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5. Social Simulation of Stock Markets: Taking it to the Next Level 1 2

5.1 Introduction

In recent years, agent-based computational finance has developed into a growing field in which researchers rely on computational methods to overcome the inherent limitations of analytic methods (LeBaron, 2000). Amongst the advantages that agent-based computational models have to offer are (1) the ease with which it is possible to limit agent rationality, (2) the facilitation of heterogeneity in the agent population, (3) the possibility of generating an entire dynamical history of the processes under study, and (4) the ease with which it is possible to have agents interact in social networks (Axtell, 2000). Furthermore, agent-based computational models as a specific instance of the broader field of social simulation are well adapted to developing and exploring theories concerned with social processes and are well able to represent dynamic aspects of change. Using these models can help to increase our understanding of the relationship between micro level attributes and individuals’ behavior and macro level aggregate effects (Gilbert & Troitzsch, 1999). An example of the latter would be

1 This chapter is a slightly modified version of:


2 We like to thank conference participants of the First World Conference on Social Simulation WCSS 06 in Kyoto, Japan and the Artificial Economics 2006 Conference in Aalborg, Denmark as well as two anonymous referees of the JASSS for their valuable comments on this paper. This paper greatly profited from all of their feedback.
to study the influences of different types of investors’ behavior on the fluctuations of asset prices as described in e.g., Takahashi and Terano (2003).

Moreover, the application of multi agent models for financial markets research has been promoted by a number of empirical puzzles or stylized facts (e.g., time series predictability, volatility persistence/clustering, and heavy or ‘fat’ tails in the asset returns distribution) that are difficult to explain using traditional representative agent structures (LeBaron et al., 1999). The latter structures assume rational agents who make optimal investment decisions and have rational expectations about future developments (Hommes, 2006). In addition, using the representative agent assumes that the choices made by all the diverse agents in a sector - e.g., consumers, investors, or producers - can be considered as the choices of one ‘representative’ standard utility maximizing individual whose choices coincide with the aggregate choices of the heterogeneous individuals. However, this reduction of the behavior of a group of heterogeneous agents is criticized as not simply being an analytical convenience but as being unjustified and leading to conclusions that are usually misleading and often wrong (Kirman, 1992).

An excellent overview of early and influential models in agent-based computational finance is given by LeBaron (2000; 2005). These models range from relatively simple models like the one of Lettau (1997) to very complicated models like the Santa Fe Artificial Stock Market (Arthur et al., 1997; LeBaron et al., 1999). It is beyond the scope of this chapter to discuss them all in detail here, but for those unfamiliar with them, we have provided a summary in appendix 1 of this chapter.

The myriad number of different agent-based models that are applied in finance makes this a complex research field. To even further complicate matters, finance itself is not a uniform entity. Rather, the finance literature distinguishes two fields, which are related, but differ in their main axioms and principles. These fields can be called ‘traditional finance’ versus ‘behavioral finance’, respectively.

Traditional finance literature is based on the assumption of rational and omniscient investors who optimize the risk/return profile of their portfolios (Olsen, 1998). This approach has merits in the development of theoretical foundations like the Capital Asset Pricing Model and the Arbitrage Pricing Theory for a stylized world with efficient markets. However, treating investors as being utility optimizing, omniscient, and unboundedly rational individuals, sets limits to understanding and explaining real-life investors’ behavior. The limitations of traditional finance are well-known in the field of behavioral finance and the extant literature in the latter field has contributed to understanding many facets of both micro level individual investor as well as macro level stock market behavior that were inexplicable from a traditional finance perspective (for a brief overview of behavioral finance, see e.g., Nofsinger (2002), Schleifer (2000), and Shefrin (2002)). However, the connections between micro level investor behavior and macro level stock market (price) dynamics - which are an essential part of the artificial stock market that will be presented in this chapter - remain an underdeveloped field of research according to Van der Sar (2004: 442). With respect to this topic, in his review of behavioral finance he argues that:
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“... there still is a gap to be bridged between the individual investor and the market, and the question of aggregation has not been settled yet.”

Traditional finance has a long history and the majority of the agent-based computational models mentioned in previous sections and that are discussed in the literature as outlined in appendix 1 of this chapter- either explicitly or implicitly - spring from this history. Consequently, unlike the literature on agent based computational finance in general, the literature on agent-based computational behavioral finance is still scarce. Takahashi and Terano (2003) are amongst the first who explicitly aim to apply behavioral finance theories in agent-based computational models. They state, that the decision-making rules of investors based on behavioral finance are much more complicated than the ones in traditional finance. Moreover, they note that it is difficult to analytically derive asset prices under these assumptions which motivates their choice to use an agent-based model (Takahashi & Terano, 2003: 2). However, they also critically observe that most research in artificial markets makes the micro level agent rules as simple as possible in accordance with Axelrod’s ‘Keep It Simple, Stupid’ principles (1997b). Takahashi and Terano (2003) furthermore argue that this results in rules that are sometimes too mechanical and emphasize that these micro level rules are different from investors’ behavior in real markets. It is stated, that one of the novelties of their paper is that their models of bounded rational agents are grounded in real theories, such as those of trend chasers, overconfident investors and the Prospect Theory as introduced by Kahneman and Tversky (1979). Another novelty these authors claim to introduce is that they analyze, based on their investor models, how the behavior of each investor type (i.e. fundamentalists, trend chasers and overconfident investors) are associated with overall asset price fluctuations.

This paper of Takahashi and Terano (2003) is an important step forward in explicitly applying behavioral finance theories and concepts in agent-based models and argues with good reason for agent rules that are more thoroughly based on theoretical work. Notwithstanding the contribution of these authors, we consider our approach and the artificial stock market model we present in this chapter, to be both novel and to offer a number of contributions to the field.

The first contribution is that our model - like the model of Takahashi and Terano (2003) - is explicitly based on real theories. However, in contrast to these

3 We note that there are authors (see e.g., Lux (1998: 148)) who argue that distinguishing between groups of trend following and fundamentalist investors is already an application of behavioral finance.
authors, we apply an interdisciplinary approach, in which (behavioral) finance, social-psychological and consumer behavior theories are used in combination. Amongst the theoretical concepts that are used in developing this model is the general notion of boundedly rational investors as propagated by the behavioral finance literature (Nofsinger, 2002; Schleifer, 2000; Shefrin, 2002) as well as more specific concepts like the Prospect Theory of Kahneman and Tversky (1979). Other theories that are utilized are theories on the different personal needs people may strive to satisfy (Maslow, 1954; Max-Neef, 1992), conformity behavior (Bikhchandani et al., 1998; Burnkrant & Cousineau, 1975; Cialdini & Goldstein, 2004; Cialdini & Trost, 1998), and the way decision-makers deal with uncertainty and risk (Knight, 1921; Mitchell, 1999; Taylor, 1974; Tversky & Kahneman, 1974). Moreover, theories on different social network topologies and the interactions within these networks are used in developing the model (Barabasi, 2002; Newman, 1999; Wellman & Berkowitz, 1997).

The second contribution lies in the fact that we not only based the model on the theories as introduced in the previous section, but furthermore, using these theories, we have developed specific hypotheses with respect to the individual investors’ trading and interaction behavior and performed empirical studies in which these hypotheses were tested. In chapters 2 and 3, we have reported on the results of these studies in which we investigated e.g., to what extent individual investors have needs that deviate from a risk/returns perspective. Moreover, differences in the amount of investment-related knowledge and experience of these investors were studied. Furthermore, the effect of these differences - which result in different levels of confidence - on the conformity behavior of these investors was examined.

In the empirical studies of the before mentioned chapters it was found that individual investors do have other, more social needs apart from their financially oriented needs. In fact, investors that gave a higher importance to social needs and/or who had lower levels of investment-related knowledge and experience, displayed more informational and normative conformity behavior. Informational conformity behavior is the expression of an individual’s tendency to accept information from others as evidence about reality (Deutsch & Gerard, 1955). This is expressed by e.g., asking the members of one’s reference group for information and subsequently using this information in one’s own decision-making. Normative conformity behavior is the expression of an individual’s desire to comply with the positive expectations of others (Deutsch & Gerard, 1955). This expression of compliance takes the form of e.g., performing similar actions as the individuals to whose norms one wishes to conform.

Subsequently, the results of these studies have been used to develop empirically plausible agent trading and interactions rules and to parameterize the model in order to achieve an artificial stock market that is a closer match with reality than existing models. In case we were unable to collect data to empirically ground a model part, we conformed to well-accepted trading and behavioral principles as reported in the agent-based computational finance literature or relied on information provided by investment practitioners like investment consultants and brokers. In the section where the model will be described, we will make explicit
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how and where empirical findings have been incorporated in the model, and where we adhered to the agent-based computational finance literature’s ‘best practices’. Moreover, more (technical) details on how the empirical findings are incorporated in the artificial stock market will be presented there.

The third and last contribution is that we not only investigate how the aggregation of micro level investor behavior results in macro level stock market results, but we will also estimate the empirical plausibility of the macro level stock market price and returns data that are generated by the model in the spirit of LeBaron (1999). Yet, we will go one step further as we do not only investigate the occurrence of possible stylized financial market facts (Cont, 2001) in the model’s returns time series, but we also evaluate the extent to which these stylized financial market facts in the returns time series of the simulation experiments agree with those of a representative empirical stock market. In this case, we will compare the returns time series of our artificial stock market that uses empirical data of Dutch investors, with the returns time series of the overall Dutch stock market using several statistical techniques.

To summarize, in this chapter we introduce the artificial stock market SimStockExchange™ (from here on ‘SSE’) by outlining the main design of this model. Moreover, we report on a number of simulation experiments and a comparison is made between the results of the SSE and those of real stock markets. The remainder of this chapter is organized as follows. In section 5.2, the SSE model will be presented. In section 5.3, the results of two typical simulation runs of the SSE using two different network types as well as empirically valid parameter settings for the various agent rules are presented and these results are subsequently compared to those of the overall Dutch stock market. Section 5.4 concludes, outlines the limitations of the current model, and discusses perspectives for related future research.

5.2 The SSE

We built an artificial stock market called SSE on a personal computer using a multi agent social simulation approach. The program is written in Java and both the Eclipse and Repast software packages are used in developing, error-testing and running the model. To succinctly show how the model works, we have included a brief sample of the model’s pseudo code in appendix 2 of this chapter.  

A demonstration version of the SSE is downloadable from www.simstockexchange.com.
The SSE is capable of simulating markets with any desirable number of investors. In the SSE, different types of investors exist who conduct transactions based on the investment rules that are formalized for each type. At the beginning of each simulation run, each investor agent is allocated a number of stocks in its portfolio as well as a cash budget. The investors can decide to invest all or part of their budget or to keep all or part of their budget in cash. Investors that are so unsuccessful that they lose their entire budget, are declared bankrupt by the SSE. The SSE offers the possibilities of either replacing these bankrupt agents by similar new agents or to let them remain bankrupt and let them no longer participate in the market interactions of the model. In the simulation experiments of this chapter, bankrupt agents are replaced.

The SSE operates in the following four steps: (1) every investor in the market receives a personal signal (information on the next period’s expected price) and observes the current market price, (2) depending on the confidence of the investor, the personal signal is weighted to a greater or lesser extent with the signal that neighboring agents have received, and based on this an order is forwarded to the stock market, (3) a new market price is calculated based on the crossing of orders in the SSE’s order book, and (4) the agent’s rules can be updated according to their results. In the following sections, the SSE will be explained in more detail.

Step 1: News Enters the Stock Market

Each time period \( t \), investors observe for a stock \( s \) a current market price \( P_{st} \) (which is the same for every investor) and they also have an expectation of the next period’s price \( E_{st} \) (which may differ among investors). The expectation of the next period’s price is based on news that enters the market. Although there is an important body of literature on the impact of different types of news on the stock markets, like announcements on company takeovers (see e.g., Keown and Pinkerton (1981)), announcements on quarterly earnings (see e.g., Bernard and Thomas (1989)), and announcements of stock splits (see e.g., Fama, Fisher, Jensen and Roll (1969)), this literature provides little information on how one could actually model a news arrival process in an artificial stock market. Nevertheless, the literature seems to agree that one important feature of such a news arrival process would have to be that neither fat tails, nor volatility clustering nor any kind of non-linear dependence in the returns time series of the model is caused by the news arrival process, but rather that the occurrence of any of these stylized financial market facts is due to the actual trading and interaction of the investors in the market (Chen et al., 2001). Inspired by the work of Chen, Lux and Marchesi
(2001: 5-6), we therefore model our news arrival process as normally distributed noise with a user-specified standard deviation around the current price. This property of the simulation model represents the fact that in real markets, different investors get different pieces of news, process the news in different ways, and consequently differ in their valuation of shares. This process leads to differences in the prices that the investors expect for the next period. The agents do not know that the signal they receive is random noise and therefore they are bounded in their rationality.\(^5\)

Yet, the current formalization of the news arrival process and corresponding decision-making by the investor agents differs in one important aspect from those that can be observed in real markets. Agents do not take news from previous periods into account for their decision-making. Rather, the agents forget the information signal of the previous period at the start of each new period. However, as we will see in later sections of this chapter, as the agents make their decisions, information is spread through the social network. Depending on the agent’s position in the social network and the agent’s moment of trading, the information it receives within a time period will be aggregated to a greater or lesser extent. So, although there is no aggregation of dispersed information beyond subsequent time periods, within a time period, there are processes of information aggregation.

**Step 2: Agents Make Investment Decisions**

In every time step \(t\), each agent has to decide how much of its budget to invest in the stock \(s\) and how much to keep in cash. To determine what proportion of its available cash budget an agent is willing to invest in the risky asset or what proportion of its portfolio of stocks it is willing to divest, formulas 5.1, 5.2, and 5.3 as depicted below are used. These formulas are based on the well-accepted principle in the field (see e.g., Lux (1998: 148) and references therein) of making a comparison between some kind of fundamental value and current market value, or in our case comparing the expected price of a stock with its current market price.

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\(^5\) In the current formalization of the SSE, the latter characteristic implies that the market interactions are a zero-sum game. Furthermore, the current formalization of the SSE features no transaction costs.
If \( -1 \leq \frac{(E_{st} - P_{st})}{P_{st}} \leq 1 \), then this proportion is \( \frac{(E_{st} - P_{st})}{P_{st}} \) (5.1)

If \( \frac{(E_{st} - P_{st})}{P_{st}} < -1 \), then this proportion is \( \frac{(E_{st} - P_{st})}{P_{st}} \) (5.2)

If \( \frac{(E_{st} - P_{st})}{P_{st}} > 1 \), then this proportion is \( \frac{(E_{st} - P_{st})}{P_{st}} \) (5.3)

\( E_{st} \) = Expected price for stock \( s \) at time \( t \)

\( P_{st} \) = Current market price for stock \( s \) at time \( t \)

That is, an agent weighs the deviation of the current market price from the expected price for the next period by the current market price. When the expected price is higher than the current price it is attractive to invest, when the expected price is lower than the current price, it is more attractive to divest. The agents react stronger as the expected price deviates more from the current market price.6

6 This mechanism can be interpreted as a form of symmetrical, linear loss aversion and is comparable to the mechanisms used in e.g., Day and Huang (1990). It is also possible to include a more elaborate loss aversion mechanism, like the type as assumed in the prospect theory (Kahneman & Tversky, 1979). Asymmetrical loss aversion types, however, call for more elaborate methods of analysis that take this asymmetry into account, like EGARCH (Brooks, 2004). For an example of the application of a loss aversion mechanism as proposed by prospect theory, see e.g., Takahashi and Terano (2003).
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Depending on the standard deviation that is chosen for the news, it would be possible that the above formula returns values that would imply an agent to invest more than its current cash budget allows or to sell more stocks than it has in portfolio. In these instances - of which the chances of occurring are extremely small using the parameter settings of the experiments that are discussed in this chapter - the proportion is limited to the agent’s available cash budget and portfolio of shares as can be seen in the formulas above. So, investors are not allowed to borrow money or short-sell stocks.

Agents with lower levels of confidence \( C \) in the correctness of their own signal perform risk reducing strategies (RRS) in order to reach an investment decision. Using regression analyses, it was found in our empirical studies that investors with lower levels of investment related knowledge and experience who therefore have lower levels of confidence \( C \), perform both more informational and more normative conformity behavior, which are two specific instances of the more general concept of RRS. In general (see e.g., Mitchell & McGoldrick (1996)), RRS may be either more individual or more socially oriented and either of a clarifying or simplifying kind. In table 5.1 below, a number of examples of these different types of RRS that are relevant for an investing context are displayed.

Table 5.1. Risk Reducing Strategies.

<table>
<thead>
<tr>
<th></th>
<th>Individual</th>
<th>Social</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simplifying</td>
<td>Use a simple heuristic, e.g., the P/E ratio of a stock.</td>
<td>Copy the behaviour of other investors in one’s social network.</td>
</tr>
<tr>
<td>Clarifying</td>
<td>Collect more information about the stock.</td>
<td>Ask other investors for more information, e.g., their expectations of the stock value.</td>
</tr>
</tbody>
</table>

In the SSE, we focus on social RRS. The parameter \( C \) weights the extent to which an agent trusts on its individual signal on the expected price for the next period versus the extent to which it uses information obtained from its social network. \( C \) is bounded between 0 (no confidence in the correctness of their individual signal, i.e. only using information from the network) and 1 (complete confidence in the correctness of their individual signal, i.e. only using individual information on the expected next period’s price). The values for \( C \) for each individual investor that will be used in the simulation experiments of this chapter are derived from the empirical studies of the previous two chapters. In order to be able to incorporate this empirical data in the simulation model, two transformations were necessary. First, for each individual respondent, the average of his or her scores on the two questions that were used to measure \( C \) was calculated. Second, this overall score, which was on a five-point Likert scale, was transformed to a score on a one-point scale so that we only had values for \( C \) between 0 and 1. These two steps resulted
in an empirically validated set of estimates of C for a group of 167 investors that subsequently could be loaded into the model.

The SSE also offers the possibility to use different distributions, e.g., uniform and normal distributions, as well as fixed values for the level of confidence C in case one would like to perform other types of experiments. The experiments presented in this chapter, however, use a replication of the distribution of C as found in the empirical study, based on the empirically derived estimates of C as found for each individual investor.

The extent to which an agent uses a clarifying versus a simplifying strategy is weighed by a parameter named R (risk reducing strategy). R is bounded between 0 (only using a clarifying strategy) and 1 (only using a simplifying strategy). The values for R that will be used in the experiments are derived from the empirical studies in the following way. First, for each individual respondent, we calculated his or her average score for his or her propensity to use clarifying strategies $S_c$, which was measured using two questions, and their propensity to use simplifying strategies $S_s$, which was measured using three questions. Second, we again transformed these overall scores - which were measured on a five-point Likert scale - to a score on a one-point scale, so that we also had values for $S_c$ and $S_s$ between 0 and 1. Third, in order to convert these two scores to one value for R that indicated the relative importance of either of these two types of strategies, we used the following formulas:

\[
\text{If } S_s \neq S_c, \quad R = \frac{S_s}{(S_s + S_c)}
\]  

(5.4)

\[
\text{If } S_s = S_c, \quad R = 0.5
\]  

(5.5)

These three steps resulted in an empirically validated set of estimates of R for a group of 167 investors. A schematic overview of the above is given in figure 5.1 below. Although in this overview, only the extreme situations ($C = 0$, $C = 1$, $R = 0$, $R = 1$) are displayed, investors in our model can also trust only partly on their own signal and partly on information obtained from their social network ($0 < C < 1$) and/or use a combination of both clarifying and simplifying strategies ($0 < R < 1$) as can be seen in appendix 3 of this chapter.
In case one would like to perform other types of experiments, the SSE also offers the possibility to use different distributions, e.g., uniform and normal distributions, as well as fixed values with regard to the relative proportion $R$ of using clarifying versus simplifying strategies. The experiments presented in this chapter, however, only use the empirically derived set of estimates of $R$ as found for the individual respondents.

In appendix 3, the empirically derived values as well as some sample statistics for $C$ and $R$ for the before discussed investor sample are displayed in a table.

![Fig. 5.1. Simplified Overview of the SSE’s Agents’ Trading Behavior.](image-url)
The previously introduced clarifying strategy is a form of informational conformity behavior. When performing this strategy, the agent asks other agents in its social network to which it is connected by a single link what prices they expect for the next period and calculates the unweighted average of these expectations. Subsequently, the agent will weigh the value of its individual signal of the expected next period’s price and the signal of the next period’s expected price obtained from its social network by its value for C (confidence). For example, if an agent’s value for C is 0.2, the agent will weigh its own expectation for 20% and the expectation obtained from its social network for 80%.

The previously introduced simplifying strategy is a form of normative conformity behavior. When performing this strategy, the agent will perform similar actions as the agents in its social network to which it is connected by a single link. The agent observes the investment behavior of its neighbors and evaluates whether there are more selling or more buying agents. It will decide whether to buy or to sell depending on what action is dominant among its neighbors. After it has identified the dominant action, it will conform to this action. In order to decide how many shares to buy or to sell, the agent will take the average value of the expectations of the next period’s price of the group of investors (either buyers or sellers) it decided to copy. Then, it again weighs this average value with its own expectation according to its level of confidence (C) to arrive at an average expected value for the next period’s price. This value is subsequently used to arrive at the decision how much of its remaining cash budget or stock portfolio to invest or divest. When the number of buyers and sellers in the market is equal, the investor will simply take the average of all their expectations of the next period’s stock price and weigh this with its own expectation to arrive at a decision. In this one and exceptional case, the decision will thus be made in a similar way as in the clarifying strategy.

All agents in the SSE are connected to each other in a social network. Depending on the specific properties of this social network topology, agents will differ in the number of other agents to which they are connected and information on the expectations of the next period’s stock price will diffuse in different ways and at different speeds through the social network. This may offer some agents an informational advantage in comparison to other agents in the social network.

7 The SSE also offers several other options of weighting the expectations of the agents’ neighbors, like weighting the neighbors’ expectations according to the importance of their network position or their past success in investing.
In every time step, all agents receive news at the same time, but the order in which the agents are allowed to decide to trade and forward their order to the order book, is random. So, depending on whether an agent is amongst the very first or last agents which are selected to trade, the information this agent collects from its direct neighbors may already contain information that these neighbors have collected from their neighbors and the neighbors’ neighbors for their decision-making or not. Notwithstanding this simplification, different social network topologies will vary in their information diffusion characteristics (Cowan & Jonard, 2004). In order to investigate to what extent the stock market returns time series’ dynamical features are affected by these differences, in our simulation experiments we compare the results of two different network topologies. The network topologies that we use are the torus network (regular lattice) and the Barabasi and Albert (1999) scale free network, respectively. A torus network is simply a lattice whose edges are connected, and a scale free network is a network in which the distribution of the connectivity of the nodes of the network follows a power law, i.e. there are many nodes with only a few connections to others, and there are only a few nodes with many connections to others. Many networks in real life, like those of websites and scientific citations, behave like a scale free network and also have small world characteristics (Amaral et al., 2000; Barabasi, 2002; Barabasi & Albert, 1999; Buchanan, 2002; Watts & Strogatz, 1998).

Figures of the two different network topologies that are used in the experiments are displayed in figure 5.2 and figure 5.3, respectively.
Fig. 5.2. Torus Network.
Step 3: a New Market Price is Calculated

To determine the market price, an order book is used to which agents forward their orders that either consist of a maximum price and the number of stocks that the agent wishes to buy for this price or a minimum price and the number of stocks the agent is willing to sell for this price. The general rule is that the processing of orders follows the FIFO (first in, first out) principle and in the
situation that it is not possible to cross a complete order, an order will be partly executed.\(^8\) The non-executed part of the order remains in the order book until it is (1) eventually crossed by another order or (2) the respective agent decides to issue a new order.

**Buying Stocks**

The *maximum price* that buying agents are willing to pay for their shares is the price they expect for the next period. At prices lower than this expectation, these agents expect to gain money. At prices higher than their expectation, the agents expect to lose money. This maximum price is used to determine the number of shares the investor wants to buy. This is done by dividing the budget to invest by the expected price for the next period, rounded off downward to the closest whole number. So, each buying investor forwards a limit order to the market stating the number of shares it wants to buy and the limit price that it is willing to pay for these shares. In case its limit order crosses another agent that is willing to sell the requested number of shares at the indicated limit price, the order is executed and removed from the order book. In case the order crosses another agent that is willing to sell at a price below the limit, the order is executed for the average of the bid and ask price. This system of ‘splitting the difference’ is inspired by Beltratti and Margarita (1992). In case there is no agent willing to sell at the limit order price of the agent, the order stays in the order book until it eventually crosses another ask order or the agent issues a new order, in which case the old order will be deleted from the order book.

**Selling Stocks**

The *minimum price* selling agents are willing to sell their shares for is the price that they expect for the next period. For them it is no problem to sell below the current price, as they expect the price in the next period to (downwardly) move towards their own expected price, which is lower than the current price. As long as the selling price is above the expected price for the next period, these agents expect to minimize their losses. However, they will not sell for less than the price they expect for the next period. So, these agents calculate the number of shares they want to sell as the budget they want to sell divided by the expected price for

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\(^8\) We use the in the financial literature common phrase ‘crossing of orders’ to indicate that a buy or sell order matches another sell or buy order, respectively, and subsequently a transaction takes place.
the next period, also rounded off downward to the nearest whole number. In case this limit order crosses another agent that is willing to buy the offered number of shares at the indicated limit price, the order is executed and removed from the order book. In case this order crosses another agent that is willing to buy at a price above the limit, the order is executed for the average of the ask and bid price. In case there is no agent willing to buy at the limit order price of the agent, the order stays in the order book until it finally crosses another bid order or the agent issues a new order, in which case the old order will be deleted from the order book.

The Market Price of Stocks

The market price that is realized in each time step is calculated as the average of the bid and ask prices that are present in the order book, weighted by the number of asked and offered shares. In this way, we account for the price pressure that is put on the market by the bid and ask orders. Even when not all the orders are executed, they will still influence the market sentiment and therefore the price level.

Although in the simulation experiments described in this chapter, only the shares of one stock are traded amongst the investor agents, the SSE offers the possibility to incorporate a number of different stocks.

Step 4: Updating the Agents’ Rules

Most if not all artificial stock market models, including the SSE, have a feedback loop from the macro to the micro level in the sense that the individual agent’s orders are influenced by an aggregate variable such as the stock price. This type of feedback effect can be characterized as feedback influencing the input that is used in the decision-making of the individual agents. However, this type of feedback does not change the way in which the agents make their decisions. Investors for example do not change the type of strategy they use, only the input that is used to determine e.g., the type of order (buy or sell) and the order size, is affected. A general point of critique on these type of models made by e.g., Arthur (1995) is therefore that the market dynamics are generated by the actions of the investors, but the cognition of the investors is never affected by the evolution of the market.

One of the contributions of the SSE is that we have incorporated the possibility to include a feedback mechanism that influences the decision-making of the investor. Investors can change their strategies according to the returns they get. Using this updating mechanism, the rules which the agents use will depend on the successfulness of these agents. Agents with higher returns, who are more successful, get higher levels of confidence C in the correctness of their own signal and therefore in the correctness of their own rules. Yet, it should be noted, that with the current formalization of the news arrival process, there is little in these news signals except from the variance - that could make decisions based on them more or less effective than other agents’ decisions. Since agents cannot ‘choose’ this variance themselves, the current formalization of the model gives little room for the agents’ decisions to be improved consistently. In future versions of the
model, experiments could be performed with news arrival processes using other
distributions, giving more learning opportunities for the agents and so the
possibility to create truly superior strategies that provide these agents with
consistently higher payoffs.

5.3 Experiments

Using the settings for both C and R as were empirically found for 167 investors,
the experiments that will be discussed in this chapter compare the time series
behavior of the simulated price and returns series in two different network
situations, a torus network and a scale free network. 9 Due to the superior
information diffusion characteristics of the latter, we expect random shocks (news
in the form of random noise) to dampen out more quickly and therefore expect
these networks to display less volatility clustering than networks with poorer
information diffusion characteristics, like the regular torus network.

In table 5.2 below, an overview of the settings of the two simulation
experiments can be found.

9 As the 167 investors for which we have obtained empirical data do not constitute a social
network by themselves, but rather are a sample of the overall population of investors with
direct investments in the Dutch stock market, the exact positions of the investor agents in
the two different social networks is arbitrarily chosen. In future research, one might try to
rebuild an actual social network of investors and incorporate it in an artificial stock
market.
Table 5.2. Parameter Settings.

<table>
<thead>
<tr>
<th>Parameter description</th>
<th>Experiment 1:</th>
<th>Experiment 2:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of agents</td>
<td>167</td>
<td>167</td>
</tr>
<tr>
<td>Initial wealth of agents</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Bankrupt agents</td>
<td>Are replaced</td>
<td>Are replaced</td>
</tr>
<tr>
<td>Network type</td>
<td>Torus</td>
<td>Scale free network</td>
</tr>
<tr>
<td>News distribution</td>
<td>Normal</td>
<td>Normal</td>
</tr>
<tr>
<td>News average μ</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>News standard deviation σ</td>
<td>0.020</td>
<td>0.020</td>
</tr>
<tr>
<td>News frequency</td>
<td>Every time step</td>
<td>Every time step</td>
</tr>
<tr>
<td>Number of stocks</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Number of time steps</td>
<td>929</td>
<td>929</td>
</tr>
<tr>
<td>Initial number of stocks in portfolio</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Initial stock price</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Updating of confidence of agents according to their returns</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Level of confidence C</td>
<td>See appendix</td>
<td>See appendix</td>
</tr>
<tr>
<td>Risk Reducing Strategy R</td>
<td>See appendix</td>
<td>See appendix</td>
</tr>
<tr>
<td>Seed to generate network</td>
<td>1159791325531</td>
<td>1159791325531</td>
</tr>
</tbody>
</table>

To generate the time series of both experiments, the market was initially run for 500 time steps to allow eventual early transients to die out. Subsequently, the market was run for another 929 time steps for which the returns time series were calculated. The number of 929 time steps was chosen to accommodate with the availability of data on the real stock market that would be used as a benchmark. In order to avoid problems of missing data due to weekends, holidays, and other special occasions, we decided to use weekly data. This resulted in 929 observations for the overall Dutch stock market from the seventh of January of 1987 until the twentieth of October 2005. ¹⁰

¹⁰ This was the longest time frame available from DataStream at the time of collecting the data for this chapter (October 2005).
In figures 5.4 through 5.7 and 5.8 through 5.11, the price and returns time series, returns distribution and autocorrelation graphs for experiment 1 and 2 with the torus network and the scale free network, respectively, are shown. Figures 5.12 through 5.15 show the same weekly information for the overall Dutch stock market.

Fig. 5.4. Price Time Series Experiment 1 (Torus Network).

Fig. 5.5. Returns Time Series Experiment 1 (Torus Network).
The impression given by figure 5.4 and figure 5.5 - although it is more difficult to notice in the latter figure - is that the price swings in the market 'arrive in clusters'. Periods of relative tranquility in which the price changes remain small are alternated by periods of increased volatility with larger price changes. Volatility clustering in the returns time series of this experiment, as can also be observed in real stock markets, is therefore expected to be present in the results of this first experiment. However, more thorough statistical analyses like GARCH
(1,1) estimates are necessary to prove this to be actually the case. In table 5.3, one can observe the results of such a GARCH model.

Evaluating figure 5.6 suggest the returns distribution of the first experiment to be relatively close to a normal distribution, while more formal results on this are also presented in table 5.3.

Figure 5.7 shows significant linear autocorrelation in the returns for many lags. In general, weekly and monthly data on real stock markets have also been found to exhibit linear autocorrelation (Cont, 2001) and it can be seen in figure 5.15, that the weekly returns of the overall Dutch stock market also show significant autocorrelation for many lags. For more high frequency data, like hourly or daily stock data, no significant linear autocorrelation in both price increments and asset returns are reported (Fama, 1970; Pagan, 1996). Absence of autocorrelation means that it is impossible to consistently achieve positive expected earnings with a simple strategy that uses statistical arbitrage. An investor cannot be expected to be able to predict tomorrow’s stock prices or asset returns using today’s stock prices or asset returns data. This can be seen as support for the efficient market hypothesis (Fama, 1991). However, as stated above, it has been proven (see e.g., Cont (2001)) that when the time scale on which the linear autocorrelation is measured is increased to e.g., weekly data, this absence of autocorrelation does not systematically hold anymore.

Fig. 5.8. Price Time Series Experiment 2 (Scale Free Network).
Fig. 5.9. Returns Time Series Experiment 2 (Scale Free Network).

Fig. 5.10. Returns Distribution Experiment 2 (Scale Free Network).
Fig. 5.11. Autocorrelation Graph of the Returns of Experiment 2 (Scale Free Network).

It is difficult to detect systematic differences between the two experiments when comparing figures 5.4 through 5.6 with figures 5.8 through 5.10. On first sight, one might be led to believe, that the price and returns time series of the experiments performed with the two different social network topologies display rather similar characteristics. Only when comparing the autocorrelation graphs of figures 5.7 and 5.11, a clear distinction between both experiments appears. While the returns of experiment 1 (torus network) show significant autocorrelation in 17 lags, this is only true for 9 lags in experiment 2 (scale free network). Using statistical tests makes it easier to detect systematic differences between the results of experiment 1 and 2. The results of such tests are depicted in table 5.3.

Fig. 5.12. Price Index Overall Dutch Stock Market 1987–2005.
Fig. 5.13. Returns Time Series Overall Dutch Stock Market 1987–2005.

Fig. 5.14. Returns Distribution Overall Dutch Stock Market 1987–2005.
Fig. 5.15. Autocorrelation Graph of the Returns of the Overall Dutch Stock Market 1987–2005.

Table 5.3. Summary Statistics of Experiment 1, 2, and the Overall Dutch Stock Market.

<table>
<thead>
<tr>
<th>Description</th>
<th>Experiment 1: torus network</th>
<th>Experiment 2: scale free network</th>
<th>Dutch Stock Market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std. σ</td>
<td>0.051</td>
<td>0.063</td>
<td>0.025</td>
</tr>
<tr>
<td>Karimova</td>
<td>3.149</td>
<td>2.919</td>
<td>8.466</td>
</tr>
<tr>
<td>Durbin Watson Statistic</td>
<td>2.184</td>
<td>2.117</td>
<td>2.194</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>7.795**</td>
<td>0.020</td>
<td>4.819</td>
</tr>
</tbody>
</table>

**Conditional Variance Equation**

<table>
<thead>
<tr>
<th>Description</th>
<th>Experiment 1: torus network</th>
<th>Experiment 2: scale free network</th>
<th>Dutch Stock Market</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.000</td>
<td>0.000</td>
<td>0.002*</td>
</tr>
<tr>
<td>Residuals (-1)∧2 (ARCH)</td>
<td>0.015</td>
<td>0.511</td>
<td>0.061</td>
</tr>
<tr>
<td>GARCH (q=1)</td>
<td>0.827*</td>
<td>0.004</td>
<td>0.078</td>
</tr>
</tbody>
</table>

- Values marked with an asterisk (*) are significant at the 0.01 level.
- Values marked with two asterisks (**) are significant at the 0.05 level.
- Please note that all statistics are performed on the continuously compounded returns as a perumage (i.e., returns \( r_t = \ln (p_t / p_{t-1}) \)).
Table 5.3 reports on the results of a number of common statistical tests performed on the overall market returns time series of both experiments and of the weekly returns data of the overall Dutch stock market.

The first row shows the volatility of the returns time series as defined by the standard deviations of the returns for the two simulation experiments as well as for the overall Dutch stock market. The standard deviation of the news in the two experiments was set at 0.02, a value close to the standard deviation that can be observed in the returns time series of the overall Dutch stock market. However, both experiments show a higher variance in the returns time series than the before mentioned 0.02 and the difference between the two cases is relatively small. Social interaction amongst investors in the SSE is a possible reason for this increased level of variance. Investors partly reacting on news and partly reacting on each other might create self-reinforcing dynamics, thereby pushing the standard deviation of returns to higher levels than can be justified by the news that hits the market only.

The second row shows the kurtosis of the simulation experiments as well as that of the overall Dutch stock market. The two simulation experiments show a kurtosis that approximates that of a normal distribution - which has a kurtosis of 3.00 - while the empirical stock market shows a significant amount of excess kurtosis, which can also be seen in figure 5.14. The returns distribution in the overall Dutch stock market is leptokurtotic; a pattern that is common for real asset returns distributions. Using the parameter settings from table 5.2, the SSE does not replicate this empirically found fact. This can be explained by the fact, that the news generation process in the SSE is currently based on a normal distribution. As argued in a previous section, this was a necessary simplification as the current literature provides no information on how one could incorporate real-life news arrival processes in a model.

The third row shows the Durbin Watson statistic, testing for autocorrelation in the residuals. We can observe from table 5.3, that the two simulation

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11 Autocorrelation is the correlation of a process X_t against a time-shifted version of itself. The efficient markets hypothesis of modern finance literature assumes that the residuals of today are uncorrelated with the residuals of tomorrow. That is, today’s news is completely and immediately absorbed in today’s stock prices and has no effect on tomorrow’s stock prices. When the Durbin Watson statistic takes on the test value of two, this corresponds to the case where there is no autocorrelation in the residuals. When this statistic takes on a test value of zero, this corresponds to the case of perfect positive autocorrelation in the residuals. In case the test statistic takes on the value of four, this corresponds to the case where there is perfect negative autocorrelation in the residuals.
experiments take on test values between 2.12 and 2.18, which lines up well both qualitatively and quantitatively with the test value of 2.19 as found for the empirical stock market. These values indicate, that both the two simulation experiments as well as the overall Dutch stock market display a very small amount of negative autocorrelation.

The next row shows the coefficients and probabilities of the Jarque-Bera test, which is a goodness-of-fit measure of departure from normality which is based on both the sample kurtosis and skewness (Bera & Jarque, 1980; 1981). When the coefficient of this test is significant, the returns time series depart from normality, and the higher the value of this coefficient, the greater the departure from normality. As can be seen in table 5.3, the returns distribution of the overall Dutch Stock Market strongly departs from normality, while the returns distribution of the first experiment (torus network) do so to a lesser extent. The returns distribution of the second experiment (scale free network) does not significantly deviates from normality.

The next three rows show a common test in finance for volatility persistence or ‘volatility clustering’, namely the (Generalized) ARCH test. A typical pattern observed for real asset returns is that the coefficients on all three terms in the conditional variance equation are highly statistical significant, with a small value for the variance intercept term C, a somewhat larger ARCH term, and an even larger GARCH term. The ARCH term represents the lagged squared error, while the GARCH term represents the lagged conditional variance. For real asset returns, both terms summed together are generally found to be close to 1 (unity). This indicates that shocks to the conditional variance will be highly persistent, i.e. there is volatility persistence or volatility clustering. Qualitatively, the results of both simulation experiments line up relatively well with the empirical stock market with regard to the relative proportions of the three terms of the conditional variance equation. Only in the first experiment, however, we observe a statistically significant GARCH term that also lines up well quantitatively with the terms found for the real asset returns of the overall Dutch stock market. For experiment 2, with the scale free network, none of the terms in the conditional variance equation is statistically significant, while experiment 1, with a regular torus network, displays a highly significant GARCH effect. So, for these two experiments, we observe ceteris paribus that in artificial stock markets with scale

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12 ARCH is the test for conditional heteroscedasticity as developed by Engle (1982). GARCH is a generalized model for conditional heteroscedasticity as developed independently by Bollerslev (1986) and Taylor (1986).
free networks, there is no statistically significant proof of volatility clustering, but artificial stock markets with a torus network do display volatility clustering. A possible explanation for this is that the superior information diffusion capacities of the scale free network facilitate an immediate absorption of the news by all network members and inhibit news shocks of yesterday to have much effect on the returns of today. Moreover, one could argue that in spite of today’s ubiquitous information through e.g., mass medial devices and the Internet, which have lowered the cost of information drastically, and despite the fact that theoretically and empirically there is a good case for the society as a scale free network, in reality (at least for the investing population of society) the society is more likely to behave like a torus network with regard to the information diffusion capacities, where information sometimes takes long to travel to remote corners of the network and shocks of the past continue to influence the present for a considerable period of time.

5.4 Conclusions and Limitations

In this chapter we have presented the SSE and given a practical example of the possible combination of empirical micro and macro level data, theoretical micro and macro level perspectives, and a multi-agent based social simulation approach in the development of an artificial stock market. It was shown, how artificial stock markets can be used to explore how different micro level behavioral processes aggregate to macro level phenomena and, in turn, how these aggregated outcomes affect individual investors’ behavior. In the SSE, investor agents make investment decisions using empirically estimated decision rules and socially interact in different social network structures. From these market interactions, macro level price and returns time series result, which are subsequently compared to empirical macro level data. First comparisons showed limited qualitative and quantitative resemblances of the simulated data with the real data and therefore a number of opportunities to improve this fit. In the following, we will outline a number of limitations of the current study, which provide opportunities for future research and which are expected to improve the fit between the results of the model and the real world when they would be overcome.

First, the artificial stock market SSE is a model and therefore remains only a simplified reproduction of reality. Especially the question on how one should model the news arrival process poses a number of difficult questions which are hard to overcome for a modeler of an artificial stock market. It is precisely this difficulty, and the chances to be heavily criticized for the way one models the news arrival process if one does try to incorporate such a process, that can be understood as a rationale for many modelers in agent-based finance to omit news arrival processes from their artificial stock markets altogether (Lux, 2006). We, however, have chosen to model the news arrival process in a simplified way as normally distributed noise around the current price, which forms a limitation of the current study but poses a challenge for future research.
Second, the respondents from the empirical studies that were used to formalize the agents’ trading and interaction rules, were mainly individual investors. Only a small percentage (approximately 5%) of the respondents indicated to be either a professional investor, a broker or to work at a large investment company. It could be argued, that in reality, large institutional investors are the main determinants of price dynamics and therefore the composition of our sample might represent a limitation. There are a number of reasons, why we expect the implications of this possible limitation to be limited. First, individual investors constitute an important group in the financial marketplace and their decision-making behavior is likely to have an impact on the stock market as a whole (De Bondt, 1998). The latter argument becomes even more pronounced taking into consideration that even a small country as The Netherlands already accommodates 2,300,000 individual investors that invest directly and indirectly in the stock market (VEB, 2002). Second, it seems safe to assume, that the stock price expectations of most large institutional investors and investment banks - who are well-connected to the most important stock analysts - are predominantly of the correct magnitude. This would imply that most market disturbances are caused by individual investors and that the large institutional investors merely form a stable force in the market that has to react to or is affected by disturbances caused by the individual investors. This then again makes it highly interesting and relevant to investigate the effect of different types of individual investors on the stock market dynamics.

Third, the empirical benchmark in this study was the overall Dutch stock market, rather than specific shares of a company that are traded on this market. We expect, however, important differences between the time series behavior of specific companies and that of the aggregate market. It seems a realistic assumption, that certain shares are traded more by investors with a low confidence or who are highly socially oriented and other shares are traded more by high confidence, experienced investors who are only interested in certain fundamental characteristics of the company and make their decisions in a more individual way. In future research, one might therefore compare the results of the SSE to the returns time series of several individual publicly traded companies.

Fourth, the time horizons of the simulation-generated data and the real market data might be incompatible. It is not yet evaluated whether one time step of the simulation model with its accompanying data point is comparable to one data point in the real market data, but in future studies we intend to experiment with different time horizons.

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13 We thank one of the anonymous referees of the Journal of Artificial Societies and Social Simulation for bringing this issue to our attention.
Appendix 1: Overview of Agent-Based Models in Finance.

The framework of Lettau (1997) is an agent benchmark that implements many of the ideas of evolution and learning in a population of traders in a very simple setting. In this setting, agents have to decide how much of a risky asset to purchase, which is sold at a price $p$ and which issues a random dividend $d$ that is paid in the next period. There are two main simplifications in this framework: the price is given exogenously and the agents are assumed to have myopic constant absolute risk aversion preferences. The objective of Lettau is to investigate how close evolutionary learning mechanisms can get to the optimal solution in deciding how much of the risky asset to hold in comparison to the risk free bond paying zero interest. The results of this framework demonstrate that the genetic algorithm is able to learn the optimum distribution between the risky asset and risk free bond, but is biased towards holding more of the risky asset.

The framework of Gode and Sunder (1993) is presented as an early benchmark paper in which the effect of zero intelligence traders is investigated. In their experiments, Gode and Sunder compared the results of a population of non-learning, randomly trading agents with the results of real trading experiments. In this framework, a double auction market is used in which the efficiency of the market is evaluated by comparing the profits earned by the traders to the maximum possible profit. Two types of experiments are performed, one in which the agents trade randomly, but with a budget constraint and one in which the agents behave completely random, without any budget constraint. The population of budget-constrained traders displays relatively calm price series that are close to equilibrium and the market efficiency of 97% is comparable to that of populations of human traders. The population of completely random traders that are not limited by a budget, however, displays very volatile price series and the market efficiency ranges from 50% to 100%. The message from this paper is that it is very important to distinguish between features of artificial (stock) markets that are due to learning and adaptation and those which are caused by the market structure itself.

An example of a more extensive framework that attempts to simulate more complicated market structures is that of Arifovic (1996). This author considers a general equilibrium foreign exchange market inspired by Kareken and Wallace (1981). LeBaron (2000) notes that a crucial aspect of this model is that it is underspecified in its price space which causes it to contain infinitely many equilibria. The economy that this model aims to represent is based on a simple overlapping generations economy in which two period agents have to decide which of two currencies to use for their savings. This framework introduces a number of important issues for artificial markets (LeBaron, 2000). First, it considers the equilibrium in a general equilibrium setting with endogenous price formation. Second, it compares the model learning dynamics to results obtained from actual experimental markets as in Gode and Sunder (1993). Third, it is able to replicate certain features from these experiments which other learning environments are unable to replicate.
A framework with an even more complex model structure is the one of Routledge (1994), which focuses on ideas of uncertainty and information in financial markets. It implements a version of Grossman and Stiglitz’s (1980) model with agents that use genetic algorithms for learning. The model is based on a repetition of a one period portfolio decision problem between a risky asset and a risk free asset in which agents can decide to purchase a costly information signal on the dividend payout. The informed agents develop their forecasts based on the signal which they have bought and the uninformed develop their forecasts using the only piece of information available to them, the price. This model illustrates the finite number problem, i.e. how many agents are needed for good learning to occur in a population; a problem which is suggested to be only really addressable using a computational framework (LeBaron, 2000).

The Santa Fe Stock Market has been claimed as “one of the most adventurous artificial market projects” (LeBaron, 2000: 690) and is described in detail in Arthur et al. (1997) and LeBaron et al. (1999). This market attempts to combine a well-defined economic structure in the market trading mechanisms with inductive learning using a classifier-based system. As with the previously described frameworks, the market setup utilizes concepts from existing work, such as Bray (1982) and Grossman and Stiglitz (1980). In the Santa Fe Stock Market, the one-period, myopic, constant absolute risk averse agents have to compose a portfolio of holdings of a risk free bond which is in infinite supply and pays a constant interest rate $r$ and a risky stock which pays a stochastic dividend $d$. The before mentioned complexity of this market brings both advantages and disadvantages (LeBaron, 2000). An advantage of this market is that it allows agents to explore a wide range of possible forecasting rules and they are flexible in deciding whether to use or ignore different pieces of information. Moreover, the interactions that cause trend following rules to persist are endogenous. A disadvantage is that the market as a computer study is relatively difficult to track and it is sometimes difficult to establish what causalities are functioning inside this market. The foregoing makes it more difficult to draw strong theoretical conclusions about the reflections of this market on real markets.

Beltratti and Margarita (1992) present another interesting framework which differs from the ones previously described in that trade takes place in a random matching environment and agents forecast future prices using an artificial neural network. Agents forecast future prices by using a network that is trained with e.g., several lagged prices and the average trade prices from earlier periods. Then, agents are randomly matched and trade occurs whenever two agents have different expected future prices. The trades are executed at the average of their two respective expected future prices, i.e. they split the difference. Another way in which this framework differs from some of the previously discussed frameworks is that trade is decentralized. An interesting result from this model is that when one varies the cost to agents of buying more complicated neural networks (that are able to give better forecasts) and/or the stages of a market’s development, different types of traders either can coexist or dominate the other type and the value of buying a more complicated neural network differs.
Many other models of artificial stock markets exist (for an overview see e.g., the review article of LeBaron (2000: 695-696)). Many of these markets are based on research that distinguishes several kinds of traders (e.g., information traders versus noise traders or fundamentalists versus chartists) and subsequently observe the market dynamics after these groups are let to interact (e.g., Chiarella (1992) and Day and Huang (1990)). For a recent overview of the specific field that is called ‘interacting agents in finance’, which studies the effects of different proportions of fundamentalists and chartists, we refer to Hommes (2006).
Appendix 2: Model Pseudo Code.

\( E_{st} = \) Expected price for stock \( s \) at time \( t \)
\( P_{st} = \) Market price for stock \( s \) at time \( t \)
\( N_{st} = \) News for stock \( s \) at time \( t \)
\( \text{Strat} = \) Preference for an agent for simplifying risk reduction \((0 \leq \text{Strat} \leq 1)\)
\( \text{NEst} = \) Aggregated expected price for stock \( s \) at time \( t \) from an agents neighbours
\( \text{SimplNEst} = \) Aggregated expected price for stock \( s \) at time \( t \) from an agents neighbours, based solely on simplifying risk reduction.
\( \text{ClarNEst} = \) Aggregated expected price for stock \( s \) at time \( t \) from an agents neighbours, based solely on clarifying risk reduction.
\( \text{Conf} = \) The agent’s confidence level \((0 \leq \text{Conf} \leq 1)\)
\( O_s = \) The amount of shares owned in stock \( s \) by the agent
\( L = \) Loss aversion type.

for \( t=1 \) to timespan
  step agents
    for each agent update expected values of stocks
      if there is news
        \( E_{st} = P_{st-1} + (P_{st-1} \times N_{st}) \)
      end
    end
  end
  for each agent get expected prices from neighbours
    \( \text{NEst} = (\text{SimplNEst} \times \text{Strat}) + (\text{ClarNEst} \times (1 - \text{Strat})) \)
    \( E_{st} = (E_{st} \times \text{Conf}) + (\text{NEst} \times (1 - \text{Conf})) \)
  end
  for each agent: place trade orders
    if \( E_{st} > P_{st} \) then
      if \( L = \) linear
        \( B = \) \((\text{cash} \times (E_{st} - P_{st})/P_{st} \times E_{st})\)
      if \( L = \) kahneman/tversky
        \( B = \) \((\text{cash} \times ((E_{st} \times 0.88) - P_{st}/P_{st}) \times E_{st})\)
    end
    else if \( E_{st} < P_{st} \) then
      if \( L = \) linear
        \( S = \) \((O_s \times (E_{st} - P_{st})/P_{st} \times E_{st})\)
      if \( L = \) kahneman/tversky
        \( S = \) \((O_s \times (P_{st} + (P_{st} \times (-2.25 \times (- (E_{st} - P_{st}) \times 0.88))))) \)
    end
  end
end

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Chapter 5: SimStockExchange

step market
for each stock
    for each order (in the order placed)
        if order is buy order
            match with the lowest priced sell order
            subtract the amount of shares needed to satisfy the buy order with the amount in the sell order, repeat matching sell orders until the buy order is satisfied
            the trading price for each transaction is the average of the limits of the two orders
        if order is sell order
            match with the highest priced buy order
            subtract the amount of shares needed to satisfy the sell order with the amount in the buy order, repeat matching sell orders until the sell order is satisfied
            the trading price for each transaction is the average of the limits of the two orders
    end
end

Set $P_{st}$ for each stock to the average trading price

end
# Appendix 3: Empirical Values and Sample Statistics C and R.

<table>
<thead>
<tr>
<th>Agent ID</th>
<th>C</th>
<th>R</th>
<th>Agent ID</th>
<th>C</th>
<th>R</th>
<th>Agent ID</th>
<th>C</th>
<th>R</th>
</tr>
</thead>
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<td>0.35</td>
<td>0.15</td>
<td>93</td>
<td>0.5</td>
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