Chapter 3

Do firms sell forward for strategic reasons? A test based on the theory*

3.1 Introduction

As mentioned in Chapter 1, electricity and gas industries used to consist of vertically integrated monopolies, state-owned or not, operating under regulatory constraints. In each country or region there was a single monopolistic importer in place, which typically owned the transmission network and either sold directly to consumers or to downstream distribution monopolies. One-to-one negotiations was the standard of trade throughout the value chain and the different parties were typically subject to long-lasting contractual relationships.

Short- and long-run efficiency considerations have led energy policy makers worldwide to gradually restructure energy markets. A key feature has been the vertical separation of production and transportation activities. This separation has enabled the creation of spot commodity markets. The Pool in the UK, ISO in California, the real-time PJM market in Pennsylvania, New Jersey and Maryland, ERCOT in Texas, EPEXSPOT in Austria, France, Germany and Switzerland are examples in electricity; NBP in the UK, the Henry Hub in the US, the Zeebrugge Hub in Belgium and TTF in The Netherlands are examples in natural gas.

*This chapter is based on van Eijkel and Moraga-González (2010).
The 2000-2001 electricity crisis in California has revealed that the combined prevalence of price risks and market power in energy markets may have fatal consequences when risk-hedging mechanisms are absent (Bushnell, 2004). As a consequence, in nowadays restructuring, it is widely held that spot markets must necessarily be complemented with forward markets (Ausubel and Cramton, 2009). In an attempt to aid firms to contract forward, platforms have been created where property rights can more easily be transferred among the participants. In addition, in some markets we have witnessed the creation of futures exchanges. Examples of markets for electricity futures are CALPX in California and EEX Power Derivatives in Austria, France, Germany and Switzerland; ENDEX runs a market for natural gas futures in The Netherlands, as well as markets for UK and Dutch electricity futures.

Facilitating forward transactions has the potential to deliver social benefits on two accounts. First, forward markets address the need of a firm to hedge risks. Forward contracts typically specify fixed delivery prices so risk-averse market participants can mitigate their exposure to price shocks in the spot market by acquiring a portfolio of futures. Central results in the literature relate to the decisions of a competitive risk-averse firm facing price uncertainty (see e.g Baron, 1970; Holthausen, 1979; and Sandmo, 1971). In the absence of a futures market, this type of firm turns out to restrict its output relative to what the firm would produce under certainty. The opening of a forward market restores the level of output that would prevail if uncertainty were removed.

Forward markets can deliver further social benefits in situations where firms wish to sell forward for strategic reasons. In their influential paper, Allaz and Vila (1993) show that forward contracts confer competitive advantages to Cournot firms so, even when there is no uncertainty at all about future market conditions, firms have incentives to engage in forward trading. By selling futures contracts at a pre-specified price, a firm ends up attaching a lower value to a high spot market price. As a result, a firm that sells forward is indirectly committing to an aggressive behavior in the spot market. This raises firm profitability, since competitors respond by adopting a compliant spot market strategy. Selling forward exhibits however the characteristics

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1 One development in natural gas has been the creation of virtual hubs, for example the NBP and TTF, as opposed to the more traditional physical hubs, like the Henry Hub and the Zeebrugge Hub. A physical hub is a location where several pipelines come together, so that total physical throughput is delivered at this point. By contrast, virtual hubs contain several entry and exit points that are interconnected, which implies that not all the gas traded has to flow through a single point in the pipeline system.
of a prisoner’s dilemma. Because every seller has incentives to sell (part of) its output forward, the resulting equilibrium aggregate production is higher (and the price lower) than in the absence of a futures market\footnote{The model of Allaz and Vila has been adapted to suit the particular organization of power markets in the UK by von der Fehr and Harbord (1992), Powell (1993), Newbery (1998) and Green (1999).}

To the best of our knowledge, whether the forward market institution by itself is successful on these two fronts at a time is not well understood yet. One obvious reason for this lack of knowledge is that a great deal of the contracts we have observed in gas and electricity markets has not been dictated by market forces but imposed by the regulators in the form of gas release programs or vesting electricity contracts (Borenstein, 2002; Wolfram, 1999). A second reason is that disentangling the two rationales motivating firms to sell forward – strategic commitment and risk-hedging – from the field data is, at least, methodologically challenging; in addition, it requires a wealth of data on forward transactions. In the core of this chapter we propose an empirical strategy to separate the various incentives behind the contract cover of a firm. In the next chapter of this thesis we will apply this strategy to the Dutch natural gas market.

Our methodology to test whether firms use forward contracts for strategic reasons and/or for risk-hedging motives builds on the idea that commitment has value only if it is (imperfectly) observable (Allaz and Vila, 1993; Kao and Hughes, 1997). Inspired from this idea, we develop a model of the interaction of asymmetric risk-averse firms that compete in a forward market before they set quantities in a spot market. The key aspect of the modelling is that it introduces the extent to which forward positions are observable (or correctly inferred from the forward price) as a structural parameter that can be estimated provided there is variation in the number of participating firms.

In real-world energy markets, it is reasonable to assume that rivals’ forward positions may be difficult to observe. First, where they exist, markets for natural gas and electricity futures are designed to be anonymous and this anonymity puts impediments for the forward positions of each individual rival firm to be observed with a reasonable precision. Second, even if in principle observation of the forward price may help firms to infer the rivals’ aggregate forward position, the process of (forward) price discovery is far from simple. This is because some (or all) of the transactions in these markets are made over the counter (OTC) as organized exchanges are often bypassed by the traders or totally lacking\footnote{Chapter 5 discusses in more detail the coexistence of OTC marketplaces and centralized ex-}.
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relatively non-transparent and price indices for these markets, typically provided
by broker associations of by specialized agencies, are based on a limited pool of
recent transactions. By construction, price indices are complex statistics so it is
unclear how much a participant in the market can learn about rivals’ deviations
from equilibrium play. Even if centralized futures markets are more transparent,
seen that arbitrage opportunities across the exchanges and the OTC markets are
not always fully exhausted, the existence of conflicting price signals (as the law of
one price fails) further troubles the quality of the inferences market participants can
make (see e.g. Anderson, Hu and Winchester, 2007; Bushnell, 2007).

Our model bridges between the extreme cases of full transparency (perfect ob-
servability of rivals’ forward positions) and complete opacity of the forward market
(no observability at all). By doing so, the model gains in flexibility to fit the data
and helps us understand the extent to which the forward market provides the players
with commitment possibilities.

We show how to estimate the model using data on total sales, forward sales,
churn ratios and numbers of producers and wholesalers. Identification of the key
parameter requires variation in the number of active wholesale firms. In the data
set we use in the following chapter this variation comes from entry of new players
in the market but in other studies the analyst could exploit variation in the number
of producers and wholesalers across regional/separated markets. The empirical test
exploits the effect that changes in the number of players has on the so-called inverse
hedge ratio –defined as the total-to-forward-sales ratio. Interestingly, for the linear
model this ratio is independent of demand intercept and marginal cost so firms with
similar aversion to risk hedge in the same way no matter their marginal costs of
production and the state of the demand.

In a nutshell, the identification arguments are as follows. The incentives of a firm
to trade forward are shaped by three forces. The first two, the risk-hedging effect and
the strategic effect, are pro-contracts. The third is a price effect that arises because
offering forward contracts lowers the spot price and, by arbitrage, the forward price
too. This price effect actually puts a downward pressure on firm’s (expected) profits
and therefore makes forward contracting less attractive.

When the forward market is relatively opaque so that players have a difficult
time to infer deviations from equilibrium play, the strategic effect is hardly present
and the contract cover of a firm is the outcome of trading off the risk-hedging effect
against the price effect. In that situation, as the number of competitors rises, the

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4To be precise, inverse hedge ratio = (spot sales + forward sales) / forward sales.
residual demand of an individual firm becomes less susceptible to demand shocks. Therefore, the incentive to hedge becomes weaker if more suppliers enter the market. The price effect, by contrast, stays constant if the number of suppliers in the market increases. As a result, by virtue of the first force, the inverse hedge ratio increases in the number of competitors.

By contrast, if the forward market is relatively transparent so that price and/or rival firms’ forward positions are regularly and precisely observed, in addition to the risk-hedging and the price effects, the strategic effect plays a significant role. This strategic effect turns out to be stronger as more players are around. This is because the (marginal) gains from affecting the rivals’ spot market strategy rise with the number of competitors. We show that the strategic effect may have a dominating influence in which case inverse hedge ratios turn out to be decreasing in the number of firms. This different impact of the number of firms on inverse hedge ratios in case forward positions can correctly be forecast by the market participants constitutes the source of identification of the strategic effect.

This chapter contributes to the literature that studies forward contracting of (essentially) non-storable goods. Allaz (1992) and Allaz and Vila (1993) show that not only risk-hedging is important for firms to sell contracts in Cournot markets. Kao and Hughes (1997) discuss the role of the observability-of-forward-positions assumption. Mahenc and Salanié (2006) demonstrate that selling forward may have anticompetitive effects when firms compete in prices. Ferreira (2003) also encounters anticompetitive effects when firms are able to sell in a futures market at infinitely many moments prior to the opening of the spot market. Our model extends previous work by examining an n-firm game with risk-averse players that are heterogeneous in their marginal costs of production and in their aversion to risk. We explore two variants of the model, one where firms observe demand shocks before the spot market opens, and one where the spot market remains uncertain. Although we cannot find closed-form solutions for the forward and spot sales of an individual firm, we derive its equilibrium (mean) inverse hedge ratio. These ratios turn out to have similar properties, the most important for our purposes that they fall in the number of players when the forward market is sufficiently transparent.

\footnote{Natural gas can be stored more easily than electricity but the costs of doing so are relatively high, so that for its most part production, delivery and consumption take place contemporaneously.}

\footnote{See also Newbery (1998) and Green (1999), who study models where firms compete in the spot market using supply schedules. These papers show that, owing to the multiplicity of equilibria in supply functions, the optimal contract cover of a firm is intimately linked to the spot market equilibrium actually played.}
There is also work that has focused more on forward and spot price differences in energy markets than on a firm’s optimal contract cover. Longstaff and Wang (2004) document significant risk-premia in electricity forward prices. Bessembinder and Lemmon (2006) derive the equilibrium forward risk premium in a competitive model of the interaction of producers and retailers in the absence of speculators. Our work differs from these papers in that we consider market power on the supply side. This allows us to address the issue of whether firms sell futures contracts to hedge or to gain market power in the spot market.\footnote{An alternative, and complementary, approach to address this issue is to use controlled experiments. In a recent paper, Brandts, Pezanis-Christou and Schram (2008) set up laboratory experiments to study the efficiency effects brought about by the possibility of forward contracting. Working with deterministic demand and cost parameters, risk-hedging can be, if not fully eliminated, at least significantly reduced. Brandts et al. observe that significant price decreases and efficiency gains are obtained compared to the case in which only spot market trading is possible. Additional experimental evidence on the pro-competitive effects of futures contracts is provided by Coq and Orzen (2006). Ferreira, Kujal and Rassenti (2009) challenge this view and present experimental evidence to suggest that forward contracts make collusion more likely. Their results are in line with Liski and Montero (2006).}

The rest of the chapter is organized as follows. The next section presents a two-period model of competition. In the first period, the forward market opens and firms sell futures contracts. In the second period, the spot market opens, firms sell quantities and delivery of all contracted and spot quantities takes places. This section also presents the main empirical prediction of our model. We exploit the equilibrium restrictions to develop an empirical strategy in Section 3.3. After having presented how to test for the existence of a strategic motive to contract forward, we study various extensions to our main framework. In Section 3.4 we generalize our model by allowing for nonlinear demand and general increasing and concave firm utility functions. Section 3.5 analyzes the magnitude of the error when applying the mean-variance approach to solve for firms’ optimal inverse hedge ratio. In Section 3.6 we study a variation of the basic model by assuming that firms cannot condition their spot strategies on the demand realization. The chapter closes with a discussion of the main results and some concluding remarks. The proof for the main proposition of this chapter can be found in the Appendix.

### 3.2 A model of forward and spot contracting

Consider an oligopolistic market with \( n \) asymmetric risk-averse firms selling a homogeneous good. Firms are asymmetric on two accounts: they differ in their degree of
risk-aversion and have different marginal costs of production. Firms can sell output in the spot market; in addition, they can also sell (or buy) the good in a forward market. Let \( s_i \) and \( x_i \) be, respectively, firm \( i \)'s total spot and forward market sales; the total output firm \( i \) supplies on the market will be denoted \( q_i = s_i + x_i \). The marginal cost of production of a firm \( i \) is denoted \( c_i \).

We assume that market demand is random and given by the linear-normal specification

\[
p = a - bQ + \epsilon, \quad \epsilon \sim N(0, \sigma^2),
\]

where \( Q = \sum_{i=1}^{n} q_i \) denotes the aggregate output delivered to consumers and \( \epsilon \) is a zero-mean random shock normally distributed with standard deviation \( \sigma \). We assume that the realization of \( \epsilon \) is observed when the spot market opens. At that stage, a firm \( i \) chooses its spot sales \( s_i \) to maximize its spot market profits.

At the forward market stage, by contrast, firms are uncertain about the price.

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8Forward contracts are contracts traded in the OTC market, while futures contracts change hands in centralized exchanges. Since both types of contracts serve the same objectives, we use them interchangeably throughout the chapter.

9In the natural gas industry, on the supply side of the market we typically encounter producers and wholesalers. The constant marginal cost assumption is probably a good approximation for the costs of extraction and shipping incurred by producers. European wholesalers typically import gas from producing countries such as Norway and Russia. Wholesalers have long-term take-or-pay contracts with foreign producers. Take-or-pay contracts stipulate that the buyer pays for a pre-specified minimum amount of gas, irrespective of whether the gas is actually taken (Masten and Crocker, 1985). Take-or-pay contracts also include a variety of (daily and yearly) flexibility clauses (Asche et al., 2002; IEA, 2002). These clauses provide wholesalers with the necessary flexibility to adjust supply to demand shocks. Take-or-pay contracts are typically indexed to the oil price so the constant marginal cost assumption is also reasonable. Moreover, because wholesalers book import capacity assuming extreme weather conditions, transport capacity is typically not binding. Pipelines also allow for line-pack, that is, for the increase in the amount of gas the system can carry by temporarily raising its pressure.

10Even though producers and wholesalers may differ in that the latter face both price and cost risk, in what follows we will not make a distinction between their economic problems. Note also that in our linear model randomness of the demand is equivalent to randomness on the marginal cost.

11This assumption is not necessary. It reflects the idea that firms are typically well informed about the state of the demand when the spot market opens and use such a market to balance their portfolios. Later in Section 3.6 we develop the case in which at the spot market stage firms are still uncertain about the realization of \( \epsilon \). The main theoretical insights of the model generalize to such case (see Proposition 3.2).

12In our application, the demand side of the market is made of domestic retailers and exporting shippers. We do not model risk aversion on the demand side because retail prices in the Netherlands and the surrounding countries are non-regulated and a great deal of the retail contracts have variable prices (see e.g. von der Fehr and Hansen, 2010).
that will prevail in the market. As a result, at that stage, a firm views its monetary flow of profits as a random variable. We assume firms are risk averse and have constant absolute risk aversion (CARA) utility functions. Let $\pi_i$ be the realized (forward and spot) monetary profits of a firm $i$. The utility function of a firm $i$ is $u(\pi_i) = -e^{-\rho_i \pi_i}$, where $\rho_i > 0$ denotes firm $i$’s degree of risk aversion. Denote by $F(\pi_i)$ the distribution of the monetary profits $\pi_i$ of a firm $i$. (Note that this distribution is endogenous and will be determined later.) Then a firm $i$ will choose its forward sales $x_i$ to maximize its expected utility:

$$E[u(\pi_i)] = \int -e^{-\rho_i \pi_i} dF(\pi_i).$$

(3.2)

For simplicity, future spot market profits are not discounted.

Next, and central to our empirical strategy, we assume that whether firms observe each other’s forward positions is uncertain. To model this idea, we introduce a Bernoulli random variable, denoted $I$, with parameter $\gamma$. If $I = 1$ forward positions become observable to the players and then we get the standard Allaz and Vila (1993) setting. By contrast, if $I = 0$ we obtain the case of unobservable forward trading, as discussed by Kao and Hughes (1997). The parameter $\gamma$ can then be interpreted as the degree of transparency of the forward market. As mentioned in the Introduction, another, and perhaps more compelling, interpretation of this parameter is that it reflects the ability of firms to infer deviations from the equilibrium path by looking at forward price changes.

We further assume there is a fringe of outside speculators. These traders, which are assumed to be risk-neutral, do not have transmission capacity rights so they cannot physically deliver the commodity to the final customers. Speculators compete

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13 The case of risk-neutrality obtains when $\rho_i \to 0$ for all $i$.

14 As it will become clear later, monetary profits are not normally distributed so maximizing (3.2) is not equivalent to maximizing the corresponding mean-variance specification. Section 3.5 analyzes the magnitude of the error when a firm’s optimal contract cover is determined using the mean-variance approach.

15 We model imperfect observability by assuming that firms either observe the (correct) forward price or they miss it altogether. An alternative way to model imperfect observability is by assuming that firms may observe forward prices that are wrong. In that case Bagwell (1995) shows that the value of commitment is fully destroyed; van Damme and Hurkens (1997) show that Bagwell’s striking result relies on an unnecessary restriction to pure strategies.

16 Forward markets are to some extent opaque. Many transactions occur over the counter and therefore are invisible to market participants. Forward price indices are published by brokers and specialized information agencies. Firms may be able to infer rivals’ forward positions upon observation of those indices but the question is how well. We can think of $\gamma$ as the fraction of times the firms are able to forecast how deviations from the equilibrium forward sales will affect the spot market price. In this sense, firms can be seen as being ignorant, or naive, $1 - \gamma$ of the times.
à la Bertrand for the quantities offered in the forward market. We shall also assume that pure financial traders observe sellers’ deviations from the equilibrium path. This assumption is based on the idea that speculators take positions that increase their exposure to risk in an attempt to (weakly) increase their wealth. As such, they have perhaps the strongest incentives to follow upon the forward market developments. For the model at hand, this assumption implies that a strong version of arbitrage (off and on the equilibrium path) between forward and spot markets holds.\footnote{\textsuperscript{17} Ferreira (2006) studies the role of the observability assumption at length. He argues that it is hard to reconcile the assumption that firms are not informed of the rivals’ forward positions with the assumption that speculators observe deviations at the forward stage. As mentioned earlier, in the real-world forecasting quantities sold upon observing forward price indices is far from trivial, specially if firms are heterogeneous. Speculators are not gamblers but highly specialized investors whose profit critically depends on the quality of the forecasts they make.}

The timing of the game is as follows. In the first stage, firms put quantities in the forward market. Then the Bernoulli variable $I$ is realized and forward positions become observable or not. Next, firms learn the demand shock $\epsilon$. Finally, firms compete in quantities in the spot market and total sales are delivered. We now solve the game by backward induction.

### 3.2.1 Spot market stage

At the time the spot market opens, demand is certain and firm $i$ chooses its spot sales $s_i$ to maximize its spot market profits:

$$\pi^s_i = (p - c_i)s_i,$$

where $p$ is the realized spot market price given in (3.1). For this, a firm $i$ takes the spot market strategies and, when observed, the forward positions of the rival firms as given. In case of an opaque forward market, firm $i$ makes a conjecture about the forward sales of its competitors.\footnote{\textsuperscript{18} We require the conjectures to be correct in equilibrium.} The spot market equilibrium turns out to be in linear strategies. Putting the linearity of the strategies up front, we write the spot market strategy of a firm $i$ as

$$s_i = A_i + B_i\epsilon, \ i = 1, \ldots, n. \quad (3.3)$$

When $I = 1$, which occurs with probability $\gamma$, firms observe each other forward positions. In that case, the realization of the spot market price can be written as follows:

$$p = a + (1 - b \sum_{j \neq i} B_j)\epsilon - b \sum_{j \neq i} x_j - b s_i - b \sum_{j \neq i} A_j,$$
where $\sum_{j \neq i} x_j$ is the sum of the actual forward positions of firm $i$’s competitors.

Profit maximization at the spot market stage gives

$$s_i = \frac{1}{2b} (a + (1 - b \sum_{j \neq i} B_j) \epsilon - c_i - bx_i - b \sum_{j \neq i} x_j - b \sum_{j \neq i} A_j).$$

Using (3.3), we can solve for $A_i$ and $B_i$:

$$A_i = a + \sum_{j \neq i} c_j - n c_i - bx_i - b \sum_{j \neq i} x_j \quad \text{and} \quad B_i = \frac{1}{b(n+1)}.$$

Therefore, conditional on the forward positions being observable, the equilibrium spot market output of firm $i$ becomes

$$s_i^{I=1} = \frac{a + \epsilon + \sum_{j \neq i} c_j - n c_i - bx_i - b \sum_{j \neq i} x_j}{b(n+1)}$$

(3.4)

and the equilibrium spot market price equals

$$p_i^{I=1} = \frac{a + \epsilon + \sum_{i} c_i - bx_i - b \sum_{j \neq i} x_j}{n + 1}.$$

The conditional reduced-form profits are then given by

$$\pi_i^{I=1} = b(s_i^{I=1})^2 + (f - c_i)x_i,$$

where $f$ denotes the forward price.

When $I = 0$, which occurs with probability $1 - \gamma$, firms do not observe each other’s actions in the forward market. This implies that deviations of a firm $i$ from the equilibrium path go undetected by the rival players. In that case, the price in the spot market is given by

$$p = a + (1 - b \sum_{j \neq i} B_j) \epsilon - bx_i - b \sum_{j \neq i} \hat{x}_j - b \sum_{j \neq i} A_j,$$

where $\sum_{j \neq i} \hat{x}_j$ is firm $i$’s conjecture about the rivals’ aggregate forward position.

The first order condition (FOC) for the spot market stage yields

$$s_i = \frac{1}{2b} (a + (1 - b \sum_{j \neq i} B_j) \epsilon - c_i - bx_i - b \sum_{j \neq i} \hat{x}_j - b \sum_{j \neq i} A_j).$$

Because firm $i$ does not observe deviations from the conjectured (equilibrium) forward sales of the rival firms, its spot market strategy is only affected by (a change in) its own forward position. We can solve for $\sum_{j \neq i} A_j$ and $\sum_{j \neq i} B_j$:

$$\sum_{j \neq i} A_j = \frac{(n - 1)(a + c_i - b \hat{x}_i - b \sum_{j \neq i} \hat{x}_j) - 2 \sum_{j \neq i} c_j}{b(n+1)} \quad \text{and} \quad \sum_{j \neq i} B_j = \frac{n - 1}{b(n+1)}.$$

(3.5)
As a result, conditional on the forward positions not being observed, the spot market sales of firm $i$ are
\[
s_i^{I=0} = \frac{a + \epsilon + \sum_{j \neq i}^n c_j + b(n - 1)\hat{x}_i/2 - nc_i - b(n + 1)x_i/2 - b\sum_{j \neq i}^n \hat{x}_j}{b(n + 1)}
\] (3.6)
and the realized market price becomes
\[
p^{I=0} = \frac{a + \epsilon + c_i + \sum_{j \neq i}^n c_j + b(n - 1)/2\hat{x}_i - b(n + 1)x_i/2 - b\sum_{j \neq i}^n \hat{x}_j}{n + 1}.
\]
The conditional profit of firm $i$’s equals
\[
\pi_i^{I=0} = b(s_i^{I=0})^2 + (f - c_i)x_i.
\]

### 3.2.2 Forward market stage

At the forward market stage, firms sell (or buy) part of their total output in the futures market to maximize their expected utility\(^{19}\). Note that $u(\pi_i) = Iu(\pi_i^{I=1}) + (1 - I)u(\pi_i^{I=0})$. Using the expressions for the conditional profits derived above, the expected utility of firm $i$ is thus given by
\[
E[u(\pi_i)] = -\gamma \int e^{-\rho_1 \pi_i^{I=1}} f(\epsilon) d\epsilon - (1 - \gamma) \int e^{-\rho_0 \pi_i^{I=0}} f(\epsilon) d\epsilon,
\]
where $f(\epsilon)$ is the density function of the normal distribution with zero mean and variance given by $\sigma^2$.

A firm $i$ picks its amount of forward sales $x_i$ to maximize its expected utility. It is instructive to write the FOC as follows:
\[
\gamma \int \rho_i e^{-\rho_1 \pi_i^{I=1}} \left( \frac{\partial \pi_i^{I=1}}{\partial x_i} + \sum_{j=1}^n \frac{\partial \pi_i^{I=1}}{\partial s_j} \frac{\partial s_j^{I=1}}{\partial x_i} \right) f(\epsilon) d\epsilon + (1 - \gamma) \int \rho_i e^{-\rho_0 \pi_i^{I=0}} \left( \frac{\partial \pi_i^{I=0}}{\partial x_i} + \frac{\partial \pi_i^{I=0}}{\partial s_i} \frac{\partial s_i^{I=0}}{\partial x_i} \right) f(\epsilon) d\epsilon = 0.
\]
In equilibrium $\hat{x}_i = x_i$ for all $i$ and so is $\pi_i^{I=1} = \pi_i^{I=0} = \pi_i$ and $\partial \pi_i^{I=1}/\partial x_i = \partial \pi_i^{I=0}/\partial x_i = \partial \pi_i/\partial x_i$; therefore, this FOC can more compactly be written as follows:
\[
\int \rho_i e^{-\rho_1 \pi_i} \left( -\frac{bx_i}{2} + (1 - \gamma) \frac{bx_i}{2} + \gamma \frac{bx_i + (n - 1)b(s_0 + x_i)}{n + 1} - \frac{\epsilon}{n + 1} + \frac{\gamma(n - 1)\epsilon}{(n + 1)^2} \right) f(\epsilon) d\epsilon = 0.
\]
\(^{19}\)We do not a priori restrict firms’ level of forward trading to be positive. However, in equilibrium each firm will sell a non-negative amount in the forward market.
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\[ \int \rho_i e^{-\rho_i \pi_i} \left( -\frac{bx_i}{2} - \frac{\epsilon}{n + 1} + \frac{b(n - 1)(x_i/2 + s_0)}{(n + 1)} + \frac{\gamma(n - 1)\epsilon}{(n + 1)^2} \right) f(\epsilon) d\epsilon = 0, \]

(3.7)

where

\[ s_0 = \frac{a + \sum_{j \neq i} c_j - nc_i - bx_i + b \sum_{j \neq i} x_j}{b(n + 1)}. \]

The first term of this equation (after the integral sign) represents marginal utility from (monetary) profit, while the term between parentheses is the marginal monetary profit from selling futures contracts. A firm \( i \) chooses its amount of futures \( x_i \) to make the expected value of the product of marginal utility and marginal monetary profit equal to zero.

The incentives of a firm \( i \) to sell forward are shaped by three forces. There is a risk-hedging effect, a strategic effect and a price effect. The first two forces are pro-contracts; the third one dampens the incentives to sell futures. These three forces can actually be seen after taking a closer look at Equation (3.7). To see them more clearly, it is useful to consider the two extreme cases of complete opacity (\( \gamma = 0 \)) and complete transparency (\( \gamma = 1 \)) of the forward market.

When forward positions cannot be observed by the rivals the strategic effect is absent and Equation (3.7) simplifies to

\[ \int \rho_i e^{-\rho_i \pi_i} \left( \frac{\partial \pi_i}{\partial x_i} + \frac{\partial \pi_i}{\partial s_i} \frac{\partial s_i}{\partial x_i} \right) f(\epsilon) d\epsilon = 0, \]

\[ \int \rho_i e^{-\rho_i \pi_i} \left( -\frac{bx_i}{2} - \frac{\epsilon}{n + 1} \right) f(\epsilon) d\epsilon = 0. \]

(3.8)

In parenthesis we see the direct effect of selling forward on a firm’s monetary profits, along with a second effect that goes via its own spot market strategy, \( s_i \). This joint effect is clearly negative and has a deterministic component and a random component. The deterministic component, \(-bx_i/2\), constitutes a negative price effect that is independent of the number of firms. A firm that puts one unit more in the forward market cuts its spot sales by half a unit (see Equation (3.6)), so its total sales increase. This results in a fall in the spot market price, which is anticipated by the speculators and therefore the forward price falls too.\(^{20}\) This own price effect dampens the incentives to put futures in the market, which explains why a risk-neutral monopolist would choose not to engage in forward contracting at all (see

\(^{20}\)The fall in the forward price originates from the assumption that speculators observe the forward quantities (Ferreira, 2006). Since they anticipate a higher total quantity to be delivered when the spot market closes, they correspondingly lower the prices they bid for the quantities on sale in the futures market.)
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Since this effect is always negative no matter the number of firms, it is also true that a risk-neutral oligopolist would opt out of the contract market (Hughes and Kao, 1997).

The random component explains the risk-hedging motive. Note that a firm that increases its contract cover away from zero lowers its exposure to demand shocks by $\varepsilon/(n+1)$ (see Equation (3.8)). This random component is negatively correlated with $\varepsilon$ and so is the marginal utility from monetary profit (since profit increases in $\varepsilon$ and the utility function is concave). As a result, the covariance between marginal utility and marginal profit is positive, giving rise to the risk-hedging incentive of selling forward contracts. When contracts cannot be observed by the market participants, the optimal contract cover of a firm $i$ is the outcome of trading off the (positive) risk-hedging motive against the (negative) price effect.

When forward positions are easily observable by the firms, or can easily be inferred from the forward price, $\gamma = 1$ and (3.7) simplifies to

$$\int \rho_i e^{-\rho_i \pi_i} \left( -\frac{bx_i}{2} - \frac{\varepsilon}{(n+1)} + b(n-1)(x_i/2 + s_0) + \frac{(n-1)\varepsilon}{(n+1)^2} \right) f(\varepsilon) d\varepsilon = 0. \quad (3.9)$$

In this case there is a third, strategic, effect of selling futures. This effect also has a deterministic component and a random component. Suppose the firms are risk-neutral. By the strategic effect, a firm that puts units in the forward market positively affects its profit via the spot market strategies of the rival firms. This gives firm $i$ an incentive to sell forward, since by doing so it benefits from rival firms’ cuts in their spot sales (Allaz and Vila, 1993). The additional random term has a positive sign and this implies that it works counter to the risk-hedging effect discussed above. This is because, since the rival firms cut their spot sales, the effect of putting one unit forward is less effective at lowering exposure to price shocks. However, the aggregate random component in Equation (3.9) is still negative and decreasing in $n$. This tells us that firms have an incentive to hedge against risk also in a transparent forward market, but that the marginal gains from doing it become smaller the more firms are around.

Obviously, the importance of the strategic motive depends on the likelihood forward positions are learnt by the players. Moreover, because the risk-hedging and the strategic effects of selling forward on a firm’s expected utility are intertwined, it is now clear why it is difficult to disentangle them using data from the field. In the remainder of this chapter, our main focus will be on the equilibrium inverse hedge ratio of an individual firm $i$, defined as total-to-forward-sales (or $q_i/x_i$) ratio. Though the forward and the spot sales of an individual firm cannot be computed explicitly, we now show how one can easily derive the stochastic process of a firm’s inverse
hedge ratio. This ratio has useful properties that we will exploit in the empirical application.

After some algebra, the FOC given by (3.7) simplifies to

$$\left( a + \sum_{j \neq i} c_j - nc_i - bx_i - b \sum_{j \neq i} x_j \right) \left( \frac{1}{n + 1} - \frac{2b\gamma + b(n + 1)(1 - \gamma)}{2\rho_i\sigma^2 + b(n + 1)^2} \right) - \frac{x_i}{2(n + 1)} = 0.$$ 

Solving for $x_i$ gives

$$x_i = \frac{2(b(n^2 - 1)\gamma + 2\rho_i\sigma^2)(a + \sum_{j \neq i} c_j - nc_i - b \sum_{j \neq i} x_j)}{b(b(n + 1)^3 - b(n - 1)^2(n + 1)\gamma + 2(3 + \gamma + n(1 - \gamma))\rho_i\sigma^2).}$$

Using equation (3.4), we can write firm $i$’s total output $q_i = s_i + x_i$ as

$$q_i = \frac{n}{n + 1} x_i + \frac{a + \sum_{j \neq i} c_j - nc_i - b \sum_{j \neq i} x_j}{b(n + 1)} + \frac{\epsilon}{b(n + 1)}.$$ 

It will prove convenient to measure the relationship between forward and spot sales by the inverse hedge ratio:

$$\frac{q_i}{x_i} = \frac{b(n + 1)^2(n + 1 + (n - 1)\gamma) + 2(3 + \gamma + (3 - \gamma)n)\rho_i\sigma^2}{2(n + 1)(b(n^2 - 1)\gamma + 2\rho_i\sigma^2) + 1} \epsilon.$$ (3.10)

As can been seen from Equation (3.10), the inverse hedge ratio is normally distributed. The following proposition discusses some more properties of the equilibrium inverse hedge ratio.

**Proposition 3.1** In equilibrium, the mean of the inverse hedge ratio of a firm $i$, defined as total-to-forward-sales ratio, is given by

$$\Gamma_i \equiv \frac{(n + 1)^2(n + 1 + \gamma(n - 1)) + 2(3(n + 1) - \gamma(n - 1))\rho_i\sigma^2}{2(n + 1)(\gamma(n^2 - 1) + 2\rho_i\sigma^2)}.$$ (3.11)

This mean of the inverse hedge ratio, $\Gamma_i$, satisfies the following properties:

(i) It is independent of the demand intercept parameter $a$ and of the firm marginal cost $c_i$, but increases in the demand slope parameter $b$.

(ii) It decreases as the risk-aversion parameter of the firm $\rho_i$ goes up, or as demand volatility $\sigma^2$ increases.

(iii) It decreases as the probability that forward positions are observed $\gamma$ increases.

(iv) There exists a critical parameter $\tilde{\gamma}(n)$ such that: For all $\gamma \leq (\geq)\tilde{\gamma}(n)$, $\Gamma_i$ increases (decreases) in the number of firms $n$.

Moreover, if firms are symmetric ($c_i = c, \rho_i = \rho$ for all $i$), the variance of the inverse hedge ratio of the firms decreases in $a$, $\rho$ and $\sigma^2$, and increases in $c$, $b$ and in $\gamma$. 
The proof is in the Appendix.

The main properties of the inverse hedge ratio as stated in Proposition 3.1 need some further explanation. First, the inverse hedge ratio does not depend on the demand parameter $a$ and the cost parameter $c_i$. This means that firms with similar risk aversion hedge in the same way on average, no matter how much they differ in their marginal cost of production. In addition, it is interesting to see that inverse hedge ratios in periods of demand expansion are similar to those in periods of demand contraction. This result, of course, rests on the linearity assumptions of the demand and cost functions. However, it should be seen as a reasonable approximation that is useful because it allows us to estimate the model without cost and demand data.

Inverse hedge ratios go down when firms becomes more risk averse, when demand uncertainty increases, or when the transparency of the forward market goes up. The former two results are driven by the risk-hedging rationale: the higher the degree of risk aversion (or the greater the uncertainty), the more a firm wants to hedge in the forward market instead of selling spot. The latter is explained by the strategic motive, since a high level of contract cover is worth more to a firm the more convincing the commitment is.

The most interesting feature of the inverse hedge ratio, at least for our purposes, is that whether firm entry/exit has an upward or downward effect on this ratio depends on the extent to which the forward market is transparent. If the futures market is relatively opaque and rivals’ futures contracts go often unobserved, the strategic effect is hardly present and the contract cover of a firm trades off the the risk-hedging effect against the price effect. In that situation the incentive to hedge against demand shocks becomes weaker (see Equation (3.8)) if more suppliers enter the market, which pushes up the inverse hedge ratio of an individual firm. This is because demand uncertainty is revealed before the spot market opens and therefore the demand shock is partly absorbed by the rivals’ spot strategies. As a result, the residual demand of a particular firm at the spot stage is less susceptible to demand shocks the higher the number of competitors. By contrast, the price effect appears not to depend on the number of competitors (see Equation (3.8)). Therefore, it is clear that when the forward market is relatively obscure, inverse hedge ratios increase in the number of suppliers. For the extreme case of $\gamma = 0$ we in fact get

$$\Gamma_i(\gamma = 0) = \frac{6\rho_i \sigma^2 + b(n + 1)^2}{4\rho_i \sigma^2},$$

which goes up in $n$.

If the futures market is more transparent and the contract positions of the rivals
are regularly observed, in addition to the risk-hedging effect and the price effect, the strategic effect plays a significant role. Note from (3.9) that the strategic effect becomes more prominent as more players are active in the market. This is because for a firm the marginal gains from affecting its rivals’ spot market strategies are higher the more competitors it faces. While the risk-hedging and the price effect (weakly) decrease as the number of competitors increases, we show that the strategic effect may have a dominating influence. For the extreme situation $\gamma = 1$, we obtain

$$\Gamma_i(\gamma = 1) = 1 + \frac{1}{n + 1} + \frac{2b}{b(n^2 - 1) + 2\rho_i\sigma^2},$$

which clearly decreases in $n$.

As shown in Proposition 3.1, for intermediate cases, the relationship between the number of firms and the inverse hedge ratio depends on how likely it is that forward positions will be observed. This is illustrated in Figure 3.1.

![Figure 3.1: The inverse hedge ratio and the number of firms ($\rho_i = 4$, $\sigma^2 = 1$, $b = 1$)](image)

### 3.3 Empirical strategy

We seek to answer the question whether firms sell futures for strategic reasons, for risk-hedging considerations, or for both. If firms’ forward positions were totally opaque to the agents in the market, strategic reasons would not play any role and therefore the observed inverse hedge ratios in the data would only be explained by risk-hedging considerations. Answering this inquiry thus amounts to finding out the extent to which firms can observe each other’s forward sales. Or, in different words,
figuring out the ability of the players to forecast correctly the aggregate position of the rivals’ upon observing the forward price/index.

If one were provided with firms’ forward and spot sales data corresponding to various levels of observability and risk, one could estimate the effects of these two factors on the hedge ratio. The problem with this approach is that these variables are hard to measure. One could for example gather data from different regional markets. Within-market price dispersion could be used as a measure of price risk. However, it is not clear how to measure the extent to which firms are capable of deducing deviations from changes in the forward price. The analyst would have to make a priori assumptions about the observability parameter \( \gamma \). Instead, we propose to estimate the model in Section 3.2 structurally. In the next chapter, we do this for the Dutch wholesale market for natural gas. For this market, we have obtained the minimal set of data: forward and spot sales, churn ratios and number of producers and wholesalers. We will now discuss the empirical approach that can be applied when one collects a data set similar to the one we have for the Dutch gas market.

The variable of interest is the inverse hedge ratio of an individual firm \( i \). As shown above, the inverse hedge ratio of an individual firm \( i \) has a random generating process given by

\[
\frac{q_i^*}{x_i^*} = \frac{b(n+1)^2(1+n+(n-1)\gamma) + 2(3+\gamma+(3-\gamma)n)\rho_i \sigma^2}{2(n+1)(b(n^2-1)\gamma + 2\rho_i \sigma^2)} + \frac{1}{b(n+1)x_i^*}\epsilon. \tag{3.12}
\]

We can rewrite equation (3.12) as

\[
(n+1)q_i^* = \frac{(n+1)^2(1+n+(n-1)\gamma) + 2(3+\gamma+(3-\gamma)n)\rho_i \sigma^2}{2((n^2-1)\gamma + 2\rho_i \sigma^2)}x_i^* + \frac{1}{b}\epsilon. \tag{3.13}
\]

Using individual firm level data on total quantities brought to the market, forward sales and number of wholesalers, the system of equations in (3.13) can be fitted to the data. Identifying assumptions are that the slope of the demand function \( b \) and the demand volatility parameter \( \sigma \) are constant over the sample period; moreover, we assume that variation in the number of players \( n \) is exogenous.\(^{21}\) Since these equations are non-linear in the parameters of interest, one has to apply non-linear least squares (NLS). Notice that the parameters \( \rho_i, \sigma \) and \( b \) cannot be separately identified. However, with the appropriate (firm-level) data, one could estimate relative risk-aversion parameters across firms, i.e. \( \rho_i/\rho_j \). The identification of these

\(^{21}\)It may be argued that the number of players is determined jointly with the hedge ratios. Economic theory suggest the demand and cost parameters as clear determinants of \( n \). If this dependence is not clearly controlled for by the variation in the forward sales of a firm \( x_i \), which clearly depend on \( a \) and \( c_i \), then an endogeneity problem would arise. In the following chapter, where we apply the empirical strategy to the Dutch gas market, we deal with this issue.
parameters would stem from variation in the hedge ratios of the different firms. If some firms were systematically seen to hedge more than others, this would provide information on the extent to which the former firms have greater risk-aversion than the latter.

The identification of the observability parameter \( \gamma \) stems from variation in the number of wholesalers \( n \). As shown above, when forward positions are relatively transparent to the firms, the model predicts that an individual firm responds to entry by decreasing its inverse hedge ratio. By contrast, if firms’ forward sales are relatively opaque and rivals’ rarely observe them, then an individual firm increases its inverse hedge ratio as a response to entry. It is precisely this differential effect of entry that enables the identification of the observability parameter \( \gamma \) using data containing variation in the number of players.

Unfortunately, due to confidentiality reasons, individual firm level data are often not publicly available so one has to rely on aggregate forward and spot sales. To work with this type of data, we proceed by aggregating at the market level. For this we need to assume that all firms have similar risk aversion parameters, i.e., \( \rho_i = \rho \) for all \( i = 1, 2, ..., n \). Summing up for all firms in (3.13), we get

\[
\frac{n+1}{n} \sum_{i=1}^{n} q_i = \frac{b(n+1)^2(1+n+(n-1)\gamma) + 2(3+\gamma+(3-\gamma)n)\frac{\sigma^2}{b}}{2n((n^2-1)\gamma + 2\frac{n\sigma^2}{b})} \sum_{i=1}^{n} x_i + \frac{1}{b} \epsilon.
\]

(3.14)

Note that \( \sum_{i=1}^{n} q_i = \sum_{i=1}^{n} s_i + \sum_{i=1}^{n} x_i \), so that equation (3.14) can be rewritten as

\[
\frac{n+1}{n} \sum_{i=1}^{n} s_i = \frac{n+1}{n} (\Gamma(\cdot) - 1) \sum_{i=1}^{n} x_i + \frac{\epsilon}{b},
\]

(3.15)

where \( \Gamma(\cdot) \) is the hedge ratio as defined in Proposition 3.1 for symmetric firms. This equation can be fit to the data by applying NLS. In Chapter 4, we apply the empirical strategy described above to the Dutch wholesale market for natural gas.

### 3.4 A general model of forward contracting

In this section, we show that the effects of forward contracting discussed in the previous section carry over to a more general version of our basic framework. We continue to have \( n \) firms producing a homogeneous good at constant (firm-specific) marginal cost \( c_i \). Sellers now face the following general inverse demand function:

\[
p = P(Q) + \epsilon, \quad \epsilon \sim (0, \sigma^2),
\]

where \( Q = \sum_{i=1}^{n} q_i \) and \( \epsilon \) is a, not necessarily normally distributed, additive random shock with zero mean and variance \( \sigma^2 \). It is assumed that the deterministic part of
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the inverse demand function, $P(Q)$, is twice differentiable, monotonically decreasing in $Q$ and (weakly) concave. Further, we maintain the assumption that firms get to know the realization of the demand shock before the spot market opens.

Firm $i$ chooses its forward position and spot sales so as to maximize its expected utility, where utility is now represented by the function $u_i = U_i(\pi_i)$, with $U' > 0$ and $U'' < 0$. This function captures a wide range of risk preferences, including constant absolute risk aversion (CARA) and constant relative risk aversion (CRRA). Again, forward positions are observed only with probability $\gamma$.

3.4.1 The spot market stage

At the time the spot market opens, demand uncertainty is resolved and firm $i$ chooses its spot sales $s_i$ to maximize its profits, given its own forward position and, if observed, the forward strategies of its rivals. In case of observability, the system of $n$ spot market FOCs is given by

$$P \left( s_i + x_i + \sum_{j \neq i}^n (s_j + x_j) \right) + \epsilon - c_i + P' s_i = 0, \quad i = 1, 2, \ldots, n. \quad (3.16)$$

Let $\{s_i^{I=1}\}_{i=1}^n$ denote the firms’ spot sales that solve this system of FOCs. When the forward market institution is not transparent, the optimal spot outputs are given by $\{s_i^{I=0}\}_{i=1}^n$ and solve the following system of FOCs:

$$P \left( s_i + x_i + \sum_{j \neq i}^n (s_j + \hat{x}_j) \right) + \epsilon - c_i + P' s_i = 0, \quad i = 1, 2, \ldots, n, \quad (3.17)$$

where $\hat{x}_j$ denotes firms’ conjectures about the forward position of producer $j$.

To see how firm $i$ affects its own spot output and the spot strategies of rival firms by selling in a transparent forward market, we first differentiate the system of FOCs, given by (3.16), to get

$$\text{M}x = -\text{b},$$

where

$$\text{M} = \begin{pmatrix}
2P' + P'' s_1 & P' + P'' s_1 & \cdots & P' + P'' s_1 \\
P' + P'' s_2 & 2P' + P'' s_2 & \cdots & P' + P'' s_2 \\
& \vdots & \ddots & \vdots \\
P' + P'' s_n & P' + P'' s_n & \cdots & 2P' + P'' s_n
\end{pmatrix},$$

$$x = \begin{bmatrix}
\frac{\partial s_1}{\partial x_i} & \frac{\partial s_2}{\partial x_i} & \cdots & \frac{\partial s_n}{\partial x_i}
\end{bmatrix}'$$
Do firms sell forward for strategic reasons? A test based on the theory

and

\[ b = [P' + P''s_1 \ P' + P''s_2 \ \cdots \ P' + P''s_n]' . \]

Note that \( M \) can be written as \( M = G + H \), where \( G = P'I \) (\( I \) being the identity matrix) and \( H \) is a matrix of rank one with columns \([P' + P''s_1 \ P' + P''s_2 \ \cdots \ P' + P''s_n]'\). Then from Miller (1981) we know that the inverse of \( M \) can be written as

\[ M^{-1} = (G + H)^{-1} = G^{-1} - \frac{1}{1 + g}G^{-1}HG^{-1}, \tag{3.18} \]

where \( g = trHG^{-1} \). Substituting \( G^{-1} \) and \( H \) into (3.18) then yields

\[ M^{-1} = \frac{1}{P'} \begin{pmatrix} 1 - \alpha_1 & -\alpha_1 & \cdots & -\alpha_1 \\ -\alpha_2 & 1 - \alpha_2 & \cdots & -\alpha_2 \\ \vdots & \vdots & \ddots & \vdots \\ -\alpha_n & -\alpha_n & \cdots & 1 - \alpha_n \end{pmatrix}, \]

where

\[ \alpha_j \equiv \frac{P' + P''s_j}{(n+1)P' + P'' \sum_{i=1}^{n} s_i}, \quad j = 1, 2, \ldots, n. \]

It thus follows that

\[ \frac{\partial s^I_{j=1}}{\partial x_i} = -\frac{P' + P''s_j}{(n+1)P' + P'' \sum_{i=1}^{n} s_i} < 0, \quad j = 1, 2, \ldots, n, \tag{3.19} \]

which tells us that firm \( i \)'s forward position has a negative effect on both firm \( i \)'s own spot sales and the spot output of its competitors.

When the hedging decisions remain hidden, firm \( i \)'s forward output has no influence on the rivals’ spot strategies. To see how firm \( i \) adjusts its spot supply due to a change in its own forward position, we differentiate (3.17) with respect to \( x_i \) to obtain

\[ \frac{\partial s^I_{j=0}}{\partial x_i} = -\frac{P' + P''s_i}{2P' + P''s_i} < 0. \tag{3.20} \]

### 3.4.2 The forward market stage

Moving one stage back, firm \( i \) chooses the amount of contracts to maximize its expected utility:

\[ \max_{x_i} E \left[ U \left( \gamma \pi^I_{i=1} + (1 - \gamma)\pi^I_{i=0} \right) \right], \]

where

\[ \pi^I_{i=1} = P \left( s^I_{i=1}(x_i) + \sum_{j \neq i} s^I_{j=1}(x_i) + x_i + \sum_{j \neq i} \hat{x}_j \right) + \epsilon - c_i \]

\[ s^I_{i=1} + (f - c_i)x_i \]
and
\[ \pi^I_i = 0 = \left( P \left( s^I_i(x_i) + \sum_{j \neq i} s^I_j(\hat{x}_i) + x_i + \sum_{j \neq i} \hat{x}_j \right) + \epsilon - c_i \right) s^I_i + (f - c_i) x_i \]
are the full profit expressions at the forward stage. We retain the assumption that the forward market is efficient, so \( f = E(p) = P \left( \sum_{i=1}^{n} E s_i + \sum_{i=1}^{n} x_i \right) \). The optimal level of forward contracting solves
\[ E [U'(\cdot) \Upsilon(\cdot)] = 0, \]
where
\[ \Upsilon(\cdot) \equiv \gamma \left( \frac{\partial \pi^I_i}{\partial x_i} + \frac{\partial \pi^I_i}{\partial s_i} \frac{\partial s^I_i}{\partial x_i} + \sum_{j \neq i} \frac{\partial \pi^I_i}{\partial s_j} \frac{\partial s^I_j}{\partial x_i} \right) + (1 - \gamma) \left( \frac{\partial \pi^I_i}{\partial x_i} + \frac{\partial \pi^I_i}{\partial s_i} \frac{\partial s^I_i}{\partial x_i} \right). \]
Applying the equilibrium property \( \hat{x}_i = x_i \) and \( s^I_i = s^I_i = s_i \) for all \( i \) yields \( \pi^I_i = \pi^I_i = \pi_i \) and \( \partial \pi^I_i / \partial x_i = \partial \pi^I_i / \partial x_i = \partial \pi_i / \partial x_i \); therefore, the FOC simplifies to
\[ E \left[ U' \left( \frac{\partial \pi_i}{\partial x_i} + \gamma \left( \frac{\partial \pi^I_i}{\partial s_i} \frac{\partial s^I_i}{\partial x_i} + \sum_{j \neq i} \frac{\partial \pi^I_i}{\partial s_j} \frac{\partial s^I_j}{\partial x_i} \right) + (1 - \gamma) \left( \frac{\partial \pi^I_i}{\partial x_i} + \frac{\partial \pi^I_i}{\partial s_i} \frac{\partial s^I_i}{\partial x_i} \right) \right) \right] = 0. \]
Suppose that the strategic effect is not present, so \( \gamma = 0 \). Then, firm \( i \) trades forward contracts up to the point where
\[ Cov \left( U', \left( \frac{\partial \pi_i}{\partial x_i} + \frac{\partial \pi^I_i}{\partial s_i} \frac{\partial s^I_i}{\partial x_i} \right) \right) + E(U')E \left( \frac{\partial \pi_i}{\partial x_i} + \frac{\partial \pi^I_i}{\partial s_i} \frac{\partial s^I_i}{\partial x_i} \right) = 0. \quad (3.21) \]
Notice that
\[ \frac{\partial \pi_i}{\partial x_i} + \frac{\partial \pi^I_i}{\partial s_i} \frac{\partial s^I_i}{\partial x_i} = P' s_i + f - c_i + \frac{\partial f}{\partial x_i} x_i + (p - c_i + P' s_i) \frac{\partial s^I_i}{\partial x_i} + \frac{\partial f}{\partial E s^I_i} E \frac{\partial s^I_i}{\partial x_i} x_i. \quad (3.22) \]
where the last term yields the effect of forward sales on the forward price via the change in speculators’ expectation of the spot output. Using the FOC at the spot stage and taking into account that \( \partial f / \partial E s^I_i = \partial f / \partial x \), (3.22) simplifies to
\[ \frac{\partial \pi_i}{\partial x_i} + \frac{\partial \pi^I_i}{\partial s_i} \frac{\partial s^I_i}{\partial x_i} = f + \frac{\partial f}{\partial x_i} \left( 1 + E \frac{\partial s^I_i}{\partial x_i} \right) x_i - p. \]
Equation (3.21) can then be rewritten as follows:
\[ Cov(U', -p) + E(U') \frac{\partial f}{\partial x_i} \left( 1 + E \frac{\partial s^I_i}{\partial x_i} \right) x_i = 0. \quad (3.23) \]
It is easy to see that the covariance term in equation (3.23), which represents the risk-hedging effect, is strictly positive. First note that  \[ \frac{\partial U'}{\partial \epsilon} = U'' \frac{\partial \pi_i}{\partial \epsilon}. \] Since by assumption \( U'' < 0 \) and

\[ \frac{\partial \pi_i}{\partial \epsilon} = \frac{\partial p}{\partial \epsilon} s_i + (p - c_i) \frac{\partial s_i}{\partial \epsilon} > 0, \] \hfill (3.24)

we get  \[ \frac{\partial U'}{\partial \epsilon} < 0. \] Now, provided that the spot price increases in the demand shock (see Footnote 22), one obtains that  \[ \text{Cov}(U', -p) > 0. \]

Next, the second term of equation (3.23) represents the price effect. To find the sign of this term, first notice that from equation (3.20) we get

\[ \frac{\partial s_i}{\partial x_i} = -\left( \frac{P'}{P'} + \frac{(\sum_{j \neq i} s_j - (n-1)s_i)}{P'} \right) \frac{\partial s_i}{\partial x_i} > -1 \quad \forall s_i \geq 0. \]

This also implies that

\[ E \left[ \frac{\partial s_i}{\partial x_i} \right] = -\int \frac{P'}{2P'} + \frac{P'' s_i(\epsilon)}{P'} f(\epsilon) d\epsilon > -\int f(\epsilon) d\epsilon = -1. \]

Now, given that an increase in the forward obligations pushes down the forward price, so \( \partial f/\partial x_i < 0 \), the second term of Equation (3.23) becomes negative. This shows the utility-reducing price effect of selling forward. Concluding, in case of an opaque forward market firms sell forward only for risk-hedging reasons, though this incentive is weakened by the negative price effect.

In case the strategic effect is present, we have \( \gamma = 1 \) and firm \( i \) then chooses the level of forward sales that solves

\[ E \left[ U' (\cdot) \left( \frac{\partial \pi_i}{\partial x_i} + \frac{\partial s_i^{l=1}}{\partial x_i} + \frac{\partial s_i^{l=1}}{\partial x_i} + \sum_{j \neq i} \frac{\partial \pi_i^{l=1}}{\partial x_i} \frac{\partial s_j^{l=1}}{\partial x_i} \right) \right] = 0. \] \hfill (3.25)

Taking into account the FOC at the spot stage, equation (3.19) and \( \partial f/\partial s_i^{l=0} = \partial f/\partial x \), we get

\[ \frac{\partial \pi_i}{\partial x_i} + \frac{\partial s_i^{l=1}}{\partial x_i} = f + \frac{\partial f}{\partial x} \left( 1 + E \frac{\partial s_i^{l=1}}{\partial x_i} \right) x_i - p. \]

Footnote 22: To find the effect of the demand shock on the spot output, it is easily obtained that \[ \frac{\partial s_i}{\partial \epsilon} = \left[ M_i^{-1} - \left( \frac{1}{P'} \sum_{j \neq i} s_j - (n-1)s_i \right) \frac{P''}{P'} \right] \frac{\partial s_i}{\partial \epsilon}, \] where \( M_i^{-1} \) is the \( i \)th row of the matrix in (3.18) and \( \iota \) is the unit vector (of size \( n \)). Thus, the demand shock has a positive impact on firm \( i \)'s spot sales as long as firms are relatively symmetric or demand is not too concave. Next, we obtain

\[ \frac{\partial p}{\partial \epsilon} = 1 + \sum_{i=1}^{n} \frac{P'}{(n+1)P'} \frac{\partial s_i}{\partial \epsilon} = \frac{p' + P'' \sum_{i=1}^{n} s_i}{(n+1)P'} \sum_{i=1}^{n} s_i > 0. \]
This implies that equation \((3.25)\) can be rewritten as follows:

\[
Cov(U', -p) + E(U') \frac{\partial f}{\partial x_i} \left( 1 + E \frac{\partial s_i^\leq 1}{\partial x_i} \right) x_i + E \left[ U'(\cdot) \sum_{j \neq i}^n \frac{\partial \pi_i^\leq 1}{\partial s_j^\leq 1} \frac{\partial s_j^\leq 1}{\partial x_i} \right] = 0.
\]

(3.26)

The first and second term of (3.26) again represent the risk-hedging motive and negative price effect, respectively. The third term of the FOC represents the strategic motive of forward selling and equals

\[
E \left[ U'(\cdot) \sum_{j \neq i}^n \frac{\partial \pi_i^\leq 1}{\partial s_j^\leq 1} \frac{\partial s_j^\leq 1}{\partial x_i} \right] = -E \left[ U'(\cdot) \sum_{j \neq i}^n \left( P's_i \Theta + \frac{\partial f}{\partial Es_j} x_i E \Theta \right) \right],
\]

(3.27)

where

\[
\Theta \equiv \frac{P'' + P''s_j}{(n + 1)P' + P'' \sum_{i=1}^n s_i}.
\]

Expanding this term is far from trivial, since it depends on more than two random variables and, on top of that, some of these random variables enter in a rather complicated way. However, if we assume that \(P'' = 0\) and that \(P'\) is non-random, as is the case with linear demand, Equation (3.27) reduces to

\[
E \left[ U'(\cdot) \sum_{j \neq i}^n \frac{\partial \pi_i^\leq 1}{\partial s_j^\leq 1} \frac{\partial s_j^\leq 1}{\partial x_i} \right] = -E \left[ U'(\cdot) \sum_{j \neq i}^n \left( P's_i \Theta + \frac{\partial f}{\partial Es_j} x_i E \Theta \right) \right] = -\frac{(n - 1)P'}{n + 1} Cov(U', s_i) - E[U'] \frac{(n - 1)P'(Es_i + x_i)}{n + 1},
\]

(3.28)

where we have used \(\frac{\partial f}{\partial Es_j} = P'\).

We already obtained that \(\frac{\partial U'}{\partial \epsilon} < 0\) and (under weak assumptions, see Footnote 22) \(\frac{\partial s_i}{\partial \epsilon} > 0\), so the covariance term in (3.28) becomes negative. Now, given that

\[
-E[U'] \frac{(n - 1)P'(Es_i + x_i)}{n + 1} > 0,
\]

the sign of the strategic effect is ambiguous. We get that (3.28) becomes positive as long as marginal utility and spot output are not highly negatively correlated, which at least holds for linear demand. If this is so, the strategic effect becomes positive and provides firms an additional incentive to sell forward.

-\(^{\text{23}}\)Note though that the price effect is less strong when forward positions are observed, as \(|\partial s_i^\leq 1/\partial x_i| < |\partial s_i^0/\partial x_i|\).
3.5 Mean-variance approximation

A commonly used approach to study an agent’s optimal behavior under risk aversion is the mean-variance model. Its attractiveness lies in its relative simplicity and its property to give linear demand functions for financial assets. Furthermore, as long as an agent’s payoff is normally distributed the mean-variance approach yields identical results as when determining an agent’s optimal choices under CARA utility. Unfortunately, in case the monetary payoff is not normally distributed, like in our framework, using the mean-variance model results in incorrect equilibrium conditions if agents maximize a CARA utility function. In this section, we study the magnitude of this error in the expected inverse hedge ratio, as this is our main variable of interest.

In order to find the size of the error, we first derive the equilibrium results under the mean-variance approach. Suppose firm $i$ maximizes the mean-variance criterion:

$$W_i = E(\pi_i) - \frac{\rho_i}{2} V(\pi_i), \tag{3.29}$$

where $\rho_i$ is again firm $i$’s risk-aversion parameter and $E(\pi_i)$ and $V(\pi_i)$ are the expected value and variance of firm $i$’s profit, respectively. Since we still assume that the realization of the demand shock takes place at the time the spot market opens, firms’ spot strategies are the same as in equations (3.4) (when forward sales become observable) and (3.6) (when forward sales remain unobserved). This implies that, when moving one stage back, the expected profit of firm $i$ can be written as

$$E(\pi_i) = \gamma E\left(b(s^I_i=1)^2\right) + (1 - \gamma) E\left(b(s^I_i=0)^2\right) + (f - c)x_i$$

$$= \gamma b(s^I_0=1)^2 + (1 - \gamma) b(s^I_0=1)^2 + \frac{\sigma^2}{b(n+1)^2} + (f - c)x_i, \tag{3.30}$$

where

$$s^I_0 = \frac{a + \sum_{j \neq i} c_j - nc_i - bx_i + b \sum_{j \neq i} x_j}{b(n+1)}$$

and

$$s^I_0 = \frac{a + \sum_{j \neq i} c_j + b(n-1)\hat{x}_i/2 - nc_i - b(n+1)x_i/2 - b \sum_{j \neq i} \hat{x}_j}{b(n+1)}$$

are, respectively, the deterministic parts of firm $i$’s spot strategy in case of observability and non-observability. The variance of the profit then becomes

$$V(\pi_i) = E\left[(\pi_i - E(\pi_i))^2\right] = E\left[\gamma \left(\pi^I_i = 1 - E(\pi_i)\right)^2 + (1 - \gamma) \left(\pi^I_i = 0 - E(\pi_i)\right)^2\right]$$

$$= \gamma(1 - \gamma) (b(s^I_0=1)^2 - b(s^I_0=0)^2)^2$$

$$+ \frac{4\sigma^2}{(n+1)^2} (\gamma(s^I_0=1)^2 + (1 - \gamma)(s^I_0=0)^2). \tag{3.31}$$
Substituting (3.30) and (3.31) into Equation (3.29) and maximizing the mean-variance criterion with respect to \( x_i \) yields the optimal forward sales for firm \( i \):

\[
x_{i \text{MV}} = \frac{2(b\gamma(n-1)(n+1)^2 + 2(n+1 - (n-1)\gamma)\rho_i \sigma^2)}{b(b(n+1)^4 - b\gamma(n^2 - 1)^2 + 4(1 + \gamma + (1-\gamma)n)\rho_i \sigma^2)} a + \sum_{j \neq i} c_j - nc_i - b \sum_{j \neq i} x_j.
\]

The expected equilibrium inverse hedge ratio becomes

\[
\Gamma_{i \text{MV}} = \frac{(n+1)^2(n+1 + (n-1)\gamma) + 4(n+1 - (n-1)\gamma)\rho_i \sigma^2}{2(n-1)(n+1)^2(\gamma + 2\rho_i \sigma^2) + 2(n+1 - (n-1)\gamma)\rho_i \sigma^2}.
\] (3.32)

The (mean of the) inverse hedge ratio under the mean-variance criterion has similar properties as the ratio obtained in our basic framework: it decreases in \( \gamma \), \( \rho_i \) and \( \sigma^2 \), increases in \( b \) and goes down in \( n \) if \( \gamma \) is sufficiently large. To see the magnitude of the error from using the mean-variance framework, we subtract (3.32) from (3.11) to get

\[
\Delta \Gamma_i = \Gamma_i - \Gamma_{i \text{MV}} = \frac{2((n+1 - (n-1)\gamma)\rho_i \sigma^2)^2}{(n+1)((n^2 - 1)\gamma + 2\rho_i \sigma^2)((n-1)(n+1)^2\gamma + 2(n+1 - (n-1)\gamma)\rho_i \sigma^2)} \in (0, 1/2].
\]

In case of no observability at all (\( \gamma = 0 \)), the error becomes 1/2, irrespective of the other parameter values. Our data reveals that this loss of accuracy seems to be quite large, given that the inverse hedge ratio is usually between 1 and 2. However, the size of the error goes down in \( \gamma \). When there is full observability in the forward market (\( \gamma = 1 \)), the accuracy loss becomes

\[
\Delta \Gamma_i(\gamma = 1) = \frac{8(\rho_i \sigma^2)^2}{(n+1)((n^2 - 1) + 2\rho_i \sigma^2)((n-1)(n+1)^2 + 4\rho_i \sigma^2)}. \tag{3.33}
\]

It can be seen from Equation (3.33) that in case forward positions are observed with certainty, the error is smaller than \( 1/(n+1) \) and rapidly converges to zero when \( n \) becomes large (relative to \( \rho_i \sigma^2/b \)). For \( \gamma \) being strictly positive, we also notice that \( \Delta \Gamma_i \) increases in \( b \) and decreases in \( \rho \) and \( \sigma^2 \). Finally, to see how the loss of accuracy changes in the number of firms, we add to Figure 3.1 the relationship between the inverse hedge ratio and \( n \) for different values of \( \gamma \) under the mean-variance approach (displayed by the dashed curves in Figure 3.2). Figure 3.2 then shows that the error becomes smaller when more firms enter the market; this effect is especially apparent when there is a high degree of observability.
Do firms sell forward for strategic reasons? A test based on the theory

Figure 3.2: The inverse hedge ratio and the number of firms ($\rho = 4$, $\sigma^2 = 1$, $b = 1$)

We can compare our outcomes with the results obtained by Newbery (1988), who investigates the loss of accuracy when futures sales and prices are derived using the mean-variance model in case firm’s income is the product of two normally distributed variables (i.e. a random spot price and random output). He shows that the error in computing the hedge ratio is quite large (near 30 percent) when producers are rather heterogeneous, the correlation between price and production is low and the futures market is biased. We notice that Newbery studies a model where a firm’s total output is exogenously given, which implies that it is not affected by the firm’s forward position. Conversely, when determining the error in the (inverse) hedge ratio in our case both the numerator and the denominator change. Furthermore, another difference between Newbery’s analysis and our’s is that he considers a competitive framework where producers cannot influence prices nor their rivals’ strategies while our suppliers have market power.

3.6 Uncertain spot market

Until now, we have assumed that firms operate in the spot market in the absence of uncertainty. Therefore, we have modeled a market where firms observe the demand shocks before they decide how much gas to put in the spot market. Arguably, the demand may still remain uncertain at that moment. In such a case, firms cannot condition their spot strategies on the demand realizations. To see whether our results depend on this assumption, we now develop a model in which firms do not observe price shocks before the spot market opens. Of course, as before, the difference be-
tween the forward and the spot market is that at the forward stage the firms can lock-in a price for their forward quantities, while the price received for their spot quantities is random.

In this new setting, a firm $i$ that chooses its spot market strategy $s_i$ aims at maximizing the expected utility from its spot market profits:

$$E[u(\pi^s_i)] = \int -e^{-\rho(p-c_i)s_i} f(\epsilon) d\epsilon.$$  

To simplify the derivations, let us focus right away on the symmetric case where all firms are equally risk averse and have similar marginal costs of production. Given a forward strategy profile, when $I = 1$ the equilibrium spot market output of a firm $i$ solves the first order condition:

$$s^I=1_i = \frac{a - c - bx_i - b(n - 1)x_{-i}}{b(n + 1) + \rho \sigma^2},$$

where $x_i$ denotes firm $i$’s forward sales and $x_{-i}$ refers to the forward sales of firms other than $i$. In this case, the conditional reduced-form equilibrium profit is

$$\pi^I=1_i = (b + \rho \sigma^2) s^I=1_i + \epsilon s^I=1_i + (f - c)x_i,$$

where $f$, as above, denotes the forward price.

When forward positions of the rival firms are not observable, $I = 0$, the equilibrium spot market output of firm $i$ is given by

$$s^I=0_i = \frac{(2b + \rho \sigma^2)(a - c - b(n - 1)\hat{x}_{-i}) + b^2(n - 1)\hat{x}_i - b(b(n + 1) + \rho \sigma^2)x_i}{(2b + \rho \sigma^2)(b(n + 1) + \rho \sigma^2)},$$

where $\hat{x}_i$ denotes rival firms’ conjecture about the position of firm $i$, and $\hat{x}_{-i}$ refers to the conjectures about the forward positions of firms other than $i$. The equilibrium payoff in this case of no observability equals

$$\pi^I=0_i = ((b + \rho \sigma^2) s^I=0_i + \epsilon) s^I=0_i + (f - c)x_i. \quad (3.34)$$

At the forward market stage, a firm $i$ picks its amount of forward sales $x_i$ to maximize its expected utility. The equivalent to the FOC in equation (3.7) is given by

$$\int \rho e^{-\rho \pi} \Lambda(\cdot) f(\epsilon) d\epsilon = 0,$$

where

$$\Lambda(\cdot) \equiv -bx + \rho \sigma^2 s + (1 - \gamma) \frac{b(bx - \rho \sigma^2 s - \epsilon)}{2b + \rho \sigma^2} - \gamma \frac{b(\rho \sigma^2 s - bx - (n - 1)b(s + x) + \epsilon)}{b(n + 1) + \rho \sigma^2}.$$

Notice that the results from maximizing CARA utility coincide with outcomes under the mean-variance method, since the a firm’s payoff is now normally distributed.
Do firms sell forward for strategic reasons? A test based on the theory

and where we have imposed the symmetry of equilibrium strategies and the correctness of equilibrium conjectures conditions, i.e. \( x_i = x_{-i} = \hat{x}_{-i} = \hat{x}_{-i} \). Solving for the equilibrium forward sales we obtain

\[
x = \frac{(a - c) \left(2 + \frac{\rho \sigma^2}{b}\right) \left(\gamma(n - 1) + (n + 1) \frac{\rho \sigma^2}{b} + \rho^2 \sigma^4\right)}{b \Phi},
\]

where

\[
\Phi \equiv \gamma(n - 1)^2 + (n + 1)^2 + (3(n + 1)^2 - 2\gamma(n - 1)) \frac{\rho \sigma^2}{b} + (n(n + 5) - \gamma(n - 1) + 3) \frac{\rho^2 \sigma^4}{b^2} + (n + 1) \frac{\rho^3 \sigma^6}{b^3}.
\]

Building on this expression, we can state the following result.

**Proposition 3.2** Suppose that firms do not observe the demand shocks before the spot market opens. Then the inverse hedge ratio of a firm \( i \) is deterministic and given by

\[
\hat{\Gamma} \equiv \frac{\gamma(n - 1) + (n + 1 + \frac{\rho \sigma^2}{b})(1 + 3 \frac{\rho \sigma^2}{b} + \left(\frac{\rho \sigma^2}{b}\right)^2)}{2 + \frac{\rho \sigma^2}{b}(\gamma(n - 1) + \frac{\rho \sigma^2}{b}(n + 1 + \frac{\rho \sigma^2}{b}))}.
\] (3.35)

The inverse hedge ratio \( \hat{\Gamma} \) is independent of the demand intercept parameter \( a \) and of the firm marginal cost \( c \), increases in the demand slope parameter \( b \), decreases in \( \rho \) and in \( \sigma^2 \), decreases in \( \gamma \) and is (weakly) monotonically decreasing in the number of firms \( n \).

Compared to the inverse hedge ratios in Proposition 3.1, we see that when spot market strategies cannot be adapted to accommodate the demand shocks, inverse hedge ratios are always (weakly) decreasing in the number of players. The intuition behind this result is as follows. As before, the incentives of a firm to sell forward are governed by a price effect, a risk-hedging effect and a strategic effect. The price effect is again constant in the number of firms. Since firms cannot adapt their spot strategies to demand fluctuations, the risk-hedging effect turns out to be independent of the number of firms too. In fact, when forward positions are not observable, the inverse hedge ratios are constant in the number of players:

\[
\hat{\Gamma}(\gamma = 0) = 1 + \frac{1}{2} \left(\frac{1}{\rho \sigma^2} + \frac{1}{2 + \rho \sigma^2}\right).
\]

When the forward market is not totally opaque so that the positions of the firms are somewhat observable, the strategic commitment role of selling forward leads
firms to sell a higher output in the forward market the higher the number of players is. As a result, by the latter effect, inverse hedge ratios fall in \( n \). When the market is fully transparent, in fact we obtain

\[
\hat{\Gamma}(\gamma = 1) = 1 + \frac{1 + \frac{\rho \sigma^2}{b}}{n - 1 + (n + 1) \frac{\rho \sigma^2}{b} + \left(\frac{\rho \sigma^2}{b}\right)^2},
\]

which clearly decreases in \( n \).

### 3.7 Concluding remarks

This chapter has proposed a methodology to investigate whether oligopolistic firms sell futures for strategic reasons, for risk-hedging motives, or for both. Our empirical test builds on a theoretical model of the interaction of risk-averse firms that compete in futures and spot markets. We find that the effects of an increase in the number of players on the equilibrium hedge ratio depend on the strategic role played by forward contracts. If forward sales play no strategic role whatsoever, the inverse hedge ratio increases as more firms enter the market; otherwise, the hedge ratio decreases. Firms hedge less if demand is very elastic and, as expected, more risk averse firms have a greater propensity to hedge. These results serve to structure the empirical research conducted in the next chapter.

Addressing the question whether firms trade forward for strategic purposes is relevant from a social point of view. To increase transparency in energy forward markets, policymakers all over the world have facilitated the creation of organized exchanges where energy futures can be traded. The costs of operating a futures market are non-negligible even in a time where virtual market places have displaced the more traditional physical hubs. In fact, personnel and ICT costs, along with the insurance and financial costs of dealing with default and other risks involved may render a marketplace unprofitable. For instance, ENDEX, the Dutch exchange for power and gas derivatives, has been making losses for 5 out of its 6 or 7 years of existence\(^\text{25}\). When (partly) financed by public funds, loss-making exchanges constitute a loss for society. If it turns out that firms trade forward contracts strategically, subsidizing a futures market may be validated if futures trade is relatively transparent and therefore has high commitment value for firms.

Forward markets exist for a number of commodities, including electricity, natural gas, emission trading permits, copper, iron ore, aluminium, steel etc. Though in

\(^{25}\text{At the end of 2009, ENDEX was taken over by the APX Group, which owns various platforms for spot trading of natural gas and electricity in the NL, Belgium and the UK.}\)
the following chapter we apply our model to the natural gas market in the Netherlands, we believe the general message of this chapter is broader. Our insights, and in particular our methodology to address the question whether firms trade futures for strategic motives, should be applicable to other markets where firms have significant market power.
3.A Appendix

Proof of Proposition 3.1 It is clear that $\Gamma_i$ depends neither on $c_i$ nor on $a$. Given that
$$ \frac{\partial \Gamma_i}{\partial \rho_i} = - \frac{(n+1-\gamma(n-1))^2 b \sigma^2}{(b\gamma(n^2 - 1) + 2\rho_i \sigma^2)^2} < 0, $$
the inverse hedge ratio decreases as the firm is more risk averse. From this it readily follows that inverse hedge ratios decrease in the uncertainty of the demand parameter and increases in slope of the demand. Since
$$ \frac{\partial \Gamma_i}{\partial \gamma} = - \frac{(n-1)(b(n+1)^2 + 2\rho_i \sigma^2)^2}{2(n+1)(b\gamma(n^2 - 1) + 2\rho_i \sigma^2)^2} < 0, $$
the inverse hedge ratio decreases in the probability forward positions are observed.

Finally, we notice that
$$ \frac{\partial \Gamma_i}{\partial n} = \frac{2b(n+1)((n+1)^2 + (n-1)^2 \gamma^2 - 2\gamma n(n+1)) \rho_i \sigma^2 - b^2 \gamma(n+1)^4 - (2\rho_i \sigma^2)^2}{(n+1)^2(b\gamma(n^2 - 1) + 2\rho_i \sigma^2)^3}.$$ Note that $\partial \Gamma_i/\partial n$ is differentiable w.r.t. $\gamma$, which implies that $\partial \Gamma_i/\partial n$ is continuous in $\gamma$. Next, we observe that
$$ \left. \frac{\partial \Gamma_i}{\partial n} \right|_{\gamma=0} = \frac{b(n+1)}{2\rho_i \sigma^2} > 0 $$
and
$$ \left. \frac{\partial \Gamma_i}{\partial n} \right|_{\gamma=1} = -\frac{1}{(n+1)^2} - \frac{4b^2 n}{(b(n^2 - 1) + 2\rho_i \sigma^2)^2} < 0, $$
for all $n \geq 2$, $b > 0$, $\rho_i > 0$ and $\sigma^2 > 0$. Given that $\partial \Gamma_i/\partial n$ is continuous in $\gamma$, this implies that $\partial \Gamma/\partial n = 0$ for at least one $\gamma \in (0, 1)$. Further, $\partial \Gamma/\partial n = 0$ has two solutions for $\gamma$, denoted by $\tilde{\gamma}(n)$ and $\tilde{\gamma}_2(n)$:
$$ \tilde{\gamma}(n) = \frac{b^2(n+1)^4 + 4bn(n+1)^2 \rho_i \sigma^2 + 4\rho_i^2 \sigma^4 - (b(n+1)^2 + 2\rho_i \sigma^2) \sqrt{Z}}{4b(n-1)^2(n+1) \rho_i \sigma^2} $$
and
$$ \tilde{\gamma}_2(n) = \frac{b^2(n+1)^4 + 4bn(n+1)^2 \rho_i \sigma^2 + 4\rho_i^2 \sigma^4 + (b(n+1)^2 + 2\rho_i \sigma^2) \sqrt{Z}}{4b(n-1)^2(n+1) \rho_i \sigma^2}, $$
where $Z = b^2(n+1)^4 + 4bn(n+1)^2(2n-1) \rho_i \sigma^2 + 4\rho_i^2 \sigma^4$. Now for all $n \geq 2$, $b > 0$, $\rho_i > 0$ and $\sigma^2 > 0$, we have:
$$ \tilde{\gamma}_2(n) > \frac{2(b(n+1)^2 + 2\rho_i \sigma^2)^2 + 4bn(n-1)(n+1)^2 \rho_i \sigma^2}{4b(n-1)^2(n+1) \rho_i \sigma^2} $$
$$ > \frac{2(b(n+1)^2 + 2\rho_i \sigma^2)^2 + 4bn(n-1)(n+1)^2 \rho_i \sigma^2}{4bn(n-1)(n+1) \rho_i \sigma^2} $$
$$ = \frac{n+1}{n} + \frac{(b(n+1)^2 + 2\rho_i \sigma^2)^2}{2bn(n-1)(n+1) \rho_i \sigma^2} $$
$$ > 1.$$

Proof of Proposition 3.2: First, we notice that $\Gamma_i$ depends neither on $c_i$ nor on $a$.
Since $\tilde{\gamma}_2(n) > 1$ and $\partial \Gamma_i / \partial n = 0$ for at least one $\gamma \in (0, 1)$, it must be true that $\tilde{\gamma}(n) \in (0, 1)$. Now, because $\frac{\partial \Gamma_i}{\partial n} \bigg|_{\gamma=0} > 0$ and $\frac{\partial \Gamma_i}{\partial n} \bigg|_{\gamma=1} < 0$ and by continuity in $\gamma$, we have $\frac{\partial \Gamma_i}{\partial n} < 0$ if $\gamma < \tilde{\gamma}(n)$ and $\frac{\partial \Gamma_i}{\partial n} > 0$ if $\gamma > \tilde{\gamma}(n)$. Finally, imposing symmetry across firms we can readily obtain the equilibrium forward sales of a firm:

$$\hat{x} = \frac{2(a - c)(n^2 - 1)\gamma + 2\rho\sigma^2}{(n + 1)((n + 1)^2 + \gamma(n - 1)^2 + 2(1 + \gamma - n(\gamma - 3))\rho\sigma^2)}.$$

Observing the expression in Equation (3.10), it is clear that the variance of the inverse hedge ratio is $\sigma^2/b^2(n + 1)^2\hat{x}^2$, which clearly decreases in $a$, $\rho$ and $\sigma^2$, and increases in $c$, $b$ and in $\gamma$. 

$\blacksquare$