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Distributed Coordination and Partial Synchronization in Complex Networks

Yuzhen Qin

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The research described in this dissertation has been carried out at the Faculty of Science and Engineering, University of Groningen, the Netherlands.

disc

The research reported in this dissertation is part of the research program of the Dutch Institute of Systems and Control (DISC). The author has successfully complete the educational program of DISC.

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and in accordance with
the decision by the College of Deans.

This thesis will be defended in public on

Friday 6 December 2019 at 14:30 hours

by

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To my family

献给我的家人

妻子葛杉杉、
母亲董红、父亲秦有清

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