Chapter 4

Socially Responsible Investment in an Overlapping Generations Model

4.1 Introduction

A problem of growing concern is the threat to the environment resulting from polluting economic activity. From economic theory we know that there is an externality associated with the conservation of the environment. This externality exhibits two dimensions. First, there is an intra-generational dimension. The environment is a public good and as such its conservation suffers from the standard free rider problem. Second, there is an inter-generational dimension. Since pollution typically accumulates, future generations bear the costs of the actions of the current generation. Various studies have proposed fiscal policy measures to manage the long-term threat of pollution to the environment in order to achieve sustainable development. This chapter proposes an alternative mechanism to deal with the inter-generational aspect of the pollution externality.

In recent years, not only policy makers, but also large corporations have put sustainable development on their agenda. Corporations publicly report that they engage in corporate social responsibility (CSR) or sustainability programs. This attitude creates the possibility of socially responsible investment (SRI). In 2005, about one out of every ten dollars under professional management in the United States...
was involved in socially responsible investing.\textsuperscript{1} The idea is that shareholders do not only care about the cash flows of a project, but also about how these cash flows are generated. For instance, an investor might oppose to use child labor or heavily polluting technologies in production processes. Socially responsible investment funds, or “green funds”, allow the stock market to function as a tool in dealing with environmental externalities. Typically agents are short-lived, so they do not internalize the long-term effects of pollution. However, in the presence of a forward looking stock market, we show that proper valuation can resolve the coordination failure between current and future generations.

To capture the conflict between generations, we study the environment in a Diamond type overlapping generations (OLG) model, in line with John and Pecchenino (1994, JP). Agents live for two periods. They work when they are young, retire and derive utility from consumption and environmental quality when they are old. We adapt the model of JP such that, instead of choosing between consumption and environmental maintenance, agents choose between investing in bonds and corporate shares. The novelty of our model is that investors acknowledge that as owners of the firm they are also responsible for the generation of the externality. The change from a consumption into an investment decision allows us to introduce and analyze the role of a stock market. Magill and Quinzii (2003) point out that when corporate ownership rights are traded separately on a stock market, externalities or frictions can push the value of equity away from the value of real capital goods. The introduction of this “missing market” can potentially deal with the negative externality of pollution in a natural way, as especially the stock market can be characterized by its forward-looking nature.

We are not the first to study the threat to the environment in a Diamond-type OLG model (John and Pecchenino, 1994; John et al., 1995; Guruswamy Babu et al., 1997; Zhang, 1999; Seegmuller and Verchère, 2004; Wendner, 2006). This literature shows that a social optimum will arise if 1) market failures are corrected using Pigovian taxes or environmental regulations and 2) an optimal distribution of welfare is achieved using lump-sum transfers or the accumulation/repayment of public debt. The proposed tax programs are usually not straightforward, because they often require the use of various instruments. The reason is that even without environmental externalities the decentralized economy need not be Pareto-optimal.\textsuperscript{2} However, the fact that in the presence of market frictions the value of financial


\textsuperscript{2} This is the well known result of Diamond (1965) that agents can over- or under invest in physical capital compared to the Golden Rule solution.
equity is not necessarily equal to the replacement value of physical capital is a possibility that has not been explored in the literature mentioned above. This chapter is also closely related to Mäler (1994) in which property rights on renewable resources are traded between generations. Mäler (1994) shows that in such a setting, the market solution is optimal in the first-best sense, which is in accordance with the Coase (1960) Theorem.

The introduction of a stock market in an OLG model brings some technical complications that we address in this chapter. Various studies discuss the indeterminacy of asset prices in OLG models (See, for example, Woodford, 1984; Tirole, 1985; Huffman, 1986; Magill and Quinzii, 2003). However, we show that our model does not suffer from this indeterminacy.

In section 4.2 we present the core of the model. We describe preferences and technology and calculate the benchmark equilibrium for the case of a central planner. In section 4.3 we turn to the discussion on socially responsible investment and its consequences for corporate valuation and reporting on firm value. We show that when introducing a stock market, proper valuation resolves the coordination problem. We briefly discuss dynamics. We conclude in section 4.4.

4.2 A two sector environmental OLG model

We introduce environmental quality in a standard Diamond (1965) OLG model. Environmental quality is modeled as a renewable resource. Pollution due to production decreases the ‘stock’ of environmental quality. In this section we discuss technology, consequences for environmental quality, and household preferences.

4.2.1 Technology, preferences and environmental quality

Output is represented by a linear homogeneous production function $F(K_t, L_t)$ where $K_t$ denotes the capital stock and $L_t$ labor used at time $t$. Capital invested at $t$, denoted $I_t$, becomes productive at $t+1$. Firms depreciate capital at a uniform rate $\delta$:

$$K_{t+1} = (1 - \delta)K_t + I_t$$ (4.1)

Because of constant returns to scale, we can rewrite output as a function of per capita capital $k_t$: $F(K, L) = f(k_t)L_t$ where $f(k_t)$ is production per capita. We use lower case letters, $i_t, k_t, c_t$, to denote per capita investment, capital, and consumption. Ca-
Capital $K_t$ creates contemporaneous pollution. We assume there is a linear relation between capital and pollution and as a consequence we are free to choose our unit of account of pollution. We normalize such that one unit of capital creates one unit of pollution.

Environmental quality $E_t$ is modeled as a renewable resource (see JP, 1994):

$$E_{t+1} = (1 - \beta)E_t - K_{t+1}$$

with $0 < \beta < 1$ representing the rate of natural recovery. Without pollution, environmental quality will return to its virgin value which is equal to zero. Note that $E_t$ takes only non-positive values; $E_t \leq 0$. Basically, environmental quality is the negative of a stock of pollution.

At each date $t$ a generation of finitely-lived consumers of fixed size $L$ is born. Consumers live for two periods and have preferences defined over per capita consumption $c_{t+1}$ and environmental quality $E_{t+1}$ at old age characterized by a utility function $u(c_{t+1}, E_{t+1})$. This simplification is quite common in OLG models which include environmental quality (see Guruswamy Babu et al., 1997 and JP, 1994). Since we focus on intergenerational conflicts due to investment choice (how are savings used), and not due to savings behavior (how much is saved), we can make this simplifying assumption without loss of generality.

### 4.2.2 A centrally planned economy

We calculate both the optimal transition path and the long-run efficient steady state benchmark equilibrium for the case of a central planner. Consider a central planner that maximizes a social welfare function that assigns a fixed weight $1/(1 + R)$ to the utility of each generation, with the planner’s discount rate $R > 0$. The planner maximizes:

$$\max \sum_{t=0}^{\infty} (1 + R)^{-t} u(c_t, E_t)$$

subject to

$$c_t = f(k_t) + (1 - \delta)k_t - k_{t+1}$$

$$E_t = (1 - \beta)E_{t-1} - k_t$$
and given initial values \( k_0, E_0 \). Along the optimal path the following first-order condition must hold:

\[
\frac{1 - \beta}{1 + \rho_{t+1}} \left[ f'(k_{t+1}) - (\rho_{t+1} + \delta) \right] = f'(k_t) - (\rho_t + \delta) - \frac{u_{E_t}}{u_{c_t}}, \tag{4.6}
\]

with \((1 + \rho_{t+1}) \equiv (1 + R) \frac{u_{E_t}}{u_{c_t+1}}\) the inverse of the marginal rate of intertemporal substitution. Equation (4.6) provides the planner with a simple investment rule. A hat on a variable denotes its steady state value. The steady state associated with equation (4.6) reads:

\[
f'(\hat{k}) = R + \delta + \frac{1 + R}{R - \beta} \frac{u_{E}}{u_{c}}. \tag{4.7}
\]

We see that when the discount rate \( R \) goes to infinity, \( f'(k) \) goes to infinity, implying (assuming Inada conditions) a steady state with zero production. In this case the planner allocates all capital in the first period to the old generation to use for consumption.

Next we turn to steady state efficiency. We substitute the steady state values of capital, consumption, and environmental quality in the utility function and choose the level of capital that maximizes utility \( u(f(\hat{k}) - \delta \hat{k}, -\hat{k}/\beta) \). A steady state \((\hat{k}, \hat{E}, \hat{c})\) is steady state optimal if it satisfies the following first-order condition:

\[
f'(\hat{k}) = \delta + \frac{1}{\bar{\beta}} \frac{u_E}{u_c}. \tag{4.8}
\]

We can see that the optimal path will lead to the efficient steady state if the planner’s discount rate \( R = 0 \), since then the steady state solution of (4.7) is equal to (4.8), which is not very surprising since there is no time preference nor uncertainty in the model.

### 4.3 Competitive economy

In this section we study a stock market economy with socially responsible investors and compare the outcome to the social planner’s allocation. Note that when we discuss the portfolio selection problem, we slightly change the interpretation of the second argument in the utility function. This does not imply, however, that we are comparing apples with oranges. We argue that \textit{ex post} the equilibrium outcome can be compared to the benchmark case.
4.3.1 Consumers

Consumers inelastically supply one unit of labor when young at a real wage rate \( w_t \), save all their wages, invest in either bonds or shares, yet to be defined, and consume when old. The price of the consumption good is the numeraire. For simplicity, we assume that firms do not issue new equity. This may seem quite restrictive. However, as we will show the Modigliani-Miller theorem holds. We can thus normalize the number of shares to one. A young agent \( j \) at time \( t \) takes as given the interest rate \( r_t \) and environmental quality \( E_t \) at time \( t \), the price \( p_t \) per share and dividends \( d_t \) per share. He constructs a portfolio of \( b^j_t \) bonds and \( n^j_t \) shares to maximize his utility:

\[
u(c^j_{t+1}, e^j_{t+1}),
\]

with

\[e^j_{t+1} = n^j_t E_{t+1}.
\]

The second argument \( e^j_{t+1} \) in the utility function captures two things. First, as before it reflects the level of environmental quality. Second, it measures to what extent consumer \( j \) feels that he is actually responsible for the level of environmental quality. The more shares \( n^j_t \) he owns of the polluting firm, the more he will feel responsible for the stock of the pollution.  

The consumer maximizes his utility (4.9) subject to:

\[
c^j_{t+1} = b^j_t(1 + r_{t+1}) + n^j_t(p_{t+1} + d_{t+1})
\]

\[w_t = b^j_t + n^j_t p_t.
\]

Equation (4.12) is the budget constraint. Socially responsible investment is modeled through equation (4.10). An investor acknowledges that, by buying shares of the firm, he is also partly responsible for the state of environmental quality. In fact, he behaves as if property rights on environmental quality are defined via his shareholdings. Nyborg et al. (2006) use a comparable approach in the context of socially responsible consumers and present a detailed discussion of the psychological background of “green consumerism”. The investor has to make a trade-off between investing responsibly (in bonds) or irresponsibly (in the polluting firm);

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\[ In fact, we overcome public good issues by assuming that each investor privately obtains a fraction of environmental pollution in proportion to his shareholdings, so that we can focus on the intergenerational externality. \]
to a socially responsible investor it matters how the cash-flows are generated. This type of modeling is standard in models of vertical differentiation where goods have a quality dimension (see, e.g. Tirole, 1988, p. 296-298) and has been applied to model green consumers (see e.g., S. Bansal and Gangopadhyay, 2003; Cremer and Thisse, 1999). We simply push this type of modeling in the direction of investment behavior.

We assume that consumers have perfect foresight. The first-order optimality condition of the consumer problem takes the form of a pricing equation:

\[
p_t = \frac{p_{t+1} + d_{t+1} + \Delta_{t+1}E_{t+1}}{1 + r_{t+1}}. \tag{4.13}
\]

The current price equals the discounted future price plus dividends plus the stock of pollution times the marginal rate of substitution between environmental quality and consumption

\[
\Delta_{t+1}E_{t+1} \equiv \frac{u_{E_{t+1}}}{u_{c_{t+1}}} E_{t+1}
\]

which we define as the “externality” premium. We label this a premium since in the steady state the firm has to deliver a return equal to \(d/p = r - E\Delta/p \geq r\) (remember that \(E_t \leq 0\)). With social damage, the return on an asset depends on characteristics other than direct financial gain.

We can also give an alternative interpretation to these non-financial characteristics, for instance, the investor might consider externalities to be liabilities such as potential environmental scandals or consumer boycotts. Hence, the externality premium represents any liability or negatively valued characteristic of the firm, or any subjective ethical concerns of investors, that cannot be directly observed in financial statements. This implies that even if the investor himself does not have ethical concerns, the social liabilities associated with irresponsibility give rise to an additional risk factor and premium.

In equilibrium the demand for shares equal the supply. Since the population size and the number of shares are normalized to one, each investor owns exactly one share and equation (4.10) reads \(e_{t+1} = E_{t+1}\). This means that ex post, society acknowledges responsibility exactly in accordance with actual total pollution. This consistent equilibrium property allows us to compare the market outcome to the social optimum.

As the novelty of our model lies in how we approach socially responsible investment, we elaborate on this preference structure with an example. Suppose an
investor enjoys utility from living in or near a forest and has the opportunity to invest in either bonds or in a firm that uses wood to fuel its production. As owner of this firm, an investor acknowledges responsibility for the state and degradation of the forest and is therefore only willing to invest in the firm if there is a premium on the return on investment compared to the interest rate. Effectively, a socially responsible investor acts as if she privately acquired a parcel of the forest and requires payments whenever the firm decides to cut down some of her trees. By analogy, investing in bonds is not associated with gaining control over the firm and is therefore free of this externality-premium.

The broad interpretation of property and control rights plays a crucial role in classifying this type of investment as either behavioral or rational. Whether socially responsible investment is rational or not is subject of discussion, but it is certainly distinct from traditional behavioral economics. To conclude, socially responsible investment fits with theories such as compensating wage differentials (see e.g. Rosen, 1974) or vertical differentiation as used in environmental economics as mentioned above, but we can also interpret social responsibility as an additional risk factor, which is more in line with asset pricing theory.

4.3.2 Corporate behavior

At time $t$ a firm issues corporate bonds $B_t$. For simplicity, we assume that firms do not issue new equity. We have:

$$F(K_{t+1}, L_{t+1}) - w_{t+1}L_{t+1} - (1 + r_{t+1})B_t + B_{t+1} = I_{t+1} + D_{t+1}$$

(4.14)

A firm can use its production net of labor payments and net interest payments to finance its real capital investments or to pay out dividends $D_{t+1}$. We choose a particular financing policy where firms issue one period bonds to finance investments, i.e. $B_t = I_t$. Rewriting (4.14), in per capita form using $d_t = D_t / L$, and rearranging we find:

$$d_{t+1} = f(k_{t+1}) - w_{t+1} - (1 + r_{t+1})i_t$$

(4.15)

where we have implemented the financing policy. We normalize the number of consumers and shares to one so that in equilibrium we find for the stock market
value of the firm:

\[ v_t = \frac{v_{t+1} + d_{t+1} + \Delta_{t+1} E_{t+1}}{1 + r_{t+1}} \]  

(4.16)

The total value of the firm \( m_t \) is equal to its share value plus debt value:

\[ m_t \equiv b_t + v_t = \frac{f(k_{t+1}) - w_{t+1} - i_{t+1} + \Delta_{t+1} E_{t+1} + m_{t+1}}{1 + r_{t+1}} \]  

(4.17)

which depends only on output and the financial structure does not make a difference (Modigliani and Miller, 1958).

A firm makes investments in real capital to maximize shareholder value according to (4.16). We let the optimal investment \( i_t^* \) at time \( t \) depend on the state variables \( k_t \) and \( E_t \), such that for the firm’s market value \( v_t^* = v^*(k_t, E_t) \) we have:

\[ v_t^* = \frac{f(k_{t+1}) - w_{t+1} - (1 + r_{t+1})i_t^*(k_t, E_t) + \Delta_{t+1} E_{t+1} + v_{t+1}^*}{1 + r_{t+1}} \]  

(4.18)

which is a Bellman Equation. The maximum principle then gives the following first-order conditions\(^4\):

\[ \frac{1 - \beta}{1 + r_{t+1}} \left[ f'(k_{t+1}) - (r_{t+1} + \delta) \right] = f'(k_t) - (r_t + \delta) - \Delta_t \]  

(4.19)

\[ f(k_t) - f'(k_t)k_t = w_t \]  

(4.20)

We can see immediately that equation (4.19) is equivalent to the planner’s solution (4.6), provided that the interest rate is equal to the marginal rate of intertemporal substitution of the planner, i.e. \( 1 + r_t = (1 + R) \frac{u_c'}{u_c} \). If this is the case, than the stock market economy is both dynamically and steady-state efficient.

Iteratively substitute (4.19) and find:

\[ f'(k_t) = (r_t + \delta) + \sum_{\tau=0}^{\infty} \frac{(1 - \beta)^\tau}{\prod_{i=0}^{\tau}(1 + r_{t+i})} \Delta_{t+\tau} \]  

(4.21)

which states that the marginal product of one unit of capital today should equal the familiar \( (r_t + \delta) \) plus the discounted sum of the externality premia \( \Delta_t \) of of all future generations. Since pollution due to investment today yields an externality

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\(^4\)To solve the maximization problem, it is useful to rewrite (4.2) as \( E_{t+1} = (1 - \beta)E_t - (1 - \delta)k_t - i_t \) and note that the firm takes into account the direct effect on the externality premium \( \Delta_t E_t \), but not second-order effects, i.e. it treats \( \Delta_t \) as a price.
flow of \( (1 - \beta) \) one period ahead we have a discount rate equal to \( \frac{1-\beta}{1+r_t} \). If firms adopt this investment policy, firm value is maximized and the externality is fully internalized.

For comparison reasons, we show that the possibility of the market value of the firm differing from its replacement value matters for corporate behavior. Imposing that the market value of the firm should equal its replacement value\(^5\), i.e. \( v_t = (1 - \delta)k_t \), yields first order conditions equivalent to JP, namely

\[
f'(k_t) = r_t + \delta - \Delta_t \tag{4.22}
\]

and (4.20). Now the marginal product of capital covers only the externality premium of the current generation. This is equivalent to JP who assume that there is a form of intragenerational coordination to establish optimal provision of the public good for agents alive at time \( t \), but no intergenerational coordination. Finally we point out that pure profit maximization, e.g. maximizing discounted cash flows -not firm value- yields the familiar conditions:

\[
f'(k_t) = r_t + \delta \tag{4.23}
\]

and (4.20). Production factors are rewarded their marginal productivity, but the externality is not internalized by the firm.

4.3.3 Equilibrium and dynamics

In equilibrium we assume factor markets clear, and utility and firm value are maximized. Note that we have not dealt yet with the ambiguity of role of the interest rate \( r_t \). The return on equity requires an externality-premium relative to the interest rate, but it does not fix the level of the interest rate. We should clear the bond market to find an endogenous interest rate. However, this makes the dynamic analysis less straightforward and adds little to the core of the analysis. We choose not to blur the focus of this chapter and keep the model tractable. We therefore take the interest \( r_t \) rate as given and constant. One can think of our economy as a small, open economy that faces full capital mobility. Alternatively, there can be trade in government bonds. Since bonds are risk-free externality-free assets and there is no growth, one can view the interest rate as a rate of pure time preference. Since the rate of pure time preference is equal to zero in our model, it would imply that

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\(^5\)Young agents buy the depreciated capital stock from the old.
bonds are simply a storing technology.

In equilibrium we can write wages, consumption, and the externality premium in terms of the state variables capital and environmental quality, i.e. \( w_t = w(k_t) = f(k_t) - f'(k_t)k_t, c_t = c(k_t, k_{t+1}) = f(k_t) + (1 - \delta)k_t - k_{t+1}, \) and \( \Delta_t = \Delta(k_t, k_{t+1}, E_t) = \frac{u_t^{E+1}}{u_{t+1}^{E+1}}. \) Equations, (4.2), (4.16) and (4.19) can then be used to study dynamic behavior. The paths of the state variables \( k_t, E_t, \) and \( v_t \) fully determine all other variables. In a steady state we have:

\[
\begin{align*}
    f'(\hat{k}) &= r + \delta + \frac{1 + r}{r + \beta} \hat{\Delta} \quad (4.24) \\
    \hat{E} &= -\frac{\hat{k}}{\beta} \quad (4.25) \\
    \hat{\vartheta} &= (1 - \delta - \frac{1 - \beta}{r + \beta} \hat{\Delta})\hat{k} \quad (4.26)
\end{align*}
\]

A hat on a variable denotes its steady state value. Equation (4.24) and (4.25) uniquely\(^6\) determine the steady state values for \( k_t \) and \( E_t \), from which the steady state value for \( v_t \) follows directly. Here we require that the standard transversality condition holds; \( \lim_{T \to \infty} \prod_{\tau=0}^{T} \frac{v_t}{r_t \cdots r_{t+\tau}} = 0. \)

To study the stability of the steady state and the dynamics of the economy, we first note that the system of three difference equations is decomposable. Equation (4.2) and (4.19) define an independent subsystem that can be studied separately, since there is no feedback from \( v_t \) on \( k_t \) and \( E_t \). Before we study the independent subsystem we focus on the difference equation in \( v_t \), the pricing equation for the stock market value of the firm.

Impose the steady state values for capital and environmental quality and substitute these in (4.16) and rewrite:

\[
v_{t+1} = (1 + r)v_t - \hat{d} - \hat{\Delta}\hat{E} \quad (4.27)
\]

and we see that \( 1 + r > 1 \) is an unstable root of the system. Therefore, provided that the independent subsystem in \( k_t \) and \( E_t \) is stable, the whole system is saddle-point stable.\(^7\)

\(^6\) Equation (4.25) is a downward sloping curve and using implicit differentiation we find for (4.24) that \( dE/dk = \left[ f'(k) - \delta \left| \frac{u_t^{E+1}}{u_t^{E+1}} \right| + \frac{r + \beta}{u_t^{E+1}} \right] \) which is positive for all \( k \geq 0 \) and \( E \leq 0 \) satisfying (4.24), so that (4.24) implicitly defines \( \hat{E} \) as a strictly increasing function in \( \hat{k} \). The implied single crossing property of the two functions defines a unique steady state.

\(^7\) Formally, since (4.16) is a second order difference equation we need to rewrite the linearized system in four first-order difference equations and calculate the four eigenvalues of the associated matrix. One can show that these are equal to the two eigenvalues of the independent subsystem, \( 1 + r, \) and zero.
For given initial values $k_0$ and $E_0$, the firm value jumps to the saddle-point stable path and hence $v_0$ is determinate. Often OLG models, in which assets are traded suffer from indeterminacy of asset prices. The system is then determinate in the sense that for given initial values the whole path of the economy can be derived. However, the initial asset price is not an equilibrium result, but must be exogenously given to the model. In our model, however, if the transversality condition is met, asset prices are fully determined.

In the steady state the total value of the firm is equal to $\dot{m} = \dot{\vartheta} + \dot{\beta} = \dot{\vartheta} + \delta \dot{k} = (1 - \frac{1 - \beta}{r + \beta}) \dot{k}$. The market value of the firm is lower than its replacement value $\dot{k}$ because of the externality it generates. Note, however, that this discrepancy between market value and replacement value does not imply that there are arbitrage opportunities. If capital goods are to be used for consumption, production is stopped and so is future pollution. Then, immediately the market value of the firm will jump to its replacement value.

We turn to the stability of the independent subsystem by log-linearizing equations (4.2) and (4.19) around the steady state. A variable with a tilde denotes a percentage change from its initial value e.g. $\tilde{k}_t = d \log k_t$

\[
\begin{bmatrix}
-f'(\hat{k}) \frac{1 - \beta}{1 + r} \epsilon_{kl} - \sigma_c \hat{\Delta} \hat{k} & 0 \\
\hat{k} & \hat{\Delta}
\end{bmatrix}
\begin{bmatrix}
\tilde{k}_{t+1} \\
\tilde{E}_{t+1}
\end{bmatrix}
= \begin{bmatrix}
-f'(\hat{k}) \epsilon_{kl} - \sigma_c \hat{\Delta} \left[ f'(\hat{k}) + (1 - \delta) \right] & -\sigma_E \hat{\Delta} \\
0 & (1 - \beta) \hat{E}
\end{bmatrix}
\begin{bmatrix}
\tilde{k}_t \\
\tilde{E}_t
\end{bmatrix}
\] (4.28)

with $\epsilon_{kl} = \frac{f''(\hat{k}) \hat{k}}{f'(\hat{k})}$ the elasticity of substitution between capital and labor, $\sigma_c = \frac{u''_c}{u'_c}$ the elasticity of marginal utility of consumption, and $\sigma_E = \frac{u''_E}{u'_E}$ the elasticity of marginal utility of environmental quality. Since it is always possible to find an interest rate such that the system is stable, we analyze stability in the case where the economy is steady-state efficient, $r = 0$. The loglinearized system can be rewritten as:
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\[
\begin{bmatrix}
\tilde{k}_{t+1} \\
\tilde{E}_{t+1}
\end{bmatrix} = \frac{1}{A} \times \\
\begin{bmatrix}
-f'(\tilde{k})\epsilon_{kl} - \sigma_c \hat{\Delta} \frac{k}{\epsilon_{kl}} [f'(\tilde{k}) + (1 - \delta)] & -\sigma_E \hat{\Delta} \\
-\beta (f'(\tilde{k})\epsilon_{kl} + \sigma_c \hat{\Delta} \frac{k}{\epsilon_{kl}} [f'(\tilde{k}) + (1 - \delta)]) & (1 - \beta) A - \beta \sigma_E \hat{\Delta}
\end{bmatrix} \\
\begin{bmatrix}
\tilde{k}_t \\
\tilde{E}_t
\end{bmatrix}
\]

(4.29)

with \( A = -f'(\tilde{k}) \frac{1 - \beta}{1 + \beta} \epsilon_{kl} - \sigma_c \hat{\Delta} \). The absolute value of the determinant of this matrix is less than one if \( f'(\tilde{k}) - \delta < \frac{\beta}{1 - \beta} \); a necessary condition for stability since in general the determinant of a matrix is equal to the product of its eigenvalues. In a steady state this is equivalent to \( \frac{\Delta}{\beta} < \frac{\beta}{1 - \beta} \) which implies that for we require that the marginal rate of substitution between environmental quality and consumption should not be too high. Furthermore we need that \( \sigma_c \) and \( \sigma_E \) should not be too large.

In figure 4.1 the curve \( E_{min} - A \) is associated with equation (4.24), the line \( 0 - A - JP - B \) with equation (4.25), and point \( A \) with the steady state of the independent subsystem. We also depict the JP steady state as defined by equation (4.22); an equilibrium in which one requires that the value of the firm is equal to its replacement value at all times. In such an equilibrium environmental quality is too low and invested capital is too high. Finally, if firms maximize pure profits instead of value -which is equivalent to an economy in which investors are not socially responsible- the economy will end up in point \( B \) and environmental quality will be even lower, naturally. As mentioned before, point \( A \) is also the first-best optimal steady state equilibrium. Finally, since the social planner finds the same allocation rule as the competitive economy, we argue that the introduction of a stock market does not bring additional restrictions in terms of stability requirements.

4.4 Conclusion

One of the key issues in achieving sustainable development is managing the impact of economic activity on the environment. In the last decade, corporations have increasingly put sustainable development on their agendas, creating the possibility of socially responsible investment. This chapter argues that the stock market can play a role in achieving sustainable development.
Figure 4.1. Comparison of steady state equilibria.

The line $0 - A - JP - B$ is associated with equation (4.25) and any point on this line can be a steady state economic outcome. The curve $E_{\text{min}} - A$ is associated with equation (4.25). Point $A$ is the steady-state equilibrium in a stock-market economy with socially responsible investors. The stock-market assures intra and intergenerational coordination with respect to environmental quality. The arrows reflect the dynamic forces of this equilibrium. Point $JB$ is the steady-state equilibrium of the economy of John and Pecchenino (1994) in which there is only coordination within each generation with respect to environmental quality. Point $B$ reflects an economy without any coordination.

We analyze this in an Diamond-type overlapping generations model with short-lived consumers that care about environmental quality, comparable to John and Pecchenino (1994). A lack of coordination between old and young agents leads to overaccumulation of pollution. We show that introducing an equity market that allows for trade of property rights can resolve the coordination failure. The intuition is straightforward: since the stock market is forward looking, equity allows for trade in future valued capital, incorporating the welfare loss of pollution of future generations.

The novelty of this chapter lies in how socially responsible investment is modeled in a dynamic setting. Such behavior will only lead to the social optimum if the stock of externalities is considered in firm valuation, not the flow. As such, a socially responsible investor acts as if property rights are assigned to the firm as well as to
the stock of pollutants.

Finally, we have focused on a specific externality, namely the intergenerational problems associated with short-lived agents and a long-lived public good. The emphasis has been on environmental issues. However, the idea that proper firm valuation can incorporate negative externalities can be generalized.
4.A Appendix

Derivation of Social Optimum

The social planner maximizes

$$\max \sum_{t=0}^{\infty} (1 + R)^{-t} u(c_t, E_t)$$

subject to

$$c_t = f(k_t) + (1 - \delta)k_t - k_{t+1}$$
$$E_t = (1 - \beta)E_{t-1} - k_t$$

and given initial values $k_0, E_0$. Define the value function as:

$$V_t = V(k_t, E_t) \equiv \max_{i_t, \tau \geq 0} \sum_{\tau=t}^{\infty} (1 + R)^{\tau-t} u(f(k_\tau) - i_\tau, E_\tau),$$

where we use $i_t = k_{t+1} - (1 - \delta)k_t$. Let the optimal value of the control variable $i_t$ at time $t$ be a function of the state variables $k_t$ and $E_t$, so $i_t^* = i_t(k_t, E_t)$. We have the following Bellman equation:

$$V_t = u(f(k_t) - i_t^*, E_t) + \frac{1}{1 + R} V(k_t(1 - \delta) + i_t^*, E_t(1 - \beta) - k_t(1 - \delta) - i_t^*),$$

where we directly substituted the constraints and rewrite (4.2) as $E_{t+1} = (1 - \beta)E_t - (1 - \delta)k_t - i_t$. Taking the derivative with respect to the control variable $i_t^*$ gives the first order condition for optimality:

$$\frac{dV_t}{di_t^*} = -u_{c_t} + \frac{1}{1 + R} \left[ \frac{dV_{t+1}}{dk_{t+1}} - \frac{dV_{t+1}}{dE_{t+1}} \right] = 0.$$ (4.A.6)

To solve we take the derivative of the value function with respect to the state variables:

$$\frac{dV_t}{dk_t} = u_{c_t} f'(k_t) + \frac{1 - \delta}{1 + R} \left[ \frac{dV_{t+1}}{dk_{t+1}} - \frac{dV_{t+1}}{dE_{t+1}} \right]$$
$$\frac{dV_t}{dE_t} = -u_{E_t} + \frac{1 - \beta}{1 + R} \frac{dV_{t+1}}{dE_{t+1}}.$$ (4.A.7)
where we have applied the envelope theorem. Combine (4.A.6) and (4.A.7) to find:

\[
\frac{dV_t}{dk_t} = u_{c_t}(f'(k_t) + (1 - \delta)) \tag{4.A.9}
\]

Substitute (4.A.9) led in (4.A.6) and rewrite:

\[
\frac{dV_{t+1}}{dE_{t+1}} = -u_{c_t}(1 + R) + u_{c_{t+1}}(f'(k_{t+1}) + (1 - \delta)). \tag{4.A.10}
\]

Substituting (4.A.10) and (4.A.10) lagged in (4.A.8) and rearranging gives

\[
\frac{1 - \beta}{(1 + R)} u_{c_t} \left[ f'(k_{t+1}) - ((1 + R) \frac{u_{c_{t+1}}}{u_{c_t}} - 1 + \delta) \right] = f'(k_t) - ((1 + R) \frac{u_{c_{t-1}}}{u_{c_t}} - 1 + \delta) - \frac{u_{E_t}}{u_{c_t}}, \tag{4.A.11}
\]

which is the difference equation that characterizes optimality (4.6).

**Consumers maximization problem**

The Lagrangean for the problem is given by

\[
\max_{b_i, n_i, \lambda} L = u(b_i^t(1 + r_{t+1}) + n_i^t(p_{t+1} + d_{t+1}), n_i^tE_{t+1}) - \lambda(b_i^t + n_i^tp_t - w_t), \tag{4.A.12}
\]

where \(\lambda\) is the Lagrange multiplier and we have substituted the expressions for \(e_{t+1}\) and \(c_{t+1}\). The first order conditions for optimality are:

\[
\begin{align*}
    u_c[1 + r_{t+1}] - \lambda &= 0, \tag{4.A.13} \\
    u_c[p_{t+1} + d_{t+1}] + u_E - \lambda p_t &= 0, \tag{4.A.14} \\
    b_i^t + n_i^tp_t - w_t &= 0. \tag{4.A.15}
\end{align*}
\]

Substituting (4.A.13) in (4.A.14) and rearrange to find the pricing equation (4.13).

**Firm’s maximization problem**

The optimal investment \(i_t^*\) at time \(t\) depends on the state variables \(k_t\) and \(E_t\). The value function \(v_t^* = v^*(k_t, E_t)\) yields the following Bellman Equation:

\[
v_t^* = \frac{f(k_{t+1}) - w_{t+1} - (1 + r_{t+1})i^*(k_t, E_t) + \Delta_{t+1}E_{t+1} + v_{t+1}^*}{1 + r_{t+1}}, \tag{4.A.16}
\]
with \( i_t = k_{t+1} - (1 - \delta)k_t \) and \( E_{t+1} = (1 - \beta)E_t - (1 - \delta)k_t - i_t \). Taking the derivative of the value function with respect to the control variable gives the first order condition:

\[
\frac{dv^*_t}{dt^*} = \frac{1}{1 + r_{t+1}} \left[ f'(k_{t+1}) - (1 + r_{t+1}) - \Delta_{t+1} + \frac{dv^*_{t+1}}{dk_{t+1}} - \frac{dv^*_{t+1}}{dE_{t+1}} \right] = 0. \tag{4.A.17}
\]

Note that the firm takes into account the direct effect on the externality premium \( \Delta_t E_t \), but not second-order effects, i.e. it treats \( \Delta_t \) as a price. To solve we take the derivative of the value function with respect to the state variables:

\[
\frac{dv^*_t}{dk_t} = \frac{1 - \delta}{1 + r_{t+1}} \left[ f'(k_{t+1}) - \Delta_{t+1} + \frac{dv^*_{t+1}}{dk_{t+1}} - \frac{dv^*_{t+1}}{dE_{t+1}} \right], \tag{4.A.18}
\]

\[
\frac{dv^*_t}{dE_t} = \frac{1 - \beta}{1 + r_{t+1}} \left[ \Delta_{t+1} + \frac{dv^*_{t+1}}{dE_{t+1}} \right], \tag{4.A.19}
\]

where we have applied the envelope theorem. Combining (4.A.17) and (4.A.18) gives:

\[
\frac{dv^*_t}{dk_t} = (1 - \delta), \tag{4.A.20}
\]

which can be led one period and substituted in (4.A.17) to find:

\[
\frac{dv^*_{t+1}}{dE_{t+1}} = f'(k_{t+1}) - (r_{t+1} + \delta) - \Delta_{t+1}. \tag{4.A.21}
\]

Substituting (4.A.21) and (4.A.21) lagged in (4.A.19) and rearranging gives

\[
\frac{1 - \beta}{1 + r_{t+1}} [f'(k_{t+1}) - (r_{t+1} + \delta)] = f'(k_t) - (r_t + \delta) - \Delta_t, \tag{4.A.22}
\]

which is the implicit difference equation that characterizes the optimal path, equation (4.19).