BETTER ESTIMATES OF DOLLAR GROSS DOMESTIC PRODUCT

FOR

101 COUNTRIES: EXCHANGE RATE BIAS ELIMINATED

by

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Better Estimates of Dollar Gross Domestic Product for 101 Counties: Exchange Rate Bias Eliminated

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The objective of this paper is simply to provide a better set of estimates of relative gross domestic product (GDP) for 101 countries of the world in 1970 and (with a few exceptions) in 1972 than exist in official statistical sources today. The claim is made that Table 2 in Section II provides such estimates.

The paper is organized as follows: Section I describes a bias in the official statistics and lays out a strategy for can be expected along these lines.

I. Exchange Rate Derived GDPs: How to Correct Them

A. What is wrong with exchange rate derived GDPs?

Most countries of the world have national income accounting systems which permit them to estimate their gross domestic products (GDPs). Since the estimate for any country is stated in terms of its domestic currency unit, comparing the GDPs of any pair of countries is “simply” a matter of converting one country's figure into the currency unit of the other by using the purchasing power parity (PPP) of the first currency relative to the other, and then comparing directly the two GDPs now stated in the same units. To facilitate comparisons among many countries, all the domestic currency GDPs are converted into the currency unit of a single numeraire country through PPPs where the numeraire country's currency is the base. The United States is normally the numeraire country and the U.S. dollar is the PPP base.

Regrettably, the PPPs can be estimated only with considerable difficulty and they are available for a limited number of countries.[1] In the absence of the detailed price and expenditure information required for estimating PPPs, the normal practice is to use exchange rates stated relative to the United States dollar as proxies for PPPs. Unfortunately, there are reasons for expecting that exchange rates deviate in a systematic way from PPPs. These reasons are set out in a number of articles by Balassa (1964, 1973, 1974), David (1972, 1973), and Samuelson (1974). The theoretical argument for expecting the currency of a poor country to have a purchasing power parity relative to that of a rich country which is less than the exchange rate between the currencies implies that a comparison of the two countries' national incomes based upon the exchange rate will overstate the level of the affluence of the rich country relative to the poor one. Furthermore, the greater the true disparity between the affluence of the countries, the greater will be this upward bias.

All of this can be written symbolically as follows. Let e^j and PPP^j be the exchange rate and purchasing power parity respectively of the j'th country relative to a base country, which without loss of generality we may take to be the United States; let y^j be the per capita national income of the j'th country, measured in the j'th country's domestic currency unit; let Y^j be the ratio of exchange rate derived national income per capita of the j'th country to the U.S. national income per capita; and let y^j be the ratio of the real national income per capita of the j'th country to the U.S. real national income per capita. Then

(1) $Y^j = \dfrac{y^j/e^j}{y^{us}}$ and (2) $y^j = \dfrac{y^j/PPP^j}{y^{us}}$

Therefore

$$(3)\ Y^j = \frac{PPP^j}{e^j} \cdot y^j$$

The verbal argument above asserts that if the United States is the richest country of all, then $y^j < 1$ and

$$(4)\ \frac{PPP^j}{e^j} = f(y^j) < 1$$

where $\frac{df(y^j)}{dy^j} > 0$. Since $y^{us} = Y^{us} = 1$, $f(1) = 1$.

Combining Equations (3) and (4) gives Equation (5):

$$(5)\ Y^j = g(y^j)$$

where (i) $g(1)=1$, (ii) $g(y^j) < y^j$, and (iii) $\frac{d[g(y^j)/y^j]}{dy^j} < 0$. The downward bias in the exchange rate derived comparison (from the standpoint of the poorer country) is what is asserted in (ii) and (iii).

B. What can be done about this bias?

Observe that if the function $g(y^j)$ were known exactly, it would be possible to adjust all of the biased national income comparisons that are based upon exchange rates. One would merely renormalize Equation (5) as in Equation (6):

$$(6)\ y^j = g^{-1}(Y^j)$$

and estimate a country's real income relative to the United States by $g^{-1}(Y^j)$.

But is it plausible to expect that the relationship of Equation (5) is a stable one, either over time or across countries? The various international comparison studies cited above as well as more fragmentary studies, provide strong empirical support for the theoretical argument that national income estimates using exchange rates are biased. (In fact, perhaps the theoretical argument was developed to explain the empirical finding.) But saying $Y^j = g(y^j)$ is stable is a much stronger statement. It surely is to be expected that inter-country differences (for example, in factor endowments) would give rise to different price structures from country to country which would make the degree of bias differ even among countries of equal affluence. Clearly, the function g of Equation (5) should have more arguments than just y^j, and Y^j should be more carefully estimated than simply as the currently observed exchange rate. Still, despite the theoretical shortcomings of Equation (5)—and the statistical problems to be discussed shortly—it is worthwhile considering the question: is there enough structure in Equation {5), given the noise which is there even though it has not been made algebraically explicit, to justify trying to carry out the debiasing method using Equation (6)?

David's answer "Yes" to the question led him to attempt to quantify Equation (5) using data drawn from the previously cited studies. For present purposes, it is not necessary to review his specific procedure or the criticisms that have been leveled at it. Suffice to say, the present authors would have thought the answer "No" if David had not shown the way. While not accepting his empirical results at all, we pay him the compliment of imitating his general method.

C. <u>The approach to estimating a debiasing equation</u>.

In order to introduce explicitly into Equation (5) a term signaling its stochastic character and also to indicate how the noise in the relationship might be reduced, we rewrite Equation (5) as Equation (5'), a form almost suitable for regression analysis.

$$(5') \quad Y^j = g(y^j, x^j_1, \ldots x^j_k; u)$$

Here $x^j_1, \ldots, x^j_k$ represent variables that help to explain Y^j given y^j, and u is a random variable with mean zero and variance σ^2 . Economic theory suggests variables that might serve as x's. For example, at any level of income the more a country engages in international trade—that is, the more open its economy— the closer the country's equilibrium exchange rate will be to its PPP. Similarly, the greater the proportion of the country's output concentrated in service industries—where output is unlikely to be traded internationally—the more its equilibrium exchange rate will diverge from its PPP. Thus, variables measuring the openness of the economies and the structure of their production illustrate candidate x's.

II. The Estimation of a Debiasing Equation

A. Data

Only a small number of observations is available for estimating the parameters of the debiasing equation. There is no shortage of Y^j's, but y^j's of adequate precision have been computed for only 13 countries. y^j has been computed for more than one year for 12 of the countries so in all, 30 observations on (y^j, Y^j) are used in this study. Eight for the year 1950, eight for 1955, five for 1967, and nine for 1970 cover three less-developed countries, one eastern bloc country, one very rapidly developing country, and eight advanced western European countries. Table 1 lists the countries and associated data[4,5] used in the final debiasing equation. Not presented in Table 1 are data on various x's. Anticipating discussion of the point below, it may be remarked that these data are not reproduced here because no such variable was found significant in the debiasing regression equation.

The-two relative income variables $\overline{y^j}$ and $\overline{Y^j}$ require comment. $\overline{y^j}$ is an index of relative national income per capita for the j'th country with the United States as the base for the index. The index type that was used was the Fisher ideal index which depends upon the price structures of the j'th country and the United States. In David (1972) a Laspyeyres, U.S. weighted index was used. David's arguments for his choice, presented both there and in David (1973), are unconvincing to us, and incidentally, to Samuelson and Belassa also. In fact, we would prefer to use the so-called multilateral quantity indexes of Kravis, Kenessey, Heston, and Summers (1975) which use price weights that reflect average world tastes. However, these indexes are available only for the 1957 and 1970 observations.

We have used official exchange rate in computing Y^j though we appreciate that these are not necessarily equilibrium exchange rates. Over short periods of time there sometimes are variations in observed rates; indeed, there may even be multiple exchange rates for some countries. Better estimates of a country's equilibrium rate might be devised by averaging over time its observed exchange rates in some appropriate way. The World Bank Atlas[6] makes use of just such a set of average exchange rates. The United Nations GDP estimates stated in dollars[7], however, are based simply upon the official exchange rate. United Nations numbers were adopted for this study instead of the Atlas ones. This was because though Atlas numbers were available for 1967 and 1970, no satisfactory method was available for establishing properly averaged Y^j values for the 1950 and 1955 cross-sections.

B. Estimation of the relationship between Y^j and y^j

The data of Table 1 appear in scatter-diagram form in Chart 1. The points range from (2.0%, 6.0%) in the lower left hand corner to (64.1%, 73.5%) in the upper right part of the graph. With the exception of two points, Hungary 1970 and France 1955, the points are fairly closely clustered around a curve that might easily be drawn free-hand with negative curvature starting at the origin and climbing upward to turn points in the vicinity of (62%, 74%)[8]. But what is the equation of the curve? What is its functional form?

The difficulties in renormalizing Equation (5) may not be simple if Equation (5) is stochastic. The point can be illustrated with a linear equation. If $\hat{Y}^j = \hat{a}\ y^j + \hat{a}$ is the regression equation that results from the application of either the Least Squares or Maximum Likelihood method, then we interpret this to mean **[**E Y^j | y^j] = $\sigma_1 y^j + \sigma_0$. We would like [E $\hat{y}^j$ | Y^j] but, unfortunately, that is not equal in general to $\frac{1}{\hat{a}_1} Y^j - \frac{\hat{a}}{\hat{a}_1}$. The treatment of this problem is easy enough if one may regard the data pairs as having been generated by random drawings from a bivariate population. In that case, we would get [E y^j | Y^j] by regressing y^j against Y^j. However , the structural causation surely runs from y^j to Y^j and the countries are by no means randomly selected. It is a happy fact, however, that the error in crudely ignoring the niceties and regressing y^j against Y^j is small if the data points are closely clustered about a straight line or curve. In fact, the simple correlation between y^j and Y^j is .973; the simple correlation between the logs is greater than .99; and the simple correlation between $\frac{1}{Y^j}$ and $\frac{1}{y^j}$ is .971.

With such high correlations the renormalization problem becomes unimportant; rationalizing what one must do in any case turns out to be easy. To illuminate the theoretical argument about the bias, Y^j should be regressed against y^j. But for our purpose, getting a debiasing equation, y^j must be regressed against Y^j, and having such a high correlation makes this tolerable.

Four functional forms were examined: linear, quadratic (in order to provide for curvature possibilities), linear-in-the-logs, and linear-in-the-reciprocals. The linear-in-the-logs form fitted best in a predictive sense to be discussed below.

A number of variables were introduced into the regression with a view to reducing the noise in the (y^j, Y^j) relationship. Despite the very high correlation coefficients, there still is some variability in the points around any single curve. (Observe that the Indian and Kenyan points have virtually the same ordinates but not the same abscissas; the same is true of the 1970 German and French points.) For example, (Exports plus Imports)/GDP, as a measure of openness of the economy, was included on the right side but its coefficient has a negligible "t" value. Similarly, (Income Originating in Services/GNP) was tried as explanatory variable without success. It too was not statistically significant.

The form of the relationship called for by Equation (6) deemed best was:

$$(7) \quad \log y^j = \underset{(.0206)}{.7818} \log Y^j + \underset{(.0360)}{.0845} \qquad ; \qquad \overline{R^2} = .9802$$

(The numbers in parentheses are standard errors.)[9] Equation (7) can be written in non-log form as Equation (8).

$$(8) \quad \hat{y}^j = 1.0882\, [Y^j]^{.7818}.$$

It should be noted that y^j is the median, not mean, of the conditional distribution of y^j _Y^j. If y^j and Y^j are expressed as percentages then Equation (8) becomes:

(8') $\bar{\hat{y}} = .29724\,[\overline{Y^{j}}]^{.7818}$

This equation, in percentage form, is the lower branch of the curve in Chart 1. With the minor exception of the Hungary 1970 and France 1955 points, all of the points are close indeed to the curve.

There are no points to the right of $\overline{Y^{j}}$ = 64.1, however. The extrapolation of the curve above 64.1 is dictated entirely by its rather rigid form and the curvature as determined by the points at the low end of the curve. While the data provide no direct information about the curve above 64.1, we would like it to go through the point (100,100). In the discussion of Equation (5) above, it was pointed out that as the j'th country's real income per capita approaches that of the United States, its exchange rate derived income also approaches the United States' income. (Statement (i) below Equation (5) asserts that g(l) = 1). Equation (8') when extrapolated to the right of where the points end misses (100,100) by almost 9%. If the extrapolation were drawn in Chart 1 instead of being truncated at (62,74), the reader would see that its slight curvature causes it to overshoot (100,100) and hit the $\overline{Y^{j}}$ = 100 vertical at 109. On the other hand, if the linear-in-the-logs relationship is constrained to go through (100,100) then the fit is disturbed significantly at the low end. (Notice how nicely it passes now between the two awkward pairs of points of (India, Kenya) and (Germany, France)). The constraint is equivalent to suppressing the constant term in the linear-in-the-logs regression; but with a "t" coefficient of 2.35, the constant term is certainly significantly different from zero. Incidentally, in each of the other regression forms (linear, quadratic, and linear in the reciprocals), the (100,100) constraint is not acceptable statistically either. The problem here is caused by the rigidity of any two-parameter family. The solution adopted in this research is to fit as an upper branch in the region 62 < $\overline{Y^{j}}$ the parabola going through (62, 74) and (100,100) which has a slope at (62, 74) equal to the slope of $y^{j} = 1.0882\,[Y^{j}]^{.7818}$. While this is arbitrary, the upper branch flows smoothly from the lower branch and is well-anchored at its ends. The synthetic relationship above $\overline{Y}$ = 62 is $\overline{y}^j$ = -.00651 $\overline{Y}^2$ + 1.740 $\overline{Y}$ - 8.9. It has been drawn in Chart 1 as the upper branch.

C. Criteria for the choice of the linear-in-the-logs regression

Since the three candidate regression forms involved different dependent variables, they could not be compared on the basis of their $\overline{R}^2$'s. Instead, a variety of "prediction" comparisons were made. Regressions were run with subsets of the data and then the unused observations were predicted by the estimated regressions. In most cases, e.g., using 1950- 55 data to predict 1967- 70 y^j's or 1950- 55 - 67 data to predict 1970 y^j's, or all but LDC data to predict the LDC's, the linear-in-the-logs predictions were best. The linear-in-the-logs equations were always more accurate in predicting y^j than Y^j by itself. It really is better to debias!

D. A Digression: Another Short Cut Method

The approach of this paper is a short-cut method of estimating a country's national income. Another short-cut method of estimating national income uses statistical indicators of various sorts which are believed to correlate strongly with national income. The usual justification for this method is based upon a reverse-Engle curve argument: If the number of automobiles per capital, or letters per capita, or telephones per capita depend at all closely upon income, then one should be able to predict national income from a knowledge of these variables.

Would an estimate based upon one or reverse-Engle curve variables be sufficiently accurate and independent of one based upon Y^j to make an average of the two kinds of estimates better than either one by itself. Experimentation with a variety of statistical indicators gave one variable, telephones per capita, which had a significant role in explaining y^j in the sense of having a statistically significant coefficient when introduced into the standard regression. It did not improve predictions, however.

III. The Debiased Gross Domestic Product Estimates

A 1970 GDP Estimates

Equation (8') and the synthetic parabola has been applied to the $\bar{Y}^j$'s of 99 countries for which GDP per capita in dollars is given in the United Nations Yearbook of National Accounts Statistics, 1972. The results appear in column(4)of Table 2. In column (5) the dollar equivalent of $\bar{y}^j$ is given. No effort as yet has been made to assess the precision of these GDP per capita estimates but we are confident they are significantly better than the exchange rate derived estimates. 92 of the 99 estimates were arrived at using the lower branch of Chart 1 and 7 required the use of the upper branch. In addition to the 99 GDPs supplied by the United Nations for 1970, Table 1 gives the GDPs of India and Hungary which come from the International Comparison Project.

B. 1972 GDP estimates

We offer very tentatively in column (6) a set of estimates of $\bar{y}^j$ for 1972 and in column (7) estimates of GDP per capita for almost the full set of 101 countries. These were not estimated in the debiasing way that the 1970 ones were. Instead of applying the debiasing formula(s) directly to 1972 exchange rate derived GDPs, the 1972 y^j's were obtained by multiplying the 1970 y^j's by $(1 + g^j)^2$ where g^j is the average real, per capita growth rate, relative to the United States, of the j'th country between 1970 and 1972[11]. This method was adopted because the substantial changes in exchange rates between 1970 and 1972 make it unlikely that a good estimate of the 1972 equilibrium exchange rates can be obtained. The individual country growth rates themselves appear in column (8).

IV. The Future

Where do we go from here? If instead of 30 observations on 13 countries we had say 45 observations on 18 countries we would be able to pin the debissing relationship down much more firmly. This data availability is a real prospect within a fairly short time. However, the scatter diagram is already easy to track. Most of the inaccuracy in Table 2 comes from scatter around the regression curve rather than from inaccuracy in estimating the coefficients of the regression equation. Being off by a few percentage points in estimating $\bar{y}^j$ is not serious for advanced countries, but it gives rise to substantial errors in the case of low-income LDC's. Is it too much to hope that when the new data come in, particularly data on low-income countries, some noise-screening variables will be found which will sharpen our estimates particularly at the low end of the world income distribution?

FOOTNOTES

*University of Pennsylvania, and the International Bank for Reconstruction and Development respectively. This research flows from the work of the International Comparison Project (ICP), a joint undertaking of the United Nations Statistical Office, the International Bank for Reconstruction and Development, and the International Comparison Unit of the University of Pennsylvania. Though the data obtained in Phase I of the ICP has been an indispersable input, the present paper represents the views of the authors alone, and not any of the participating organization of the ICP. Valuable discussions with Alan W. Heston and Irving B. Kravis of the ICP staff are gratefully acknowledged.

[1] The major studies that have developed estimates of PPPs are Gilbert and Kravis (1954), Gilbert and Associates (1958), Braithewaite (19), and Kravis, Kennessey, Heston, and Summers (1975).

[2] This abstracts from the obvious point that capital movements enter into the determination of the exchange rate.

[3] David (1972).

[4] In the 1950 and 1955 cross-sections the gross product concept used was Gross National Product while in the 1967 and 1970 ones, the concept used ws Gross Domestic Product. We assume that

$$\frac{GDP^j}{GDP^{US}} = \frac{GNP^j}{GNP^{US}} .$$

[5] For ease of presentation, y^j and Y^j are presented in Table 1 in percentage form. $y^j = 100 * y^j$ and $Y^j = 100 * Y^j$.

[6] World Bank Atlas (1970).

[7] United Nations (1972).

[8] As one looks at the scatter of points in Chart 1, the classic examination question of an introductory regression analysis course comes to mind: why is the curve on gets from a regression equation really better than a carefully drawn free-hand curve?

[9] When the data set is divided into 1950-55 observations and the 1967-70 observations, two sets of regression coefficients are obtained:

$$\text{1950-55:}\quad \log y^j = \underset{(.0734)}{.8011}\ \log Y^j + \underset{(.0833)}{.0782} \qquad \bar{R}^2 = .8873$$

$$\text{1967-70:}\quad \log y^j = \underset{(.0271)}{.7954}\ \log Y^j + \underset{(.0611)}{.1410} \qquad \bar{R}^2 = .9851$$

An analysis of covariance shows the two sets of regression coefficients are not significantly different. There does seem to be temporal stability!

[10] Beckerman [196]; Beckerman and Bacon [196].

[11] These growth rates were obtained from an unpublished memorandum of the World Bank.