CHAPTER 3
REDUCTION OF LAWS AND THEORIES

3.1 Introduction

In this chapter, and the next, I will discuss the meaning of the terms ‘to reduce’ and ‘reduction’ in science and I will discuss the various types of reduction occurring in science. The next chapter deals with the reduction of concepts, this one with the reduction of laws and theories.21

I will start by discussing Nagel’s (1961/82) general model of the reduction of laws and theories. Although this model has been criticized, adapted and elaborated upon by later authors, it is still of great value, because Nagel argued extensively that reduction is a kind of explanation and hence pertains to the logical relations between statements or systems of statements (theories). In this sense, reduction is an epistemological issue and should not be confused in any way with ontological reduction, that is, the idea that things, or properties or other attributes of things, are being ‘reduced’. This insight, which in my view has not been sufficiently stressed by most later authors, will prove to be of major importance in later chapters.

Next, I will discuss Kuipers’s (1990) comprehensive model of explanation and reduction, which may in fact be considered an integration of earlier models. Kuipers’s model shows that there are many different types of reduction, to wit deductive and approximative reductions, iso-reductions and micro-reductions, homogeneous and heterogeneous reductions, and, within the last type, correlative reductions and identificatory reductions. The natures of these types depend on the kinds of auxiliary hypotheses and so-called bridge principles being used in a reduction (in addition to the reducing theory). I will discuss each of these types, and the reduction steps of which they are composed, and I will illustrate them by some examples (all based on Kuipers 1990). In chapter 5 the distinctions between these types will prove to be extremely important in illuminating the so-called doctrine of emergence (or the concept of ‘emergent’ properties at higher levels of organization).

Finally, I will discuss the difference between the replacement and the reduction of some law by a theory, or of a theory by another theory. Though both have been seen as kinds of reduction, the former being called ‘replacement reductions’ and the latter ‘explanation reductions’, I think it is preferable to use the term ‘reduction’ only in the latter, explanatory, sense. The major difference is that, whereas in ‘replacement reductions’ the ‘reduced’ law or theory is being eliminated (replaced) by the ‘reducing’ theory, in explanatory reductions the former is (through its being explained and not explained away) consolidated or even reinforced by the latter. Thus, along with the conclusion that reduction is an epistemological (and not an ontological) issue, the major conclusion of this chapter will be that reductions in science should not be confused in any way with the elimination of (higher level) laws or theories and least of all with the elimination of the ontologies (‘things’ and ‘attributes’ of things) to which these laws or theories refer. In chapter 4 I will show that, contrary to the claims of many philosophers, the same applies to reduction of concepts, and in chapter 5 I will show that this conclusion can be linked in a very interesting and illuminating way to the doctrine of emergence.

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21 For a different, though fully compatible approach to reduction, called the structuralist approach, see Balzer, Moulines and Sneed 1987.
3.2 Nagel’s general reduction model

The *locus classicus* in the field of reduction of laws and theories is Ernest Nagel (1961/82). According to Nagel, a reduction is "the explanation of a theory or a set of experimental laws established in one area of inquiry [called by Nagel the secondary science; RL], by a theory usually though not invariably formulated for some other domain [the primary science; RL]" (Nagel 1982: 338). Thus, in the view of Nagel, reduction is a kind of explanation and, more in particular, a kind of deductive-nomological explanation.

Nagel made a distinction between two types of reduction, namely *homogeneous* and *heterogeneous* reduction. The difference is that in heterogeneous reductions the law or theory to be reduced generally employs one or more terms which do not appear in the reducing theory, whereas in homogeneous reductions this is not the case. The difference is related to the fact that in homogeneous reductions both the reduced law or theory and the reducing theory apply to the same domain of phenomena, whereas in heterogeneous reduction they apply to different domains (and often, though not always, to different levels of organization). For that reason, these types of reduction have also been called *domain-preserving* and *domain-combining* reductions, respectively (Nickles 1973).

3.2.1 Homogeneous reductions

Nagel considered homogeneous reductions unproblematical and payed little or no attention to them: "In reductions of the sort so far mentioned, the laws of the secondary science employ no descriptive terms that are not also used with approximately the same meanings in the primary science. Reductions of this type can therefore be regarded as establishing deductive relations between the two sets of statements that employ a homogeneous vocabulary. Since such "homogeneous" reductions are commonly accepted as phases in the normal development of a science and gives rise to few misconceptions as to what a scientific theory achieves, we shall pay no further attention to them" (Nagel 1982, p. 339).

This has proved to be incorrect. There are indeed cases of homogeneous reduction where the one theory (or experimental law) can be derived deductively from the other (such as the reduction of rigid body mechanics to classical particle mechanics; see Sneed 1971). However, several authors, in particular Popper (1957, 1962), Feyerabend (1962, 1965) and Kuhn (1962), have argued that many, if not most, major cases of reduction in the history of science were not accomplished through the deductive derivation of the one theory from the other, but through the *replacement* of one theory by another. These cases concern the replacement of a former theory, T2, by a later one, T1, thus of a preceding theory and its successor, where it is established that T2 is in certain respects untrue and in these respects is *corrected* by T1. Examples of this are the replacement (reduction) of Kepler’s and Galilei’s laws by Newtonian particle mechanics and the replacement of Newtonian particle mechanics by Einstein’s special theory of relativity.

Because it is established, in the light of the succeeding (reducing) theory (or in the light of the reduction) that the preceding theory is untrue, both theories are actually incompatible in these cases, so that one cannot say that the one, T2, is being deductively derived from the
other, T1. At most, one can say that T2 follows *approximately* from T1 (Feyerabend 1962).\(^{22}\) For this reason, this type of reduction has also been called *approximative reduction* (Schaffner 1967; Sklar 1967; Mayr 1981). The main characteristic of this type of reduction seems to be some sort of assumption that a certain term in the reducing theory is negligibly small or unimportant in comparison to the law or theory to be reduced and may therefore be eliminated. For example, Galilei’s law of free fall states that the acceleration \(a(p)\) of an object \(p\) that is falling freely to the earth, equals \(M/R^2\), where \(M\) and \(R\) indicate respectively the mass and the radius of the earth. However, application of Newton’s theory of gravitation, assuming that the earth’s gravitational force is the only force operating on the object, leads to the statement that \(a(p) = M/[R + h(t)]^2\), where \(h(t)\) indicates the height of \(p\) at time \(t\). Galilei’s law can be derived from this statement only by approximation, by assuming that the height of the object is negligibly small when compared to the earth’s radius, whence the term \(h(t)\) may be eliminated.

Thus, in replacement reductions T2 is not so much being (deductively) explained as corrected by T1. Several authors (Schaffner 1967; Sklar 1967; Nickles 1973) have asked themselves what then T1 does explain if it doesn’t explain T2. The answer appears to be that T1 explains, among other things, the same as did T2, namely in those areas of its domain where T2 remains (in the light of T1) adequate, and hence why T2 *seemed* correct for such a long time, why it had been successful for such a long time, and that T1 also explains why eventually T2 appears to be incorrect or untrue (in those areas of its domain where T2 is being corrected by T1). One may think of, for instance, the prolonged success of Newtonian mechanics which even after its replacement by Einstein’s relativity theory (or by quantum mechanics) is still being used (that is, still applies in those areas where it has not been corrected; compare note 22). Sklar (1967, pp. 112-113) notes in this connection that “if the distinction between explaining a theory and explaining its apparent success is drawn there is no reason whatever why we cannot admit that T2 is reducible to T1, admit that T2 is incompatible with T1, and yet deny that T1 explains T2”. In other words, there is no reason why we shouldn’t uphold the deductive nature of explanations (and hence of other types of

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\(^{22}\)Actually, Feyerabend (1962, 1965) and also Kuhn (1962) claim much more than just the non-deductive, approximative nature of this type of reduction. They also argue that the transition from the one theory to the other is associated with a change of conceptual schemes (or paradigms) such that the same terms get a completely different meaning in the reducing theory than they had in the reduced theory. As a result, so the authors claim, both theories are actually incommensurable (incomparable). However, according to other authors (Achinstein 1964; Shapere 1966; see also Sklar 1967; Schaffner 1967; Nickles 1973), this is going much too far. They claim first of all that if theories are incommensurable (as claimed by Feyerabend and Kuhn) they cannot be incompatible (as also claimed by Feyerabend and Kuhn): for in order to establish that theories are incompatible one must be able to compare them. In the second place, the meaning change is generally restricted to only one or at most a few terms in both theories and is usually not as drastic as to involve a complete change of conceptual schemes, which is the reason why the theories can be compared, why it can be established that they are in certain respects incompatible, and why the former theory can be derived approximately from its successor. See also Davidson (1984) for arguments against the idea of incommensurable theories or conceptual schemes.
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reduction). Schaffner (1967) has incorporated both types of reduction, that is, ‘replacement reductions’ and ‘explanation reductions’, in one general reduction model. He argues that although one cannot say, in cases of replacement reduction, that T2 is being deductively explained by T1, it is possible to derive from T1 a corrected version of T2, T2*, from which T2 can then be derived by approximation. Thus, according to Schaffner, replacement reductions can be reconstrued as the correction of T2 by T2* (approximative reduction) and the subsequent reduction (deductive explanation) of T2* to (by) T1 (see also Schaffner 1993a,b). I will return to this later, when discussing Kuipers’s reduction model.

Though one might get the impression that replacement reductions are always of the homogeneous, domain-preserving type, this is not the case. For there are also many heterogeneous reductions in which, besides deductive steps, an approximation step occurs (see below). Thus, the difference between replacement reductions and explanation reductions is not linked to the distinction between homogeneous and heterogeneous reductions, but to whether or not an approximation step occurs in the reduction, and this may happen in both homogeneous and heterogeneous reductions. I will return to the terminology of ‘replacement’ reductions versus ‘explanation’ reductions in the final section of this chapter.

3.2.2 Heterogeneous reductions

As mentioned above, Nagel (1961/82) payed attention almost exclusively to heterogeneous reductions. He asserted that reductions of this type must satisfy a number of formal and informal conditions. The main informal conditions are that both the law or theory to be reduced and the reducing theory must be sufficiently supported by empirical evidence, and that the reducing theory must be more general than the (law or) theory to be reduced. The reason for the latter condition is that it is only when this condition is met that one can speak of the reduction of the one theory, T2, to the other, T1, and not the other way around. (Incidentally, this condition applies to all reductions, whether heterogeneous or homogeneous.)

Nagel made two major formal demands on reductions, which he called the condition of derivability and the condition of connectability. The former relates to his conviction that a reduction is a deductive-nomological explanation and, hence, boils down to the requirement that the law or theory to be reduced be logically derivable from the reducing theory. However, in heterogeneous (domain-combining) reductions, the law or theory to be reduced generally contains one or more terms not being employed by the reducing theory. In such cases, the reducing theory must be supplemented with one or more auxiliary hypotheses

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23 There are also cases of replacement of the one theory by another, such as the replacement of Priestley’s phlogiston theory by Lavoisier’s oxygen theory or the replacement of the theory of vital forces by various mechanistic theories in biology, which neither involve deductive (explanatory) reduction nor approximative reduction. These cases involve the total elimination of an inadequate or untrue theory by a theory which, for the moment, is considered more adequate. The reduction model of Kemeny and Oppenheim (1956), in which ‘reductions’ are not accomplished through logical relation between theories but through their explained observational domains, is considered to apply to such cases (Schaffner 1967; Kuipers 1997). I will return to this in section 3.4.2.
Reduction of laws and theories connecting the specific terms of the law or theory to be reduced with the theoretical terms of the reducing theory. Hence Nagel’s second condition of connectability. This second condition may actually be seen as a derivative of the first condition, but was stated explicitly by Nagel to stress the importance of auxiliary hypotheses in accomplishing reductions. He called such auxiliary hypotheses rules of correspondence, but they are also called bridge principles (in distinction from the internal principles of the reducing theory; see Hempel 1965, 1967; Schaffner 1967).

According to Nagel, these correspondence rules or bridge principles should not be regarded as analytical statements (in which terms are defined equivalent) but as empirically determined 'correlatory laws'. Somewhat confusingly, Nagel had in mind examples of what later authors agreed upon must be ontological identity relations (Schaffner 1967, 1969; Sklar 1967; Hempel 1969; Causey 1972, 1977; Nickles 1973; Girill 1976; Pluhar 1978; Kuipers 1990). These are supposed to be statements to the effect that the objects or attributes represented by terms in the law or theory to be reduced are ontologically identical to the objects or attributes represented by the theoretical terms of the reducing theory. Examples of such ontological identity relations are (supposedly) 'light waves are (identical to) electromagnetic waves', 'genes are (identical to) pieces of DNA', or 'the macroscopic temperature of a gas is identical to the mean kinetic energy of its molecules'. On these grounds, it has been thought for some time that there is only one type of heterogeneous reduction, namely reduction with (ontological) identity or identificatory reduction. This has spurred much discussion, because examples were found of what were considered heterogeneous reductions which did not, however, contain ontological identity relations, or in which the nature of the bridge principles as ontological identity relations was disputed. In time, however, it has become clear that there are in fact several types of (heterogeneous) reduction relating to the fact that besides or instead of ontological identity relations there may also be bridge principles in the form of correlations or, by definition

\[24\text{In this context the term 'correlation' means indeed, by definition, causal relation. It should not be confused, therefore, with the meaning of the term 'correlation' in for instance statistics, where correlations say nothing about causal relations.}\]

3.3 Kuipers’s reduction model

We have already seen that there are various types of reduction: homogeneous and heterogeneous reductions, deductive and approximative reductions, and within heterogeneous reductions more in particular identificatory reductions (to be distinguished from correlative reductions; see below). Also, there is an important type of reduction, which pertains to part-whole relationships and which is therefore of special interest in this book, called micro-reduction (Causey 1972a,b; Girill 1974, 1976). All these types are integrated in Kuipers’s (1990) reduction model, called the five steps model of explanation and reduction.

Kuipers claims that every explanation of a law by a theory or of a theory by another theory
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can be decomposed into one or more of five possible steps, to wit (1) application, (2) aggregation, (3) identification, (4) correlation, and (5) approximation. Based upon the literature, he also claims that in order for an explanation to be also called a reduction at least one of the steps aggregation, identification or approximation must occur. In the following sections I will shortly discuss each of the five steps, then indicate the types of explanation or reduction to which they, or their combinations, lead, and illustrate these types by some examples. These sections are based almost entirely on Kuipers’s (1990) exposition. To make things a bit less abstract and to provide some insight into the nature of these steps, I will start with an example of the type of reduction which is of most interest in the context of this book, viz. *heterogeneous micro-reduction*. The example concerns the reduction of the ideal gas law to the kinetic theory of gases (Kuipers 1982, 1990; Nagel 1982, pp. 338-366)

### 3.3.1 The reduction of the ideal gas law

The ideal gas law pertains to standard, isolated amounts of gases in closed spaces (vessels) and states that there is always a proportional relationship between the pressure $P$, the volume $V$ and the temperature $T$ of such a gas, according to the equation $PV = RT$ ($P$ times $V$ equals $R$ times $T$). $R$ is a constant called the ideal gas constant. The reduction of this law proceeds as follows.

In the kinetic gas theory it is assumed that a gas consists of molecules and that these molecules are constantly moving: they have a certain motion energy or kinetic energy $u$ which equals $\frac{1}{2}mv^2$, where $m$ represents the mass of a molecule and $v$ its velocity. It is assumed thereby that these molecules may be regarded as particles in the sense of Newtonian particle mechanics and that therefore Newton’s laws of motion may be applied to them. Accordingly, in the first reduction step, an *application step*, Newtonian mechanics is being applied to a single molecule and it is assumed that this molecule has a mass $m$ and a velocity $v$. Because it is constantly moving, the molecule is also constantly colliding with other molecules and with the wall of the vessel containing the gas. An extra assumption in this first step is that these collisions are elastic, that is to say that the velocity of the molecule before and after the collision is the same. The result is an *individual law* stating that the momentum exchange between the molecule and the wall, $q$, equals twice the mass times the velocity of the molecule: $q = 2mv$.

In the second step, an *aggregation step*, the total effect of the collisions (that is, of momentum exchanges) of a large number, $N$, of molecules against the wall is being aggregated. $N$ is the number of Avogadro and is the number of molecules in a standard amount (a mole) of gas. This step consists of an ingenious aggregation of individual momentum exchanges, whereby a number of statistical auxiliary hypotheses are being used. With the help of these extra hypotheses the *aggregated law* can be derived that the product of the total momentum exchange between the molecules and the wall, their kinetic pressure $p$, and the volume $V$ of the vessel is equal to the mean kinetic energy $\bar{u}$ of the molecules times a constant, $(2/3)N$. Thus: $pV = (2/3)N\bar{u}$.

In the third and final step, a *transformation step*, two bridge principles (or, in Kuipers’s terminology, transformation rules) are being used, which are both, according to Kuipers, *ontological identity relations*. The first one states that the kinetic pressure $p$ of the molecules is exactly the same as what is measured as the macroscopic pressure $P$ of the gas: $p = P$. The second principle states that the mean kinetic energy of the molecules is directly proportional...
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to what is measured as the macroscopic temperature T of the gas: \( \hat{u} = \frac{3}{2} \left( \frac{R}{N} \right) T \). Here, R is again the ideal gas constant. Because N is also a constant, the right-hand side of this equation contains a constant times T, hence the direct and constant relationship between \( \hat{u} \) and T. When both bridge principles are filled in into the equation for the aggregated law, \( pV = (\frac{2}{3})N\hat{u} \), we get \( PV = (\frac{2}{3})N(\frac{3}{2})(\frac{R}{N})T \), or, simplifying, \( PV = RT \), the ideal gas law.

In this reduction three of the five steps of Kuipers’s model occur: application, aggregation and identification. Because of the aggregation step it is an example of micro-reduction. Because of the identification step it is an example of heterogeneous reduction and more in particular of identificatory reduction. And because with these three steps the ideal gas law can be derived deductively from the kinetic theory of gases, it is also an example of deductive reduction. In sum, it is an example of deductive, heterogeneous, identificatory micro-reduction. I will return to this terminology later.

The remaining steps in Kuipers’s model are correlation and approximation. I will now shortly discuss each of the five steps.

### 3.3.2 Explanation and reduction steps

a) application

Since reductions are the deductive or approximative derivation of a law or theory by another, more general theory, reconstructions of them generally start with the application of the reducing theory. Thus, the first step in every explanation, and hence in every reduction, is an application step\(^{25}\), so called because in this step the explanatory theory is applied, ‘tailored’ or, to use Cartwright’s term, ‘modelled’ to the kind of object or system to which the law or theory to be explained pertains, or to parts thereof. Various auxiliary hypotheses are generally being used in this step, in particular hypotheses specifying the initial conditions and boundary conditions of the ‘system’. When this step pertains to the component parts of some whole and when it is followed by an aggregation step, as in the reduction of the ideal gas law, it leads to a regularity or law which, in the light of this following step, is called an individual law or regularity. It is also possible, however, that the application step is the only step occurring in an explanation, such as when Newtonian mechanics is used to explain or predict construction work or the motion or collision of objects (for example, biljard balls), or when thermodynamics is used to explain or predict what happens when things are under put pressure (the tea kettle, a bicycle tube). In these cases, the application step leads directly to the law to be explained. Such explanations are never called reductions, however, for reasons to be specified below.

b) aggregation

The second possible step is an aggregation step, in which case we are invariably dealing with part-whole relationships. In this step the total result of the application step at the level of the parts (that is, of individual regularities) over the relevant number or set of parts is then being

\(^{25}\) Though this step is in itself not sufficient to speak of a reduction. See below in the main text.
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assessed. This step may consist of a *simple addition* or summation of individual regularities (such as when masses of parts are being added up), in which case it is called *trivial aggregation*, but also be a more or less complex or ingenious step (as in the reduction of the ideal gas law), in which case it is called *non-trivial aggregation*. This distinction will later prove to be important in explicating and defining the concept of ’emergence’ (chapter 5). In the latter case, this step also generally involves the use of auxiliary hypotheses, often of a statistical nature. In both cases the resulting law or regularity is called an aggregated law or regularity.

c) identification

The next possible step is an identification step. When this step occurs, we are dealing with a heterogeneous ’jump of language’ and therefore with heterogeneous reductions. As mentioned earlier, such reductions require bridge principles or rules of correspondence to connect terms in the law or theory to be reduced, which are not being employed by the reducing theory, with the theoretical terms of the reducing theory (see 3.2.2). Kuipers calls these rules transformation rules. In the case of an identification step, this rule is an *ontological identity relation* (or hypothesis), stating that a thing or attribute referred to by a term in the reduced law or theory is ontologically identical to a thing or attribute referred to by a term in the reducing theory. In the reduction of the ideal gas law we have already met two examples of such ontological identity relations and I will discuss some other examples later.

d) correlation

The fourth possible step is a correlation step. This step is, like an identification step, a transformation step and thus also involves a heterogeneous jump of language. However, instead of ontological identities, a correlation step expresses correlations or, by definition, *causal relations* between whatever is referred to by terms in the reduced law or theory and whatever is referred to by terms in the reducing theory. It will be intuitively clear what the distinction between ontological identities and correlations amounts to, but its justification has proved to be an extremely hard nut to crack (if it has been cracked at all). However, since the distinction is of crucial importance in the reduction of concepts, I will postpone a discussion of it to the next chapter. I will provide some examples of a correlation step in the following section.

26The same distinction has been made by Girill (1976) in the form of ’homogeneous’ parts (that is, parts which are all of the same type) and ’heterogeneous’ parts (of different types). However, Girill associated the distinction with different types of identity relations between (attributes of) parts and a whole. He paid no attention to the aggregation step and therefore seems to have missed a major point with regard to identificatory reduction: there are no known examples of identificatory reduction (of a law or theory!; not of concepts, of course) without an aggregation step (Kuipers 1990; see below in the main text) and the distinction Girill made between different types of ’identification’ generally plays a role in the aggregation step, not in the identification step.
The final possible step in Kuipers’s model is an approximation step. I have already mentioned this step in 3.2.1, when discussing replacement reductions. Contrary to the other steps, the approximation step is not a deductive step, but a step in which the law or theory to be reduced is derived approximately. As noted above, approximative reductions involve some sort of correction of the reduced law or theory. In terms of Schaffner’s (1967, 1993a,b) model, one may deductively derive from T1 a corrected version of T2, T2*, from which T2 can be derived only by approximation. From Newton’s theory of gravitation (T1) one can deductively derive a corrected version of Galilei’s law of free fall (L2*: a(p) = M/[R + h(t)]²) which reduces to Galilei’s law (L2: a(p) = M/R²) by approximation.

Kuipers notes that approximation is more or less the opposite of concretization in the sense of the model of idealization and concretization (Krajewski 1977; Nowak 1980). Conversely, then, approximation is more or less the same as idealization: by approximation, T2* is being simplified (idealized) to T2. Thus, the correction of T2 by T2* is the same as the concretization of T2 to T2*, and, conversely, the approximation of T2* by T2 is the same as the idealization of T2* to T2. The correction of Galilei’s law of free fall (L2) by the law derived from Newton’s theory (L2*) is the same as the concretization of Galilei’s law (L2) to Newton’s version (L2*), and the approximation of Newton’s version (L2*) by Galilei’s law (L2) is the same as the idealization of Newton’s version (L2*) by Galilei’s law (L2). I will discuss some more examples of this in the following section, and in chapter 12 I will provide an example of idealization and concretization in ecology.

3.3.3 Types of reduction

The first step, application, occurs in every explanation and hence in every reduction. Therefore, it plays no role in distinguishing between types of reduction. By reference to the other steps, however, Kuipers distinguishes between the following basic types of reduction. Reductions with an aggregation step are called micro-reductions, without this step they are called iso-reductions. Reductions with a transformation step are called heterogeneous reductions, without this step homogeneous reductions. Heterogeneous reductions may be correlative reductions or identificatory reductions or a combination of both. Finally, reductions with an approximation step are called approximative reductions, without this step they are called deductive reductions.

As has already become clear, however, all sorts of combinations of steps, and hence of these basic types of reduction, may occur. All in all, three times two alternative steps may occur, leading to 2 x 2 x 2 = 8 possible combinations (types a to h below). Also, I have already

27 According to the model of idealization and concretization, growth of knowledge often starts with the forming of a global theoretical concept or model (a first approximation of reality called an idealization, such as the Hardy/Weiberg model in genetics) which is then gradually factualized or concretized. Such concretizations generally involve some sort of correction of the idealization by adding factors to it that were first left out but later appeared to be significant (such as adding mutation and migration to the Hardy/Weiberg model).
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mentioned that, according to Kuipers, there are some combinations of steps yielding explanations that cannot be called (or are never called) reductions, and that one speaks of reduction only if at least one of the steps aggregation, identification or approximation occurs. I will now turn to these possible combinations of steps and to the different types of explanation or reduction to which they lead. I will start with the more simple types and then work towards more complicated types. Once again, this section is based almost entirely on Kuipers’s exposition.

a) deductive homogeneous iso-explanations

When the application step is the only step occurring, we get a first example of a type of explanation that is never called a reduction. The reason for this is that such explanations neither involve part-whole relationships (aggregation step), nor a heterogeneous jump of language (transformation step), nor some form of correction of the law or theory to be explained (approximation step). Given the lack of these other steps, it is easy to check that in Kuipers’s terminology such explanations are called *deductive homogeneous iso-explanations*. In my view, they may be regarded as the simplest form of ‘horizontal’ explanation, phenomenological explanation or even ‘holistic’ explanation (because of the absence of reduction steps). To say that this type of explanation is never called reduction is tantamount to saying that there is no such thing as deductive homogeneous iso-reduction.

b) deductive homogeneous micro-reduction

When the application step is followed only by an aggregation step, we get examples of *deductive homogeneous micro-reduction*. In such cases, the ‘wholes’ are usually relatively simple composites and the aggregation step consists of a trivial addition. An example is the reduction of rigid body mechanics to classical particle mechanics. In this reduction the laws of motion of a rigid body are derived by application of Newton’s laws of motion to the component parts of a rigid body and by aggregation of the results of this application step for all parts of the body, where it is assumed that the mutual distances between these parts are constant (which is by definition the case in rigid bodies).

c) approximative homogeneous iso-reduction

When, instead of an aggregation step, the application step is followed by (only) an approximation step, we are dealing with cases of *approximative, homogeneous iso-reduction*, homogeneous, because it doesn’t involve a heterogeneous jump of language (transformation step), and iso-reduction, because it doesn’t involve an aggregation step. This may be called the simplest or the ‘pure’ type of ‘replacement’ reduction or, better (in my view; see 3.4.2), of approximative reduction (reduction2 in the terminology of Nickles 1973). Examples are the afore-mentioned reduction of Galilei’s law of free fall to Newton’s theory of gravitation and the reduction of Newtonian particle mechanics to Einstein’s relativity theory. As mentioned before, this step generally involves some sort of correction of the reduced law or theory and this correction may also be seen as the concretization of that law or theory by the reducing theory.
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d) approximative homogeneous micro-reduction

The next possible combination of steps is application, aggregation and approximation, leading to *approximative homogeneous micro-reductions*. An example of this is the reduction of the macro-economic consumption function, which expresses a linear relationship between national income and national consumption demand, to individual utility theory. Interestingly, this reduction starts as the deductive homogeneous micro-reduction of the probabilistic version of the macro-consumption function to utility theory, by application of this theory to the behaviour of a single individual and aggregation of the resulting individual regularity over a large number of individuals. Next, it is transformed into approximative homogeneous micro-reduction by approximation of the aggregated, probabilistic regularity to the standard, non-probabilistic version of the macro-consumption function (see Janssen 1987 for details).

e) deductive heterogeneous micro-reduction

When the application step is followed by both an aggregation step and a transformation step, we get cases of *deductive heterogeneous micro-reduction* and this can be more in particular *identificatory reduction* or *correlative reduction* (or a combination of both; see below). I have already discussed an example of deductive identificatory micro-reduction in the form of the reduction of the ideal gas law. An example of *deductive correlative micro-reduction* is the reduction of the law (or hypothesis) of Olson to utility theory (as all the examples I present, this example can be found in Kuipers 1990, but it is worked out in detail in Kuipers 1984). Olson’s hypothesis states that the chance of a group realizing some collective good decreases with increasing group size. According to the central principle of utility theory, the principle of utility maximization, individuals who are faced with a choice between alternative actions will do whatever they expect has the highest utility (and neglect to do whatever they expect has lower utility). By application of this principle to the choice between participation or non-participation in the realization of a collective good, at variable group size, and using some auxiliary hypotheses about probability and utility assessments by individuals, one can derive the following individual regularity: each individual has his own switch group size, that is, the size where he or she switches from participation to non-participation. Next, by aggregation of this individual regularity and using some statistical hypotheses about the comparability of groups with respect to the mean and variance of this individual switch group size, one can derive the aggregated regularity that the larger the group the smaller the degree of participation. Finally, by using the *correlation hypothesis* that the smaller the degree of participation the smaller the chance of realization of the collective good, one can derive Olson’s hypothesis: the larger the group the smaller the chance of realization of a collective good.

f) approximative heterogeneous micro-reduction

A still more complex type of reduction is *approximative heterogeneous micro-reduction*, consisting of the steps application, aggregation, transformation and approximation. An example of this, which is at the same time another example of the model of idealization and concretization, is the reduction-with-detour of the ideal gas law to the kinetic theory of gases via Van der Waals’s law. Using essentially the same reduction steps as in the direct reduction
of the ideal gas law, it is also possible to derive from the kinetic theory of gases first the kinetic (microscopic) and next the macroscopic version of Van der Waals’s law, from which the ideal gas law can then be derived by approximation (by neglecting the pressure and volume constants; see Kuipers 1985 for details). That is to say, the macroscopic version of Van der Waals’s law is a concretization of the ideal gas law and, conversely, the ideal gas law is, as the name indicates, an idealization which can be derived from Van der Waals’s law only by approximation. The example is also another illustration of Schaffner’s claim (or reduction model): the macro-version of Van der Waals’s law (T2*) is a concretization (correction) of the ideal gas law (T2) and this concretization can be derived deductively from the kinetic theory of gasses (T1). Thus whereas the direct reduction of the ideal gas law to the kinetic gas theory is an example of deductive, identificatory micro-reduction, the reduction-with-detour via Van der Waals’s law is an example of approximative, identificatory micro-reduction (including, however, the same deductive steps as the direct reduction!).

Kuipers also provides an example in which all five steps of his model occur. This is the reduction of the periodic table, or the periodic law, of Mendeleev to quantum atomic theory. Apart from an application, aggregation and approximation step, this reduction also contains both a correlation step and an identification step. In the former the correlation hypothesis is being used that chemically similar behaviour of elements is caused by equal atomic structure. In the latter the ontological identity hypothesis is being used that the atomic number (weight) of an element is identical to its number of protons or electrons (see Hettema & Kuipers 1988 for details). Based upon its steps, it will be clear that this too is a case of approximative heterogeneous micro-reduction, but spelled out more fully it is an example of approximative, heterogeneous, correlative, identificatory micro-reduction. I only mention this to indicate that reductions can be quite complicated indeed. In chapter 6 I will provide an example of heterogeneous micro-reduction in biology which also involves both a correlation step and an identification step.

g) deductive heterogeneous iso-reduction?

In the first paragraph of this section I mentioned that there is no such thing as deductive homogeneous iso-reduction. There are also no examples of deductive heterogeneous iso-reduction, that is, reduction by only application and transformation, but the reason for this is a bit more complicated.

In the first place, the transformation step could in principle be an identification step, in which case we could speak of (identificatory) reduction. However, there are no known examples of such reductions. In all known cases of identificatory reduction, the identification step is always preceded by an aggregation step, whence they are always examples of micro-reduction. As Kuipers remarks, one may consider it an interesting challenge to find counter-examples.

In the second place, the transformation step could also be a correlation step, but in that case we would again get examples of explanation which, according to Kuipers, are never called reduction. I must confess, however, that I have my doubts as to why explanations of this type cannot be called reductions. For after all they involve a heterogeneous jump of language similar to the one in an identification step and therefore would seem to fall under Nagel’s heading of heterogeneous reduction.

To make things more complicated, the (only) example Kuipers provides of this type of
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The explanation of the laws of interbreeding in biology by Mendelian genetics, as Kuipers claims, but not as my view does not involve an aggregation step. For instance, under certain well-defined starting conditions one crosses a population of green peas with an equally large population of white peas, the next generation yields approximately 75% green peas and 25% white peas. The explanation provided by Mendelian genetics is that peas are heterozygous for a gene determining their colour, that is to say that there are two alleles of this gene, say A for green peas and a for white peas, and that allele A is dominant over allele a. This means that of four possible combinations after fertilization, AA, Aa, aA and aa, three lead to green peas while only the combination aa leads to white peas. When sufficient numbers of peas are being crossed, one can determine, using some statistical auxiliary hypotheses, that roughly 75% of the next generation of peas will be green and 25% white. According to Kuipers, this is a typical case of correlative explanation: whole chains of causal processes between genes and pea colour are being assumed. However, "because only the steps (...) application and (...) correlation are involved", it "is, as far as I know, never called a reduction" Kuipers 1990, p. 256).

I have the strong suspicion, however, that this is really a case of reduction, firstly because it consists of the explanation of phenotypical variation in traits of peas (wholes) in terms of a theory about some of their component parts (alleles, genes), and secondly because the example actually involves four levels of organization, to wit the level of alleles, the level of genes (combinations or pairs of alleles), the level of individual peas, and the level of populations of peas. I think that one needs at least one aggregation step to come from the lowest level (alleles) to the highest level (populations) and perhaps even two. For in the application step Mendel’s theory is actually being applied to the level of alleles. In particular, the only law needed here, Mendel’s first law of random segregation, states that each descendant receives one allele from its one parent and another allele from its other parent and that the probabilities of receiving one or the other of either parent’s alleles are equal. Next, however, we need an aggregation step to come from the level of alleles to the level of genes (combinations or pairs of alleles) and to arrive at the four possible combinations of alleles after fertilization (depending upon whether the parents are homozygous AA or aa or heterozygous Aa): AA, Aa, aA and aa. Then follows the correlation step linking the level of genes with the level of individual peas, where, with the help of the dominance hypothesis, it is derived that the combinations AA, Aa, aA lead to a green pea, while only the combination aa leads to a white pea. And finally I think we need another aggregation step to come from the level of individual peas to the level of populations of peas and in which the results of crossing large numbers of green and white peas are being aggregated. As mentioned above, one needs sufficient numbers of peas and a number of statistical auxiliary hypotheses to derive the result that approximately 75% of the peas in the next generation will be green and 25% white. (More specifically, if one starts with a ‘pure’, that is homozygous AA, population of green peas and a population of white peas, which is always homozygous aa, the next generation, F1, yields 100% heterozygous Aa green peas, and random crossing within this generation yields approximately 75% green and 25% white peas in the next F2 generation. Starting with a random mixed population yields the same results.)

Thus, if my suspicions are correct, the correlation step is preceded (and followed) by an aggregation step, making the example a case of (deductive correlative) micro-reduction and not of iso-explanation.
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To make things even more complicated, Kuipers also provides no examples of other (theoretically) possible types of correlative reduction without an aggregation step, that is, reductions by (a) application, correlation and approximation; (b) application, correlation and identification; and (c) application, correlation, identification and approximation. Although this does not mean that such examples do not or could not exist, and I would certainly not preclude the possibility, it does lead to the interesting question whether a correlation step is not also, like an identification step, always preceded by an aggregation step and, hence, whether correlative reductions are not also always micro-reductions. Perhaps one should it consider a challenge to find counter-examples to this claim too.

h) approximative heterogeneous iso-reduction?

The final combination of steps I will discuss is the combination application, transformation and approximation, yielding approximative heterogeneous iso-reductions. After what I said in the previous paragraph, it should come as no surprise that this is a highly dubious combination. The discussion of this type is doubly complicated by the fact that the example provided by Kuipers, the reduction of Mendelian genetics to molecular genetics, is still incomplete and highly controversial. According to Kuipers, after the application of molecular genetics to the relevant type of organisms, an auxiliary hypothesis is being used in which Mendelian genes are identified with molecular genes (or pieces of DNA molecules), making this at least a case of heterogeneous and more in particular identificatory reduction. However, the reduction is extremely problematical, because in Mendelian genetics genes are being defined in three different ways, namely as units of mutation (the so-called muton), as units of recombination (the recon), and as units of phenotypic expression (the cistron; see Benzer 1957). However, these units do not correspond to identical pieces of DNA molecules. The unit of mutation (that is, the minimum number of nucleotides within which a base substitution will produce a mutation and a change in the genetic code) is 3 or more nucleotides. The unit of recombination may be a single nucleotide in a DNA strand, but also any higher number of nucleotides that can be recombined between DNA strands. And the unit of phenotypic expression may be hundreds of nucleotides spread over various DNA strands. Thus there is no one-to-one correspondence between Mendelian genes and molecular genes, and not even a many-to-one correspondence (which would still not be that problematic), but a many-to-many correspondence, which makes the identification extremely problematic (see Hull 1974; Ruse 1974; Schaffner 1974; Kitcher 1982; and Rosenberg 1985 for a further discussion of this problem; see also Balzer and Dawe 1986). The conclusion which should probably be drawn from this is that it is not so much Mendelian genetics which is being reduced to molecular genetics, but a corrected version of it (transmission genetics), and that Mendelian genetics can be derived from this corrected version only by approximation. Hence, according to Kuipers, this is probably a case of approximative, identificatory iso-reduction. He adds, however, that "Whether something like aggregation occurs, and hence whether it is a case of micro-reduction, has still to be investigated" (Kuipers 1990, p. 253). Notice that if this were not the case, the example would be a counter-example to Kuipers’s own claim that an identification step is always preceded by an aggregation step.

To this should be added that, apart from the many-to-many correspondence between Mendelian genes and molecular genes, one may have serious doubts as to whether the relationship is one of identity at all. For according to present molecular theory, there are all
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sorts of molecular genes which do not code directly for phenotypical traits but play an intermediary role in the way of controlling or regulating other genes that do (repair genes, operator genes, repressor genes, promotor genes). Since this involves all sorts of causal interactions at the molecular level, it is more likely that there are causal relations (correlations) between molecular genes and Mendelian genes. In that case it is even more likely that this is indeed an example of micro-reduction, since one will have to aggregate over the interactions between individual molecular genes in order to arrive at the ‘products’ of Mendelian genes, the phenotypical traits of organisms.

Whatever the correct view on this matter, it will be clear both that the example is extremely problematic as an example of identificatory iso-reduction and that this type of reduction is highly dubious. That the example is a case of heterogeneous reduction, and that it is probably a case of approximative reduction, seems beyond doubt, however.

3.4 Concluding remarks
3.4.1 Epistemological versus ontological reduction

I have now discussed all (eight) types of explanation and reduction resulting from the relevant combinations of the five step in Kuipers’s model. It is extremely important to realize that all these types pertain to logical relations between statements or systems of statements (theories). That is to say that reduction is an epistemological issue and should not be confused in any way with ontological reduction or the idea that in scientific reductions not only laws or theories (or concepts, see the next chapter) are being reduced but also the ontologies, that is, the objects or attributes of objects, to which these laws or theories refer. With the reduction of some law or theory nothing changes at the level of the ontology of that law or theory. Reduction is a kind of explanation, not of explaining-away. Take, for example, the reduction of the ideal gas law. Has this resulted in there not being gases any more with (macroscopic) pressures and temperatures? Of course not. I will return to, and extend upon, this issue in chapter 5.

3.4.2 Eliminative versus non-eliminative reduction

What was said above applies even to so-called replacement reductions, or, better, approximative reductions. For even in approximative reductions the reduced law or theory most often still stands after its reduction and is not actually being replaced in the sense of eliminated. The ideal gas law is still being used even though it has been shown to be an idealization (approximation) of Van der Waals’s law. Newtonian particle mechanics is still being used even though it has been approximatively reduced to Einstein’s relativity theory. Mendelian genetics is still being used even though it is in the process of being approximatively reduced to molecular genetics.

It seems to me, therefore, that the term replacement reduction is not a particularly fortunate term to use for approximative reductions. The only cases in which a law or theory is actually being replaced to the extent that it is no longer being used or applied are cases of so-called ‘instrumentalist’ or ‘observational’ reductions (Kuipers 1997), where a former law or theory has been found totally inadequate or untrue and has been replaced by a more successful successor. Examples of this are the replacement of Ptolemy’s geocentric theory by Copernicus’s heliocentric theory of astronomy, the replacement of Priestley’s phlogiston
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theory by Lavoisier’s oxygen theory, and the replacement of vitalism by various mechanistic theories in biology. The reduction model of Kemeny and Oppenheim (1956), in which reduction does not involve logical relations between theories but occurs through the explained observational domains of theories, is thought to apply to such cases of total replacement (Schaffner 1967; Kuipers 1997). These cases neither involve the deductive explanation (in one of the senses discussed above) nor the approximative explanation (correction) of one theory by another, but the total elimination of a inadequate or untrue theory by another theory which, for the moment, is considered more adequate. If such cases are to be called reductions at all, they are, in my view, best called eliminative reductions. In all explanatory types of reduction, but especially of course in the deductive types, the reduced law or theory is not being replaced or eliminated but, on the contrary, consolidated or even reinforced by the reducing theory. At most, some correction due to an approximation step occurs.

A major difference between eliminative and non-eliminative (consolidatory) reductions is that in examples of the former, but not of the latter, one can say indeed that also the ontology of the reduced theory is being replaced or even eliminated: today we don’t believe any more that there are such things as phlogiston or vital forces. Once again, however, in all the other types of reduction discussed in this chapter reduction has nothing to do with the replacement or elimination of the reduced law or theory and least of all with the replacement or elimination of the ontologies of those laws or theories. On the contrary, through the consolidation or reinforcement of the reduced law or theory by the reducing theory their ontologies are also being consolidated or reinforced. I will return to this issue too in chapter 5.