A Theory of Rational Hedging

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Abstract

The apparently low demand for consumer hedging instruments, and particularly weather derivatives, is explained as a rational response to deadweight costs and the risk of contractual nonperformance. Rational demand for hedging instruments does not correspond with economist’s intuition about indemnity insurance. In particular, optimal demand is zero for infinitely risk averse individuals, and is nonmonotonic in risk aversion, wealth and price. For CARA, demand for an actuarially unsubsidised hedging instrument is hump-shaped in the coefficient of absolute risk aversion. Upper bounds are derived for the optimal demand from rational individuals. These bounds appear to be low or zero for many of the unsubsidised weather derivative products currently being sold to poor farmers. The level of observed demand, and the relationship with measures of risk aversion, appear to be consistent with a model of rational demand.

Keywords: Microinsurance; index insurance; derivative; basis risk; hedge.

JEL Classification Numbers: D14, D81, G20, O16.

1 Introduction

When should consumers use financial contracts to hedge against a potentially material loss, and when should they not? The question is not a trivial one to answer. The net transfer under a hedging instrument may be correlated with a consumer’s loss but, unlike under an indemnity insurance contract, the realised net transfer cannot be written as a monotonic function of the incurred loss. Weighed against any benefit from hedging is therefore both the deadweight cost of such contracts, typically passed on to the purchaser through an increased premium, and any risk that the net income from the financial contract will not accurately reflect the incurred loss. As compared to indemnity insurance, hedging instruments offer the advantage of a significant reduction in costs and information asymmetries, but may be considered to be of lower quality in the sense that the net transfer to the purchaser will not always fully reflect the incurred loss.

One particularly instructive hedging product is that of the weather derivative, which over the last ten years has begun to be sold by a variety of well-meaning institutions to poor farmers. The rationale typically given is quite convincing: agriculture is an uncertain business, leaving households vulnerable to serious hardship (Dercon 2004, Collins et al. 2009), and traditional indemnity-based approaches to crop insurance were unsustainably expensive, plagued by moral hazard, adverse selection and high loss adjustment costs (Hazell 1992, Skees et al. 1999). By
comparison, contracts conditional only on weather indices can be fairly cheap whilst still offering much needed protection against extreme weather events such as droughts (Hess et al. 2005).

The potential for weather derivatives to improve the lives of the rural poor has led to careful empirical studies by academic economists who have in turn noted two puzzles. First, demand for such products has been lower than expected by the researchers. For example, Cole et al. (2009) report on a series of rainfall derivative trials in two Indian states in which only 5–10% of households in study areas purchased cover, despite rainfall being overwhelmingly cited as the most important risk faced. Moreover, the vast majority of purchases were for one policy only, which the authors estimate would hedge only 2–5% of household agricultural income. This puzzle of apparently low demand for weather derivatives has prompted attention from empirical economists, hoping to disentangle various candidate causes of low demand such as financial illiteracy, lack of trust, poor marketing, credit constraints, basis risk, and price (Giné et al. 2008, Giné and Yang 2009, Cole et al. 2009, Cai et al. 2009).

The second empirical puzzle is that demand seems to be particularly low from the most risk averse (Giné et al. 2008, Cole et al. 2009). Guided by theoretical predictions arising from the mean variance model of Giné et al. (2008), both these sets of authors attribute this result to an unwillingness to experiment or a lack of understanding on the part of farmers. Karlan and Morduch (2009) summarises this view in the recent entry to the Handbook of Development Economics:

‘The most likely explanation [for demand falling with risk aversion] is that it is uncertainty about the product itself (Is it reliable? How fast are pay-outs? How great is basis risk?) that drives down demand.’ (Karlan and Morduch 2009)

This paper takes a different approach. Our model is one of rational demand, where the consumer is assumed to be a price taking risk averse expected utility maximiser with, for some results, decreasing absolute risk aversion (DARA). Our model is able explain both of the above puzzles as rational responses to the risk of contractual nonperformance, or basis risk as it is referred to in financial markets, and actuarially unfair premiums, that is where the expected net transfer to the purchaser is negative.

One critical aspect of the model is the nature of the joint probability structure of the hedging product and the consumer’s loss. The net transfer from the available hedging instrument is assumed be imperfectly affiliated with the consumer’s net loss, with purchase of a hedge assumed to both worsen the worst possible outcome and improve the best possible outcome; a consumer might incur a loss but receive no net income from the hedge or incur no loss but receive a positive net income from the hedge. This model of basis risk is fundamentally different to the independent, additive, uninsurable background risk often considered by insurance theorists, under which purchase of indemnity insurance results in a contraction of net wealth in the sense of Rothschild and Stiglitz (1970). However, the present model shares many similarities with Schlesinger and Schulenburg’s (1987) and Doherty and Schlesinger’s (1990) models of indemnity insurance with insurer default.

Under the assumed joint probability structure, the shape of rational demand for the hedging product is fundamentally different to rational demand for indemnity insurance. For example, to understand the rational behaviour explanation for the puzzle that purchasing of hedging products is decreasing in risk aversion consider an infinitely risk averse, maximin, consumer. If offered indemnity insurance such an individual would rationally purchase full insurance, since would maximise the minimum possible wealth. However, if offered a hedge with basis risk it would be rational to purchase zero units of the hedge, since hedging decreases the minimum possible wealth; the individual could incur the maximum possible loss but receive a negative net income from the hedge. In general, rational demand for a hedging product is non-monotone in the level of risk aversion. Moreover, when the hedging product is actuarially unfair rational demand is shown to be hump-shaped in the coefficient of absolute risk aversion for the class of CARA utility functions.
Table 1: Joint probability structure

<table>
<thead>
<tr>
<th>Loss</th>
<th>Index = 0</th>
<th>Index = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1 − q − r</td>
<td>q + r − p</td>
</tr>
<tr>
<td>p</td>
<td>1 − p</td>
<td>p</td>
</tr>
<tr>
<td>1 − q</td>
<td>r</td>
<td>p − r</td>
</tr>
<tr>
<td>q</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the key theoretical contribution of this paper, we derive upper bounds for rational purchase of unsubsidised hedging instruments. For the case of indemnity insurance, that is insurance without basis risk, risk aversion and DARA alone cannot bound the purchase of indemnity insurance below full insurance; an infinitely risk averse individual would rationally purchase full insurance. However, tighter bounds may be derived for hedging products with basis risk, both under the restriction of risk aversion alone, and that of risk aversion and DARA. These bounds appear to be zero or very small for many of the weather derivatives currently being sold to poor farmers, suggesting that the low demand observed by Cole et al. (2009) is not prima facie evidence of irrationality.

The rest of the paper is organised as follows. Section 2 outlines the key results of the paper using a simple 4-state model. (The full model is omitted from this version of the paper.) Section 3 concludes.

2 A simple example

To capture the essence of basis risk consider the following four state model. An individual begins with wealth $w$ and suffers a loss which can take the value $L$ with probability $p$ or zero with probability $1 − p$. There is also an index which can take the value $I$ with probability $q$ or zero with probability $1 − q$. The index is not necessarily perfectly correlated with the loss and so there are four possible states $s \in \{00, 0I, L0, LI\}$.

For the purpose of talking about an increase or decrease in basis risk it is perhaps natural to consider changes in basis risk, whilst holding the marginal index and loss distributions fixed. In our four state model with $p$ and $q$ fixed there is only degree of freedom in the joint probability distribution and so we may, without loss of generality, define basis risk parameter $r$ as the joint probability that the index is 0 and the loss is $L$, and interpret an increase in $r$, without any change in $p$ or $q$, as an increase in basis risk. Denoting the probability of each state $s$ by $\pi_s$, we therefore have $\{\pi_{00}, \pi_{0I}, \pi_{L0}, \pi_{LI}\} = \{1 − q − r, q + r − p, r, p − r\}$ (see Table 1).\(^1\) For an index realisation of $I$ to be a signal that the loss has been $L$, we require that $\frac{\pi_{0I}}{\pi_{00}} > \frac{\pi_{L0}}{\pi_{00}}$, that is $r < p(1 − q)$. We will also assume that basis risk $r$ is strictly positive and so together

$$0 < r < p(1 − q). \quad (1)$$

If all $\pi_s$ are to have nonnegative probability equation 1 in turn implies $0 < p < 1$ and $0 < q < 1$.\(^2\)

The loss is observable to the individual but not to the insurer, and so there is no market for indemnity insurance, with claim payment conditional only on the loss. However, the individual can purchase an indexed security that pays a proportion $\alpha \geq 0$ of the potential loss $L$ when the index realisation is $I$. The indexed product is priced with a multiple of $m \geq 1$; that is the

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1. Other specifications of basis risk are possible. One alternative specification would be that the probability of a loss was $p$ and that the probability that the index was ‘wrong’, that is $\frac{\pi_{0I}}{\pi_{L0} + \pi_{LI}} = \frac{\pi_{0I}}{\pi_{0I} + \pi_{00}}$ was some $\rho$. However, under this specification $P[\text{Index} = I] = p + (1 − 2p)\rho$ and so for $p \neq 1/2$ a change in basis risk parameter $\rho$ would change the marginal distribution of the index in addition to increasing basis risk.

2. Using (1), $0 < p(1 − q)$ implies $p > 0$ and $q < 1$. Combining the restriction $\pi_{0I} = q + r − p \geq 0$ and (1) leads $p = 1$ and $q = 0$ to imply contradictions $1 − q < 1 − q$ and $p < p$ respectively.
Table 2: Four state framework

<table>
<thead>
<tr>
<th>State $s$</th>
<th>$L_0$</th>
<th>$LI$</th>
<th>00</th>
<th>01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability $\pi_s$</td>
<td>$r$</td>
<td>$p-r$</td>
<td>$1-q-r$</td>
<td>$q+r-p$</td>
</tr>
<tr>
<td>Wealth, no indexed cover</td>
<td>$w-L$</td>
<td>$w-L$</td>
<td>$w$</td>
<td>$w$</td>
</tr>
<tr>
<td>Wealth, indexed cover of $\alpha L$</td>
<td>$w - P - L$</td>
<td>$w - P - L + \alpha L$</td>
<td>$w - P$</td>
<td>$w - P + \alpha L$</td>
</tr>
</tbody>
</table>

individual pays a premium of $P = qm\alpha L$ for cover of $\alpha L$, receiving a claim payment of $\alpha L$ if the index realisation is 1 or zero if the index is 0.\footnote{Using the standard insurance terminology, the corresponding loading would be $m-1 \geq 0$.} For $m > 1$, $m = 1$ and $m \leq 1$ the premium will be said to be actuarially unfair, actuarially fair, and actuarially subsidised, respectively. We will ignore the case in which $qm \geq 1$ for which zero indexed coverage is trivially optimal.

The parametrisation of basis risk now has a natural interpretation: $r$ is the probability that an individual who has purchased indexed cover will incur a loss but receive no claim payment. For example, a farmer could lose her entire crop due to pestilence or localised weather conditions, but receive no claim payment from a weather derivative contract due to good weather having been observed at the contractual weather station.

Although it has come under attack as a positive theory of decision under uncertainty, the subjective expected utility model has stood the test of time as a normative framework; people do not always act as though they were maximising the expectation of some utility function, but economists still typically believe that, absent computational difficulties, they should. As is usual in models of insurance demand we assume that the individual holds strictly risk averse preferences over wealth, with utility of wealth denoted $u$, where $u’ > 0$, $u'' < 0$. This is equivalent to assuming that, endowed with certain wealth, such an individual would never accept a non-degenerate lottery if the expected net gain from the lottery was nonpositive.

The individual’s objective is therefore to choose a level of indexed coverage $\alpha \geq 0$ to maximise expected utility:

$$EU = (p-r)u(w-\alpha qmL-(1-\alpha)L) + (q+r-p)u(w-\alpha qmL+\alpha L) + (1-q-r)u(w-\alpha qmL-L) + ru(w-\alpha qmL-L)$$

(2)

The first-order condition for an interior solution to equation (2), after cancelling $Lq(1-qm) > 0$, is:

$$Au’_{LI} + (1-A)u’_{0I} - B \times [Cu’_{00} + (1-C)u’_{L0}] = 0,$$

(3)

where for the sake of notational convenience,

$$A = \frac{p-r}{q}, \quad B = \frac{m-qm}{1-qm}, \quad C = \frac{1-q-r}{1-q},$$

(4)

and $u’_s$ denotes marginal utility in state $s$.\footnote{The second-order condition is trivially satisfied.} The parameter restrictions above ensure that $p < A < 1$, $B \geq 1$ and $1-p < C < 1$, and therefore that $A + C > 1$.

We begin by stating a basic result, for which we claim no novelty.

**Proposition 1.** Full indexed coverage is never optimal, $\alpha^* < 1$. If the premium is actuarially fair then positive, partial indexed coverage is always optimal, $0 < \alpha^* < 1$.

**Proof.** If $\alpha \geq 1$ and $m \geq 1$ we have $B \geq 1$, $u’_{00} \geq u’_{LI}$ and $u’_{L0} > u’_{0I}$ and so the LHS of (3) is strictly negative, violating first-order condition (3). For $\alpha = 0$ and $m = 1$ the LHS of (3) is
strictly positive since \( B = 1 \) and \( A + C > 1 \). By continuity and the intermediate value theorem there is some solution \( 0 < \alpha^* < 1 \) when \( m = 1 \). Moreover, by the strict concavity of objective function (2), this is the only solution.

In the model with \( r > 0 \) it is impossible to eliminate all uncertainty, due to the existence of an uninsurable basis risk, negatively correlated with the loss. This may be compared to the well known results that for actuarially fair indemnity insurance, that is when \( m = 1, p = q \) and \( r = 0 \), full insurance is optimal (Mossin 1968, Smith 1968), and remains optimal on addition of an independent background risk.

For \( 0 \leq \alpha < 1 \), objective function (2) has strictly decreasing differences in \((\alpha; r)\), that is \( \frac{\partial^2 E}{\partial \alpha \partial r} < 0 \), yielding the following proposition:

**Proposition 2.** The optimal level of coverage \( \alpha^* \) is decreasing in basis risk \( r \), and strictly decreasing if \( \alpha^* > 0 \).

Whilst it is somewhat intuitive that risk aversion is sufficient to ensure that an increase in basis risk reduces demand, it bears mentioning that such a result doesn’t hold for Doherty and Schlesinger’s (1990) model of one-sided basis risk. In their model demand is nonmonotonic in the probability of insurer default, expect for the special cases of constant absolute risk aversion and quadratic utility. Whilst in both models an increase in basis risk reduces the correlation between the available cover and the loss, in our model it does so in a symmetric fashion, resulting in a mean preserving spread of wealth for any fixed \( \alpha \) (Rothschild and Stiglitz 1970); the movements of probability mass from inner state \( 00 \) to outer state \( L0 \) and from inner state \( LI \) to outer state \( 0I \) each reduce the incentive to purchase indexed cover without changing the insurance premium. By comparison, in Doherty and Schlesinger’s (1990) model, an increase in basis risk moves probability mass from inner state \( LI \) to outer state \( L0 \) and decreases the insurance premium, the combined effect of which is ambiguous.

In the remainder of this section we use the four state model to demonstrate that hedging products with basis risk are fundamentally different to those of indemnity insurance. First, following Schlesinger and Schulenburg (1987) and Doherty and Schlesinger (1990), the shape of optimal indexed coverage is shown to be substantially different to that of optimal purchase of indemnity insurance in terms of risk aversion, wealth, price and size of loss. Second, limits for optimal purchase by individuals satisfying risk averse or risk averse and DARA are derived.

### 2.1 Risk aversion

To appreciate the effect of basis risk on optimal demand for different levels of risk aversion, first consider an infinitely risk averse, maximin, individual. Such an individual’s objective is to maximise the lowest possible utility and therefore full purchase of indemnity insurance \((r = 0)\) is optimal so long as the net transfer to the individual from the contract is positive in at least one state \((qm < 1)\); full insurance purchase increases the lowest possible wealth from \( w - L \) to \( w - qmL \).

In stark contrast, an infinitely risk averse individual would optimally purchase zero indexed cover \((r > 0)\) at any positive premium; purchase of cover of \( \alpha \) would decrease the minimum wealth from \( w - L \) to \( w - L - \alpha qmL \).

For actuarially unfair premiums, \( m > 1 \), indexed cover decreases mean wealth and so risk neutral individuals would also optimally purchase zero cover. Combining this with the observation that infinitely risk averse individuals would not purchase any indexed cover, it is clear that only individuals with intermediate levels of risk aversion might wish to purchase material amounts of cover.

For the case of constant absolute risk aversion the optimal cover may be derived as an explicit function of \( p, q, r, m, L \) and the coefficient of absolute risk aversion \( \gamma \), and can be shown to be hump shaped in \( \gamma \) for \( m > 1 \) and monotonically decreasing in \( \gamma \) for \( m < 1 \).
Proposition 3. For any individual with constant absolute risk aversion of $\gamma > 0$ the optimal level of indexed cover is:

$$\alpha^{\ast}_{\text{CARA}}(\gamma) = \begin{cases} 0 & \text{if } \frac{A+(1-A)e^{-\gamma L}}{BCe^{-\gamma L}+B(1-C)} \leq 1 \\ \frac{C}{(1-A)e^{-\gamma L} + A} & \text{otherwise} \end{cases}$$

Proof. Substituting $u'(x) = e^{-\gamma x}$ into first order condition (3) and rearranging gives equation (5).

Proposition 4. For individuals with constant absolute risk aversion the optimal level of actuarially unfair indexed cover is hump shaped in the coefficient of absolute risk aversion. The optimal level of actuarially subsidised indexed cover is decreasing in the coefficient of absolute risk aversion.

Proof. Consider $\tilde{\alpha}_{\text{CARA}}(\gamma) = \frac{1}{2L} \ln \left( \frac{A+(1-A)e^{-\gamma L}}{BCe^{-\gamma L}+B(1-C)} \right)$, defined such that $\alpha^{\ast}_{\text{CARA}}(\gamma) = \max(0, \tilde{\alpha}_{\text{CARA}}(\gamma))$ for $\gamma \in (0, \infty)$.

Differentiating the equation for $\tilde{\alpha}_{\text{CARA}}(\gamma)$ with respect to $\gamma$, and evaluating it at some $\gamma_0 \in (0, \infty)$ such that $\tilde{\alpha}_{\text{CARA}}'(\gamma_0) = 0$, gives:

$$\tilde{\alpha}_{\text{CARA}}(\gamma_0) = \frac{(A + C - 1)}{(1 - A)e^{-\gamma_0 L} + A}(Ce^{-\gamma_0 L} + (1 - C))$$

$\tilde{\alpha}_{\text{CARA}}(\gamma)$ is continuously differentiable on $(0, \infty)$ and the RHS of equation (6) is strictly monotonically decreasing from $A + C - 1 > 0$ to 0 on $\gamma_0 \in (0, \infty)$.

For $m > 1$, L’Hospital’s rule gives $\lim_{\gamma \to 0^+} \tilde{\alpha}_{\text{CARA}}(\gamma) = -\infty$ and $\lim_{\gamma \to \infty} \tilde{\alpha}_{\text{CARA}}(\gamma) = 0$. The RHS of equation (6) is strictly monotonically decreasing in $\gamma$ and so can only cross the LHS at most once in $(0, \infty)$. There can therefore be at most one $\gamma_0 \in (0, \infty)$ such that $\tilde{\alpha}_{\text{CARA}}'(\gamma_0) = 0$.

For $0 < m < 1$, L’Hospital’s rule gives $\lim_{\gamma \to 0^+} \tilde{\alpha}_{\text{CARA}}(\gamma) = +\infty$ and $\lim_{\gamma \to \infty} \tilde{\alpha}_{\text{CARA}}(\gamma) = 0$ and so the LHS and RHS of (6) can only meet in the limit as $\gamma \to \infty$.

Figure 1 plots the optimal purchase of indexed cover for constant absolute and relative risk averse expected utility maximisers with respect to the coefficients of absolute and relative risk aversion, respectively.

![Figure 1: Rational hedging and risk aversion (p = q = 1/3, r = 1/9)](image)

CRRA demand ($w = 2L$)  CARA demand ($L = 1$)

As one might expect from the above discussion, an increase in risk aversion, either in the sense of Arrow-Pratt or even in the stronger sense of Ross (1981), does not necessarily lead to an
increase in demand for indexed cover, echoing the earlier results of Schlesinger and Schuleenburg (1987) and Doherty and Schlesinger (1990) for the case of one-sided basis risk.

The Arrow-Pratt ordering of utility functions is weaker than that of Ross, so we need only demonstrate the lack of a monotonic relationship for the latter. Recall that if $V$ is a strongly more risk averse preference ordering than $U$ in the sense of Ross (1981) then there exists $G : \mathbb{R} \to \mathbb{R}, G' < 0, G'' < 0$ and $\lambda > 0$ such that $V(W) = \lambda U(W) + G(W)$. If $\alpha^*$ is the optimal level of cover for $U$ then the first order condition for $V$, evaluated at $\alpha^*$ becomes:

$$\frac{1}{q(1-q)} \frac{dEV}{d\alpha} \bigg|_{\alpha^*} = AG''(\alpha) + (1 - A)G'(\alpha) - B \times [CG''(\alpha) + (1 - C)G'(\alpha)]$$

The sign of equation (7) can be positive or negative and therefore the optimal level of cover under $V$ can be higher or lower than $\alpha^*$.

2.2 Wealth and Price

Other comparative statics results do follow through from Doherty and Schlesinger (1990), as would be expected since their model is a reparameterised, special case of ours. To begin with, there is no monotonic relationship between demand and wealth $w$, pricing multiple $m$ or loss $L$, even if one restricts preferences to satisfy increasing absolute risk aversion (IARA) or decreasing absolute risk aversion (DARA).

The first result stands in direct contrast to Mossin’s (1968) observation that indemnity insurance is an inferior good under DARA. In models without basis risk, an increase in insurance purchase transfers wealth from high to low wealth states, subject to some deadweight cost. However, in the presence of basis risk an increase in indexed cover transfers wealth from the lowest and intermediate wealth states to the highest and other intermediate wealth states, and the restriction of DARA is no longer relevant for determining whether indexed cover is an inferior good. For the case of CRRA and actuarially subsidised cover ($m < 1$), rational demand is increasing in wealth and decreasing in the level of risk aversion, implying that for subsidised voluntary hedging programs, subsidies would be captured by wealthier, less risk averse individuals.

The second result, that indexed cover may be a Giffen good, stands in direct contrast to the result of Mossin (1968) and Smith (1968) who show that demand for indemnity insurance is lower for $m > 1$ than for $m = 1$. This result also arises from indexed cover not being an inferior good, even under DARA. Although an increase in premium increases $B$, acting to decrease demand, it also uniformly decreases wealth, which has an ambiguous effect on optimal demand for reasons described in the previous paragraph. If one restricts preference to satisfy constant absolute risk aversion (CARA) this wealth effect disappears, ruling out the possibility that indexed cover is a Giffen good. However, without the restriction to CARA the effect on demand of an increase in $m$ is ambiguous. Figure 2 plots the optimal purchase of indexed cover for constant absolute and relative risk averse expected utility maximisers with respect to initial wealth.

The third result follows trivially from the observation that demand is nonmonotone in loss $L$ for CARA (see equation (5)), and therefore by extension for DARA or IARA. Although this result was not mentioned in Doherty and Schlesinger (1990), it also holds in the case of one-sided basis risk. By way of comparison, demand for indemnity insurance under DARA is monotonically increasing in loss $L$.

2.3 Bounds for rational demand

It is also possible to derive some fairly strong results about the welfare implications of hedging. A financial advisor advising on the purchase of indemnity insurance cannot reasonably rule out
any level of purchase without understanding a client’s preferences; zero purchase may be advisable for a risk neutral client, and nearly full cover advisable for a risk averse client. As we will show in this section, this intuition does not follow through to the case of indexed cover with basis risk.

First, there are indexed products for which zero coverage is optimal for any risk averse expected utility maximiser.

**Theorem 1.** For any risk averse individual the optimal level of indexed coverage is zero if

\[ r \geq p \left( 1 - q m \right). \]

**Proof.** 0 ≤ α∗ < 1 from Proposition 1 and w" < 0, and so w′′L0 ≥ w′L and w′00 ≥ wQ0. Since C > 1 − A and B ≥ 1, (1 − A)u′Q1 = BCu′00 < 0 and so first order condition (3) cannot hold unless Au′L1 > B(1 − C)u′L0. This in turn implies that A > B(1 − C) which can be rearranged to give the restriction \( r \geq p \left( 1 - q m \right) \), using the definitions of (4).

Theorem 1 may be understood as follows. Purchase of indexed cover does not increase average wealth, and so for positive purchase to be optimal for a risk averse individual it must at least result in an increase in average wealth in low wealth states, that is the states in which a loss has occurred. Purchase of cover of α increases the premium by αqmL and results in average claim income, conditional on a loss L having occurred, of \( \alpha L p - r p \). The condition ensuring that the average net gain in these low wealth states is positive on purchase of indexed cover is therefore

\[ \alpha L \left( \frac{p}{p} - q m \right) > 0, \]

or equivalently \( r < p \left( 1 - q m \right) \).

Note that when m = 1, theorem 1 reduces to \( r \geq p \left( 1 - q \right) \), which we have ruled out by assuming that the index and loss are strictly positively correlated. When basis risk is low, for example when the only source of basis risk is a 1% chance of insurer default, the restriction of theorem 1 is not much tighter than the restriction that the maximum possible claim payment \( \alpha L \) is larger than the premium \( \alpha q m L \). However, when there is both a sizeable basis risk \( r \) and premium multiple \( m \), the restriction of theorem 1 is tighter.

This unfortunate combination of premium loading and basis risk seems to be particularly relevant for the weather derivatives currently being sold to poor farmers. Both fixed costs and variable costs are high for small products marketed to poor farmers, and such expenses are typically passed back to the farmer through the premium multiple. Giné et al. (2007) and Cole et al. (2009) use historic weather data to estimate objective premium multiples \( m \) for products sold to poor farmers in India, with the former quoting an average premium multiple of 3.4 and the latter calculating multiples individually for seven products, ranging from 1.7 to 5.3. Neither paper
directly quotes the frequency of claim payments \( q \), but it is likely to be between \( \frac{1}{10} \) and \( \frac{1}{3} \).\(^5\) Moreover, it is quite possible that \( \frac{r}{p} \), the probability of a farmer receiving no material claim payment despite having suffered a severe loss, is between 25% and 75%; regardless of how clever the insurance design, weather derivatives make payments conditional only on recorded weather at contractual weather stations and provide little protection for other perils such as insects of disease, or localised weather events that can occur on the farmer’s land without being observed at the contractual weather station.

Now, Indian farmers are not exposed to binary shocks, nor do the products described in Giné et al. (2007) and Cole et al. (2009) offer binary claim payments. However, the rationale of theorem 1 still holds; if, conditional on each potential loss, the marginal premium rate is larger than the marginal expected claim income, zero purchase will be optimal for any risk averse expected utility maximiser. The summary statistics from the previous paragraph, suggest that for at least some of the products being offered, this bound will bind (see Table 3).

Table 3: Examples of products for which zero cover is optimal for any risk averse expected utility maximiser.

| \( P[\text{Loss}] = P[\text{Claim}] \) \((p=q)\) | \( P[\text{No Claim}|\text{Loss}] \) \((r/p)\) | Premium Multiple \((m)\) |
|---|---|---|
| 1/10 | 1/2 | 5.0 |
| 1/4 | 1/4 | 3.0 |
| 1/4 | 1/2 | 2.0 |
| 1/3 | 1/3 | 2.0 |
| 1/3 | 1/2 | 1.5 |
| 1/2 | 1/4 | 1.5 |

For \( r < p(1 - qm) \), risk aversion alone is not sufficient to justify an upper bound for demand tighter than full cover, since we cannot rule out the possibility that the individual is approximately risk neutral except for some interval between \( w - aqmL - (1 - \alpha)L \) and \( w - aqmL \) in which she is very risk averse. However, we may rule out such contrived cases by assuming that absolute risk aversion decreases with wealth. This assumption of decreasing absolute risk aversion (DARA) is equivalent to assuming that if the decision maker would accept some lottery given certain endowment of \( w \) she would accept the same lottery given certain endowment of \( w' > w \). Regardless of whether this restriction is appropriate from a positive point of view, that is whether individual’s actual decisions violate this assumption, we agree with Arrow (1965) and Pratt (1964) who argued that it is normatively sound. Moreover we consider DARA, and by extension the upper bound of the following theorem, to be an appropriate basis for generic financial advice about products designed to reduce risk.

Under DARA we may derive the following upper bound on rational demand for indexed cover, identical to the former upper bound for products with \( r \geq p(1 - qm) \) and tighter for products with \( r < p(1 - qm) \).

**Theorem 2.** For any strictly risk averse individual with decreasing absolute risk aversion the optimal level of indexed coverage is zero if \( r \geq p(1 - qm) \), or otherwise bounded above by the unique \( \bar{\alpha} \) that solves

\[ A\bar{\alpha}^\alpha(1 - \bar{\alpha})^{1-\alpha} = (A + BC - 1)^\alpha \times (B(1 - C))^{1-\alpha}, \quad \bar{\alpha} \in \left(0, \frac{A + BC - 1}{A + B - 1}\right) \tag{8} \]

\(^5\)Giné et al. (2007) report on a three phase product that pays claims in 10.7% of phases, suggesting a claim payment frequency between 11% and 29%.
Although we relegate the full proof to the appendix, a sketch is as follows. Conditional on the marginal utility of wealth in states $LI$ and $00$, $u'_{LI}$ and $u'_{00}$, we first show that demand is bounded above a utility function which is risk neutral above $u'_{00}$ and has constant absolute risk aversion of $\gamma$ below $u'_{00}$. The maximisation problem is then to choose $\gamma \geq 0$ to maximise demand $\alpha$, conditional on demand satisfying first order condition (3). Equation (8) defines this largest optimal $\alpha$ for any $\gamma \geq 0$.

Loosely speaking, the logic of the DARA upper bound of Theorem 2 is as follows: if an individual cares enough about the risk between states $LI$ and $00$ to want to purchase a sizeable hedge, the individual must care enough about the downside basis risk between states $L0$ and $LI$ and the deadweight cost of hedging to limit the size of the hedge.

Table 4 presents values of $\bar{\alpha}$ when the probability of no claim being paid conditional on a loss having occurred is taken to be $\frac{1}{3}$. For the stated parameter values, the bounds for rational demand are close to zero.

| $P[\text{Loss}] = P[\text{Claim}] (p = q)$ | $P[\text{No Claim}|\text{Loss}] (r/p)$ | $\text{Premium Multiple} (m)$ | $\bar{\alpha}$
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{10}$</td>
<td>$\frac{1}{3}$</td>
<td>6.0</td>
<td>$3.9%$</td>
</tr>
<tr>
<td>$\frac{1}{8}$</td>
<td>$\frac{1}{3}$</td>
<td>4.5</td>
<td>$6.7%$</td>
</tr>
<tr>
<td>$\frac{1}{5}$</td>
<td>$\frac{1}{3}$</td>
<td>3.0</td>
<td>$4.5%$</td>
</tr>
<tr>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{3}$</td>
<td>2.4</td>
<td>$4.8%$</td>
</tr>
<tr>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{3}$</td>
<td>1.8</td>
<td>$5.4%$</td>
</tr>
</tbody>
</table>

When premiums are actuarially fair, that is $m = 1$, the formula for the upper bound may be simplified.

**Corollary 1.** For any strictly risk averse individual with decreasing absolute risk aversion the optimal level of actuarially fair indexed coverage is bounded above by $\bar{\alpha} = \frac{p(1-q)-r}{(p-r)(1-q)}$.

*Proof.* For $m = 1$, $\bar{\alpha} = \frac{A+C-1}{A} = \frac{p(1-q)-r}{(p-r)(1-q)}$ is a solution to equation (8), and by theorem 2 it is the unique solution.

It is perhaps also worth pointing out that theorems 1 and 2 are robust to the introduction of an uninsurable background risk. Although we have introduced the insurance purchase decision in terms of a four state model, the intuition follows through if there are other uninsurable risks, so long as they are statistically independent of the loss and index.

**Corollary 2.** Theorems 1 and 2 are robust to the introduction of a background risk which is independent of the joint distribution of the loss and index.

*Proof.* Denote random background risk as $\bar{x}$ and indirect utility function $v(y) = \mathbb{E}u(\bar{x} + y)$ for all $y \in \mathbb{R}$. The optimal decision of agent $u$ with background risk of $\bar{x}$ is the same as for agent $v$ who faces no background risk. $v^n(y) = \mathbb{E}u^n(\bar{x} + y)$ where $u^n$ is the $n$th derivative of $u$, and so $v$ inherits the properties $v' > 0$ and $v'' < 0$. Moreover $v$ inherits decreasing absolute risk aversion from $u$ (Gollier 2001, p116). Theorems 1 and 2 therefore follow through replacing $u$ with $v$.

Theorems 1 and 2 also have implications for product design. Cole et al. (2009) propose the idea that demand for weather derivatives from poor farmers is low due to the learning process and suggest that ‘A potential contract design improvement to facilitate this learning would be to amend the contract to pay a positive return with sufficient frequency’. Increasing the frequency of claim payments, $q$ in our model, is unlikely to decrease basis risk $r$ and will not change the
loss distribution \( p \). Although rational demand is nonmonotone in \( q \) in general, an increase in \( q \) makes it easier for the zero demand bound restriction of Theorem 1 to be satisfied and leads to a decrease in rational demand for the case of CARA.

Corollary 2 allows the upper bounds on demand from theorems 1 and 2 to be tested in experimental settings. So long as the randomisation device used to determine net transfers in an experiment is statistically independent of participant’s background wealth, demand from participants in excess of the respective upper bound may be taken as a violation of risk aversion for products with \( r \geq p(1 - qm) \) or risk aversion and DARA for other products. For example, Clarke and Macchiavello (2010) report on the results of one particular decision problem with \( p = q = \frac{1}{2}, r = \frac{1}{8}, m = 1.2 \) and \( L = 50 \) Ethiopian Birr, played with 258 rural Ethiopians. 66% of participants purchased a hedge that exceeded the rational upper bound of \( \bar{\alpha} = 21\% \), suggesting that, as for decision makers in rich countries, poor rural Ethiopians could benefit from robust generic financial advice and consumer protection legislation for complex financial products.

2.4 Hedging and the poverty headcount measure

There are, of course, sets of preferences which violate risk aversion or DARA, for which optimal purchase of indexed cover may exceed the upper bound of Theorem 2. Whilst we have argued above that such sets of preferences can reasonably be ignored for the purpose of providing generic financial advice to individuals, there is one particular class of preferences worthy of particular attention.

Suppose that policymakers choose policies to minimise the expected number of individuals below some poverty line \( \mathcal{P} \). For individuals with wealth \( w \) and subject to loss \( L \) such that \( \mathcal{P} \in [w - L, w - qmL] \), then purchase of cover \( \alpha L \in \left( \frac{P - (w - L)}{1 - qmL}, L \right) \) would reduce their expected contribution to headcount poverty from \( p \), the probability of a loss occurring, to \( r \), the probability of a loss occurring and no indexed claim payment being made. This result arises from a well documented feature of the headcount poverty measure, whereby it is possible to reduce expected headcount poverty by transferring wealth from states below the poverty line to states also below the poverty line but with higher wealth (Sen 1979). Providing targeted subsidies for indexed financial products may therefore meet the objectives of policymakers, even if all rational individuals would strictly prefer the subsidy to be in the form of an actuarially equivalent unconditional cash transfer.

3 Concluding remarks

Recent work on weather derivatives for the poor by academic economists seems to have been lacking a sound theoretical basis. This paper attempts to address this by presenting a model of rational demand for indexed products. The presence of basis risk is shown to alter or reverse many of the key results arising from the theory of rational demand for indemnity insurance (see Table 5, which extends Table 1 of Doherty and Schlesinger (1990)). Both the level of and the effect of risk aversion on observed demand for weather derivatives from poor farmers seem to be consistent with a model of rational demand, thereby offering explanations for two outstanding empirical puzzles.

Weather derivatives have changed the face of agricultural insurance for the poor by focusing the minds of academics and practitioners on a problem which, if suitably addressed, could lead to a substantial increase in welfare for many of the world’s rural poor (Banerjee 2002, Collins et al. 2009, Karlan and Morduch 2009). Experimenting with financial innovations is costly and outcomes are difficult to evaluate, particularly for insurance contracts where payouts are typically expected to be made in only one year out of five. However, financial innovations in developed markets have often been theory-led and the recent advances in positive microfinance theory leave economists well placed to make suggestions for improvements.

One promising suggestion, made in a slightly different context by Doherty and Richter (2002),
Table 5: Indemnity and indexed insurance: a summary of key results

<table>
<thead>
<tr>
<th>Result without basis risk</th>
<th>Result with basis risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Indemnity insurance, $r = 0$)</td>
<td>(Indexed cover, $r &gt; 0$)</td>
</tr>
<tr>
<td><strong>Shape of rational hedging:</strong></td>
<td><strong>Shape of rational hedging:</strong></td>
</tr>
<tr>
<td>• More risk averse $\Rightarrow$ buy more cover</td>
<td>• $\alpha$ decreasing in basis risk</td>
</tr>
<tr>
<td>• More risk averse $\Rightarrow$ buy more cover</td>
<td>• More risk averse $\Rightarrow$ buy more cover</td>
</tr>
<tr>
<td>• Positive loading ($m &gt; 1$) $\Rightarrow$ buy less insurance</td>
<td>• Positive loading ($m &gt; 1$) $\Rightarrow$ buy less insurance</td>
</tr>
<tr>
<td>• Insurance is inferior for DARA utility</td>
<td>• CRRA/CARA cover hump shaped in coefficient of RRA/ARA</td>
</tr>
<tr>
<td>• Infinitely risk averse $\Rightarrow$ $\alpha = 1$</td>
<td>• Infinitely risk averse $\Rightarrow$ $\alpha = 0$</td>
</tr>
<tr>
<td>• Fair price ($m = 1$) $\Rightarrow$ $\alpha = 1$</td>
<td>• Fair price ($m = 1$) $\Rightarrow$ $\alpha &lt; 1$</td>
</tr>
<tr>
<td>• Positive loading ($m &gt; 1$) $\Rightarrow$ buy less insurance</td>
<td>• Positive loading ($m &gt; 1$) $\Rightarrow$ buy less insurance</td>
</tr>
<tr>
<td>• Larger potential loss $L$ $\Rightarrow$ buy more insurance for DARA utility</td>
<td>• CRRA cover hump shaped in wealth</td>
</tr>
<tr>
<td>• Larger potential loss $L$ $\Rightarrow$ buy more insurance for DARA utility</td>
<td>• CRRA cover is normal</td>
</tr>
<tr>
<td><strong>Level of rational hedging:</strong></td>
<td><strong>Level of rational hedging:</strong></td>
</tr>
<tr>
<td>• Positive loading ($m &gt; 1$) $\Rightarrow$ DARA upper bound of $\bar{\alpha} = 1$</td>
<td>• Positive loading ($m &gt; 1$) $\Rightarrow$ Risk averse upper bound of $\bar{\alpha}<em>{RA} \in {0, 1}$ and DARA upper bound of $\bar{\alpha}</em>{DARA} \leq 1$</td>
</tr>
</tbody>
</table>

is to combine the speed and relative low cost of weather index insurance with an indemnity-based floor to offer protection against the downside basis risk. Such a product could be relatively cheap, whilst still offering protection when it is needed the most. Another approach, proposed by Clarke (2010) is to note that groups of poor individuals, such as extended families or members of close-knit communities, are typically able to support relatively inexpensive pooling for idiosyncratic shocks. The role of the formal insurer should then be to design state contingent products for which the net transfer to a group is high when the group has suffered a large aggregate loss. One such agricultural insurance product is area yield index insurance, described in Mahul (1999), where the claim payment from the insurer is contingent on the sample mean loss from a random selection of local plots audited by the insurer.

Regardless of how insurance for the poor develops it seems important for economists to be engaged with normative issues of welfare, generic financial advice and product design, in addition to positive issues such as the shape and level of observed demand. As noted by Banerjee (2002), ‘if economists can be persuaded to be more involved in suggesting other ways of doing things, perhaps the next wave of innovations [in microfinance] is not far away.

**References**


**Banerjee, A.**, “The Uses of Economic Theory: Against a Purely Positive Interpretation of Theo-


A Appendix

Proof of Theorem 2. For 0 ≤ α ≤ 1 and any risk averse utility function \( u \) that satisfies DARA there exist constants \( \gamma_1 \geq \gamma_2 \geq \gamma_3 > 0 \) such that

\[
\frac{\partial^2 u}{\partial L^2}(\gamma_1, \gamma_2, \gamma_3, \alpha) = u'_L(\alpha) e^{-\gamma_1 L}, \quad \frac{\partial^2 u}{\partial B^2}(\gamma_1, \gamma_2, \gamma_3, \alpha) = u'_L(\alpha) e^{-\gamma_3 L}, \quad \frac{\partial^2 u}{\partial L \partial B}(\gamma_1, \gamma_2, \gamma_3, \alpha) = u'_L(\alpha) e^{-\gamma_2 L}.
\]

It is possible, that \( \gamma_1 \leq \gamma_2 < \gamma_3 \), which may be rearranged to give equation (8). Now, all that remains to be proven is that there is a unique solution \( \alpha^* \).

Denoting \( \alpha = \frac{B(1 - C)}{A + B - 1} \) and the requirement that \( \Gamma \) is chosen optimally becomes

\[
0 = \frac{\alpha B(1 - C)}{(A + B - 1)(1 - \alpha)} \leq 1
\]

Rearranging equations (A-2) and (A-3) and recalling that 0 ≤ \( \Gamma \) ≤ 1 gives:

\[
\left( B(1 - C) \right)^{\frac{1}{\alpha}} = \Gamma = \frac{\alpha B(1 - C)}{(A + B - 1)(1 - \alpha)} \leq 1
\]

which may be rearranged to give equation (8). Now, all that remains to be proven is that there is a unique solution to equation (8). Taking the natural logarithm of both sides of equation (8) and rearranging gives:

\[
H(\alpha) = [\alpha \ln(\alpha) + (1 - \alpha) \ln(1 - \alpha)] - \ln \left( \frac{B(1 - C)}{A} \right) + \alpha \ln \left( \frac{A + B - 1}{1 - \alpha} \right) = 0
\]

\( H(\alpha) \) is strictly convex for 0 < \( \alpha \leq \frac{A + B - C - 1}{A + B + C - 1} < 1 \) and therefore to prove uniqueness it is sufficient to show that

\[
\lim_{\alpha \to 0^+} H(\alpha) > 0 \quad \text{and} \quad \lim_{\alpha \to 0^+} H'(\alpha) = \ln \left( \frac{A}{B(1 - C)} \right) > 0
\]

since \( r < p(1 - qm) \). Finally, \( H(\frac{A + B - C - 1}{A + B + C - 1}) = \ln \left( \frac{A}{A + B + C - 1} \right) \leq 0 \) since \( B \geq 1 \).

\( \square \)