Distributed coordination and partial synchronization in complex networks

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Summary

Coordinating behaviors in populations of interacting units have been widely observed in many natural systems. Many attempts have been made to understand these behaviors, which have also inspired a lot of applications. This thesis is devoted to: 1) the study of distributed coordination algorithms, which is one of those applications, and 2) the investigation on the underlying mechanisms of a particular type of coordinating behaviors, i.e., partial synchronization.

Part I of this thesis focuses on the study of distributed coordination algorithms. When implementing distributed coordination algorithms, the computational processes are inevitably influenced by some random factors, which can be random changes in network structures or stochastic communication delays. Besides, some randomness may also be introduced deliberately to improve global performance. Taking the randomness into account, distributed coordination algorithms can be modeled by stochastic systems. However, traditional methods cannot be directly used for stochastic stability analysis of these systems in many circumstances. There is a great need for developing new results to study the stability of stochastic systems. This is exactly the aim of Chapter 3. In Chapter 3, we develop a new Lyapunov criterion, termed the finite-step Lyapunov criterion, for discrete-time stochastic systems. From the existing Lyapunov criterion, a constructed Lyapunov function needs to decrease at every time step to ensure stability. In sharp contrast, we relax this requirement by allowing it to decrease after some finite time steps. This relaxation provides a larger range of choices when constructing a Lyapunov function. In Chapter 4, we then show how the obtained Lyapunov criteria can be utilized to solve the problems we encountered in dealing with several distributed coordination algorithms including 1) convergence of products of random sequences of stochastic matrices, 2) asynchronous-updating-induced agreement of agents coupled by periodic networks, and 3) algebraic equation solving via distributed averaging algorithms in a randomly changing network.

Global synchronization across the entire brain is always a sign of certain brain diseases, while partial synchronization usually takes place in the healthy brain. This motivates us to study partial synchronization in Part II, trying to uncover the

underlying mechanisms that could give rise to this special type of coordinating behavior. Towards this end, we employ the Kuramoto model and its variation, i.e., Kuramoto-Sakaguchi model, to describe the dynamics of oscillators. Two classes of partial synchronization are studied in this part: 1) synchronization among a set of oscillators that have direct connections; 2) synchronization among a set of oscillators that have no direct link, termed *remote synchronization*. We have studied the first class in Chapter 5. Inspired by the organization of cortical neurons in the brain, a two-level network structure is considered. The oscillators are all-to-all connected, forming local communities at the lower level; at the higher level, the communities are interconnected by a sparse network. We show that strong coupling strengths among the set of directly connected oscillators can lead to partial synchronization. Remote synchronization in star networks is investigated in Chapters 6 and 7. To prove the stability of remote synchronization, one often needs to show the partial stability of a nonlinear system. However, existing criteria for partial stability are not directly applicable in our case, which motivates us to develop some new criteria for partial stability analysis of nonlinear systems in Chapter 6. We first prove that a constructed Lyapunov function does not need to have a negative-definite time derivative. If it decreases after a finite time, asymptotic (or exponential) stability can be ensured. We then show that the exponential stability of a class of slow-fast systems can be studied by analyzing the averaged systems obtained by periodic averaging. In Chapter 7, we first consider directed star networks and show the important role that the symmetries of the connections going out from the central oscillator play in rendering remote synchronization among peripheral oscillators. Finally, we focus on an undirected star network of two peripheral oscillators. Using the new criteria developed in Chapter 6, we prove that the natural frequency detuning of the central oscillator can actually enhance remote synchronization.