Chapter 2

Entry deterrence, coordination, and finite cartel duration

Every industry or occupation that has enough political power to utilize the state will seek to control entry.

George J. Stigler (1971, p. 5)

2.1 Introduction

Why do some cartels last longer than others? This question is rarely asked in industrial organization. It is unclear why the quest for understanding the determinants of cartel duration has not received more attention in the literature. For instance, evaluating current antitrust policy requires a proper understanding of the welfare costs of collusive conduct. An important determinant of these welfare costs is the duration, or longevity, of a cartel.

There is little theory to guide the policymaker or applied economist. Conventional models of collusive behavior predict that cartels are either stable, and last forever, or are unstable, and fail to emerge at all. This prediction is at odds with cartel duration data, which shows that cartels are often short-lived. This chapter proposes a model that explains finite cartel duration.

Clearly, the mere observation of an episode of high price-cost margins followed by an episode of low price-cost margins is not sufficient to establish that cartels have finite lifetimes. In some models of collusion price wars may occur in equilibrium. This suggests that it is \textit{a priori} impossible to distinguish between a price
war and a cartel breakdown. However, Levenstein and Suslow (2006) write that “[o]ur overview of the empirical literature suggests [that] the outbreak of a price war–as opposed to the threat of a price war–is rarely a sign of cartel success [and] cartels break down in some cases because of cheating, but more frequently because of entry, exogenous shocks, and dynamic changes in the industry.” These empirical observations suggest that it is safe to assume to cartel breakdowns are more common than equilibrium price wars.

The central idea of the model is that the threat of entry entails a coordination problem that may lead to the cartel’s demise. Incumbent firms may form a cartel and increase their current profits. Every period they face the threat of entry of potential firms. If the incumbents do not take actions to deter entry, the new firms enter the market and firm profits decrease. Entry can be deterred by investing in a general entry deterrence technology. One interpretation of this technology could be lobbying the government to legally restrict entry. Entry deterrence is an indivisible public good: if at least one incumbent incurs the fixed cost of deterring entry, all other incumbents benefit. If the cartel is unable to coordinate on entry deterrence, incumbents act non-cooperatively when deciding whether to deter entry. In the symmetric Nash equilibrium, incumbents deter entry with some probability. This results in an endogenous probability that the cartel breaks down. Though the model is simple, it yields novel and sometimes counterintuitive insights about the determinants of cartel duration. The model’s predictions may be valuable in empirical studies of cartel duration.

This chapter advances coordination failures as the key determinant of finite cartel duration. However, there are some alternative explanations. A typical interpretation of cartel breakdowns is that collusion ended because a firm cheated by undercutting the cartel price. This explanation cannot be correct, or at least rational, because firms know that cheating will be met by an episode of fierce competition, and this deterrent effect of the punishment phase ensures that collusion is a stable equilibrium. Another possible explanation is that cartels are unstable because antitrust authorities have adopted leniency policies. This argument ignores that even cartels that are perfectly legal may be unstable. For instance, in his analysis of the Webb-Pomerene export cartels that were granted exemption of the Sherman act, Dick (1996) reports that the average cartel duration was between 5 and 6 years.

The outline of the remainder of this chapter is as follows. The next section discusses the related literature. Section 2.3 introduces the formal model. The main results can be found in section 2.4 and the empirical implications are discussed in
section 2.5. The model can be easily adjusted to allow for more general environments, as shown in Section 2.6. Introducing demand shocks, for instance, implies that the equilibrium features both temporary price wars and finite cartel duration. Brief conclusions can be found in section 2.7.

2.2 Related literature

Just two papers propose a formal theory of cartel duration, Jacquemin, Nambu and Dewez (1981) and Harrington (2007). Jacquemin et al. deviate from the standard equilibrium requirement of collusion, by assuming that a cartel is stable, in the sense that cartel members never cheat. Every period that a cartel is active, it incurs an organization and maintenance cost. This cost is assumed to depend on standard factors such as the industry’s concentration or the degree of homogeneity of the product, but also on less conventional and problematic ones, such as ‘the degree of encouragement by the public authority’. Moreover, the cost and benefits are time-dependent. Cartels therefore face a problem of finding the optimal cartel length that maximizes the joint profits. Thus, the problem of cartel duration is reduced to a simple cost-benefit analysis. Jacquemin et al. (1981)’s model is to a large extent ad hoc, because the problem of cartel stability is ignored and the inclusion of the various factors is merely driven by data availability than by theoretical considerations.

Harrington (2007) is perhaps closest related to the analysis in this chapter, as he requires that collusion is an equilibrium phenomenon. He studies birth and death processes for cartels to understand the proportion of cartels. Firms have an opportunity to form a cartel at irregular intervals. The cartel breaks down with some probability as demand shocks induce firms to defect from the cartel agreement. The main differences between Harrington’s model and the one in this chapter are that Harrington treats cartel dissolution as exogenous and ignores entry.

The essential ingredient in this chapter’s model is entry deterrence. This is extensively studied in the literature. Dixit (1980) shows that installing capacity enables an incumbent monopolist to credibly deter entry of a potential firm. When there are multiple incumbents, entry deterrence is a public good and a coordination problem may arise. See Gilbert and Vives (1986), Waldman (1987), Appelbaum and Weber (1992) or Kovenock and Roy (2005) for models that feature this idea.

Despite the large literature on entry deterrence, it is rarely discussed in connection with collusion. For example, in his survey about the strategic analysis of entry
deterrence, Wilson (1992) does not mention collusion at all. Hence, an additional contribution of this chapter is the introduction of collusive behavior in a dynamic model of entry deterrence.

### 2.3 A model of dynamic entry deterrence

There are \( n \geq 2 \) incumbents and \( k \geq 1 \) potential firms, who may enter the market if entry costs are sufficiently low. Every period \( t = 0, 1, 2, \ldots \), the firms play a simple two-stage game. In the first stage, the entry deterrence stage, each incumbent firm may invest a fixed amount \( K > 0 \) in erecting an entry barrier. The entry costs \( c \) for each potential firm are

\[
c = \begin{cases} 
0 & \text{if none of the incumbents invested} \\
+\infty & \text{if at least one incumbents invested.}
\end{cases}
\]  

(2.1)

It is clear that this is a very strong assumption about the entry deterrence technology.\(^1\) The main reason for imposing this particular form is to keep the model tractable. Moreover, this technology restricts the incumbents’ investments to have a purely entry deterrent effect and therefore helps to clarify how coordination failures lead to finite cartel duration. The entry deterrence technology can be interpreted as a reduced form of a lobbying game: at cost \( K \), the government is willing to impose legal entry barriers.\(^2\) This is not entirely unrealistic. According to Levenstein and Suslow (2006) “cartels have turned to the state to create regulation (e.g., salt), impose export tariffs (potash), or provide anti-dumping protection (citric acid) with the goal of excluding outsiders” (p.74).

The crucial assumption of the model is that incumbents are unable to coordinate on deterrence. One argument for this assumption starts by noting that an effective lobby is a coherent and well-organized effort to influence the policymaker. Splitting the lobbying activity into several distinct small lobbies creates confusion and makes it increasingly likely that the policymaker is not convinced. Given this indivisibility, it is attractive for incumbents to quit a joint lobby attempt when at least one incumbent still continues to lobby. Moreover, this incentive to free-ride on a single incumbent’s efforts may be even stronger because the need to deter entry

\(^{1}\) Section 2.6.2 considers a more general specification and shows that the main results are unaffected.

\(^{2}\) Other interpretations are equally valid. The firms could invest \( K \) in advertisements or R&D to keep potential firms at a lag. Or \( K \) can be thought of the ‘maintenance cost’ of a cartel.
is likely to arise infrequently. Then, the incumbents perceive the entry deterrence game as a one-shot game.

In the second stage, the market stage, the potential firms observe the actions of the incumbents and may enter the market. At the end of the second stage, profits are realized. Figure 2.1 summarizes the sequence of events in the stage game.

![Figure 2.1. Timing of the stage game.](image)

The exact nature of competition and behavior at the second stage is left implicit. The firms could be playing a Cournot or a Bertrand game, and they may be acting cooperatively or non-cooperatively. If $m$ firms are active, each active firm receives $\pi^I(m) \geq 0$, where $I$ is the mode of behavior. This chapter considers three possible types of behavior of firms; non-cooperative behavior ($N$), collusive behavior ($C$), and optimal deviation by a single firm from a cartel agreement ($D$). As is standard, it is assumed that $\pi^D(m) > \pi^C(m) > \pi^N(m)$. Consistent with many industrial organization models, it is supposed that $\pi^I(m)$ is decreasing in $m$ and $\lim_{m \to \infty} \pi^I(m) = 0$ for $I \in \{N, C\}$.

The assumption that $\lim_{m \to \infty} \pi^I(m) = 0$ is made to simplify the analysis. Without it, the potential firms may face a coordination problem that complicates the derivation of the equilibrium. In the absence of fixed costs, each potential firm enters the market and the incumbents’ profits drop from $\pi^I(n)$ to $\pi^I(n + k)$. Note that the mode of behavior may change, because the current collusive agreement may not be stable for the new number of firms.

The stage game is infinitely repeated. Each firm discounts the payoffs obtained at period $t$ with a discount factor $\beta \in (0, 1)$. The payoffs within stage games are not discounted.

All aspects of the model are common knowledge. Each firm is risk-neutral and aims to maximize expected profits. Since firms are symmetric and the game is stationary, it is natural to focus on symmetric stationary equilibria. It will be helpful to introduce some additional notation to derive the equilibrium. Let $V^I(\lambda)$ be the value, or the discounted expected sum of future profits, of an incumbent firm when
all firms deter entry with probability $\lambda$ and the mode of behavior is $I$. Furthermore, let $V^I(\lambda_i; \lambda_{-i})$ be the value of an incumbent $i$ which deters entry with probability $\lambda_i$ when all other incumbents deter entry with probability $\lambda$.

### 2.4 Results

#### 2.4.1 Equilibrium entry deterrence

To find the equilibrium, suppose first that the incumbent firms currently earn $\pi^I$ at each stage game, where $I \in \{N, C\}$. That is, the firms either act non-cooperatively or cooperatively. It is easy to see that if $\pi^I - K > \pi^J$, there cannot be an equilibrium in which no incumbent deters entry. Recall that $\pi^J$ denotes the stage profits of each firm after entry, where $J \in \{N, C\}$. If entry is not deterred, new firms enter the industry and stage profits become $\pi^J$. An incumbent can profitably deviate from this proposed equilibrium by incurring a cost $K$ every stage game. This yields a value of

$$V^I(\lambda_i = 1; \lambda_{-i} = 0) = \sum_{t=0}^{\infty} \beta^t (\pi^I - K) = \frac{\pi^I - K}{1 - \beta}. \quad (2.2)$$

Now, consider an equilibrium in which all incumbents deter entry. Then, the value of each incumbent is

$$V^I(\lambda = 1) = \frac{\pi^I - K}{1 - \beta}.$$

It is rational for an individual incumbent $i$ to deviate from this proposed equilibrium. Not deterring entry (setting $\lambda = 0$) increases its value to

$$V^I(\lambda_i = 0; \lambda_{-i} = 1) = \frac{\pi^I}{1 - \beta}.$$

These observations imply that for $\pi^I - K > \pi^J$ the remaining candidate symmetric equilibrium must be in mixed strategies. Given that the potential firms have not entered the market, each incumbent deters entry in the stage game with some probability $\lambda \in (0, 1)$. (It is clearly a dominated action to deter entry after the potential firms have entered.) In a mixed strategy equilibrium, each incumbent firm must be

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3 The argument of $\pi(\cdot)$ is dropped when confusion is unlikely.
indifferent between deterring entry and not deterring entry. This means that

\[ V^I(\lambda_i = 0; \lambda_{-i}) = V^I(\lambda_i = 1; \lambda_{-i}) \]  

holds in each period before entry. Given that each firm chooses to deter entry with probability \( \lambda \), this equilibrium condition can be written as

\[ (1 - (1 - \lambda)^{n-1})(\pi^I + \beta V) + (1 - \lambda)^{n-1} \frac{\pi^I}{1 - \beta} = \pi^I - K + \beta V, \]  

where \( V \) is the equilibrium value of \( V^I(\lambda = 0) \) and \( V^I(\lambda = 1) \). An incumbent who chooses not to deter entry obtains a profit \( \pi^I + \beta V \) if at least one of the other incumbents deters entry, and a profit \( \pi^I \) if entry occurs. These events occur with probability \( 1 - (1 - \lambda)^{n-1} \) and \( (1 - \lambda)^{n-1} \), respectively. In a mixed strategy equilibrium, this expected profit equals the profit obtained by deterring entry with certainty. Furthermore, the value of an incumbent who chooses either action must be \( V \).

It is straightforward to solve equation (2.2) for the equilibrium entry deterrence probability \( \lambda^* \):

\[ \lambda^* = 1 - \left( \frac{K - \beta K}{\pi^I - \pi^I - \beta K} \right)^\frac{1}{n}. \]  

The existence of equilibrium requires \( \lambda \in [0, 1] \) and this is satisfied for \( \pi^I - K \geq \pi^I \). This leads to the first result.

**Lemma 2.1.** The entry deterrence game has a unique symmetric equilibrium in pure strategies if \( \pi^I(n) - K < \pi^I(n + k) \) in which no incumbent deter entry. If \( \pi^I(n) - K \geq \pi^I(n + k) \), there is a unique symmetric equilibrium in mixed strategies where each firm deter entry with probability given by equation (2.5).

Suppose from now on that \( \pi^I(n) - K \geq \pi^I(n + k) \), so the symmetric equilibrium is in mixed strategies. This implies that incumbents would like to coordinate, irrespective of the mode of behavior. With positive probability, more than one firm erects an entry barrier in the mixed strategy equilibrium. Moreover, with probability \( (1 - \lambda^*)^n \) no firm deter entry and the value of each incumbent reduces to zero. It is easy to show that

**Corollary 2.1.** The probability of entry \( (1 - \lambda^*)^n \) increases in the costs of entry deterrence and the profit firms earn upon entry. Entry becomes less likely if the incumbents’ profits,
the discount factor, or the number of potential firms increases. The effect of the number of incumbents on the probability of entry is ambiguous.

Proof. The results simply follow by inspecting the relevant derivatives of \((1 - \lambda^*)^n\). These are

\[
\frac{\partial (1 - \lambda^*)^n}{\partial K} = \frac{n(\pi^I - \pi^J)(\lambda^*)^{2n-1}}{K^2(n-1)(1 - \beta)} > 0,
\]

\[
\frac{\partial (1 - \lambda^*)^n}{\partial \pi^I} = -\frac{n(\lambda^*)^{2n-1}}{K(n-1)(1 - \beta)} < 0,
\]

\[
\frac{\partial (1 - \lambda^*)^n}{\partial \pi^J} = -\frac{nK(\pi^I - \pi^J - K)(\lambda^*)^{\frac{1}{\pi^I}}}{(n-1)(\pi^I - \pi^J - \beta K)^2} < 0,
\]

\[
\frac{\partial (1 - \lambda^*)^n}{\partial \beta} = -\frac{(\lambda^*)^{\frac{2n-1}{\pi^I}}}{(n-1)^2} \left( \log(\lambda^*) + \frac{(n-1)n}{\pi^I - \pi^J - \beta K} \right) \geq 0,
\]

\[
\frac{\partial (1 - \lambda^*)^n}{\partial n} = \frac{\partial n}{\partial \pi^I} \frac{\partial \pi^I}{\partial n} < 0.
\]

The most striking result is arguably the effect of an increase in \(k\). This effect suggests a ‘paradox of potential competition’. As the number of potential firms increases, the incumbents’ profits upon entry decrease, and therefore they defend their market more vigorously. The effect of \(n\) is ambiguous because one cannot sign \(\frac{\partial \pi^I}{\partial n} - \frac{\partial \pi^J}{\partial n}\) without additional information. However, when the mode of behavior is unaltered as new firms join the industry, this sign is exactly zero. Then, an increase in \(n\) has a ‘crowding-out’ effect on investment in entry deterrence. An increase in the number of incumbents aggravates the coordination problem.

2.4.2 Cartel stability

No cartel member should have an incentive to deviate from the collusive agreement about prices, quantities, or other strategic variables. Otherwise, collusion will not occur in equilibrium Collusive behavior and the associated stage profits \(\pi^C\), can be sustained by a grim trigger strategy. Under this strategy, each firm reverts to static non-cooperative behavior ‘forever’ as soon as it observes that a cartel member deviated from the collusive agreement.
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Firms do not coordinate on entry deterrence. This form of non-cooperative behavior within a cartel seems reasonable. If a non-rational firm defects from a price agreement, the other firms may discipline the firm by temporarily revert to the static Nash equilibrium. This enables the defector to learn about the workings of a cartel ‘the hard way’. This type of learning is not possible if a firm deviates from an ‘entry deterrence agreement’. Then, if a firm deviates, potential firms enter and the industry’s profits decrease permanently.

Given that the cartel agreement is stable, each firm obtains a profit of $\pi^C$ every market stage. As $\pi^C > \pi^N$, Corollary 2.1 implies that the probability of entry is lower for a cartel than for a non-cooperative industry, all other things equal.

For a stable cartel to exist, no firm should have an incentive to defect from the collusive agreement. Suppose one firm deviates at the market stage and obtains a profit $\pi^D$. All other firms respond by non-cooperative behavior in all future periods. Then, the cartel is stable if and only if

$$\pi^D(n) + \beta V^N(n) \leq \pi^C(n) + \beta V^C(n).$$

Using the expressions of $V^N$ and $V^C$, this inequality can be rewritten as

$$\beta \geq \frac{\pi^D(n) - \pi^C(n)}{\pi^D(n) - \pi^N(n)}.$$  \hspace{1cm} \text{(2.6)}

The above inequality gives all $\beta$’s for which collusion can be sustained and is standard in the theory of industrial organization. The critical discount factor is defined to be the $\bar{\beta}(n)$ for which (2.6) holds with equality. Similarly, collusion is feasible after entry if

$$\beta \geq \frac{\pi^D(n + k) - \pi^C(n + k)}{\pi^D(n + k) - \pi^N(n + k)} \equiv \bar{\beta}(n + k).$$  \hspace{1cm} \text{(2.7)}

The cartel breaks down upon entry if $\bar{\beta}(n) < \beta < \bar{\beta}(n + k)$.

The next proposition combines the above insights.

**Proposition 2.1.** A cartel with finite expected cartel duration exists if $\pi^C(n) - K > \pi^N(n + k)$ and $\bar{\beta}(n) < \beta < \bar{\beta}(n + k)$ hold simultaneously.

Clearly, the two conditions for finite cartel duration can hold simultaneously, because the critical discount factor $\bar{\beta}(n)$ is independent of $K$. Proposition 2.1 establishes that, even in an infinitely repeated game and perfectly rational actors, finite cartel duration may arise in equilibrium.
Cartel duration and cartel stability are closely related, but conceptually different, concepts. Despite this difference, many empirical studies of cartel duration simply point to theories of cartel stability to justify their selection of the determinants of cartel duration. Furthermore, the two notions are often used interchangeably in policy circles. This belief in the (observational) equivalence of cartel duration and cartel stability seems to be validated by Corollary 2.1. If, for instance, the incumbents’ discount rate decreases (\( \beta \) goes up), the firms may be able to sustain collusion (\( \beta \) is pushed above the critical discount factor) and the probability of entry decreases. However, an increase in \( \beta \) may also have a more subtler effect. If collusion among the \( n \) incumbents is already feasible, an increase in \( \beta \) just nominally increases cartel stability, as it does not affect \( \pi^C(n) \). Still, a higher discount factor may enable the firms to collude after entry if it pushes \( \beta \) above \( \bar{\beta}(n+k) \). This implies a sharp increase in after-entry profits, from \( \pi^N(n+k) \) to \( \pi^C(n+k) \). By Corollary 2.1, this change in after-entry profits reduces the expected duration of the incumbents’ cartel and therefore also the incumbents’ expected expenditure on entry deterrence outlays. Hence, a policy that increases the stability of a cartel may increase welfare.

This example of the effects of a higher discount factor illustrates an important and surprising consequence of the model which deserves to be stated in an independent proposition.

**Proposition 2.2.** A factor that fosters collusion among \( n+k \) firms may contribute to the breakdown of a cartel consisting of \( n \) firms.

Hence, the relation between cartel stability, cartel duration and welfare is intricate. Proposition 2.2 warns antitrust authorities to be careful when designing policies to combat collusion. A policy that makes collusion among \( n+k \) firms less likely lowers \( \pi(n+k) \) and therefore induces the incumbents to deter entry more vigorously. This results in an increased duration of the incumbents’ cartel. By choosing to increase after-entry profits, but to keep before-entry profits constant, the policymaker chooses between the lesser of two evils.

The critical discount factor that sustains collusion before entry, \( \bar{\beta}(n) \), is independent of \( K \), the cost of deterring entry. This is an artifact of the model. Intuitively, one would expect that a positive endogenous probability of cartel breakdown hinders cartel stability because it requires a higher discount factor. One reason for why this intuition may be correct is that the cost of deterring entry could differ between modes of behavior. For instance, a cartel may have better political connections than a non-cooperative firm. As a result, a collusive firm would deter entry at a cost \( K^C \),
while a firm in a non-cooperative industry needs $K^N$ to achieve the same result, where $K^N > K^C \geq 0$. In this case collusion is incentive compatible if and only if

$$\beta \geq \frac{\pi^D - \pi^C}{\pi^D - \pi^N + K^N - K^C}$$

and this increases in the difference between $K^N$ and $K^C$. So, the more efficient a cartel is in deterring entry, the lower is its critical discount factor.

### 2.4.3 Entry deterrence as a participation game

The model is essentially an infinitely repeated participation game. In a participation game, players make an either/or choice, such as whether to enter a market or whether to save a drowning swimmer. In this chapter’s model, a firm ‘participates’ when it chooses to deter entry. As shown by Anderson and Engers (2007) the symmetric equilibrium of a participation game is typically inefficient. In particular, in the symmetric equilibrium the equilibrium probability of entry deterrence, $1 - (1 - \lambda)^n$, is too low from the industry’s perspective. That is, if the firms could commit to a probability $\lambda$ with which each firm deters entry, they would choose a larger $\lambda$.

Proposition 2.1 establishes that finite cartel duration may occur in an infinitely repeated game. To derive this, the above analysis focused on the unique symmetric stationary equilibrium, which is in mixed strategies. However, participation games, like the current model, admit many equilibria, even in a one-shot game. See e.g. Palfrey and Rosenthal (1983) or Heijnen (2007). It would be interesting to see whether the main result of this paper carries over to other equilibria.

An infinitely repeated game may accommodate infinitely many equilibria. To somehow limit the number of possible equilibria, it seems reasonable—given the stationarity of the static game—to restrict attention to stationary equilibria. It is fairly easy to characterize the number of these equilibria. In each equilibrium, a set of firms never deter entry, and the remaining firms deter entry with some probability. That means that there are

$$\sum_{l=0}^{n-1} \binom{n}{l} = \sum_{l=0}^{n} \binom{n}{l} - \binom{n}{n} = 2^n - 1$$

**semi-mixed strategy equilibria** in which $l$ firms never deter entry, and the remaining $n - l$ firms randomize. There are exactly $n$ pure strategy stationary equilibria, in
which one firm deters entry, while the other incumbents enjoy a free ride. In the 
\(2^n - n - 1\) remaining equilibria, at least two firms deter entry with some probabili-
ty. Therefore, coordination problems also arise in those equilibria, and finite cartel
duration does not rely on the symmetry postulate. Note that there can be many
equilibria with entry. If, for instance \(n = 10\), in 1013 of the 1024 equilibria entry is
allowed with positive probability.

Of course, the market is not entered in any of the \(n\) stationary equilibria in pure
strategies. These equilibria Pareto dominate the other equilibria, giving the incum-
bents a good reason to coordinate on that equilibrium. The firms may use a lottery
to achieve this. The lottery selects a firm that is expected to deter entry at the \(x\)
subsequent entry deterrence stages. After that, the firms organize a new lottery to
pick a new ‘volunteer’.

To study this lottery in more detail, suppose that at period 0, the cartel members
organize a lottery and the losing firm’s value \(V^l\) is

\[
V^l = \pi - K + \beta(\pi - K) + \ldots + \beta^{x-1}(\pi - K) + \frac{\beta^x V}{1 - \beta}
\]

\[
= \beta^x V + \frac{\pi(1 - \beta^x)}{1 - \beta},
\]

where \(V\) is the value of each incumbent just before the lottery. Similarly, the value
\(V^w\) of all other winning firms is

\[
V^w = \pi + \beta \pi + \ldots + \beta^{x-1} \pi + \frac{\beta^x V}{1 - \beta}
\]

\[
= \beta^x V + \frac{\pi(1 - \beta^x)}{1 - \beta}.
\]

Assuming that firms have an equal probability of losing, the expected value of each
firm \(V\) obeys

\[
V = \frac{1}{n} V^l + \frac{n - 1}{n} V^w.
\]

Solving the above equations for \(V\) gives

\[
V = \frac{\pi - \frac{K}{n}}{1 - \beta}.
\] (2.10)

The number of periods a firm needs to deter entry, \(x\), drops out. Because firms are
risk-neutral, they are indifferent between paying \(K/n\) for sure and paying \(K\) with
probability \(1/n\).
Coordination on this equilibrium can be quite valuable, as the value of each firm is strictly higher than in the symmetric mixed strategy equilibrium. It is natural to call the difference between these two values the value of coordination. Thus, the value of coordination is

$$\frac{\pi - \frac{K}{n}}{1 - \beta} - \frac{\pi - K}{1 - \beta}.$$ \hspace{1cm} (2.11)

This difference is increasing in the number of firms, suggesting that the higher the number of firms, the more firms are willing to pay to be able to coordinate on this (correlated) equilibrium. The establishment of a trade union, for instance, could be a way to coordinate on the more profitable equilibrium. The above analysis may explain why industries with many firms are more likely to form trade unions.

### 2.5 Empirical implications

In this simple model all variables are stationary and, in the symmetric stationary equilibrium, the probability that a cartel breaks down is stationary too. Therefore, the duration of a cartel follows a geometric distribution, with probability density $$(1 - (1 - \lambda)^n)^l - 1 (1 - \lambda)^n$$. The expected duration of the cartel is simply

$$\sum_{k=1}^{\infty} k(1 - (1 - \lambda)^n)^{k-1}(1 - \lambda)^n = \frac{1}{(1 - \lambda)^n}.$$ \hspace{1cm} (2.12)

This could perhaps be used in empirical studies of cartel duration. Given that the economic model is correct, the probability that a given firm deters entry, on a yearly basis, can be calculated by using the expression for the expected duration. For instance, suppose that a cartel lasted for 10 years, and included 5 members. This corresponds to an expected value of $\lambda \approx 0.37$.

Another approach is to confront the model’s comparative statics with the data. Do higher cartel profits translate to a higher cartel duration? Are more patient incumbents more inclined to deter entry? Relatedly, one could test whether cartel duration decreases in the number of incumbents. In an extensive study of 207 international cartels discovered between 1990 and 2004, Zimmerman and Connor (2005) find that the number of firms in a cartel is negatively related to cartel duration. However, other empirical studies obtain insignificant or even positive relations. For a summary of findings, see LS, Table 6.

More generally, the model’s main message is that cartel duration is a phenome-
non that can and should be modeled. By building a more sophisticated model of cartel duration, one could obtain a much richer set of empirical predictions. This is an important insight. Even though several empirical papers analyze the determinants of cartel duration, none is based on an equilibrium theory of cartel duration. As a result, the papers in this literature rather informally argue why their explanatory variables should be included in the statistical analysis. Variables that tend to facilitate collusion by increasing the critical discount factor are commonly interpreted as variables that tend to increase cartel duration.\(^4\) This is a problematic interpretation, as pointed out in the discussion of proposition 2.2.

2.6 Extensions

2.6.1 Coordination failures between entrants

In the basic model, entry costs are zero, unless the incumbents invest. It seems more realistic to assume that entry costs are always positive, irrespective of the incumbents’ behavior. When entry costs are positive, but not too large, a coordination problem between potential firms may arise. See for instance Elberfeld and Wolfstetter (1999) for a dynamic analysis of a related ‘grab-the-dollar’ game.

To study this case, adjust the model as follows. There are \(k = 2\) potential firms. Their cost of entry is

\[
c = \begin{cases} 
    e & \text{if none of the incumbents invested} \\
    +\infty & \text{if at least one incumbents invested,}
\end{cases}
\]  

\(2.13\)

where \(e > 0\). The timing is the same as in the basic model. In particular, the potential firms still decide simultaneously whether they enter. To keep the analysis interesting, assume that if there are \(n\) firms active, at most one firm can profitably enter. That is,

\[
\frac{\pi(n + 1)}{1 - \beta} - e > 0
\]

\(^4\) As an illustration of this approach, consider Zimmerman and Connor (2005), who argue that “[e]conomic theory has long posited the significance of the effects of market structure characteristics on cartel duration. Empirical studies have since shown that a significant positive relationship exists between concentration and market share and cartel duration.”, and “[c]onsistent with IO theory, markets possessing a large number of buyers allow for longer cartel duration […]”
and
\[ \frac{\pi(n+2)}{1-\beta} - e < 0. \]

As before, focus on symmetric stationary equilibria. Clearly, there is no equilibrium in pure strategies for the game between the potential firms. Let \( \mu \in (0,1) \) denote the probability that a potential firm enters, conditional on that the entry costs are low. In a mixed strategy equilibrium, the potential firms must be indifferent between entering the market and staying out of it. If a firm does not enter, it obtains zero profits. Thus, the equilibrium condition is
\[ \mu \frac{\pi(n+2)}{1-\beta} + (1-\mu) \frac{\pi(n+1)}{1-\beta} - e = 0. \]

This equation can be solved for the equilibrium \( \mu^* \):
\[ \mu^* = \frac{\pi(n+1) - (1-\beta)e}{\pi(n+1) - \pi(n+2)}. \]

Now, consider the optimal strategy for the incumbents. As before, there will not be a symmetric equilibrium in which each firm deters entry with probability one. Suppose that none of the incumbents incurs the cost of entry deterrence. Then, given the mixed strategies of the potential firms, the value function of an incumbent is
\[ V = (1-\mu^*)^2(\pi(n) + \beta V) + 2\mu^*(1-\mu^*) \frac{\pi(n+1)}{1-\beta} + (\mu^*)^2 \frac{\pi(n+2)}{1-\beta}. \]

After substitution of \( \mu^* \), the above equation implicitly defines the equilibrium value \( V^* \) of the proposed equilibrium in which entry is not deterred. Suppose an incumbent deviates by deterring entry once. Its value becomes \( \pi(n) - K + \beta V^* \). A symmetric equilibrium in pure strategies does not exist if
\[ \frac{\pi(n) - K}{1-\beta} > V^*. \]

Since \( V^* \) is (proportional to) a weighted average of \( \pi(n) \), \( \pi(n+1) \) and \( \pi(n+2) \), and profits are decreasing in \( n \), this inequality holds if \( K \) is sufficiently small. In that case, the unique symmetric stationary equilibrium is in mixed strategies. As before, let \( \lambda \) denote the probability that an incumbent deters entry in a given period. In this
equilibrium, the value of each incumbent $V^{**}$ is implicitly defined by

$$
(1 - \lambda^*)^{n-1} \left[ (1 - \mu^*)^2 (\pi(n) + \beta V^{**}) \\
+ 2\mu^* (1 - \mu^*) \frac{\pi(n+1)}{1 - \beta} + (\mu^*)^2 \frac{\pi(n+2)}{1 - \beta} \right] \\
+ (1 - (1 - \lambda^*)^{n-1}) \left[ \pi(n) + \beta V^{**} \right] = \pi(n) - K + \beta V^{**},
$$

(2.14)

where as before the equality follows from the fact that firms should be indifferent between the two actions. The first term between square brackets is the continuation value of an incumbent when no other firm deterred entry. The second term denotes the continuation value when at least one incumbent deterred entry.

The introduction of positive entry costs induces a coordination problem between entrants. In turn, this lowers the incumbents’ cost of a coordination failure. This can be seen from the first term between square brackets in (2.14), which decreases in $\mu^*$. When the coordination problem between entrants is significant ($\mu^*$ is low), the incumbents deter entry with lower probability ($\lambda^*$ goes down). Hence, positive entry costs mitigate the incumbents’ entry deterrence problem.

### 2.6.2 Underinvestment in entry deterrence

In the basic model, where entry deterrence is modeled as a participation game, the probability with which incumbents deter entry is inefficiently low. This feature of the equilibrium extends to more general entry deterrence technologies. To show this, suppose that entry is deterred with probability $F(K)$, if the incumbents invest $K = k_1 + k_2 + \ldots + k_n$ and $F' > 0$ and $F'' < 0$. The investment of firm $i$ is $k_i$.

Assume for simplicity that a symmetric equilibrium exists. Additionally, suppose that entry wipes out the profit of each incumbent. Given that firms decide non-cooperatively on entry deterrence, firm $i$ would maximize

$$
F(k_i + k^* (n - 1))(\pi + \beta V) - k_i.
$$

The first-order condition for a maximum is

$$
f(nk^*)(\pi + \beta V) = 1.
$$

If firms can coordinate on entry deterrence, they would maximize

$$
F(nk^{**})(\pi + \beta n V) - nk^{**}.
$$
The corresponding first-order condition is

\[ f(nk^{**})(\pi + \beta nV) = 1. \]

Clearly, since \( F \) is concave, \( k^{**} > k^* \). Just as in the participation game, firms in this version of the model are more likely to postpone entry when they cooperate. The main difference between the two models is that firms in this section’s model follow pure strategies.

### 2.6.3 Finite cartel duration vs. collusive price wars

Green and Porter (1984) and Rotemberg and Saloner (1986) derive conditions under which a cartel would temporarily lower its price. In their models, collusion is stable, even when the cartel seems to have collapsed. Price wars simply ensure that collusion is an equilibrium. If a model exhibits finite cartel duration, cartels truly collapse after some time.

Although cartel duration and price wars are distinct concepts, the two can be studied simultaneously. An analysis of a model that features both finite cartel duration and price wars may yield additional insight in the determinants of cartel duration. In particular, since a model with price wars is non-stationary, in the sense that the continuation values of firms change over time, firms may have different incentives to deter entry during a price war. This may lead to a higher probability of cartel breakdown when the cartel is in a punishment phase. This intuition is correct, as the following simple extension shows.

Consider Tirole’s (1988) version of the Green and Porter model with \( n \) symmetric incumbents. In Tirole’s version, current market demand is zero with probability \( \alpha \), and \( D(p) \) with probability \( 1 - \alpha \). The \( n \) incumbents compete by setting prices. Firms do not observe the price decisions of their competitors, only their realized profit. Tirole shows that, for \( \beta \) sufficiently large and \( \alpha \) sufficiently small, this model has a collusive equilibrium. In the collusive phase of this equilibrium, firms charge the monopoly price until profits suddenly drop (as a result of the demand shock, in equilibrium). The firms revert to a temporary punishment phase, in which they obtain zero profits. After \( T \) periods, the firms reenter the collusive phase. During the collusive phase firms have a value of \( V^+ \) and at the start of the punishment phase their value is \( V^- \).

Now, suppose that the incumbents in Tirole’s model face a threat of entry. As in the above section, entry reduces each firm’ profits to zero. (This can be thought of as
Entry may be deterred by investing $K$ such that entry is deterred with probability $F(K)$. Then, assuming a collusive equilibrium as described in the above paragraph exists, firm $i$ maximizes

$$F(k_i + (n - 1)k) \left( (1 - \alpha)(\pi^C + \beta V^+) + \alpha \beta V^- \right) - k_i$$

when it is in the collusive phase.

The first-order condition is

$$f(nk) \left( (1 - \alpha)(\pi^C + \beta V^+) + \alpha \beta V^- \right) = 1.$$  

This gives a unique solution $k^+$. In contrast, if a firm is in the last period of the punishment phase, it maximizes

$$F(k_i + (n - 1)k) \beta V^+ - k_i.$$  

The corresponding first-order condition is

$$f(nk) \beta V^+ = 1,$$

which has a unique solution $k^T$, where the superscript $T$ denotes the ‘terminal’ period of the punishment phase. The collusive profits of a firm in the terminal period of the punishment phase are deferred for one period. This implies that the solution $k^T$ is strictly lower than $k^+$. As a result, a cartel is more likely to break down at the end of the punishment phase than in the collusive phase.

More generally, let $\Pi_t$ be the value of a firm in period $t$ of the punishment phase, where $1 \leq t < T$. Then, at the entry deterrence stage, this firm’s value is

$$\Pi_t = \arg \max_{k_i} F(k_i + (n - 1)k) \beta \Pi_{t+1} - k_i.$$  

since $F$ and $\beta$ are strictly smaller than one, $\Pi_t < \Pi_{t+1}$. Then, concavity of $F$ implies that $k^t < k^{t+1}$. So, a firm is less eager to deter entry, the longer the remaining part of the punishment phase.

Hence, cartels are more likely to break down during price wars. In a price war, incumbents’ continuation values are low, and therefore have little incentive to deter entry. This even holds when firms can coordinate on entry deterrence, because the cartel’s continuation value is low as well during a price war.

The above insights complement Green and Porter (1984) analysis. They show
Figure 2.2. Equilibrium dynamics with unobservable demand.

that unobservable demand shocks may destabilize collusion, by increasing the critical discount factor that is necessary to sustain collusion and making price wars necessary in the optimal cartel agreement. The variation considered here demonstrates that unobservable demand shocks destabilize collusion by making entry more likely.

Figure 2.2 illustrates the dynamics of the model. During the collusive phase, firms coordinate on the high collusive price. Following a demand shock, or a defection by a cartel member, the firms enter the punishment phase and charge a price equal to marginal cost. During the punishment phase incumbents deter entry less vigorously and this implies an increase in the probability of entry. As the punishment phase nears its end, firms increase their effort to deter entry because it is more likely that they can reach a new collusive phase.

### 2.7 Concluding remarks

Cartel duration is one of the most important determinants of the welfare costs of collusion. Yet, it is also one of the least studied phenomena in IO theory. This chapter proposes a simple model to explain finite expected cartel duration. The basic framework is that of an oligopoly threatened by entry. Entry can be delayed, but it is costly to do so. The standard (static) models of entry deterrence are not
suitable to study collusive behavior. Therefore, the model uses a repeated-game framework, which allows collusion to be an equilibrium phenomenon. Given that firms do not coordinate on entry deterrence, the incumbents may form a stable cartel that nevertheless breaks down with certainty.

The model yields several predictions about the determinants of cartel duration. For instance, an increase in the number of entrants leads incumbents to deter entry more vigorously and therefore to increase cartel duration. On the other hand, an increase in the number of incumbents tends to lower cartel duration. It is often thought that cartel stability and cartel duration are closely related, if not equivalent, notions. The results in this chapter demonstrate that a policy based on that belief may lead to sub-optimal welfare outcomes. The main insights of the model are robust to several extensions and alternative specifications, as shown in section 2.6. The basic model is simple, yet flexible, and can be adjusted to study cartel duration in various environments. It is, for instance, easy to include unobservable demand shocks to show that cartels are more likely to collapse during price wars.

The analysis suggests a number of policy implications. Antitrust authorities should direct their attention to cartels with the longer expected duration. The model provides a number of clues of how to determine this. An industry with a well-functioning trade union and excellent political connections will find it relatively easy to deter entry ($K$ is relatively low). Additionally, a collusive industry which produces a good for which a large number of potential entrants exists ($k$ is relatively high) will have a big incentive to prevent entry and, accordingly, a long expected duration. Instead of trying to end collusion directly, a policymaker may also attempt to decrease the duration of a cartel. This can be done by making access of firms to politicians more difficult or lowering import tariffs.

Whether entry really explains finite cartel duration is, ultimately, an empirical issue. This chapter offers just one possible explanation. For some cases the model is clearly too stylized to offer a convincing story. Consider, as an illustration, the case of a cartel that functions well for some time, dissolves, and re-establishes itself after a while. There are many examples of cartels that seem to have distinct episodes of collusive behavior. Perhaps the most famous example is the Joint Executive Committee, as studied by Porter (1983) and many others. In the basic model of this chapter, a cartel only exists for a finite period until new firms enter. Still, this seems to be fairly consistent with empirical regularities. In Suslow (2005), 31 cartels out of 45 cartels in manufacturing and commodity industries had just one episode. The Webb-Pomerene cartels reorganized even less frequently: only 16 of the 93 cartels
had several cartel episodes (Dick, 1996).

As a suggestion for future work, it would be interesting to allow potential firms to enter the market sequentially. That presumably gives interesting dynamic patterns, in which the size of the cartel gradually increases, and suddenly disintegrates. Additionally, including a strategic antitrust authority that actively seeks to increase social welfare may help to understand the effects of leniency policies on cartel duration. The key explanation of cartel duration in this chapter is the threat of entry. Often, cartels seems to break down because of internal disagreements, as LS argue. It would be very worthwhile to explicitly include bargaining problems in a model of collusion.