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Distributed coordination and partial synchronization in complex networks

Qin, Yuzhen

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Introduction

Coordinating behaviors in large groups of interacting units are pervasive in nature. Remarkable examples include fish schooling [1], avian flocking [2], land animals herding [3], rhythmic firefly flashing [4], and synchronized neuronal spiking [5]. Extensive efforts have been made to uncover the mechanisms behind these astonishing coordinating behaviors. There have been major progresses, and many of them have also been applied to solving various problems in engineering practice. For example, distributed weighted averaging has found applications in distributed computation in robotic networks. On the other hand, the mechanisms of many coordinating behaviors remain unknown. For example, what gives rise to a variety of synchronization patterns in the human brain is still an intriguing question. In this thesis, we first study distributed coordination algorithms in stochastic settings. We then investigate partial instead of global synchronization in complex networks, trying to reveal some possible mechanisms that could render correlations across only a part of brain regions as indicated by empirical data. In this chapter, we introduce some background knowledge of distributed coordination algorithms as well as synchronization, provide a sketch of the main contributions, and explain how this thesis is structured. Some notations used throughout the thesis are also presented.

1.1 Background

In the next two subsections, we introduce some background information of distributed coordination algorithms and synchronization, respectively.

1.1.1 Distributed Coordination Algorithms

A huge number of models have been proposed to describe coordinating behaviors in a network of autonomous agents. The DeGroot model and the Vicsek model are two of the most popular models. Introduced in 1975, the DeGroot model describes how a group of people might reach an agreement by pooling their individual opinions [6]. Proposed in 1995, the Vicsek model is used to investigate the emergence of self-organized motion in systems of particles [7]. These two models have fascinated a lot of researchers in different fields because they are very simple yet revealing, and they are capable of explaining rich collective behaviors in nature. They have also inspired the development of distributed coordination algorithms in multi-agent systems. There are two key features of distributed coordination algorithms that are inherited from the Vicsek model and the DeGroot model: 1) each agent simply needs to compute the weighted average of the states of itself and its neighbors; and 2) only local information is required for computation of the weighted averages, and thus the distributed coordination algorithms are also known as the distributed weighted averaging algorithms.

Distributed coordination algorithms in complex networks have attracted much interest in the recent two decades. Just like the Vicsek's model, each agent's nearest neighbors in distributed coordination algorithms can change with time. To study this, early works have considered dynamically changing networks and provided some connectivity conditions for convergence [8–12]. Moreover, agents may not have a common clock to synchronize their update actions in practice. Thus, asynchronous events have also been taken into account, and conditions have been obtained such that the convergence can be preserved [10, 13, 14]. Distributed coordination algorithms actually serve as a foundation for a considerable number of network algorithms for various purposes such as load balancing [15, 16], information fusion [17, 18], rendezvous of robots [19, 20], placement of mobile sensors [21, 22], formation control [23, 24]. More recently, distributed coordination algorithms have also been used for many other research topics including distributed optimization [25, 26], distributed observer design [27, 28], solving linear equations distributively [29, 30], and modeling of opinion dynamics in social networks [31–33].

Most of the aforementioned studies on distributed coordination algorithms and their applications are in deterministic settings. However, in many circumstances, the implementation of distributed coordination algorithms is often under the influence of uncertainty in the environment. Some further works have shown that the convergence can still be guaranteed even with the presence of randomly changing network topologies [34–36], random network weights [37], random communication delays [38–40], and random asynchronous events [41, 42]. Much less attention has been paid to the

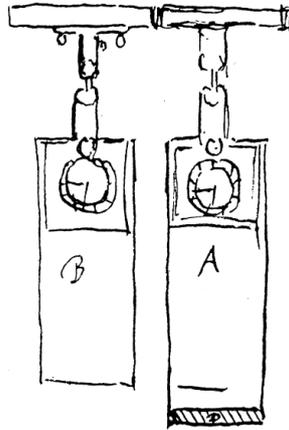


Figure 1.1: Original drawing of Christiaan Huygens: two pendulum clocks hanging side by side on a beam (source: [46])

investigation of how the presence of some randomness can be helpful for coordination in a network. Surprisingly, random noise, usually believed to be troublesome, sometimes brings benefits to a system in terms of achieving better system-level performance. For example, the survivability of a group of fish can be boosted by random schooling [43]; random deviation can enhance cooperation in social dilemmas [44]; and behavioral randomness can improve the global performance of human in a coordination game [45]. There is a great need in systematically studying stochastic distributed algorithms, which enables the analysis of coordination in networks under the influence of both detrimental and beneficial randomness.

1.1.2 Synchronization and Brain Communication

In February 1665, staring aimlessly at two pendulum clocks hanging side by side on a wooden structure (shown in Fig. 1.1), Christiaan Huygens suddenly noticed they began to swing perfectly in step. More unexpectedly, he found that they seemed to never break step. This renowned Dutch physicist, mathematician and astronomer described this surprising discovery by “*an odd sympathy*”. After more than 350 years, the interesting phenomenon is nowadays termed *synchronization*.

As another form of coordinating behaviors, synchronization has attracted attention from scientists in various disciplines due to its ubiquitous occurrence in many natural, engineering, and social systems. The snowy tree crickets are found to be able to synchronize their chirping [47]; rhythmic hand clapping often appears after theater

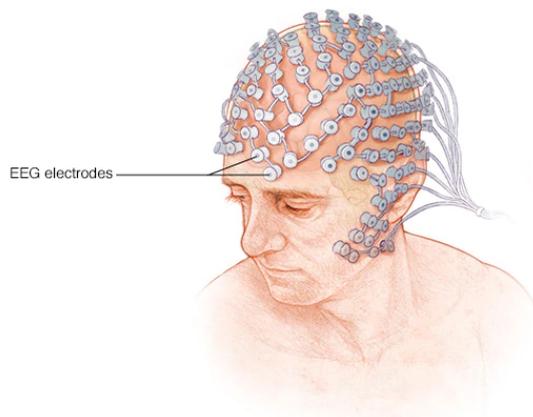


Figure 1.2: In illustration of how EEG records brain waves (source: <https://www.mayoclinic.org/tests-procedures/eeg/about/pac-20393875>)

and opera performances [48]; power generators operate synchronously to function properly [49]; and circadian rhythms of almost all land animals are often in accordance with the environment [50] (e.g., sleep and wakefulness are closely related to daily cycles of daylight and darkness).

Synchronization has also been detected pervasively in neuronal systems [51–53]. It plays a central role in information processing within a brain region and neuronal communication between different regions. Investigation on synchronization of neuronal ensembles in the brain, especially in cortical regions, becomes one of the most important problems in neuroscience. The *electroencephalogram* (EEG) is a typical method to measure brain activities, and is essential to experimentally study synchronization of the cerebral cortex. Measuring brain waves using EEG is quite simple since it is noninvasive and painless. Fig. 1.2 provides an illustration of how EEG is used to record brain signals. Several early experiments indicate that synchronization of neuron spikes in the visual cortex of animals accounts for different visual stimulus features [5, 53]. Inter-regional spike synchronization is shown to have a functional role in the coordination of attentional signals across brain areas [54, 55]. Recently, it has been shown that phase synchronization contributes mechanistically to attention [56], cognitive tasks [57], working memory [58], and particularly interregional communication [52, 59].

In fact, synchronization across brain regions is believed to facilitate interregional communication. Only cohesively oscillating neuronal groups can exchange information effectively because their communication windows are open at the same time [52]. However, abnormal synchronization in the human brain is always a sign of pathology [60, 61]. As an example, Fig. 1.3 presents the EEG recording of brain waves during

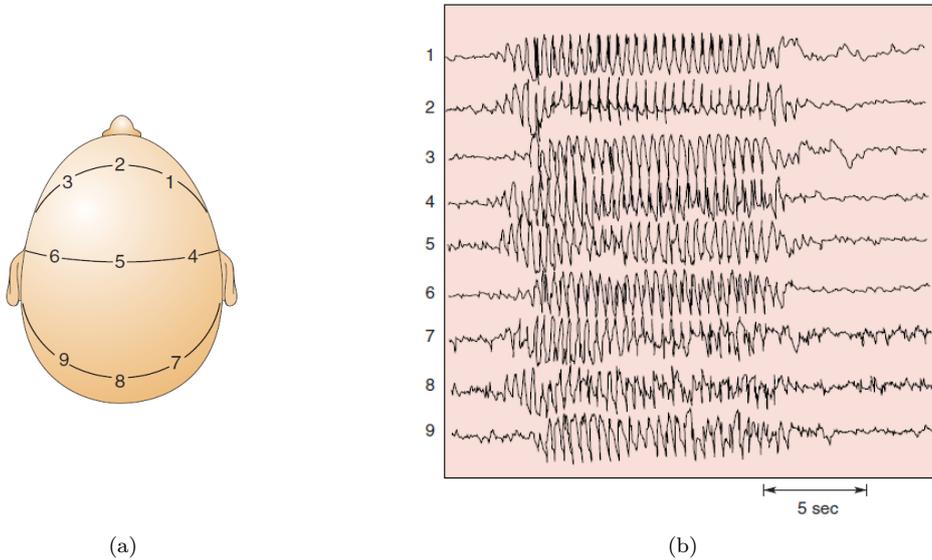


Figure 1.3: A EEG recording of an epileptic seizure (source: [50, Fig. 19.14]): (a) positions on the scalp where EEG electrodes are placed; (b) the EEG signals recorded by the electrodes.

an epileptic seizure, where synchronization across the entire brain is observed. Such strikingly abnormal behavior is never detected in a healthy brain. This suggests that there are some robust and powerful regulation mechanisms in a non-pathological brain that are able to not only facilitate but also preclude neuronal communication. Partial synchronization is believed to be such a mechanism [52]. Only necessary parts of regions are synchronized for some specific brain function. Communication between incoherent brain regions is prevented. In this case, information exchange between two neuronal groups is not possible because their communication windows are not coordinated. Synchronizing a selective set of brain regions can render and also prevent neuronal communication in a selective way.

When it comes to the study of synchronization, the Kuramoto model serves as a powerful tool. After it was first proposed in 1975 [62], the Kuramoto model rapidly became one of the most widely-accepted models in understanding synchronization phenomena in a large population of oscillators. It is simple enough for mathematical analysis, yet still capable of capturing rich sets of behaviors. Thus, it has been extended to many variations [63]. The Kuramoto model and its generalizations are also widely used to model the dynamics of coupled neuronal ensembles in the human

brain. It is of great interest to analytically study partial synchronization with the help of the Kuramoto model and its variations, trying to reveal the possible underlying mechanisms that can give rise to different synchrony patterns in the human brain.

1.2 Contributions

In the first part of this thesis, we restrict our attention to distributed coordination algorithms in stochastic settings since the implementation of them is often under random influences and the introduction of some randomness sometimes can be beneficial.

Study of stochastic distributed coordinate algorithms is often associated with stability analysis of stochastic discrete-time systems. There are some noticeable Lyapunov theories on stability of stochastic systems including Khasminskii's book [64], and Kushner's works [65–67]. Particularly in [66, 67], the expectation of a Lyapunov function is required to decrease after every time step, in order to show the stability of a stochastic discrete-time system. However, it is not always easy to construct such a Lyapunov function. Therefore, we propose some new Lyapunov criteria for asymptotic and exponential stability analysis of stochastic discrete-time systems. We allow the expectation of Lyapunov function candidates to decrease after some finite steps instead of every step. This relaxation enlarges the range of applicable Lyapunov functions and also provides us with the possibility of working on systems with non-Markovian states.

Using these new Lyapunov criteria, we then study the convergence of products of random stochastic matrices. While implementing distributed coordinate algorithms, one always encounters the need to prove the convergence of products of stochastic matrices, or equivalently the convergence of inhomogeneous Markov chains. The study of products of stochastic matrices dates back to more than 50 years ago in Wolfowitz's paper [68]. Since then, a lot of progress has been made [69–73], and many applications have been implemented [8–11, 74]. Recent years have witnessed an increasing interest in studying products of random sequences of stochastic matrices [35, 75, 76]. Nevertheless, most of the existing results rely on the assumption that each matrix in a sequence has strictly positive diagonal entries. Without this assumption, many existing results do not hold anymore. Moreover, the underlying random processes driving the random sequences are usually confined to some special types, such as independent and identically distributed (i.i.d) sequences [35], stationary ergodic sequences [36], or independent sequences [75, 76]. The new Lyapunov criteria we obtained enable us to work on more general classes of random sequences of stochastic matrices without the assumption of nonzero diagonal entries. We obtain some quite mild conditions compared to the existing results on random sequences of stochastic matrices such that convergence of the products can be guaranteed. The convergence

speed, which is believed to be quite challenging, is also estimated. We also consider some special random sequences including stationary processes and stationary ergodic processes.

As another application, we study agreement of multi-agent systems in periodic networks. Periodic networks often lead to oscillating behavior, but we show that agreement can surprisingly be reached if the agents activate and update their states asynchronously. We relax the requirement that networks need to be aperiodic, and obtain a necessary and sufficient condition for the network topology such that agreement can take place almost surely. We further apply our Lyapunov criteria to solving linear equations distributively. We relax the existing conditions in [77] on the changing network topology such that equations can be solved almost surely.

In the second part of this thesis, we study partial synchronization in complex networks. As we have discussed in the previous section, partial synchronization is perhaps more common than global synchronization in nature. Particularly, global synchronization in the human brain is often a symptom of serious diseases [60]. Unlike global synchronization, partial synchronization is a phenomenon that only a specific portion of units in a network are synchronized, while the rest remains incoherent. Unlike global synchronization, on which a lot of results have been obtained (we refer the readers to a survey paper [78]), the study on partial synchronization is much less. However, it has attracted growing interests recently. Cluster synchronization is a type of partial synchronization, which describes the situation where more than one synchronized groups of oscillators coexist in a network. It has been shown that network topology and the presence of time delays are quite important to render cluster synchronization [79–85]. The Chimera state is another interesting type of partial synchronization, which is characterized by the coexistence of both coherent and incoherent groups within the same network. Chimera states were initially discovered by Kuramoto *et al.* in 2002. Since then several investigations have been made [86–88]. We refer the readers to a survey for more details [89].

With the help of the Kuramoto model and its variations, we identify two mechanisms that can account for the emergence and stability of partial synchronization: 1) strong local or regional connections, and 2) network symmetries. Inspired by some empirical works [90,91], we show that a part of oscillators in a network can be quite coherent if they are directly connected and the connections between them are strong, while the rest that are weakly connected remain incoherent. In addition, we also show that oscillators that are not directly connected can also be synchronized, with the ones connecting them having different dynamics, if they are located at symmetric positions in a network. Such a phenomenon is called *remote synchronization*, which has also been widely detected in the human brain, where distant cortical regions without direct neural links also experience functional correlations [92].

In the first case, we utilize the incremental 2-norm and the incremental ∞ -norm based Lyapunov functions to study partial synchronization. Sufficient conditions on the network parameters (i.e., algebraic connectivity and nodal degrees) are obtained such that partial synchronization can take place. We calculate the regions of attraction and estimate the ultimate level of synchrony. The results using incremental ∞ -norm are the first known ones that are used to study synchronization in non-complete networks.

In the second case, we study remote synchronization in star networks by using the Kuramoto-Sakaguchi model. The phase shift in the Kuramoto-Sakaguchi model is usually used to model synaptic connection delays [93]. A star network is simple in structure, but has basic morphologically symmetric properties. The peripheral nodes have no direct connection, but obviously play similar roles in the whole network. The node at the center acts as a relay or mediator. As an example, the thalamus is such a relay in neural networks. It is connected to all the cortical regions, and is believed to enable separated regions to be completely synchronized [94, 95]. We show that network symmetries indeed play a central role in giving rise to remote synchronization as is predicted in some works such as [80, 96]. We reveal that the symmetry of outgoing connections from the central oscillator is crucial to shaping remote synchronization, and is possible to render several clusters for the peripheral oscillators. Note that the coupling strengths of incoming links to the central oscillator are not required to be symmetric.

Motivated by some experimental works [97, 98], we then study how detuning the natural frequency of the central oscillator in a star network with two peripheral nodes can enhance remote synchronization. To analyze this interesting problem, we obtained some new Lyapunov criteria for partial stability of nonlinear systems. Partial stability describes the behavior of a dynamical system in which only a given part of its state variables, instead of all, are stable. To show partial asymptotic or exponential stability, the time derivative of a Lyapunov function candidate is required to be negative definite according to the existing results [99–101]. We relax this condition by allowing the time derivative of the Lyapunov function to be positive, as long as the Lyapunov function *per se* decreases after a finite time. We then establish some further criteria for partial exponential stability of slow-fast systems using periodic averaging methods. We prove that partial exponential stability of the averaged system implies that of the original one. As some intermediate results, a new converse Lyapunov theorem and some perturbation theorems are also obtained for partial exponential stability systems. Finally, we use the obtained Lyapunov criteria to prove that natural frequency detuning of the central oscillator actually strengthens the remote synchronization, making it robust against to the phase shift. The proof reduces to the demonstration of the partial exponential stability of a slow-fast system.

1.3 Thesis Outline

The remainder of this thesis is organized as follows. Chapter 2 provides some preliminary concepts and theories that will be used throughout the thesis, including probability theory, graph theory, and some concepts related to stochastic matrices.

The main body of the thesis is divided into two parts. The first part consists of two chapters, i.e., Chapters 3 and 4, in which we focus on stochastic distributed coordination algorithms. In Chapter 3, we propose some new Lyapunov criteria for stability and convergence of stochastic discrete-time systems. The results in Chapter 3 provide some tests for stability analysis of asymptotic convergence, exponential convergence, asymptotic stability in probability, exponential stability in probability, almost sure asymptotic stability, or almost sure exponential stability of a stochastic discrete-time system. These criteria are then used in Chapter 4, where the convergence of products of random stochastic matrices, agreement problems induced by asynchronous events, and solving linear equations by distributed algorithms are studied. The content of Chapter 3 is based on [102], and that of Chapter 4 on [102] and [103].

The second part of the thesis consists of three chapters, i.e., Chapters 5, 6, and 7. In this part, we aim at identifying some possible underlying mechanisms that could lead to partial synchronization in complex networks. We first investigate in Chapter 5 how partial synchronization can take place among directly connected regions. We find that strong local or regional coupling is a possible mechanism. Tightly connected oscillators can have coherent behaviors, while other oscillators that are weakly connected to them can evolve quite differently. In addition, we also study how partial synchronization is possible to occur among oscillators that have no direct connections, namely remote synchronization phenomena. In order to study remote synchronization, we develop some new criteria for partial stability of nonlinear systems in Chapter 6. In Chapter 7, we analytically study remote synchronization in star networks. We employ the Kuramoto model and the Kuramoto-Sakaguchi model to describe the dynamics of the oscillators. Some sufficient conditions are obtained such that remote synchronization can emerge and remain stable. The content of Chapter 5 is based on [104] and [105], Chapter 6 on [106] and [107], and Chapter 7 on [107] and [108].

1.4 List of Publications

Journal articles

- [1] **Y. Qin**, M. Cao, and B. D. O. Anderson, “Lyapunov criterion for stochastic systems and its applications in distributed computation.” *IEEE Transactions on Automatic Control*, doi: 10.1109/TAC.2019.2910948, to appear as a *full paper*.
- [2] **Y. Qin**, Y. Kawano, O. Portoles and M. Cao. “Partial phase cohesiveness in networks of Kuramoto oscillator networks.” *IEEE Transactions on Automatic Control*, under review as a *technical note*.
- [3] **Y. Qin**, Y. Kawano, B. D. O. Anderson, and M. Cao. “Partial Exponential Stability Analysis of Slow-fast Systems via Periodic Averaging.” *IEEE Transactions on Automatic Control*, under review as a *full paper*.
- [4] M. Ye, **Y. Qin**, A. Govaert, B. D. O. Anderson, and M. Cao. “An influence network model to study discrepancies in expressed and private Opinions,” *Automatica*, 107: 371-381, 2019, *full paper*.

Conference papers

- [1] **Y. Qin**, Y. Kawano and M. Cao, “Stability of remote synchronization in star networks of Kuramoto oscillators,” in *Proceedings of the 57th IEEE Conference on Decision and Control*, Miami Beach, FL, USA, 2018, pp. 5209-5214.
- [2] **Y. Qin**, Y. Kawano, and M. Cao, “Partial phase cohesiveness in networks of communitinized Kuramoto oscillators,” in *Proceedings of IEEE European Control Conference*, Limassol, Cyprus, 2018, pp. 2028-2033.
- [3] **Y. Qin**, M. Cao, and B. D. O. Anderson, “Asynchronous agreement through distributed coordination algorithms associated with periodic matrices,” in *Proceedings of the 20th IFAC World Congress*, Toulouse, France, 2017, 50(1): 1742-1747.
- [4] A Govaert, **Y. Qin**, and M. Cao. “Necessary and sufficient conditions for the existence of cycles in evolutionary dynamics of two-strategy games on networks,” in *Proceedings of IEEE European Control Conference*, Limassol, Cyprus, 2018, pp. 2182-2187.

1.5 Notation

Sets

Let \mathbb{R} be the set of real numbers, \mathbb{N}_0 the set of non-negative integers, and \mathbb{N} the collection of positive integers. Let \mathbb{R}^q denote the real q -dimensional vector space, $\mathbf{1}_q$ the q -dimensional vector consisting of all ones, and for any $n \in \mathbb{N}$ let $\mathbf{N} = \{1, 2, \dots, n\}$. For any $\delta > 0, x \in \mathbb{R}^n$, define $\mathcal{B}_\delta(x) := \{y \in \mathbb{R}^n : \|y - x\| < \delta\}$ and $\bar{\mathcal{B}}_\delta(x) := \{y \in \mathbb{R}^n : \|y - x\| \leq \delta\}$. Particularly, let $\mathcal{B}_\delta = \{y \in \mathbb{R}^n : \|y\| < \delta\}$ and $\bar{\mathcal{B}}_\delta = \{y \in \mathbb{R}^n : \|y\| \leq \delta\}$.

Norms

Let $\|\cdot\|_p, p \geq 1$, be any p -norm for both vectors and matrices.

Comparison functions

A continuous function $h(x) : [0, a) \rightarrow [0, \infty)$ is said to belong to class \mathcal{K} if it is strictly increasing and $h(0) = 0$. It is said to belong to class \mathcal{K}_∞ function if $a = \infty$ and $h(r) \rightarrow \infty$ as $r \rightarrow \infty$.

Other Notation

Given two sets \mathcal{A} and \mathcal{B} , the union of them is denoted by $\mathcal{A} \cup \mathcal{B}$, the intersection is denoted by $\mathcal{A} \cap \mathcal{B}$, and $\mathcal{A} \setminus \mathcal{B}$ presents the difference between \mathcal{A} and \mathcal{B} , i.e., $\mathcal{A} \setminus \mathcal{B} = \{x : x \in \mathcal{A}, x \notin \mathcal{B}\}$. Given $x \in \mathbb{R}^n, y \in \mathbb{R}^m$, denote $\text{col}(x, y) = (x^\top, y^\top)^\top$. With a bit abuse of notation, we denote $\text{col}(f_1, f_2) = (f_1(x)^\top, f_2(x)^\top)^\top$ for two given functions $f_1 : \mathbb{R}^{n+m} \rightarrow \mathbb{R}^n$ and $f_2 : \mathbb{R}^{n+m} \rightarrow \mathbb{R}^m$.

In Part I of this thesis, we let x^i denote the i th element of a given vector $x \in \mathbb{R}^n$ for the purpose of notational clarity; in Part II, we denote the i th element of x in the conventional way, i.e., x_i . Given a vector $x \in \mathbb{R}^n$, let

$$\text{diag}(x) = \begin{bmatrix} x_1 & & \\ & \ddots & \\ & & x_n \end{bmatrix}.$$

For any $x \in \mathbb{R}$, Let $\lfloor x \rfloor$ denote the largest integer that is less than or equal to x , and $\lceil x \rceil$ the smallest integer that is greater than or equal to x .

