Distributed coordination and partial synchronization in complex networks

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Conclusion

In this thesis, we have studied distributed coordination algorithms and partial synchronization in complex networks in Part I and Part II, respectively.

In Part I, we focus on distributed coordination algorithms in stochastic settings. Inspired by coordinating behaviors observed in nature, distributed coordination algorithms serve as a foundation for a number of network algorithms for various purposes such as information fusion, load balancing, placement of mobile sensors, etc. When implementing those network algorithms, the stochastic influence such as random changes of network structures and communication delays and noise cannot be ignored. Besides, randomness is sometimes also deliberately introduced to improve global performance. Such stochastic network algorithms can be modeled by stochastic systems, which are often analyzed by stochastic Lyapunov theory. However, existing Lyapunov criteria are often not directly applicable especially when the stochastic processes are not confined to some certain ones such as i.i.d. Motivated by this, we have developed some new Lyapunov criteria for discrete-time stochastic systems in Chapter 3. In contrast to the existing Lyapunov theory for discrete-time stochastic systems, a constructed Lyapunov function is not required to decrease after every time step anymore. Instead, stability can be guaranteed if it decreases after finite time steps. We then use them in Chapter 4 to study the following distributed coordination algorithms: 1) the products of random stochastic matrices, 2) asynchrony-induced agreement problems in periodic networks, and 3) distributed linear equation solving. Sharper results compared to those in the literature have been obtained.

In Part II, we have uncovered some possible underlying mechanisms that could give rise to partial synchronization in complex networks. We have investigated in Chapter 5 how partial synchronization can take place among directly connected regions. We have found that strong regional (or local) connection is a possible mechanism. If some oscillators in a network are connected tightly, they can evolve in unison, while the rest that are weakly coupled remain incoherence. The Lyapunov functions based on the incremental 2-norm and the incremental ∞-norm are used to construct the proof. In addition, we have studied how partial synchronization is possible to occur among units that have no direct connections, an interesting phenomenon termed remote synchronization. This phenomenon has also been widely detected in the human brain, where distant cortical regions without direct neural links also experience functional correlations. In order to study remote synchronization, we have developed some new criteria for partial stability of nonlinear systems in Chapter 6. These criteria enable us to study the partial stability of some nonlinear systems that are not easy to analyze using existing results. Then, we study remote synchronization in simple network structures, i.e., star networks, in Chapter 7. We have found that the symmetry of outgoing connections from the central oscillator is crucial to shaping remote synchronization, and is possible to induce several clusters for the peripheral oscillators. We have further investigated how detuning the natural frequency of the central oscillator in a star network with two peripheral nodes can strengthen remote synchronization. Finally, we use the obtained Lyapunov criteria on partial stability to prove that natural frequency detuning of the central oscillator actually makes the remote synchronization more robust against the phase shifts.

Outlook

The Lyapunov theory plays a fundamental role in the control field. We are interested in further developing control Lyapunov criteria for both discrete-time and continuous-time stochastic systems since they can be applied to many practical problems including distributed optimization.

As it has been observed in the human brain, partial synchronization is perhaps more common than global synchronization. It is certainly more interesting to study partial synchronization further since it would help us to better understand the sophisticated mechanisms behind the synchrony patterns in the brain. Particularly, we are even more interested in studying remote synchronization in more complex networks than star networks. We believe that network symmetries would still play a crucial role in rendering remote synchronization. Before considering general networks, We plan to start with some simpler ones such as line works and bipartite networks. Instead of considering an identical phase shift for all oscillators, we plan to study the case when phase shifts are heterogeneous. Phase shifts are usually used to model small time delays in the Kuramoto model. When the delays are large, the Kuramoto-Sakaguchi model is not accurate anymore. In this case, it is better to employ a time-delayed Kuramoto model and investigate the role that delays play in remote synchronization.

The analysis will be quite challenging. The partial stability theory can be very helpful for the analysis. It is quite interesting to further develop partial stability theory and applied it to the study of remote synchronization. We also plan to test our theoretical findings via experiments of the brain by cooperating with neuroscientists.

We believe theoretical study using appropriate mathematical models of the brain will be very important to explain and predict brain behaviors, and also contribute to the treatment of various brain diseases in the future.