from our Schwarzschild models (see Sect. 5.4.2), we estimate the error on these ‘true’ parameters by considering those locations where $\langle \Delta v \rangle$ changes by a factor 2 with respect to its minimum value (green contour). The mass parameter is then within the range $[7.63, 7.73]$, the scale radius larger than 2.4 kpc. For the smaller scale radii ($R_s < 2$ kpc) slightly higher values for the characteristic mass parameter would be preferred, but $\langle \Delta v \rangle$ is also larger

in such cases. Note that the NFW mass value that we just estimated corresponds well to the mass enclosed within 1 kpc of a spherical Evans model with $R_c = 1$ kpc and $v_0 = 20$ km/s (as assumed in Sect. 5.2), since then $\log_{10}(M_{1\text{kpc},\text{Evans}}[M_\odot]) \simeq 7.67$.

Although we will not constrain the flattening parameter, we can investigate how the parameters of our “matched” Vogelsberger model would change if different values for c/a are taken. Setting $c/a = 0.70$ results in $\langle \Delta v \rangle = 0.32$ km/s for its minimum at $\log_{10}(M_{1\text{kpc}}[M_\odot]) \simeq 7.69$ and $R_s = 4.4$ kpc, and setting $c/a = 0.85$ results in $\langle \Delta v \rangle = 0.37$ km/s for $\log_{10}(M_{1\text{kpc}}[M_\odot]) \simeq 7.69$ and $R_s = 5.1$ kpc. The corresponding Vogelsberger mass parameter is thus not affected by the choice of c/a . The corresponding scale radius only increases slightly for larger values for the flattening parameter (i.e. rounder shapes), though the effect⁸ is rather small. In addition, the grey contours, which are drawn at fixed $\langle \Delta v \rangle$, span very similar regions for different values for c/a .

In Fig. 5.7 we compare the potential of the “matched” Vogelsberger model to the true Evans potential of our mock galaxy. In the left panel we show the gradients of the potentials along the major and minor axis. Note that the Evans model seems to have lower $|RF_R|$ and $|zF_z|$ for $R \lesssim 1$ kpc and $z \lesssim 0.75$ kpc, respectively, than the “matched” NFW model. In the panel on the right we confirm that the potential flattening parameter is matched quite well by showing isopotential contours. Both panels reveal that only in the centre (< 0.7 kpc) and at the distances larger than 3 kpc, the gradients of both potentials start to deviate from each other.

In summary, the “matched” Vogelsberger system can be described by $\log_{10}(M_{1\text{kpc}}[M_\odot]) \simeq 7.69^{+0.04}_{-0.06}$ and by $R_s \gtrsim 2.4$ kpc (with its most likely value at $R_s = 4.9$ kpc) for $c/a = 0.776$.

Fitting Vogelsberger models with the Schwarzschild method: exploring different sample sizes

Since we could not constrain the flattening parameter when the functional form of the system’s potential was known (see Sect. 5.4.1), we cannot expect to constrain the flattening parameter if we examine a different functional form for the model. We set $c/a = 0.80$, equal to the observed axial ratio in the light, and subsequently find the inferences on the mass

⁸Even for a spherical potential, i.e. $c/a = 1.0$, we find the minimum $\langle \Delta v \rangle = 0.62$ km/s to be located at $\log_{10}(M_{1\text{kpc}}[M_\odot]) \simeq 7.68$ and $R_s = 6.1$ kpc.

Figure 5.8 – Difference between the best-fit Vogelsberger model (M772Rs250, blue line in the subpanels) and the observed velocity dispersion when applying the Schwarzschild method in 9x9 kinematic-bins to our mock dataset consisting of 10^5 stars in the FOV (see Fig. 5.3 for a comparison).

$\log_{10}(M_{1\text{kpc}}[M_{\odot}])$ and scale radius R_s . We initially make a grid in $(\log_{10}(M_{1\text{kpc}}), R_s)$ -space, where R_s ranges from 1 to 8 kpc with steps of $\Delta R_s = 1$ kpc, while for the characteristic mass we take steps of 0.05 for values from $\log_{10}(M_{1\text{kpc}}[M_{\odot}]) = 7.55$ to $\log_{10}(M_{1\text{kpc}}[M_{\odot}]) = 7.85$, i.e. just spanning a factor of 2 in mass. Later, we also decided to sample $\log_{10}(M_{1\text{kpc}}[M_{\odot}]) = [7.68, 7.72]$ for $R_s \in [1.5, 7.5]$ kpc with a similar ΔR_s step.

To be more efficient we decrease the number of orbits compared to Sect. 5.4.1 and set $N_{\text{ener}} = 24$, $N_{I_2} = 24$ and $N_{I_3} = 8$, such that a total of $24 \times 24 \times 8 \times 5^3 = 576000$ suborbits are integrated and $2 \times 24 \times 24 \times 8 = 9216$ orbital weights are determined. We have found this gives good results in terms of recovery of the light profile and kinematics. In addition we also add regularisation terms to the fit in this more realistic experiment: by applying regularisation we set additional constraints such that the orbital weights are more smoothly distributed, i.e. in a more physical way (as the weights relate to the distribution function, which itself is expected to be

smooth). More details on the concept of regularisation and its effects can be found in Appendix 5.C.

We present the results following the same structure of Sect. 5.4.1 and name the Vogelsberger models by MxxxRsyyy, where:

xxx = $100 \log_{10}(M_{1\text{kpc}}[M_{\odot}])$ and yyy = $100R_s$ (R_s in kpc). We discuss how well we can recover the characteristic parameters of the Vogelsberger potential for mock datasets containing 10^5 , 10^4 and 2000 stars.

We start with the case of a kinematic dataset containing 10^5 stars. We use 9x9 kinematic-bins, but no folding. For this case, we find that model M772Rs250 provides the best fit ($\chi^2_{\text{tot}} = 275.1$). Fig. 5.8 shows that this model reproduces well the mock velocity dispersions in all kinematic-bins (since $\chi^2_{\text{kin}} = 220.9$, which results in 0.68 per kinematic constraint). The fit is of comparable quality to the best-fit Evans model (for the same case) although the light is recovered slightly less well, which may be driven by the smaller number of orbits being used now, the choice of a different (and incorrect) potential form, or by adding regularisation in the fit.

Fig. 5.9 shows the resulting $\Delta\chi^2$ -distribution in $(\log_{10}(M_{1\text{kpc}}), R_s)$ -parameter space. The scale radius of the Vogelsberger potential is constrained to $R_s = 2.5^{+0.6}_{-0.1}$ kpc and the mass parameter to $\log_{10}(M_{1\text{kpc}}[M_{\odot}]) = 7.72^{+0.01}_{-0.01}$. The Schwarzschild model thus prefers values towards the lower end for the scale radius and a mass parameter that agrees well with our expectations. The panels on the right show the $\Delta\chi^2$ -landscapes when only considering $\chi^2_{2\text{D light}}$ (top), χ^2_{kin} (top-middle), $\chi^2_{3\text{D light}}$ (bottom-middle), or χ^2_{reg} (bottom). The total $\Delta\chi^2$ -landscape is dominated by the kinematics and 2D light.

As shown in Fig. 5.10 similar best-fit parameters are obtained for a smaller mock kinematic dataset containing 10^4 stars and folding the data into 5x5 kinematic-bins. The mass and scale parameters are constrained to $R_s = 3.0^{+0.7}_{-0.4}$ kpc and $\log_{10}(M_{1\text{kpc}}[M_{\odot}]) = 7.75^{+0.05}_{-0.03}$. For the best-fit model M775Rs300, $\chi^2_{\text{tot}} = 78.0$ and $\chi^2_{\text{kin}} = 33.9$, or 0.339 per kinematic constraint on average. This χ^2_{tot} is lower than for the case of 10^5 stars, likely because we folded the data. In comparison to the best-fit Evans model, the quality of the fit of the kinematics is slightly worse but still very good.

When decreasing the sample size of the kinematics even further to 2000 stars, we find that models with low values for R_s and larger $\log_{10}(M_{1\text{kpc}}[M_{\odot}])$ are now preferred, as shown in Fig. 5.11, although the 95%-confidence region still overlaps with the values for the parameters of the “matched” Vogelsberger model. For the best-fit model M780Rs100

Figure 5.9 – Confidence intervals for the axisymmetric Vogelsberger model in $(\log_{10}(M_{1\text{kpc}}), R_s)$ (after fixing $c/a = 0.8$) for a kinematic dataset with 10^5 stars and 9×9 kinematic-bins. The $\Delta\chi^2 = [2.3, 6.18, 11.8]$ -contours are in white, grey and black respectively. The best-fit model is indicated by the white cross, while the expectations are given by the green contour (identical to that shown in Fig. 5.6). The mass parameter is well constrained and models with $R_s \leq 2.0$ kpc are strongly disfavoured, consistent with our expectations. The small panels on the right show the $\Delta\chi^2$ -landscapes when only considering $\chi^2_{2\text{D light}}$ (top), χ^2_{kin} (top-middle), $\chi^2_{3\text{D light}}$ (bottom-middle), or χ^2_{reg} (bottom).

we find $\chi^2_{\text{tot}} = 101.9$ and $\chi^2_{\text{kin}} = 67.2$, or 0.672 per kinematic constraint on average. We infer $R_s = 1.0^{+0.2}_{-0.0}$ kpc and $\log_{10}(M_{1\text{kpc}}[M_\odot]) = 7.80^{+0.02}_{-0.01}$ for the Vogelsberger parameters.

It is interesting to note that the shape of the confidence contours obtained from the Schwarzschild method for all sample sizes, follows very closely the shape of the contours of $\langle\Delta v\rangle$ depicted in Fig. 5.6. Recall that the quantity $\langle\Delta v\rangle$ is a proxy for the difference in enclosed mass between the Evans and Vogelsberger model. This implies that Schwarzschild’s method is actually very sensitive to enclosed mass, and it is identifying the set of Vogelsberger models that best follow the true underlying mass distribution. Also interesting is that the trend favouring larger values of the mass parameter when decreasing sample size, is present both for the Evans as well as for the Vogelsberger models.

Figure 5.10 – Similar to Fig. 5.9, but now after fitting mock kinematic data consisting of 10^4 stars and folding into 5×5 kinematic-bins. The decrease in sample size (by a factor 10) has led to a slight increase by the area spanned by the probability contours, although the inference on the mass parameter is still very good and only changed to slightly higher masses.

Figure 5.11 – As in Fig. 5.10, but now for a kinematic dataset with 2000 stars. Note how the confidence contours follow the shape of the green contour (derived in Fig. 5.6).

We compare the Evans and Vogelsberger best-fit models to the observed velocity dispersions in Fig. 5.12. The left and right panels compare the behaviour on the major and minor axes respectively, for different sample sizes for the kinematics: 10^5 , 10^4 and 2000 stars (in the top, middle and bottom rows respectively). The shaded areas enclose the minimum and maximum velocity dispersions for the evaluated models within the $\Delta\chi^2 = [2.3, 6.18, 11.8]$ -contours. These comparisons show that the Evans models fit the kinematics slightly better but that nearly equally good fits are provided by the Vogelsberger models (except in along the minor axis for the smallest dataset, bottom right panel).

From the analyses presented in this section we may thus conclude that the Schwarzschild modelling technique is sensitive to the mass enclosed and that it is successful in constraining well the mass parameter of the models, even if the functional form of the system's potential is not known.

5.5 Discussion and Conclusions

We explored the ability of the Schwarzschild's orbit superposition method to characterise the intrinsic properties of an axisymmetric dSph galaxy, such as its mass, scale radius and flattening. We did this by setting up an isothermal Sculptor-like mock galaxy that is flattened in both the luminous and dark components. We have shown that Schwarzschild's method applied to mock datasets with a realistic number of stars with measured radial velocities distributed following the luminosity profile of the system, is successful in recovering the characteristic mass parameter of the underlying (true) logarithmic potential, even if the potential flattening is not known. On the other hand, we find that we cannot put constraints on the flattening parameter.

Most likely, our inability to constrain the flattening is the consequence of our choice of the specific Evans model for our mock galaxy. In this model with a distribution function that is ergodic, the line-of-sight velocity profile is exactly the same everywhere and depends on the mass parameter only. This means that the kinematics are independent of the inclination and flattening, and the light alone does not contain enough information to constrain the flattening parameter.

One might also argue that it might not be optimal for a spectroscopic survey to sample stars according to the light profile of the system. In fact, slightly better results were obtained when the dataset with radial velocities

Figure 5.12 – Comparison of the results from the Schwarzschild modelling fits to the observed velocity dispersion along the major (left column) and minor (right column) axis. From top to bottom we show the best-fit Evans (red line) and Vogelsberger (blue line) models for kinematic datasets containing $N = 10^5$, $N = 10^4$, and 2000 stars, respectively. The shaded regions denote the error bands computed as described in the text. Black dotted lines indicate the input (theoretical, Eq. 5.4) velocity dispersions.

provided an equal number of stars to each kinematic-bin. All these factors, in combination with the fact that for our specific Evans model just $\sim 30\%$ of the system’s light is within our FOV, are likely playing a role. It might be possible however that better results could be obtained with a more realistic and general distribution function (i.e. non-ergodic), applied to a galaxy for which the kinematic tracers cover well the full system and sample more the outskirts.

Since in reality the functional form of the system’s potential is not known, we also explored the case in which we assume an axisymmetric NFW model. We first determined the values of the parameters of the “matched” NFW model that mimics the mock galaxy best by comparing some basic properties (potential flattening and gradients in the potential).

We found that even in this case, i.e. the orbits that form the building blocks of Schwarzschild’s method are integrated in the incorrect potential, we can retrieve the correct characteristic mass and scale parameters.

We have explored the dependencies of our results on the sizes of the data samples used, and find that a decrease in the number of stars with line-of-sight velocities, only slightly affects the determination of the characteristic parameters of the model. For the smallest kinematic sample considered, with 2000 stars, the inference on the mass of the “matched” NFW model is somewhat poorer but the true value differs by only 20% from the best-fit and also lies within the 95% confidence interval.

We have checked that our results are not strongly dependent on the choices of e.g. the number of orbits in the orbit libraries, number of kinematic- or light-bins, and the number of velocity bins. Furthermore we have also briefly investigated the distribution functions for the the best-fit models, and found that, particularly when regularisation is included, they are quite similar to the distribution function of the mock dwarf spheroidal galaxy.

In conclusion, it is promising that the mass of our flattened system can be recovered so well even if the flattening parameter is unknown. This is also aligned with the results of Kowalczyk et al. (2018), who applied their spherical Schwarzschild models on non-spherical objects. To some extent, this provides us with more confidence regarding previously reported estimates of the mass of dSph galaxies obtained assuming spherical symmetry.

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Appendix 5.A: Generating a mock kinematic dataset with realistic errors

Like in Breddels et al. (2013)⁹, we define v_i as the true line-of-sight velocity of star i and ϵ_i as the (true and unknown) measurement error on that star. Therefore $v_i + \epsilon_i$ is the observed velocity of star i . We note that the expectation values for the moments of the measurement errors, which are drawn from a Gaussian distribution with $\sigma = 2$ km/s, are given by: $E[\langle \epsilon_i^n \rangle] = E[\epsilon_i^n] = 0$ for odd n and $s_n \equiv E[\langle \epsilon_i^n \rangle] = E[\epsilon_i^n] = (n-1)!!\sigma^n$ for even n . In our terminology, $\hat{\mu}_n = E[\langle v_i^n \rangle] = E[v_i^n]$ denotes the true n^{th} moment, μ_n is its estimator and the observed n^{th} moment is $m_n = \frac{1}{N} \sum_{i=1}^N (v_i + \epsilon_i)^n$ for a sample of N stars in a given positional bin on the sky (i.e. kinematic-bin).

Since we want to know the true value of the moments, i.e. without measurement errors, we will compute the estimators of the true moments. We will also use raw moments (i.e. not taken about the mean velocities), and in what follows, we thus refer to ‘moments’ to denote ‘raw moments’. Since we can only in practise compute the estimators of the true moments, we replaced $\hat{\mu}_n$ with μ_n in the right-hand side of the following equations. The first four moment estimators are then given by:

$$\mu_1 = \frac{1}{N} \sum_{i=1}^N (v_i + \epsilon_i), \quad (5.14)$$

⁹With respect to Breddels et al. (2013), small corrections to the Equations are made. The updates concern Eqs. 5.17, 5.21, and 5.23.

$$\mu_2 = \frac{1}{N} \sum_{i=1}^N (v_i + \epsilon_i)^2 - s_2, \quad (5.15)$$

$$\mu_3 = \frac{1}{N} \sum_{i=1}^N (v_i + \epsilon_i)^3 - 3\mu_1 s_2, \quad (5.16)$$

and

$$\mu_4 = \frac{1}{N} \sum_{i=1}^N (v_i + \epsilon_i)^4 - 6\mu_2 s_2 - 3s_2^2. \quad (5.17)$$

To compute the error on these moments, we compute the square root of the variance of the moments; $Var(\hat{\mu}_n) \approx Var(\mu_n) \approx Var(m_n) = E[m_n^2] - (E[m_n])^2$:

$$\begin{aligned} Var(\mu_1) &= \frac{\mu_2 + s_2 - \mu_1^2}{N} \\ &= \frac{1}{N} \left\{ \frac{1}{N} \sum_{i=1}^N (v_i + \epsilon_i)^2 - \left[\frac{1}{N} \sum_{i=1}^N (v_i + \epsilon_i) \right]^2 \right\}, \end{aligned} \quad (5.18)$$

$$\begin{aligned} Var(\mu_2) &= \frac{1}{N} [\mu_4 - \mu_2^2 + 4\mu_2 s_2 + 2s_2^2] \\ &= \frac{1}{N} \left\{ \frac{1}{N} \sum_{i=1}^N (v_i + \epsilon_i)^4 - \left[\frac{1}{N} \sum_{i=1}^N (v_i + \epsilon_i)^2 \right]^2 \right\}, \end{aligned} \quad (5.19)$$

$$\begin{aligned} Var(\mu_3) &= \frac{1}{N} [\mu_6 + 15\mu_4 s_2 + 45\mu_2 s_2^2 + 15s_2^3 - \mu_3^2 \\ &\quad - 6\mu_3 \mu_1 s_2 - 9\mu_1^2 s_2^2] \\ &= \frac{1}{N} \left\{ \frac{1}{N} \sum_{i=1}^N (v_i + \epsilon_i)^6 - \left[\frac{1}{N} \sum_{i=1}^N (v_i + \epsilon_i)^3 \right]^2 \right\}, \end{aligned} \quad (5.20)$$

and

$$\begin{aligned} \text{Var}(\mu_4) &= \frac{1}{N} [\mu_8 + 28\mu_6 s_2 - \mu_4^2 - 12\mu_4 \mu_2 s_2 + 204\mu_4 s_2^2 \\ &\quad - 36\mu_2^2 s_2^2 + 384\mu_2 s_2^3 + 96s_2^4] \\ &= \frac{1}{N} \left\{ \frac{1}{N} \sum_{i=1}^N (v_i + \epsilon_i)^8 - \left[\frac{1}{N} \sum_{i=1}^N (v_i + \epsilon_i)^4 \right]^2 \right\}. \end{aligned} \quad (5.21)$$

where the errors on the third and fourth moment estimators also depend on:

$$\mu_6 = \frac{1}{N} \sum_{i=1}^N (v_i + \epsilon_i)^6 - 15\mu_4 s_2 - 45\mu_2 s_2^2 - 15s_2^3, \quad (5.22)$$

and

$$\mu_8 = \frac{1}{N} \sum_{i=1}^N (v_i + \epsilon_i)^8 - 28\mu_6 s_2 - 210\mu_4 s_2^2 - 420\mu_2 s_2^3 - 105s_2^4. \quad (5.23)$$

Obviously the errors on the moments decrease when the number of stars in a kinematic-bin increases.

Appendix 5.B: The effect of the sampling of line-of-sight velocities

In the main sections of this chapter we have drawn samples of line-of-sight velocities that follow the light distribution of the mock galaxy. Here we show the results of applying the Schwarzschild modelling technique to a kinematic dataset consisting of 10^5 stars, but this time distributed such that each kinematic-bin has an equal number of stars, and thus effectively increasing the relative importance of the kinematic bins near the edges of our FOV with respect to the most central kinematic bins. This is because the errors on the moments go down with increasing number of stars in a kinematic bin. In reality one might also consider probing larger radii.

Fig. 5.13 presents the inference on the mass and flattening parameter and should be compared to Fig. 5.2 and 5.3. As can be observed, we have very similar inferences with the best-fit flattening parameter slightly moved into the direction of its input/true value. Nonetheless, this remains fairly unconstrained.

Figure 5.13 – Left: The confidence intervals after fitting the Evans models to a new realization of the dataset containing 10^5 stars. This time we ensure an equal number of stars per kinematic-bin. The inference on the mass parameter remains the same. The flattening parameter remains unconstrained but has slightly shifted into the direction of the correct flattening parameter. Right: The velocity dispersion profile for the best-fit model q86v21.

Appendix 5.C: Regularisation

The solution of our minimisation problem may result in a distribution of orbital weights that is rapidly varying or shows sharp discontinuities. Such a distribution would not be physical. Therefore we make the distribution of the orbital weights smoother by adding extra terms to the χ^2 -fitting algorithm such that a new quantity $\widetilde{\chi^2_{\text{tot}}}$ is minimised:

$$\widetilde{\chi^2_{\text{tot}}} = \chi^2_{\text{tot}} + \chi^2_{\text{reg}}. \quad (5.24)$$

This procedure is called regularisation. The regularisation strength is chosen such that the orbital weights are forced to change smoothly from one neighbouring orbit to the next, while finding similar values for the best-fit characteristic parameters. In addition, the confidence contours should not be significantly shaped by the χ^2_{reg} -term. We refer the reader to van den Bosch et al. (2008) for more information about the exact implementation, in particular to Eqs. 28 and 29 of that paper. These equations require the 3-dimensional stellar density profile. For this chapter we assumed to know ρ_{lum} (see Eq. 5.2). In reality one needs the inclination angle to transform the observed surface brightness profile into the stellar density profile.

In the bottom panels of Fig. 5.14, we show the effect of adding regularisation for the Evans q80v20 model (i.e. this is the true model)

Figure 5.14 – Orbital distributions of angular momentum around the symmetry axis, i.e. L_z , for fixed energy slices corresponding to circular orbits at $x = 0.4$ (left), $x = 1.0$ (middle), and $x = 1.8$ kpc (right). Note the different axes ranges for the panels of each column. In the top row we show the distributions for the true Evans model q80v20 (blue) obtained after fitting a kinematic dataset of 10^5 stars (see Sect. 5.4.1) and for a realization of the mock galaxy (red, here containing 4×10^5 stars in total). The effect of adding regularisation to the fit is shown in the bottom panels. Adding regularisation makes the recovered distribution smoother and more similar to the true distribution.

on the distribution of angular momentum around the symmetry axis (L_z). The distributions can be compared to those of the q80v20 model without regularisation (top rows). We here show the example with 10^5 stars with line-of-sight velocities. The modelled distribution functions (blue) are generally smoother when regularisation is used. As a reference we also include the distributions for a realization of the mock galaxy (red). The model reproduces the mock distribution reasonably well, though some differences exist. The fact that only $\sim 30\%$ of the total number of stars of the mock galaxy end up in our FOV might play a role here, in addition to the fact that we have discretized the data (by using kinematic-bins, and by modelling only the first four velocity moments).

Nederlandse samenvatting

Introductie

Sterrenkunde en onze plaats in het heelal

Het heelal is onvoorstelbaar groot. Typische afstanden in het heelal worden daarom niet weergegeven in eenheden van meters of kilometers, maar in eenheden zoals lichtjaren, de afstand die een lichtstraal in één jaar tijd aflegt¹⁰. De snelheid van het licht is 299792 kilometer per seconde. En in plaats van kilogrammen worden aardmassa's of zonsmassa's gebruikt. De massa van de aarde is alleen al ongeveer 6 miljoen miljard miljard kilogram. De zon bevat naar schatting nog eens 333000 keer zoveel massa.

De gemiddelde afstand van de aarde tot de maan is 384000 kilometer. De gemiddelde afstand van de aarde tot de zon is ongeveer 400 keer zo groot. De zon zelf is een ster. Sterren stralen licht uit, een vorm van energie, vanwege het feit dat het eigenlijk grote kernreactoren zijn. In de kern van een ster zoals onze zon bedraagt de temperatuur meer dan 15 miljoen graden Celsius, zo heet dat lichte atoomkernen fuseren¹¹. Bij dit proces komt energie vrij in de vorm van licht.

¹⁰In dit proefschrift wordt veelal gebruikt gemaakt van de afstands-eenheid kilo-parsec (kpc): 1 kpc staat gelijk aan ongeveer 3263 lichtjaar.

¹¹Dit in tegenstelling tot de huidige door mensen gemaakte kernreactoren waarin zware atoomkernen worden gespleten, al zijn er experimenten gaande om ook kernfusie rendabel te maken.

Figuur 5.15 – Een impressie van hoe de Melkweg, een balkspiraalsterrenstelsel, er ongeveer uitziet. Gemaakt door NASA.

De zon is slechts één van de naar schatting honderden miljarden sterren uit ons sterrenstelsel, de Melkweg (zie figuur 5.15). De zon bevindt zich op ongeveer 27000 lichtjaar van het centrum van de Melkweg en draait hier in ongeveer 230 miljoen jaar omheen. Dit komt neer op een snelheid van ongeveer 230 km/s. De meeste sterren uit de Melkweg bevinden zich net als de zon in een schijf en roteren in dezelfde richting rondom het centrum van de Melkweg. De schijf zelf bestaat uit een dunne en een dikke component en heeft verhoogde sterdichtheden in de vorm van een spiraalstructuur. Andere sterren bevinden zich in een centrale verdikking van onze Melkweg, andere in een centrale balkvormige structuur. De overige naar schatting 1% van de sterren bevinden zich in een bolvormige structuur (ook wel ‘halo’ genoemd) welke qua omvang groter is dan de schijf. Sterren in de halo bewegen veel minder geordend en bewegen in verschillende richtingen, hoewel ze gemiddeld genomen een klein beetje tegengesteld bewegen aan de sterren die deel uitmaken van de schijf. In de halo bevinden zich ook ongeveer 150 compacte sterclusters. Rondom de Melkweg draaien vervolgens weer satellietstelsels, zoals bolvormige dwergsterrenstelsels.

Het heelal is echter nog veel groter dan ons eigen sterrenstelsel. Het dichtstbijzijnde volgende sterrenstelsel is het Andromeda sterrenstelsel op een afstand van meer dan twee miljoen lichtjaar. Net als de Melkweg heeft ook Andromeda een groot aantal satellietstelsels die om haar heen draaien. Samen met nog andere sterrenstelsels vormen de Melkweg en Andromeda een groep sterrenstelsels, genaamd de Lokale Groep. De Lokale

Groep maakt weer deel uit van een groter geheel, het Virgo supercluster, dat vervolgens ook weer deel uitmaakt van een nog grotere structuur. Astronomen schatten dat het aantal sterrenstelsels in het zichtbare heelal meer dan honderd miljard bedraagt. Er wordt daarom ook wel gezegd dat er meer sterren in het heelal zijn dan zandkorrels op aarde. En om elk van deze sterren zouden ook weer één of meerdere planeten kunnen draaien.

Modellen van het heelal

Aan de hand van waarnemingen met telescopen kunnen astronomen modellen van het heelal maken. Volgens het huidige standaard model begon ons heelal met een Oerknal (in het Engels: Big Bang). Alle materie en energie was opeengepakt in een zeer kleine en hete bol die ongeveer 13.7 miljard jaar geleden uit zijn voegen spatte. Een enorm snelle expansie van het heelal volgde. Door de uitdijng koelde het heelal langzaam af zodat geladen atomaire deeltjes konden combineren tot voornamelijk waterstof en helium. Uit deze gassen vormden vervolgens de eerste sterren. Groepen sterren vormden vervolgens sterrenstelsels. De onderlinge zwaartekracht heeft ervoor gezorgd dat veel sterrenstelsels in de loop der tijd zijn samengesmolten tot grotere sterrenstelsels, een proces dat vandaag de dag nog steeds plaatsvindt. Andromeda en de Melkweg zullen bijvoorbeeld over iets meer dan vier miljard jaar met elkaar botsen. Door deze interacties krijgen sterrenstelsels steeds meer massa en kunnen ze van vorm veranderen.

Tegelijkertijd hebben individuele sterren niet het eeuwige leven. Als de ‘brandstof’ voor kernfusie geleidelijk aan opraakt zwellen lichte sterren op waarbij ze een deel van hun buitenste lagen verliezen. Zware sterren imploderen waarna de buitenste lagen met hoge snelheid terug het heelal in worden geslingerd in een zogenaamde supernova-explosie. Van de restanten sterrenstof worden vervolgens weer nieuwe sterren, met eventueel planeten, gevormd. Omdat deze sterrenstof atomen bevat die in vorige generaties sterren zijn gevormd, zullen de nieuwe generatie sterren een iets andere chemische samenstelling krijgen. Over het algemeen zullen de latere generatie sterren steeds meer elementen zwaarder dan waterstof en helium bevatten dan de eerste generatie sterren.

Donkere materie

De zwaartekracht bepaalt voor een groot deel in welke mate sterrenstelsels van verschillende soorten groottes ontstaan, in welke mate deze naar elkaar toegetrokken worden en dus in welke mate botsingen van sterrenstelsels voorkomen. Zwaartekracht heeft daarnaast ook een grote invloed op de banen die individuele sterren in een sterrenstelsel afleggen. Een voorbeeld: voor een cirkelbaan in een sterrenstelsel met veel massa, moet een ster een hogere baansnelheid hebben dan voor eenzelfde cirkelbaan in een sterrenstelsel met aanzienlijk minder massa. Met andere woorden: als de sterrenstelsels uit het heelal nauwkeurig in kaart kunnen worden gebracht en precieze baansnelheden van sterren kunnen worden bepaald, is het mogelijk te bepalen in welke mate zwaartekracht invloed heeft op de geanalyseerde sterrenstelsels.

Veel sterrenstelsels in het heelal vertonen stersnelheden die veel groter zijn dan verwacht op grond van de hoeveelheid waargenomen massa in het sterrenstelsel (bijvoorbeeld de hoeveelheid sterren en de hoeveelheid gas tussen de sterren). Zulke snelheden kunnen worden verklaard als een deel van de daadwerkelijke massa in een sterrenstelsel onzichtbaar is. Deze onzichtbare massa wordt ook wel donkere materie genoemd. Tot op de dag van vandaag is het echter nog een raadsel wat deze donkere materie nou precies is. Als alternatief voor donkere materie is het ook mogelijk dat de zwaartekrachtswetten van Newton niet helemaal juist zijn. Het is één van de grootste onopgeloste vraagstukken binnen de sterrenkunde.

De Melkweg als testcase

Om betere modellen van het heelal te kunnen maken en om het proces van de vorming en evolutie van sterrenstelsels beter te begrijpen, kan ons eigen sterrenstelsel als testcase worden gebruikt. De Melkweg is namelijk een tamelijk gemiddeld sterrenstelsel. Bovendien kunnen waarnemingen van ons eigen sterrenstelsel modellen van het heelal verbeteren. Deze modellen moeten namelijk in staat zijn om de eigenschappen van de Melkweg te reproduceren. Het voordeel van het bestuderen van ons eigen sterrenstelsel is dat het veel makkelijker is om nauwkeurige posities en snelheden van individuele sterren waar te nemen dan voor andere, verder weg gelegen, sterrenstelsels.

Figuur 5.16 – De *Gaia* ruimtetelescoop, gelanceerd op 19 december 2013. *Gaia* meet de posities en snelheden van meer dan een miljard sterren uit ons sterrenstelsel, de Melkweg, met ongekende nauwkeurigheid. Gemaakt door ESA/ATG medialab.

De *Gaia* ruimtetelescoop

In december 2013 is de *Gaia* ruimtetelescoop gelanceerd (zie figuur 5.16). Deze telescoop neemt van ongeveer 1% van de sterren van ons sterrenstelsel nauwkeuriger dan ooit tevoren de posities en snelheden waar. *Gaia* neemt de sterren herhaaldelijk waar over een periode van meerdere jaren. Ze vergelijkt de onderlinge posities van de sterren met elkaar om zo drie-dimensionale posities (afstand en positie aan de hemel) en twee-dimensionale snelheden (bewegingen aan de hemel) af te kunnen leiden. Voor een deel van de sterren, de meest heldere sterren, bepaalt *Gaia* ook de snelheden in de radiële richting (van ons af of naar ons toe). *Gaia* geeft de resultaten vrij in verschillende ‘data releases’.

Op 14 september 2016 was de eerste data release en op 25 april 2018 volgde de tweede data release. Deze twee data releases zijn gebruikt in dit proefschrift. De komende jaren zullen volgende data releases een nog nauwkeuriger beeld van ons sterrenstelsel geven.

Dit proefschrift

In dit proefschrift zijn de gegevens van de *Gaia* ruimtetelescoop geanalyseerd met als doel om meer te weten te komen over de massaverdeling van de Melkweg en haar satellietstelsels.

In hoofdstuk 1 wordt het proefschrift geïntroduceerd aan de hand van een wetenschappelijke inleiding. Er wordt teruggeblikt op wat astronomen allemaal al wisten over de Melkweg voordat de gegevens van *Gaia* werden vrijgegeven. Daarnaast wordt een kort overzicht van de resultaten sinds de eerste twee data releases van *Gaia* gegeven.

In hoofdstuk 2 wordt de eerste data release van *Gaia* gebruikt. In deze data release zijn de posities en snelheden van ongeveer twee miljoen sterren bepaald. Echter, omdat *Gaia* nog niet lang genoeg waarnemingen had gedaan, moest ook gebruikt worden gemaakt van gegevens van de voorganger van *Gaia*, de Hipparcos ruimtetelescoop, welke van 1989 tot 1993 data verzamelde. Radiële snelheden waren überhaupt nog niet gemeten door *Gaia*. Om toch de beschikking over alle snelheidscomponenten te hebben, zijn de gegevens gecombineerd met die van andere onderzoeken die wel radiële snelheden hebben gemeten aan de hand van telescopen op de grond. Vervolgens is de dichtheid van de donkere materie bepaald ter hoogte van de positie van de zon. Hiervoor was het nodig de bewegingen van de sterren tot op duizenden lichtjaren afstand zo nauwkeurig mogelijk te bepalen. Omdat de gemeten afstanden tot de sterren nog steeds relatief grote foutmarges bevatten, is besloten een selectie te maken van een specifiek soort sterren waarvan de intrinsieke helderheid nauwkeurig bekend is. Aan de hand van de hoeveelheid flux ontvangen op aarde kan zo een nauwkeurige afstandsbepalings worden gedaan. De hoeveelheid ontvangen flux neemt namelijk (kwadratisch) af naarmate de afstand tot een object van bekende intrinsieke helderheid groter wordt.

De resultaten bevestigen het vermoeden van de aanwezigheid van donkere materie. Vervolgens is een schatting van de hoeveelheid donkere materie gepresenteerd. De statistische onzekerheden, gebaseerd op de gemeten posities en snelheden van de sterren, in deze schatting zijn klein. Echter, de aannames gemaakt om tot deze schatting te komen hebben een groot effect op de uitkomst. Als de typische dikte van de schijf van onze Melkweg 10% verkeerd is ingeschat, verandert onze schatting van de dichtheid van de donkere materie met zeker 30%. Deze systematische foutmarges zijn dus veel groter dan de statistische foutmarges. Het is echter hoogst onwaarschijnlijk dat de systematische fouten zo groot zijn dat er geen donkere materie nodig is om de snelheden van de sterren te kunnen verklaren.

In hoofdstuk 3 wordt de tweede data release van *Gaia* gebruikt. Hierin zijn van meer dan 7 miljoen sterren alle componenten gemeten die nodig voor een nauwkeurige positie- en snelheidsbepaling en dit keer geheel onafhankelijk van andere onderzoeken. Hierdoor is het mogelijk om voor het eerst van een groot deel van de Melkweg in kaart te brengen hoe sterren bewegen. In het bijzonder is het verband tussen de naar binnen en naar buiten gaande snelheidscomponent in vergelijking met de snelheidscomponent loodrecht op het vlak van de schijf van de Melkweg onderzocht. Voorgaand onderzoek heeft namelijk uitgewezen dat correlaties tussen deze snelheden in verband kunnen worden gebracht met de globale massaverdeling van ons sterrenstelsel. Hierdoor is het mogelijk te bepalen of de donkere materie in de Melkweg bol-symmetrisch is verdeeld of niet.

De resultaten laten zien dat de correlaties niet overeenkomen met een bol-symmetrische massaverdeling. De massaverdeling lijkt te veranderen als functie van afstand vanaf het centrum van de Melkweg. In een uitgebreide discussie wordt het effect van verkeerde afstanden op de gevonden correlaties geanalyseerd. Het is namelijk bekend dat de door *Gaia* gemeten afstanden tot de sterren een systematische fout bevatten. Hoe groot deze fouten voor individuele sterren zijn is niet precies bekend: deze lijken af te hangen van de locatie van de sterren, hun helderheid en het soort sterren. Het effect van het overschatten van de afstanden tot de sterren wordt gepresenteerd. De correlaties kunnen wel consistent zijn met bol-symmetrie als grotere correcties voor de afstands-bepaling worden uitgevoerd. De afstands-bepalingen zijn dan ook van groot belang voor het nauwkeurig bepalen van de correlaties tussen de snelheidscomponenten. Toekomstige *Gaia* data releases zullen nauwkeuriger afstands-bepalingen opleveren en uitsluitsel geven over hoe de correlaties veranderen als functie van positie in de Melkweg. Aan de hand hiervan kunnen betere uitspraken gedaan worden over de manier waarop de massa in de Melkweg verdeeld is.

Ook in hoofdstuk 4 worden de gegevens van de tweede data release van *Gaia* gebruikt. Dit keer worden de banen van de sterren uit de dikke schijf en de halo van de Melkweg onderzocht. Door de chemische samenstelling en de banen van sterren te onderzoeken zijn eerder al bewijzen gevonden voor satellietstelsels die zijn opgeslokt door de Melkweg. Sterren van dezelfde oorsprong hebben namelijk een soortgelijke chemische samenstelling en volgen banen die niet veel van elkaar afwijken. In de loop der tijd hebben de sterren zich echter over een groot volume verspreid waardoor ze niet meer als een groep aan de hemel geclusterd zijn.

Een manier om te bekijken of sterren soortgelijke banen afleggen is het bestuderen van de minimale en maximale afstand van de banen ten opzichte van het centrum van het sterrenstelsel. De maximale afstand loodrecht ten opzichte van de schijf van de Melkweg is een andere grootte die vergeleken kan worden. Ander onderzoek heeft uitgewezen dat er structuren te zien zijn wanneer deze baaneigenschappen tegen elkaar worden uitgezet. Het is gesuggereerd dat dit te maken zou kunnen hebben met opgeslokte sterrenstelsels. In dit hoofdstuk wordt aangetoond dat dit niet altijd het geval hoeft te zijn. De onderzochte grove structuren kunnen het directe gevolg van de massaverdeling van ons sterrenstelsel zijn. Sommige banen zijn namelijk onstabiel of ook wel chaotisch. Sterren die zich op of nabij zulke banen bevinden zullen zich daarvandaan bewegen. Ook zijn er specifieke banen waar sterren naartoe zullen migreren. Dit is typisch voor een sterrenstelsel met een schijf en een halo. Wanneer baaneigenschappen van sterren worden vergeleken zijn dus niet alle structuren per se indicatoren voor opgeslokte sterrenstelsels.

In het laatste wetenschappelijke hoofdstuk, hoofdstuk 5, wordt een bepaald type satellietstelsels van de Melkweg onderzocht: de bolvormige dwergsterrenstelsels. De bewegingen van sterren uit deze sterrenstelsels duiden op een erg grote hoeveelheid donkere materie. Het is daarom uiterst interessant om te bepalen hoeveel donkere materie deze systemen precies bevatten en hoe de massa van deze donkere materie verdeeld is.

Tot nu toe hebben de meeste onderzoekers aangenomen dat deze sterrenstelsels bolvormig zijn. Echter, observaties hebben aangetoond dat de verdeling van de sterren in sommige van deze sterrenstelsels in meer of mindere mate afgeplat is. Met andere woorden: de massa is misschien ovaalvormig verdeeld en niet bolvormig. In dit hoofdstuk wordt onderzocht hoe nauwkeurig de massa van zulke systemen kan worden bepaald en ook of het mogelijk is om te bepalen in welke vorm de massa verdeeld is.

Door middel van een testcase wordt aangetoond dat de massa van zo'n sterrenstelsel nauwkeurig kan worden bepaald, ook als de aangenomen vorm van het sterrenstelsel niet juist is. De vorm zelf, waarin de massa is verdeeld, kan echter niet worden bepaald. In de discussie worden enkele mogelijke oorzaken genoemd waardoor de vorm niet kon worden bepaald in onze testcase. Meer onderzoek naar realistischere testcases zullen nodig zijn om definitief uitspraak te kunnen doen over de massaverdeling van bolvormige dwergsterrenstelsels.

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¹²South African for 'congrats', though in Dutch it would be interpreted as 'lucky you'.

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