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# Old-Keynesianism in the New Keynesian model

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# Old-Keynesianism in the New Keynesian model<sup>\*</sup>

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#### Abstract

Yield curves on advanced economies' government debt imply that financial markets expect short-term interest rates to remain close to or at the Zero Lower Bound (ZLB) for many years or decades to come. In such a situation, conventional monetary policy can no longer be employed for macroeconomic stabilization. Therefore, I investigate an alternative 'old Keynesian' fiscal policy in which government spending endogenously responds to inflation and the output gap, while the nominal interest rate is pegged at the ZLB. I do so within a standard Representative Agent New Keynesian model (RANK), as well as a two-period Overlapping Generations New Keynesian model (OLG-NK). For both model versions, I find that the equilibrium values for inflation and the output gap under a standard Taylor rule regime can be replicated under the 'old Keynesian' regime. However, a unique stable countercyclical equilibrium is only feasible within the OLG-NK model. Finally I show that the old Keynesian policy features a 'fiscal' divine coincidence under which government spending can simultaneously ensure zero inflation and elimination of the output gap.

**Keywords:** Monetary Policy; Fiscal Policy; Zero Lower Bound; Old-Keynesianism **JEL:** E32, E52, E62, E63

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# 1 Introduction

The regular New Keynesian model is closed by formulating a Taylor rule in which the nominal interest rate endogenously responds to inflation and the output gap (Woodford (2003) and subsequent literature). However, such a rule does not provide a realistic description once the economy lands at the Zero Lower Bound (ZLB), in which case unconventional monetary policies and fiscal policy are typically employed for macroeconomic stabilization. Such a situation particularly applies to advanced economies that have been at the ZLB since the Great Financial Crisis of 2007-2009, and are expected to remain there for many years or even decades to come. Therefore, I explore an alternative 'old Keynesian' fiscal policy in which government spending endogenously responds to inflation and the output gap, while the nominal interest rate is pegged at the ZLB. I do so within the class of otherwise standard New Keynesian models.

Figure 1 provides a strong indication that financial markets expect short-term interest rates in the Eurozone, Japan, and the United Kingdom to remain at or close to the ZLB for many years to come. Specifically, Figure 1a shows that yields on government bonds increase by less than 1% when moving from a maturity of less than one year to a maturity of 30 years. At the same time, their level remains below 1% even for debt with a maturity of 30 years. Figure 1b shows the instantaneous forward rate that is implied by the yield curves in Figure 1a. This forward rate can be understood as the future short-term interest rate expected by financial markets. Figure 1b clearly shows that future short-term interest rates in the Eurozone are not expected to increase above 0.2% during the next 30 years, while they will only temporarily increase above 1% in the United Kingdom, and then revert back below 0.5%. This implies that financial markets expect these economies to remain at or close to the ZLB for the next 30 years. Ofcourse, developments between today and 30 years could easily get these economies away from the ZLB, in which case the ZLB turns out to be temporarily binding expost, a situation that is adequately described by existing models in the literature (Christiano et al., 2011; Eggertsson, 2011). What is relevant for this paper, however, is that economic agents currently *expect* short-term interest rates to be permanently at the ZLB, and given Figures 1a and 1b that case can clearly be made. In addition, the fact that Japan has actually been at the ZLB for almost 30 years provides further proof that a (almost) permanent ZLB is a realistic possibility.

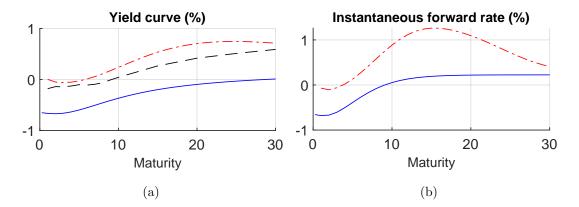


Figure 1: The left figure displays yield curves for the euro area (blue, solid), the United Kingdom (red, slotted), and Japan (black, dashed). The right figure displays the instantaneous forward curve for the euro area and the United Kingdom, which is the short-term (instantaneous) interest rate for future periods that is implied in the yield curve. Maturity is in years. *Sources:* European Central Bank, Bank of England, Ministry of Finance Japan.

In this paper I investigate within the class of standard New Keynesian models whether endogenous government spending can stabilize the macroeconomy when the nominal interest rate is pegged at the ZLB. Within such a framework, several more specific questions arise: is it possible to replicate the equilibrium that would arise under an active Taylor rule through an appropriate choice of the government spending rule? What type of spending rules generate a unique stable equilibrium? Does the answer to this question depend on whether I employ a standard representative agent version of the New Keynesian model or an overlapping generations version? Is there a fiscal equivalent to the 'monetary' divine coincidence (Blanchard and Galí, 2007; Galí, 2015)?

To answer these questions, I employ the standard Representative Agent New Keynesian (RANK) model, as well as a two-period Overlapping Generations New Keynesian (OLG-NK) model. Both model versions feature pricing rigidities a la ? and government spending that is financed by issuing one-period bonds and by levying lump sum taxes. The OLG-NK model is closest in spirit to Galí (2014),

and features a young generation that consumes, provides labor, pays lump sum taxes, and saves through government bonds, while the old generation uses gross repayment of bonds and a government transfer that is linear in output to pay for consumption and lump sum taxes. I set lump sum taxes on the old equal to their gross interest payments on government bonds, and thereby eliminate government debt as a state variable. I do so to ensure analytical tractability. As a result, the OLG-NK model also features Ricardian equivalence. The production side of the economy and the government budget constraint turn out to be equivalent under both models, except for the government transfer to the old generation in the OLG-NK model. Both model versions do not feature physical capital.

I distinguish between the familiar 'monetary' regime in which government spending is constant and the nominal interest rate follows an active Taylor rule on the one hand (Taylor (1993); Woodford (2003) and subsequent literature), and a 'fiscal' or 'old Keynesian' regime in which the nominal interest rate is pegged at the ZLB, see Figure 1, and government spending endogenously responds to inflation and the output gap on the other. The linearized version of both models can be reduced to the familiar New Keynesian Phillips curve and an aggregate demand equation which relates the (expected) output gap to the return difference between the expected real rate and the natural rate of interest, the market clearing interest rate under perfectly flexible prices (Woodford, 2000). The channel through which endogenous government spending affects the equilibrium is by changing the natural rate of interest. Crucially, the natural rate increases within the RANK model with the *difference* between today's and tomorrow's expected government spending, whereas it increases with the (weighted) *sum* of today's and tomorrow's expected government spending within the OLG-NK model.

My first contribution is to show that the equilibrium values of the output gap and inflation that arise under the monetary regime can be replicated under the fiscal regime through appropriate choice of the feedback coefficients of inflation and the output gap on government spending. The resulting government spending rule turns out to be countercyclical in inflation and the output gap: the government reduces aggregate demand by reducing spending when inflation and the output gap are positive and vice versa, just as the central bank reduces aggregate demand by raising the nominal and real interest rate under the monetary regime. Therefore, this result shows that being permanently stuck at the ZLB (in expectation) does not prevent the government from achieving the equilibrium values of the output gap and inflation that would arise when conventional monetary policy can be employed.

However, whereas the resulting countercyclical government spending rule is consistent with a unique stable equilibrium within the OLG-NK model, it turns out that this is not the case within the RANK model. To explain the intuition behind this result, consider a positive productivity shock which initially decreases the natural rate of interest. As a result, the return difference between the expected real rate of interest and the natural rate increases, inducing households to shift from spending to saving. The output gap turns negative, and the economy features (expected) deflation through the New Keynesian Phillips curve. This, in turn, raises the expected real rate everything else equal. To have a unique stable equilibrium, the natural rate must also *increase* in equilibrium. This, however, requires that the increase in the natural rate arising from endogenous government spending must be larger than the initial decrease generated by the productivity shock. This is only feasible within the OLG-NK model, where the natural rate increases with the (weighted) sum of today's and tomorrow's expected government spending. Within the RANK model, however, the natural rate only increases with the *difference* between today's and tomorrow's expected government spending. Therefore, no unique stable equilibrium with countercyclical government spending exists. This contrasts with the monetary regime, for which I show that unique stable equilibria are qualitatively very similar for the RANK and the OLG-NK model.

My final contribution consists of establishing the existence of a fiscal counterpart to the 'monetary' divine coincidence, which says that employing one policy instrument (the nominal interest rate) can achieve the double goal of zero inflation and elimination of the output gap (Blanchard and Galí, 2007). While the monetary divine coincidence is achieved by instantaneously adjusting the nominal interest one-for-one with changes in the natural rate that arise from exogenous shocks (Galí, 2015), the fiscal divine coincidence is achieved through endogenous government spending directly offsetting the changes in the natural rate that arise from exogenous shocks. Just as in the case of the monetary divine coincidence, one policy instrument (government spending) achieves the two simultaneous goals of zero inflation and elimination of the output gap.

#### Literature review

First of all my paper is related to the classic IS-LM literature that started with Hicks (1937) after publication of John Maynard Keynes' General Theory (Keynes, 1936). This framework encompasses the recommendation of Keynes that fiscal policy should be expansionary in recessions to mitigate the drop in GDP, while it should be contractionary in booms (Keynes, 1936). My model also employs fiscal policy for macroeconomic stabilization, but all stabilization is performed through changes in government spending, as Ricardian equivalence prevents government deficits from affecting the equilibrium in both the RANK model and the OLG-NK model.

Although the primary instrument for macroeconomic stabilization within the standard New Keynesian model is the nominal interest rate, this instrument is no longer available when the economy hits the ZLB like in the Great Financial Crisis of 2007-2009. In response, governments around the world resorted to fiscal policy to provide additional macroeconomic stimulus. This has inspired a whole new strand within the New Keynesian literature in which government spending is increased for as long as the economy is at the ZLB (Christiano et al., 2011; Eggertsson, 2011; Woodford, 2011; Eggertsson and Krugman, 2012). These papers differ in two respects from my paper. First, the economy is only temporarily at the ZLB, and eventually returns to a regime in which monetary policy regains full potency. Second, the level of government spending depends on the regime (ZLB vs. no ZLB) but is exogenous within a particular regime. Eggertsson et al. (2019) explicitly model how an economy can be permanently at the ZLB as a result of secular stagnation. I, however, perform my analysis within the class of standard New Keynesian models for two reasons. First, this class of models allows for analytical tractability, and second, it facilitates the comparison of my fiscal regime with the standard New Keynesian monetary regime.

A problem with the RANK model is that it features indeterminacy issues at the (temporarily binding) ZLB (Cochrane, 2017). Within heterogeneous agents models

such as Hagedorn (2016) and Hagedorn (2018), this problem is eliminated by specifying fiscal policy in *nominal* sequences for government spending, government debt, and taxes. As a result, the present value government budget constraint is satisfied at all times and for any price level, which in turn adjusts until demand and supply in the goods market, or equivalently the asset market, are equalized. Therefore, the indeterminacy of the price level and inflation when monetary policy is implemented through an interest rate target (Sargent and Wallace, 1975) is eliminated.

Leeper (1991) identifies under which monetary and fiscal policies a unique stable equilibrium is feasible within a stochastic representative agent model. He finds that when the nominal interest rate is pegged to its steady state value, fiscal policy must be active in the sense that the feedback from government debt to lump sum taxes does not respond strongly, or not at all, as in the fiscal theory of the price level (Sims, 1994; Woodford, 1995; Cochrane, 1999). Lump sum taxes in my model, however, abide by the Bohn (1998) condition, and are therefore passive in the terminology of Leeper (1991). The reason why unique stable equilibria are still possible is the fact that government spending is endogenous, unlike the constant real spending in Leeper (1991). Therefore, fiscal policy can be considered active, as the fiscal authority does not take the state of government debt into account when determining how much to spend.

There is also a literature which studies the effects of fiscal policy within endogenous growth models (Barro, 1990; Turnovsky, 1996, 2000; Agénor, 2008; Barseghyan and Battaglini, 2016). Fiscal policy is endogenous in the sense that government spending depends on the amount of taxes levied, which in turn depends on aggregate production (Barro, 1990; Agénor, 2008), or on an explicit modeling of the legislative bargaining process (Barseghyan and Battaglini, 2016). The focus of most of these papers is on optimal fiscal policy, the fiscal policy that maximizes long-run growth (Barro, 1990; Turnovsky, 1996, 2000; Agénor, 2008). One exception is Chari et al. (1994), who study optimal fiscal policy within a business cycle model, and determine the optimal tax rate on capital and labor by solving the Ramsey problem. Just as Chari et al. (1994), I focus on business cycle dynamics rather than long-run dynamics, but I refrain from looking at optimal fiscal policy.

Finally, my paper is also related to the literature with overlapping generation

models, which started with Samuelson (1958). My overlapping generations model is closest to Galí (2014), in which there is price stickiness as well. Galí (2014), however, differs in four important dimensions. First, Galí (2014) features a bubbly asset. Second, production firms operate for two periods, while my firms are infinitely lived to keep the model as comparable with the RANK model as possible. Third, labor supply is inelastic, while it is endogenous in my setup. Fourth, there is no government spending.

I describe the model in Section 2, and establish analytical results in Section 3. I present numerical simulations in Section 4, and conclude in Section 5.

# 2 Model

As the RANK model and its derivations are by now standard in the literature, I refer the interested reader to Appendix A for the full description of the model, and immediately present the linearized equations below.

The overlapping generations model is inspired by Galí (2014). Specifically, a generation lives for two periods, the size of which has mass one and is constant across time. Each member of a generation has identical preferences, and is referred to as "the young" in the first period of existence, and as "the old" in the second period of existence. The young receive income from providing endogenous labor and ownership of all production firms, which they spend on consumption, lump sum taxes, and government bonds.<sup>1</sup> The old receive a transfer from the government and the gross repayment of government bonds, which they spend on consumption and lump sum taxes. Lump sum taxes are levied on both the young and the old, with lump sum taxes on the old equal to the gross interest payments of bond holdings), while lump sum taxes on the young respond to the stock of previous period government debt, thereby satisfying the Bohn (1998) principle.<sup>2</sup>

<sup>&</sup>lt;sup>1</sup>Most OLG models would place ownership of firms with the old rather than the young. Within my model this would result in a negatively sloped New Keynesian Phillips curve, which I think is unrealistic. Instead, I place firm ownership at the young.

 $<sup>^{2}</sup>$ By choosing lump sum taxes in this way, I am capable of eliminating the beginning-of-period stock of government debt as a state variable. This is necessary for my theoretical analysis, as otherwise I am not capable of deriving a closed-form expression for the natural level of output in

The government budget constraint is the same as in the RANK-model, except that the government makes a transfer to the old that is linear in current output. The production sector is identical to that in the RANK model, except that ownership is in the hands of the young and transferred to the next generation when the young turn old. Therefore, production firms discount future profits using a stochastic discount factor that features the marginal utility of future young generations.

Both the RANK and the OLG-NK model do not feature physical capital to keep the models analytically tractable. Unless otherwise stated, the only exogenous shock in the main text is a productivity shock that follows a regular AR(1) process. A full specification of both models can be found in Appendix A and B.

# 2.1 The Representative Agent New Keynesian (RANK) model

I start by linearizing the standard representative agent New Keynesian model in Appendix A, which can eventually be described by two familiar equations. These are the New Keynesian Phillips curve and the aggregated Euler equation which I will refer to as the aggregate demand equation:

$$\hat{\pi}_t = \beta E_t \left[ \hat{\pi}_{t+1} \right] + \kappa \tilde{y}_t, \tag{1}$$

$$\sigma\left(\bar{y}/\bar{c}\right)\tilde{y}_{t} = \sigma\left(\bar{y}/\bar{c}\right)E_{t}\left[\tilde{y}_{t+1}\right] - \left(r_{t} - \hat{R}_{t}^{*}\right), \qquad (2)$$

where  $\hat{x}_t$  denotes the percentage deviation of variable  $x_t$  from its steady state.  $\tilde{y}_t \equiv \hat{y}_t - \hat{y}_t^n$  denotes the output gap, which is the difference between output under the New Keynesian model  $\hat{y}_t$  and output under perfectly flexible prices  $\hat{y}_t^n$ .  $\sigma$ denotes households' coefficient of relative risk aversion, and  $\bar{y}$  and  $\bar{c}$  denote the steady state level of output and consumption, respectively. The factor  $\bar{y}/\bar{c}$  arises because output is not only absorbed by consumption, but also by government

terms of the exogenous state variables. This, in turn, is necessary to obtain analytical expressions for my model economy that feature the output gap rather than the level of output, which is a key variable in the New Keynesian literature that studies monetary policy (Galí, 2015). However, setting lump sum taxes in this way without providing a government transfer would leave the old with zero income after lump sum taxes, and therefore with zero consumption in equilibrium. This motivates the introduction of the government transfer which ensures positive consumption by the old.

spending.  $r_t \equiv \hat{R}_t^n - E_t [\hat{\pi}_{t+1}]$  is the expected real interest rate, where  $\hat{R}_t^n$  denotes the percentage deviation of the nominal interest rate from its steady state value, while  $\hat{R}_t^*$  denotes the natural rate of interest (Woodford, 2000), which can be decomposed into two components:

$$\hat{R}_t^* = \hat{R}_t^{z,*} + \hat{R}_t^{g,*}.$$
(3)

These two components are given by:

$$\hat{R}_t^{z,*} = -\sigma \left( \bar{y}/\bar{c} \right) \left( \frac{1+\varphi}{\sigma \left( \bar{y}/\bar{c} \right) + \varphi} \right) \left( 1-\rho_z \right) \hat{z}_t, \tag{4}$$

$$\hat{R}_{t}^{g,*} = \sigma\left(\bar{y}/\bar{c}\right) \left(\frac{(\bar{g}/\bar{y})\varphi}{\sigma\left(\bar{y}/\bar{c}\right)+\varphi}\right) \left(\hat{g}_{t} - E_{t}\left[\hat{g}_{t+1}\right]\right),$$
(5)

where  $\hat{z}_t$  denotes the exogenous productivity shock,  $\varphi$  the inverse Frisch elasticity, and  $\bar{g}$  the steady state level of government spending. The term  $\hat{g}_t - E_t[\hat{g}_{t+1}]$ arises from the fact that the aggregate demand equation is derived from the households' Euler equation, which features today's and tomorrow's expected consumption. Through substitution of the (linearized version of the) aggregate resource constraint  $y_t = c_t + g_t$ , these terms introduce today's and tomorrow's expected government spending on opposite sides of the equality sign.

A key observation is that the natural rate of interest is no longer exogenous when government spending endogenously responds to inflation and the output gap, as will be the case below. In fact, changing the natural rate of interest is the key channel through which government spending affects the equilibrium of the economy (1) - (2), as government spending does not show up at other places in these equations. This marks a key difference with the textbook case, in which macroeconomic stabilization is performed through adjustment of the nominal interest rate (Woodford, 2003; Galí, 2015).

Before I continue, I discuss the intuition behind the above expressions for the natural rate of interest, where we remember that the natural rate is the equilibrium interest rate in a model with perfectly flexible prices. We see from equation (4) that a temporary positive productivity shock reduces the natural rate of interest (assuming constant government spending). Given an AR(1) process for productivity-

ity, a positive shock implies that productivity will be higher today than tomorrow. As such, households know that today's income will be higher than tomorrow's, everything else equal. To smooth consumption over time, households would like to save part of the additional income that the positive productivity shock generates today by buying additional government bonds. However, the supply of bonds does not increase, while the supply of final goods increases as a result of the productivity shock. The only way to clear both the bond market and the goods market is through a fall in the equilibrium interest rate (Walsh, 2010).

Next, we see from equation (5) that a positive government spending shock increases the natural rate of interest. Higher government spending increases the demand for final goods as well as the supply of government bonds. To induce households to reduce consumption and increase savings so that equilibrium in goods and bond markets can be achieved, the natural rate of interest must increase. However, an interesting observation is the fact that this natural rate increases with  $\hat{g}_t - E_t[\hat{g}_{t+1}]$ . As such, expected government spending tomorrow reduces the natural rate of interest today: an increase in future government spending reduces households' life-time income, everything else equal, and therefore future consumption. In response, households would like to save more today to smooth consumption over time, which increases the demand for government bonds. As today's supply of bonds is not directly affected by expected spending tomorrow, the natural rate of interest must decrease to achieve equilibrium in the bond market. As such, the fact that the natural rate depends on  $\hat{g}_t - E_t [\hat{g}_{t+1}]$  causes a persistent government spending shock to increase the natural rate by less than when the natural rate only depends on  $\hat{g}_t$  (Walsh, 2010).

# 2.2 Overlapping Generations New Keynesian Model (OLG-NK)

Next, I discuss the two-period OLG-NK model. The derivations of the nonlinear first order conditions, and the resulting set of linearized equations can be found in Appendix B. I show that the New Keynesian Phillips curve is the same as in the RANK-model (1). However, the aggregate demand equation changes. To derive it, I start from the young's (linearized) Euler equation which determines how much

to consume and how much to save through government bonds:

$$-\sigma \hat{c}_{t}^{1} = -E_{t} \left[ \sigma \hat{c}_{t+1}^{2} \right] + \hat{R}_{t}^{n} - E_{t} \left[ \hat{\pi}_{t+1} \right], \qquad (6)$$

where  $\hat{c}_t^1$  and  $\hat{c}_t^2$  denote consumption of the young and old, respectively. Both the young and old's coefficient of relative risk aversion is  $\sigma$ .

To arrive at an aggregate demand equation in terms of inflation and the output gap, I substitute a linearized version of the old's budget constraint  $c_t^2 = s_t = (\bar{s}/\bar{y}) y_t$ , where  $s_t$  denotes the government transfer to the old, and  $\bar{s}/\bar{y}$  the steady state transfer in terms of steady state output.<sup>3</sup> In addition, I employ a linearized version of the aggregate resource constraint  $c_t^1 = y_t - c_t^2 - g_t = (1 - \bar{c}_2/\bar{y}) y_t - g_t$ , and an analytical expression for the natural level of output to arrive at the following aggregate demand equation, a detailed mathematical derivation of which can be found in Appendix B:

$$\sigma\left(\bar{y}/\bar{c}_{1}\right)\left(1-\frac{\bar{c}_{2}}{\bar{y}}\right)\tilde{y}_{t} = \sigma E_{t}\left[\tilde{y}_{t+1}\right] - \left(r_{t}-\hat{R}_{t}^{*}\right),\tag{7}$$

Compared with the representative agent version of the aggregate demand equation (2), we see that the coefficients in front of  $\tilde{y}_t$  and  $E_t[\tilde{y}_{t+1}]$  are not the same anymore, which is a result of the fact that the young and old have different budget constraints. As such, the different numerical values of these two coefficients will at least quantitatively affect the young's savings decision with respect to the savings decision of the representative household in the RANK model.

Although  $\hat{R}_t^*$  can still be decomposed into the two components of expression (3), the resulting expressions change with respect to their counterparts (4) and (5) in the RANK model:

$$\hat{R}_{t}^{z,*} = -\sigma \left( \left( \bar{y}/\bar{c}_{1} \right) \left( 1 - \frac{\bar{c}_{2}}{\bar{y}} \right) - \rho_{z} \right) \left( \frac{(1+\varphi)}{\sigma \left( \bar{y}/\bar{c}_{1} \right) \left( 1 - \frac{\bar{c}_{2}}{\bar{y}} \right) + \varphi} \right) \hat{z}_{t}, \quad (8)$$

$$\hat{R}_{t}^{g,*} = \sigma\left(\bar{y}/\bar{c}_{1}\right) \left(\frac{(\bar{g}/\bar{y})}{\sigma\left(\bar{y}/\bar{c}_{1}\right)\left(1-\frac{\bar{c}_{2}}{\bar{y}}\right)+\varphi}\right) \left(\varphi\hat{g}_{t}+\sigma E_{t}\left[\hat{g}_{t+1}\right]\right),\tag{9}$$

 $<sup>^{3}</sup>$ Remember that the old generation's lump sum taxes exactly equal their gross interest payments on government bonds so that their consumption equals the government transfer.

Comparing the new expression for the natural rate of interest arising from productivity shocks with the equivalent expression in the RANK model, we see that the response will qualitatively be the same as in the RANK model. The young understand that an AR(1) productivity process implies that income today increases by more than income tomorrow ceteris paribus, which increases their desire to save. To achieve clearing in bond and goods market, the natural rate of interest must come down.

The key difference with the natural rate of interest within the RANK model, however, is the component that arises from government spending, expression (9). Compared with the equivalent expression in the RANK-model (5), today's and tomorrow's expected government spending terms are now additive ( $\varphi \hat{g}_t + \sigma E_t [\hat{g}_{t+1}]$ ), rather than subtractive ( $\hat{g}_t - E_t [\hat{g}_{t+1}]$ ). The reason for this sign switch has to do with the term relating to tomorrow's expected consumption by the representative agent and the young, respectively. Whereas tomorrow's expected consumption by the representative agent is substituted by the difference between output and government spending in the RANK model, tomorrow's expected consumption by today's young is substituted by the government transfer in the OLG-NK model, which is linear in output alone.

As a result of today's and tomorrow's expected government spending being additive, the same persistent government spending shock increases the natural rate of interest by more in the OLG-NK model than in the RANK model. In addition, a more persistent spending shock leads to a *larger* change in the natural rate, everything else equal. This sharply contrasts with the RANK model, where more persistent shocks lead to a *smaller* change in the natural rate. Therefore, government spending is more powerful in changing the natural rate of interest in the OLG-NK model, and will therefore likely be a more effective tool in stabilizing the macroeconomy.

#### 2.3 The different policy regimes

In this subsection I specify the two regimes that I study in this paper. These consist of the regular monetary regime that is typically studied in the literature (see Woodford (2003) and Galí (2015), for example), and the fiscal regime that I

define below.

Specifically, the monetary regime is defined by government spending being equal to its steady state value, i.e.  $\hat{g}_t = 0$ , and an active Taylor rule for the nominal interest rate (Taylor, 1993):

$$\hat{R}_t^n = \kappa_\pi \hat{\pi}_t + \kappa_y \tilde{y}_t, \tag{10}$$

which satisfies the Taylor principle  $\kappa_{\pi} > 1$  and  $\kappa_{y} \ge 0$ . Therefore, macroeconomic stabilization is performed by adjusting the nominal interest, which in turn changes the expected real interest rate. Both within the RANK model, as well as the OLG-NK model, the natural rate only features the exogenous productivity component, since  $\hat{g}_{t} = 0$  across time.

The fiscal regime is captured by a nominal interest rate that is equal to its steady state value, i.e.  $\hat{R}_t^n = 0$ , while government spending is given by:

$$\hat{g}_t = g_\pi \hat{\pi}_t + g_y \tilde{y}_t. \tag{11}$$

Under this regime, macroeconomic stabilization is performed by adjusting the natural rate of interest. At the same time, the nominal interest rate is no longer employed for stabilization.

### 2.4 The aggregate demand equation under the fiscal regime

I end the current section by substituting the government spending rule under the fiscal regime (11) into the aggregate demand equation and discuss the resulting expressions. This will help us understand some of the (analytical) results in the next sections.

#### 2.4.1 The RANK model

Substitution of the endogenous government spending rule (11) into the component of the natural rate of interest arising from government spending (5) gives the following expression:

$$\hat{R}_{t}^{g,*} = B\left\{g_{\pi}\left(\hat{\pi}_{t} - E_{t}\left[\hat{\pi}_{t+1}\right]\right) + g_{y}\left(\tilde{y}_{t} - E_{t}\left[\tilde{y}_{t+1}\right]\right)\right\} = \hat{R}_{t}^{\pi,*} + \hat{R}_{t}^{y,*}, \quad (12)$$

where B is given by:

$$B = \sigma \left( \bar{y}/\bar{c} \right) \left( \frac{(\bar{g}/\bar{y}) \varphi}{\sigma \left( \bar{y}/\bar{c} \right) + \varphi} \right) > 0, \tag{13}$$

while  $\hat{R}_t^{\pi,*}$  and  $\hat{R}_t^{y,*}$  are given by:

$$\hat{R}_{t}^{\pi,*} = Bg_{\pi} \left( \hat{\pi}_{t} - E_{t} \left[ \hat{\pi}_{t+1} \right] \right), \qquad (14)$$

$$R_t^{y,*} = Bg_y \left( \tilde{y}_t - E_t \left[ \tilde{y}_{t+1} \right] \right), \tag{15}$$

Expression (12) shows that an endogenous natural rate changes in response to the *difference* between inflation today and expected inflation tomorrow, as well as to the *difference* between the output gap today and the expected output gap tomorrow. This marks a significant contrast with the monetary regime, in which the nominal interest rate only responds to changes in today's *level* of inflation and the output gap.

To enhance our understanding of the results in the next sections, I substitute expression (12) into the aggregate demand equation (2), and rearrange the term of the natural rate that is related to the output gap (15):

$$\left(\sigma\left(\bar{y}/\bar{c}\right) - Bg_{y}\right)\tilde{y}_{t} = \left(\sigma\left(\bar{y}/\bar{c}\right) - Bg_{y}\right)E_{t}\left[\tilde{y}_{t+1}\right] - \left(r_{t} - \hat{R}_{t}^{z,*} - \hat{R}_{t}^{\pi,*}\right).$$
(16)

The resulting structure of the above equation is the same as under the monetary regime: we have the current output gap on the left hand side, and the expected output gap and the difference between the expected real rate and the natural rate on the right hand side. A key difference however, is the coefficient  $\sigma(\bar{y}/\bar{c}) - Bg_y$ in front of both output gaps. While this coefficient is unambiguously positive under the monetary regime (in which case B = 0 and  $\hat{R}_t^{\pi,*} = 0$ ), we see that this coefficient switches sign and becomes negative when  $g_y > \sigma(\bar{y}/\bar{c}) / B$ . In that case, the response of inflation and the output gap to a productivity shock will not only be affected quantitatively, but also qualitatively. For the moment I leave it at this observation, but I will revisit this issue after having inspected the stability properties of the fiscal regime, and its dynamic response to a productivity shock in Section 4.

#### 2.4.2 The OLG-NK model

Next, I inspect the aggregate demand equation for the OLG-NK model. Just as in the RANK model, I start by substituting the government spending rule (11) into the component of the natural rate that arises from government spending (9) within the OLG-NK model:

$$\hat{R}_{t}^{g,*} = B^{*}\left[g_{\pi}\left(\varphi\hat{\pi}_{t} + \sigma E_{t}\left[\hat{\pi}_{t+1}\right]\right) + g_{y}\left(\varphi\tilde{y}_{t} + \sigma E_{t}\left[\tilde{y}_{t+1}\right]\right)\right] = \hat{R}_{t}^{\pi,*} + \hat{R}_{t}^{y,*}, (17)$$

where  $B^* = \sigma\left(\bar{y}/\bar{c}_1\right) \left(\frac{(\bar{g}/\bar{y})}{\sigma(\bar{y}/\bar{c}_1)\left(1-\frac{\bar{c}_2}{\bar{y}}\right)+\varphi}\right) > 0$ , while  $\hat{R}_t^{\pi,*}$  and  $\hat{R}_t^{y,*}$  are given by

$$\hat{R}_{t}^{\pi,*} = B^{*}g_{\pi} \left(\varphi \hat{\pi}_{t} + \sigma E_{t} \left[\hat{\pi}_{t+1}\right]\right), \qquad (18)$$

$$\hat{R}_t^{y*} = B^* g_y \left(\varphi \tilde{y}_t + \sigma E_t \left[ \tilde{y}_{t+1} \right] \right), \tag{19}$$

Hence we see from expression (17) that the endogenous natural rate will not change in response to the *difference* between today's and tomorrow's expected inflation and output gap (as in the RANK model), but rather to the (weighted) *sum* of today's and tomorrow's expected inflation and output gap.

Next, I substitute the expression for the natural rate of interest arising from government spending (17) into the aggregate demand equation (7):

$$\left[\sigma\left(\bar{y}/\bar{c}_{1}\right)\left(1-\frac{\bar{c}_{2}}{\bar{y}}\right)-\varphi B^{*}g_{y}\right]\tilde{y}_{t}=\sigma\left(1+B^{*}g_{y}\right)E_{t}\left[\tilde{y}_{t+1}\right]-\left(r_{t}-\hat{R}_{t}^{z,*}-\hat{R}_{t}^{\pi,*}\right),(20)$$

Just as in the case of the RANK model, the structure of the equation is the same as under the monetary regime, for which  $B^* = 0$  and  $\hat{R}_t^{\pi,*} = 0$ . However, we see that endogenous government spending affects the aggregate demand equation in a fundamentally different way than in the RANK model. Whereas the coefficients in front of the output gaps are the same within the RANK model and affected by the inclusion of endogenous government spending in a symmetric way (see equation (16)), we see that these coefficients are affected in an asymmetric way within the OLG-NK model. Just as for the RANK-model, I leave further discussion for Section 4.

# 3 Analytical results

In this section I establish three analytical results. First, I show for both the RANK and the OLG-NK model that the equilibrium values for inflation and the output gap under the monetary regime can be replicated under the fiscal regime through an appropriate mapping from the monetary feedback coefficients  $\kappa_{\pi}$  and  $\kappa_{y}$  to the government spending feedback coefficients  $g_{\pi}$  and  $g_{y}$ . This shows that endogenous government spending can ensure that the same equilibrium arises as in the case where the ZLB is not binding, and conventional monetary policy is employed for macroeconomic stabilization.

Second, I show that such a countercyclical equilibrium is not unique in the RANK model and that multiple other equilibria exist. This result implies that the RANK model might not be the right framework to explore the consequences of an old-fashioned Keynesian fiscal policy.

Finally, I show the existence of a fiscal counterpart to the 'monetary' divine coincidence in both the RANK and the OLG-NK model. The divine coincidence denotes the concept that the New Keynesian Phillips curve (1) allows for one policy instrument to simultaneously achieve the two goals of i) hitting the central bank's inflation target, and ii) setting the output gap equal to zero (Blanchard and Galí, 2007).

### 3.1 Equivalence between monetary and fiscal equilibrium

I start by analytically calculating the impulse response functions to a productivity shock using the method of undetermined coefficients. I do so for both the monetary and the fiscal regime, see Appendix A.11 for the RANK model and Appendix B.10 for the OLG-NK model. Doing so will allow me to show that the equilibrium values for inflation and the output gap that arise under the monetary regime can be replicated under the fiscal regime through an appropriate choice of the government spending coefficients  $g_{\pi}$  and  $g_y$  in terms of the monetary feedback coefficients  $\kappa_{\pi}$ and  $\kappa_y$ .

**Proposition 1.** An equivalence exists between the equilibrium allocations for inflation and the output gap under the monetary and fiscal regime through an appropriate choice of  $g_{\pi}$  and  $g_{y}$  in terms of the monetary feedback coefficients  $\kappa_{\pi}$  and  $\kappa_{y}$ .

*Proof.* In Appendix A.11, I show for the RANK model that the following mapping between the monetary policy coefficients  $\kappa_{\pi}$  and  $\kappa_{y}$  and the government spending coefficients  $g_{\pi}$  and  $g_{y}$  results in the exact same equilibrium allocations for inflation and the output gap in response to productivity shocks:

$$\kappa_{\pi} = -B \left(1 - \rho_z\right) g_{\pi}, \tag{21}$$

$$\kappa_y = -B\left(1-\rho_z\right)g_y,\tag{22}$$

In Appendix B.10 I show the equivalent mapping for the OLG-NK model, which is given by:

$$\kappa_{\pi} = -B^* \left( \sigma \rho_z + \varphi \right) g_{\pi}, \tag{23}$$

$$\kappa_y = -B^* \left(\sigma \rho_z + \varphi\right) g_y, \tag{24}$$

The economic intuition behind this result is straightforward and can be explained by considering a positive productivity shock that decreases inflation and the output gap. Under the monetary regime, the central bank reduces the nominal and real interest rate to increase aggregate demand. As a result, inflation and the output gap increase with respect to the initial decrease that resulted from the productivity shock. To achieve the same equilibrium under the fiscal regime, the government also has to increase aggregate demand, which is now achieved by raising government spending.

Interestingly, the above proposition implies that countries, which are currently stuck at the ZLB and predicted to remain there for many years to come, are (theoretically) not in any way limited by their inability to employ conventional monetary policy; they can simply employ government spending to achieve their desired levels of inflation and output gap. Given these results, it is interesting to observe that Japan has employed fiscal policy much more aggressively in recent years. And although it has not been able to bring inflation back to its target of 2%, Blanchard and Tashiro (2019) suggest that it has been able to close the output gap.<sup>4</sup>

# 3.2 The impossibility of a unique stable countercyclical spending equilibrium within the RANK model

Above we saw that the equilibrium under the monetary regime can be replicated under the fiscal regime through an appropriate choice of the feedback coefficients of inflation and output gap on government spending. However, the fact that an equilibrium is feasible does not guarantee that it is stable and unique. In fact, I show in the next proposition that a unique stable equilibrium is not feasible in the RANK model for any countercyclical government spending rule with  $g_{\pi} < 0$  and  $g_y < 0$ .

**Proposition 2.** There is no unique stable equilibrium in the RANK model for countercyclical government spending  $(g_{\pi} < 0 \text{ and } g_y < 0)$ .

Proof. See Appendix A.10.1.

I postpone explaining the intuition behind this result to the next section, as it is more easily understood with the help of numerical simulations. However, I can already conclude that while the RANK model is the workhorse model for studying monetary policy (Woodford (2003) and subsequent literature), it might not be the right framework for analysis of an active countercyclical government spending policy in an economy that is (almost) permanently at the ZLB, such as, for example, Japan during the last 30 years. The reason is that there is a broad consensus among policymakers and academics since Keynes (1936) that fiscal policy should be conducted countercyclically when it is employed for macroeconomic stabilization.

# 3.3 The 'fiscal' divine coincidence

Next, I prove the existence of the 'fiscal' divine coincidence. To do so, I assume the government can instantaneously observe the productivity shock, and can directly

<sup>&</sup>lt;sup>4</sup>Ofcourse, Japan has also aggressively employed unconventional monetary policies such as quantitative easing, so this outcome cannot be attributed to fiscal policy alone.

adjust spending in response:

$$\hat{g}_t = A_z \hat{z}_t + g_\pi \hat{\pi}_t + g_y \tilde{y}_t. \tag{25}$$

Such a rule is in line with the proof of the divine coincidence under the monetary regime (Galí, 2015), in which the nominal interest rate responds directly to productivity shocks. I am now ready to prove the existence of the fiscal divine coincidence.

**Proposition 3.** There exists a 'fiscal' divine coincidence equilibrium with  $\hat{\pi}_t = 0$ and  $\tilde{y}_t = 0$  across time.

*Proof.* An equilibrium with  $\hat{\pi}_t = 0$  and  $\tilde{y}_t = 0$  across time requires that  $\hat{R}_t^* = 0$  period by period. Substitution of equation (25) into equation (3) allows me to solve for  $A_z$  such that  $\hat{R}_t^* = 0$  for any shock  $\hat{z}_t$  within the RANK model. This is the case when  $A_z$  is given by:

$$A_z = \frac{1+\varphi}{(\bar{g}/\bar{y})\,\varphi} > 0, \tag{26}$$

Similarly, I substitute the government spending rule (25) into equation (7) to find the coefficient  $A_z^{OLG}$  for which a fiscal divine coincidence exists within the OLG-NK model:

$$A_z^{OLG} = \frac{\left(1+\varphi\right)\left[\left(\bar{y}/\bar{c}_1\right)\left(1-\frac{\bar{c}_2}{\bar{y}}\right)-\rho_z\right]}{\left(\bar{g}/\bar{c}_1\right)\left(\varphi+\sigma\rho_z\right)} > 0, \tag{27}$$

since  $(\bar{y}/\bar{c}_1) \left(1 - \frac{\bar{c}_2}{\bar{y}}\right) - \rho_z > 0.5$ 

Hence we see that in both models government spending has to increase in response to a positive productivity shock to absorb the initial lack of demand for final goods that arises from households' increased desire to save, see Section 2. Simultaneously, the extra bonds issued to finance additional government spending

<sup>&</sup>lt;sup>5</sup>Taking the steady state aggregate resource constraint  $\bar{y} = \bar{c}_1 + \bar{c}_2 + \bar{g}$ , I can write  $(\bar{y}/\bar{c}_1)\left(1 - \frac{\bar{c}_2}{\bar{y}}\right) = (\bar{y}/\bar{c}_1)\left(\frac{\bar{c}_1}{\bar{y}} + \frac{\bar{y}}{\bar{y}}\right) = 1 + \frac{\bar{y}}{\bar{c}_1} > 1$ . Since  $0 \le \rho_z < 1$ , we immediately see that  $(\bar{y}/\bar{c}_1)\left(1 - \frac{\bar{c}_2}{\bar{y}}\right) - \rho_z > 0$ .

satisfy households' increased desire to save, preventing the natural rate from having to fall to clear the bond market. As a result, the natural rate does not change in equilibrium, and inflation is at target while the output gap is zero.

The divine coincidence is an important concept as it provides the theoretical underpinning for inflation targeting: by making sure that inflation is at target, central bankers automatically ensure that output is at the efficient level of output in the absence of real imperfections (Blanchard and Galí, 2007), with the efficient level of output being equal to the level of output a social planner would choose. However, most central bankers believe such real imperfections to exist, and therefore experience a short-run trade-off between hitting the inflation target and stabilization of the output gap, the gap between the actual level of output and the efficient level of output. Still, the divine coincidence is an important theoretical concept that underpins the concept of medium-run inflation targeting (Goodfriend and King, 1997).

Therefore, it is interesting to know that such a divine coincidence is not only restricted to the case where conventional monetary policy is employed for macroeconomic stabilization, but also exists when endogenous government spending is employed at the ZLB. Ofcourse, the same real imperfections that prevent the short-run divine coincidence to exist under the monetary regime in the real world, will also prevent a short-run divine coincidence to exist within the fiscal regime. At the same time, however, it tells us that it is (theoretically) possible for the fiscal authority to achieve the efficient level of output in the medium run by ensuring that the inflation target is met using government spending as the policy instrument.

# 4 Numerical results

In this section, I numerically investigate the RANK and the OLG-NK model. I begin by investigating the stability properties of the RANK model, and show the regions for which this model has a unique and stable equilibrium. Then I show the impulse response functions to a productivity shock, which will allow me to explain why a unique stable equilibrium with procyclical government spending exists in the RANK model. I will then perform the same analysis for the OLG-NK model, which will allow me to highlight the differences with the RANK model that allow for a unique stable countercyclical equilibrium to exist.

#### 4.1 Calibration

The numerical analysis is meant to illustrate the qualitative properties of the model, rather than perform a quantitative analysis for a particular country. Therefore, all parameters have numerical values that are typical within the New Keynesian literature.

I set the steady state nominal interest rate equal to zero to capture an economy that is at the ZLB in the long run. I also set steady state net inflation equal to zero. As a result, the steady state real interest rate must be zero as well, which is achieved by setting the subjective discount factor equal to 1. The coefficient of relative risk aversion and the inverse Frisch elasticity are both set to 1. The elasticity of substitution is set to 10, implying a steady state markup of 11%, while the Calvoprobability is set to 0.75. I set steady state government spending over GDP equal to 0.2. As both the RANK model and the OLG-NK model feature Ricardian equivalence, steady state debt-GDP does not affect the equilibrium allocation, and therefore does not need to be chosen. Steady state consumption of the old is 40% of steady state output within the OLG-NK model to make sure that the steady state nominal interest rate is also zero in the OLG-NK model. The AR(1) coefficient for productivity is 0.95, while the standard deviation is equal to 0.01. These parameter values result in B = 1/9 and  $B^* = 1/5$ . A table with numerical values can be found in Appendix C.

#### 4.2 Stability in the RANK model

I start by investigating for which values of  $g_{\pi}$  and  $g_y$  a unique stable equilibrium exists within the RANK model. I do so by calculating the roots for the system that consists of equations (1) and (16) with  $\hat{R}_t^n = 0$ . The results can be found in Figure 2, where the blue plus sign denotes a unique stable equilibrium, while the red cross sign represents a combination for which multiple equilibria exist.<sup>6</sup>

 $<sup>^{6}</sup>$ Note that the system consisting of equations (1) and (16) contains two forward-looking variables. As such, there are no explosive equilibria, and we either have a unique stable equilibrium

From Figure 2 we see that a unique stable equilibrium only exists for strictly positive coefficients  $g_y > 0$  and  $g_{\pi} > 0$ . In fact, it turns out that  $g_y > \sigma (\bar{y}/\bar{c})/B$  for every unique stable equilibrium, something that will turn out to be important below.

These results confirm the analytical result from Section 3.2 that a unique stable equilibrium does not exist when government spending is countercyclical in both inflation and the output gap. Before I move on to study the stability properties of the OLG-NK model, however, it is useful to investigate the impulse response functions of the RANK model to a positive productivity shock for a combination of  $g_{\pi}$  and  $g_y$  that features a unique stable equilibrium. This will eventually help to understand why a countercyclical unique stable equilibrium is not feasible.

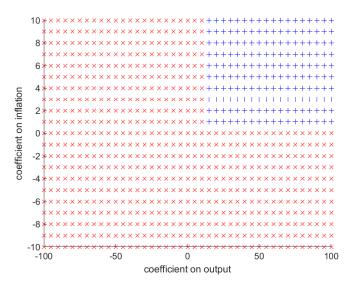


Figure 2: Stability properties of the RANK model under the fiscal regime. The horizontal axis features the output coefficient  $g_y$  and the vertical axis the inflation coefficient  $g_{\pi}$ . A unique, stable equilibrium exists for combinations of  $g_{\pi}$  and  $g_y$  that have a blue plus sign, while the combinations that feature multiple equilibria are denoted with a red cross.

or multiple equilibria (Blanchard and Kahn, 1980).

# 4.3 Dynamic response to productivity shock in the RANK model

To follow up on the theoretical analysis of Section 3 and the stability properties I found in the previous subsection, I investigate the RANK economy's impulse response functions to a productivity shock for a combination of  $g_{\pi}$  and  $g_y$  for which a unique stable equilibrium exists. Specifically, I investigate in Figure 3 the impact of a positive productivity shock of 1%, and compare the response under the monetary regime (blue, solid) with that under the fiscal regime with  $g_{\pi} = 10$  and  $g_y = 20$  (red, slotted).

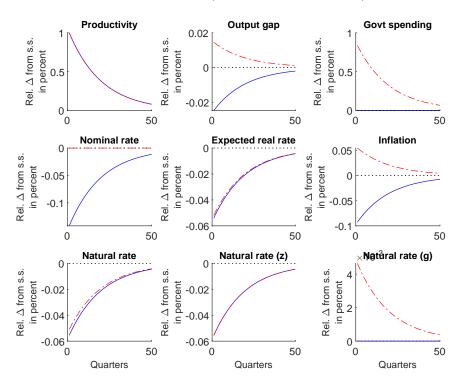
Strikingly, we see that inflation and the output gap have opposite signs under the monetary and fiscal regime. Meanwhile, the expected real interest rate  $r_t = \hat{R}_t^n - E_t [\hat{\pi}_{t+1}]$  and the natural rate of interest (3) have the same sign. They both decrease in response to the productivity shock. Also observe that there is a small quantitative difference in the natural rate of interest between the two regimes, which is caused by endogenous government spending. Therefore, changes in government spending have a relatively minor influence on the natural rate, and most of the change in the natural rate is driven by the exogenous productivity component (4).

To better understand these results, I will first revisit the aggregate demand equation under the monetary regime, and subsequently discuss it under the fiscal regime. Despite the fact that the monetary regime is well known from the literature (Galí (2015) for example), revisiting it allows me to compare and contrast it with the fiscal regime, which in turn will help us understand the counterintuitive response of inflation and the output gap.

The aggregate demand equation under the standard monetary regime is given by:

$$\sigma\left(\bar{y}/\bar{c}\right)\tilde{y}_{t} = \sigma\left(\bar{y}/\bar{c}\right)E_{t}\left[\tilde{y}_{t+1}\right] - \left(r_{t} - \hat{R}_{t}^{z,*}\right), \qquad (28)$$

In this equation, a positive productivity shock reduces the natural rate of interest  $\hat{R}_t^{z,*}$  which increases the return difference between the expected real rate  $r_t$  and the natural rate  $\hat{R}_t^{z,*}$ . As a result, households shifts from spending to saving with



## RANK model (productivity shock)

Figure 3: Figure displaying the impulse response functions to a 1% positive productivity shock in the RANK model (percentage deviation from steady state). The monetary regime with  $\kappa_{\pi} = 1.5$  and  $\kappa_{y} = 0.125$  is denoted by the blue, solid line, while the fiscal regime with  $g_{\pi} = 10$  and  $g_{y} = 20$  is denoted by the red, slotted line. Time is on the horizontal axis, and is measured in quarters. "Expected real rate" refers to the variable  $r_{t} = \hat{R}_{t}^{n} - E_{t} [\hat{\pi}_{t+1}]$ . The variable "Natural rate" refers to equation (3), "Natural rate (z)" corresponds to equation (4), while "Natural rate (g)" refers to (5).

respect to the flexible prices equilibrium, resulting in a negative output gap. The term  $\sigma(\bar{y}/\bar{c})$  determines the degree to which spending today is reduced with respect to the flexible prices equilibrium. Unsurprisingly, we see that the size of the negative output gap depends on households' intertemporal elasticity of substitution  $1/\sigma$ : the larger this elasticity (the smaller  $\sigma$ ), the larger households' desire to reduce spending today and increase savings, resulting in a more negative output gap, everything else equal. A negative output gap today results in (expected) deflation through the New Keynesian Phillips curve (1), which decreases the nominal and real interest rate through the Taylor rule. This, in turn, (partially) offsets the initial increase in the return difference between the expected real rate  $r_t$  and the natural rate  $\hat{R}_t^{z,*}$  that resulted from the productivity shock, allowing for a unique stable equilibrium to emerge.

Next, I move to the fiscal regime, and study the resulting aggregate demand equation (16) that was derived in Section 2.4:

$$(\sigma(\bar{y}/\bar{c}) - Bg_y)\,\tilde{y}_t = (\sigma(\bar{y}/\bar{c}) - Bg_y)\,E_t\,[\tilde{y}_{t+1}] - \left(r_t - \hat{R}_t^{z,*} - \hat{R}_t^{\pi,*}\right).$$

This expression allows me to explain why a shock that initially increases the return difference between the expected rate of interest  $r_t$  and the natural rate  $\hat{R}_t^*$  results in a *positive* output gap and inflation, see Figure 3. Previously, the increase in the return difference would generate a shift from spending to saving. This desire to save, however, is now more than offset by a change in the natural rate arising from a change in the output gap (captured by  $Bg_y$ ), since  $g_y > \sigma(\bar{y}/\bar{c})/B$ . As a result, households shift from saving to spending instead, resulting in a positive output gap in equilibrium. The additional purchases generate inflation through the New Keynesian Phillips curve (1), which drives down the expected real interest rate  $r_t = -E_t [\hat{\pi}_{t+1}]$ . The increase in the endogenous part of the natural rate and the decrease in the expected real interest rate offset the initial increase in the return difference between the expected real rate and the natural rate, thereby giving rise to a unique stable equilibrium.

Now that I have explained the economic intuition behind the impulse response functions of the procyclical unique equilibrium in the RANK model, I move on to investigate the OLG-NK model. In doing so, it will finally become clear why a countercyclical unique stable equilibrium is not feasible within the RANK model.

## 4.4 The OLG-NK model

In the previous section I numerically investigated the RANK model, and was able to draw two conclusions. First, government spending needs to be procyclical within the RANK model for a unique stable equilibrium to exist. Second, the sign of the response of the output gap and inflation switch with respect to the monetary regime in such a procyclical equilibrium. In this section, I turn my attention to the OLG-NK model. I will investigate for what values of  $g_{\pi}$  and  $g_y$  a unique stable equilibrium exists, as well as the impulse response functions to the same productivity shock as in the previous section.

First, I investigate in Figure 4 the stability properties of the OLG-NK model. Just as in the RANK model, the output coefficient  $g_y$  is on the horizontal axis, while the inflation coefficient  $g_{\pi}$  on the vertical. The combinations with a blue plus sign indicate a unique stable equilibrium, whereas the red crosses indicate the existence of multiple equilibria. From the figure we see that the inflation coefficient can never be positive, while the output coefficient can at most be zero. Therefore, a unique stable equilibrium is only feasible when government spending is countercyclical. As it is the current consensus among policymakers and academics since Keynes (1936) that fiscal policy should be countercyclical when employed for macroeconomic stabilization, the OLG-NK model is clearly to be preferred over the RANK model.

To understand why government spending has to be countercyclical, I turn to Figure 5, in which I investigate the impulse response functions for the OLG-NK model to the same positive productivity shock as in Figure 3. The blue solid IRFs display the monetary regime, whereas the red slotted IRFs display the fiscal regime with feedback coefficients  $g_{\pi} = -10$  and  $g_y = -10$ . Interestingly, we see that the output gap and inflation now have the same sign as under the monetary regime, a result that strongly contrasts with the RANK model. Instead, we see that the sign of the expected real interest rate  $r_t = -E_t [\hat{\pi}_{t+1}]$  and the natural rate  $\hat{R}_t^*$  switch with respect to that under the monetary regime. This also differs from the RANK model, where the expected real interest rate and the natural rate had the same

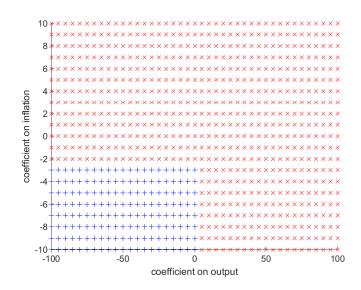


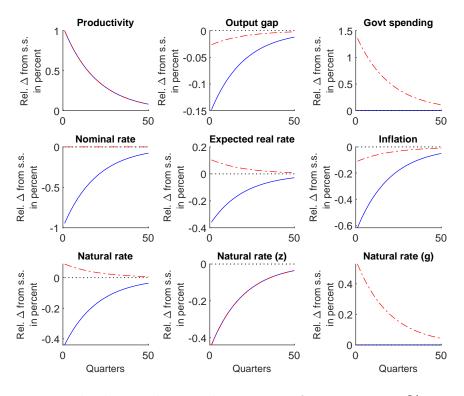
Figure 4: Stability properties of the OLG-NK model under the fiscal regime. The horizontal axis features the output coefficient  $g_y$  and the vertical axis the inflation coefficient  $g_{\pi}$ . A unique, stable equilibrium exists for combinations of  $g_{\pi}$  and  $g_y$  that have a blue plus sign, while the combinations that feature multiple equilibria are denoted with a red cross.

sign as under the monetary regime, and the quantitative difference between the natural rate of the two regimes was small.

To enhance our understanding behind these results, I first study the monetary regime. Just as in the previous section, I write down the aggregate demand equation:

$$\sigma\left(\bar{y}/\bar{c}_{1}\right)\left(1-\frac{\bar{c}_{2}}{\bar{y}}\right)\tilde{y}_{t} = \sigma E_{t}\left[\tilde{y}_{t+1}\right] - \left(r_{t}-\hat{R}_{t}^{z,*}\right),\tag{29}$$

A comparison with the aggregate demand equation under the monetary regime in the RANK model (28) shows that the two equations are qualitatively the same, and only differ to the extent that the coefficients in front of today's output gap  $\tilde{y}_t$  and tomorrow's expected output gap  $E_t[\tilde{y}_{t+1}]$  are quantitatively different. As a result, the impulse response functions of the RANK model and the OLG-NK model are qualitatively the same (compare the blue solid lines in Figure 3 on the one hand with the blue solid lines in Figure 5 on the other). Therefore the intuition behind the impulse response functions under the monetary regime of the RANK



#### OLG-NK model (productivity shock)

Figure 5: Figure displaying the impulse response functions to a 1% positive productivity shock in the OLG-NK model (percentage deviation from steady state). The monetary regime with  $\kappa_{\pi} = 1.5$  and  $\kappa_{y} = 0.125$  is denoted by the blue, solid line, while the fiscal regime with  $g_{\pi} = -10$  and  $g_{y} = -10$  is denoted by the red, slotted line. Time is on the horizontal axis, and is measured in quarters. "Expected real rate" refers to the variable  $r_{t} = \hat{R}_{t}^{n} - E_{t} [\hat{\pi}_{t+1}]$ . The variable "Natural rate" refers to equation (3), "Natural rate (z)" corresponds to equation (8), while "Natural rate (g)" refers to (9).

model is similar to that under the monetary regime of the OLG-NK model.

Now I turn my attention to the fiscal regime under the OLG-NK model, and study the aggregate demand equation (20) that was derived in Section 2.4:

$$\left[\sigma\left(\bar{y}/\bar{c}_{1}\right)\left(1-\frac{\bar{c}_{2}}{\bar{y}}\right)-\varphi B^{*}g_{y}\right]\tilde{y}_{t}=\sigma\left(1+B^{*}g_{y}\right)E_{t}\left[\tilde{y}_{t+1}\right]-\left(r_{t}-\hat{R}_{t}^{z,*}-\hat{R}_{t}^{\pi,*}\right),$$

The first observation from the above equation is that countercyclical government spending  $g_y < 0$  increases the coefficient in front of the current output gap  $\tilde{y}_t$ with respect to the monetary regime, while it decreases the coefficient in front of tomorrow's expected output gap. As a result, the same exogenous shock will lead to a smaller change in the current output gap with respect to the monetary regime, as we see in Figure 5. However, as the sign of the coefficient in front of today's output gap  $\tilde{y}_t$  does not change with respect to the monetary regime, the sign of the impulse response functions for the output gap and inflation do not change under the fiscal regime.

The economic intuition behind the smaller change in the output gap is the fact that a countercyclical government spending rule raises the natural rate when the output gap is negative, and thereby induces households to shift from saving to spending, everything else equal, which together with the extra government spending reduces the (negative) output gap.

However, the same reasoning regarding the sign of the impulse response functions applies for a countercyclical feedback coefficient  $g_y < 0$  in the RANK model. Why then, is a unique stable countercyclical equilibrium feasible in the OLG-NK model and not in the RANK model? The short answer is that the natural rate responds strongly to government spending in the OLG-NK model, where it changes with the (weighted) *sum* of today's and tomorrow's expected government spending, while the natural rate responds weakly in the RANK model, where it changes with the *difference* between today's and tomorrow's expected government spending. As a result, endogenous government spending in the OLG-NK model is capable of switching the sign of the natural rate with respect to the initial change caused by the exogenous shock.

This sign switch becomes very clear by comparing the equilibrium natural rate of interest under the monetary and fiscal regime in Figure 5 (lower left panel), since the difference between the two regimes is entirely driven by endogenous government spending under the fiscal regime. This sharply contrasts with the weak response of the natural rate in the RANK model (Figure 3), where the difference between the monetary and fiscal regime is very small.

To highlight why this endogenous sign switch of the natural rate is necessary for a unique stable countercyclical equilibrium to emerge, I consider again Figure 5. The exogenous productivity shock initially *decreases* the natural rate of interest, which in turn increases the return difference between the expected real rate and the natural rate of interest. Just as under the monetary regime, there is a shift from spending to saving. A negative output gap emerges, which leads to (expected) deflation through the New Keynesian Phillips curve (1). This expected deflation raises the expected real interest rate under the fiscal regime. In order for a unique stable equilibrium to be feasible, the increase in the expected rate must be partially offset by an *increase* of the natural rate of interest. This requires a sign switch of the natural rate, as the productivity shock initially decreases the natural rate. This sign switch can only occur within the OLG-NK model, in which the natural rate increases with the (weighted) *sum* of today's and tomorrow's expected government spending.

I end this section by observing that the key to generate unique stable countercyclical equilibria is that government spending affects the natural rate sufficiently strong. Therefore, my specific overlapping generations model will probably not be the only model that is capable of generating unique stable countercyclical equilibria; any model in which the natural rate responds sufficiently strong to government spending will probably be capable of doing so.

# 5 Conclusion

Most of the New Keynesian literature that studies episodes in which the economy lands at the ZLB, assume it is temporarily binding and that conventional monetary policy regains full potency after the ZLB-episode has ended (Christiano et al., 2011; Eggertsson, 2011). However, one can infer from yield curves for the euro area, Japan, and the United Kingdom that financial markets expect short-term interest rates to remain at or close to the ZLB for as much as 30 years to come. In that case, other instruments such as unconventional monetary policies and fiscal policy are needed for macroeconomic stabilization. In this paper I investigate one such policy, namely an 'old Keynesian' fiscal policy within the New Keynesian framework. Specifically, this policy consists of government spending endogenously responding to inflation and the output gap while I peg the nominal interest rate at the ZLB. I employ two versions of the New Keynesian framework, both of which do not feature physical capital. The first is the standard Representative Agent New Keynesian (RANK) model, while the second is a two-period Overlapping Generations New Keynesian (OLG-NK) model that is similar in spirit to Galí (2014).

Both under the standard 'monetary' regime, as well as under my 'fiscal' or 'old Keynesian' regime, government policy affects the economy through the aggregate demand equation, which relates the (expected) output gap to the return difference between the expected real rate and the natural rate of interest, the equilibrium rate of interest in an economy with perfectly flexible prices. While conventional monetary policy affects the equilibrium by changing the expected real interest rate (through adjustment of the nominal rate), government spending affects the equilibrium by changing the natural rate of interest. A crucial difference between the RANK and the OLG-NK model is that the natural rate in the RANK model increases in the *difference* between today's and tomorrow's expected government spending, whereas it increases in the (weighted) *sum* of the two in the OLG-NK model. As such, government spending has a stronger effect on the natural rate in the OLG-NK model than in the RANK model.

I subsequently find the following results. First, both under the RANK and the OLG-NK model, the equilibrium values for inflation and the output gap under the monetary regime can be replicated under the fiscal regime through an appropriate choice of the feedback coefficients of inflation and the output gap on government spending. The resulting government spending rule turns out to be countercyclical in inflation and the output gap. This can be understood by considering a positive productivity shock that decreases inflation and the output gap. Under the monetary regime, the central bank will reduce the nominal and real interest rate to increase aggregate demand, thereby increasing inflation and the output gap with respect to the initial decrease. Under the fiscal regime, the government raises

aggregate demand by increasing government spending.

My second result is that a unique stable equilibrium with countercyclical government spending is only feasible within the OLG-NK model. Government spending needs to change the natural rate sufficiently strong such that the initial change in the natural rate caused by exogenous shocks is more than offset. To understand why this is necessary, consider a positive productivity shock which initially reduces the natural rate and therefore increases the return difference between the expected real rate and the natural rate. This increase in the return difference induces a shift from spending to saving that generates a negative output gap and (expected) deflation, which raises the expected real interest rate. In order to have a unique stable equilibrium, the increase in the expected real rate must be partially offset by an *increase* in the equilibrium natural rate. For that to happen, the increase in the natural rate arising from endogenous government spending must be larger than the initial decrease caused by the productivity shock. This, however, is only feasible within the OLG-NK model, in which the natural rate increases with the (weighted) *sum* of today's and tomorrow's expected government spending.

Third, I establish the existence of a fiscal counterpart to the 'monetary' divine coincidence (Blanchard and Galí, 2007): when exogenous shocks that affect the natural rate of interest are instantaneously observed, the government can adjust government spending in such a way that the change in the natural rate arising from exogenous shocks is exactly offset by changes in the natural rate arising from government spending. That allows for an equilibrium in which inflation and the output gap are permanently equal to zero, which is the efficient allocation in the absence of real imperfections (Blanchard and Galí, 2007).

My analysis provides two lessons for policymakers. First, a properly chosen government spending rule can replicate the equilibrium values for inflation and the output gap that would arise under the conventional monetary regime in the absence of a ZLB. This implies that policymakers can employ an active government spending policy to stabilize the business cycle when the economy is (almost) permanently at the ZLB. Second, the fact that a countercyclical government spending rule only generates a unique stable equilibrium in the OLG-NK model and not in the RANK model implies that the last model might not be adequate for policymakers: ever since Keynes (1936), a broad consensus of academics and policymakers have argued that fiscal policy should be employed countercyclically when used for macroeconomic stabilization. The conclusion that the OLG-NK is to be preferred over the RANK model sharply contrasts with the standard monetary regime, for which I find that unique stable equilibria in the RANK and OLG-NK model are qualitatively very similar.

# Appendix "Old-Keynesianism in the New Keynesian model"

# A Representative Agent New Keynesian (RANK) model

I employ a standard New-Keynesian model without capital such as can be found in standard textbook treatments such as Galí (2015). Households consume, supply labor, and save through one-period nominal government bonds, which pay a nominal interest rate that is set by the central bank. Within my model, the central bank will set the interest rate equal to its steady state value. The fiscal authority raises revenue from issuing one-period government bonds and lump sum taxes, while these revenues are spent on government purchases of the final good and gross interest payments (including the principal) of government bonds issued in the previous period. Lump sum taxes satisfy the Bohn (1998) principle, which results in my model satisfying Ricardian equivalence. As is standard in the New Keynesian literature, the production sector is three-layered. Final goods producers have a production function that has a constant elasticity of substitution between different retail goods. They operate in a perfectly competitive market, and therefore take prices as given while choosing how many goods to purchase from each retail goods producer. Retail goods producers require one intermediate good to produce one retail good, and operate in a market of monopolistic competition. Therefore, they have the capacity to set prices while taking the demand schedule into account, resulting in a markup over the intermediate goods. However, they are subject to ? pricing frictions which prevents some retail goods producers to change prices in a given period. Due to their monopoly power, retail goods producers make a profit in equilibrium, which is transferred to households. Finally, intermediate goods producers operate in a perfectly competitive market in which they produce using a production function that is linear in labor. They hire labor in a perfectly competitive labor market. As a result, intermediate goods producers take prices and wages as given, and only determine how much labor to hire in equilibrium.

### A.1 Households

There is a continuum  $i \in [0, 1]$  of identical households. Each household *i* receives income from supplying labor  $W_t h_t(i)$ , where  $W_t$  is the nominal wage rate, and  $h_t(i)$ the number of hours worked. In addition, income is received from gross repayment of nominal one-period bonds, which can be decomposed in the principal  $B_{t-1}(i)$  and interest  $R_{t-1}^n B_{t-1}(i)$ , where  $R_t^n$  is the net nominal interest rate set by the central bank. Finally, households receive income  $\Omega_t(i)$  from profits of firms owned by household *i*. Income is spent on consumption  $C_t(i)$ , purchases of new government bonds  $B_t(i)$ , and lump sum taxes  $P_t \tau_t(i)$ , where  $P_t$  is the price level of the final good. This gives rise to the following nominal budget constraint for household *i*:

$$C_t(i) + B_t(i) + P_t\tau_t(i) = W_th_t(i) + (1 + R_{t-1}^n) B_{t-1}(i) + \Omega_t(i).$$
(30)

Division by the price level  $P_t$  results in the following budget constraint in terms of the final good:

$$c_t(i) + b_t(i) + \tau_t(i) = w_t h_t(i) + \left(\frac{1 + R_{t-1}^n}{\pi_t}\right) b_{t-1}(i) + \omega_t(i),$$
(31)

where  $\pi_t \equiv P_t/P_{t-1}$  is the gross inflation rate of the price level. Household *i* maximizes the expected discounted life-time utility, which is separable in consumption and labor:

$$\max_{\{c_{t+s}(i), b_{t+s}(i), h_{t+s}(i)\}_{s=0}^{\infty}} E_t \left\{ \sum_{s=0}^{\infty} \beta^s \xi_{t+s} \left[ \frac{c_{t+s}(i)^{1-\sigma} - 1}{1-\sigma} - \chi_h \frac{h_{t+s}(i)^{1+\varphi}}{1+\varphi} \right] \right\},$$

where  $\xi_t$  denotes a preference shock. The Lagrangian of household's *i* maximization problem is given by:

$$\mathcal{L} = E_t \left\{ \sum_{s=0}^{\infty} \beta^s \xi_{t+s} \left[ \frac{c_{t+s}(i)^{1-\sigma} - 1}{1-\sigma} - \chi_h \frac{h_{t+s}(i)^{1+\varphi}}{1+\varphi} \right] \right\} + E_t \left\{ \sum_{s=0}^{\infty} \beta^s \lambda_{t+s} \left[ w_{t+s} h_{t+s}(i) + \left( \frac{1+R_{t-1+s}^n}{\pi_{t+s}} \right) b_{t-1+s}(i) + \omega_{t+s}(i) - c_{t+s}(i) - b_{t+s}(i) - \tau_{t+s}(i) \right] \right\}$$

This results in the following first order conditions:

$$c_t(i) \quad : \quad \lambda_t = \xi_t c_t(i)^{-\sigma}, \tag{32}$$

$$h_t(i) \quad : \quad \chi_h h_t(i)^{\varphi} = w_t c_t(i)^{-\sigma}, \tag{33}$$

$$b_t(i) : E_t \left[ \beta \Lambda_{t,t+1} \left( \frac{1 + R_t^n}{\pi_{t+1}} \right) \right] = 1,$$
 (34)

where  $\beta \Lambda_{t,+s} = \beta \lambda_{t+s} / \lambda_t$  denotes households' stochastic discount factor.

# A.2 Production firms

I explained in the main text that the production sector consists of final goods producers, retail goods producers, and intermediate goods producers. Below I show the formal derivations of their first order conditions.

### A.2.1 Final goods producers

Final goods producers purchase retail goods  $y_t^f$  at price  $P_t^f$  from retail goods producer  $f \in [0, 1]$ , and combine these into final goods  $y_t$  using the following constant elasticity of substitution production technology:

$$y_t = \left[\int_0^1 \left(y_t^f\right)^{\frac{\epsilon-1}{\epsilon}} df\right]^{\frac{\epsilon}{\epsilon-1}},\tag{35}$$

where  $\epsilon$  denotes the elasticity of substitution between two retail goods producers. There is perfect competition among final goods producers, hence all final goods producers charge the same price  $P_t$  for their final goods. They take demand  $y_t$  for final goods as given, and only decide how many retail goods  $y_t^f$  to buy from each retail goods producer. Hence final goods producers' optimization problem is given by:

$$\max_{\{y_t^f\}} \quad P_t y_t - \int_0^1 P_t^f y_t^f df,$$

subject to the production technology (35). Taking the first order condition with respect to  $y_t^f$  results in the following demand schedule for retail good  $f \in [0, 1]$ :

$$y_t^f = \left(\frac{P_t^f}{P_t}\right)^{-\epsilon} y_t,\tag{36}$$

Substitution of (36) into final goods producers' production function (35) allows me to find the general price level  $P_t$ :

$$P_t^{1-\epsilon} = \int_0^1 \left( P_t^f \right)^{1-\epsilon} df, \tag{37}$$

### A.2.2 Retail goods producers

Retail goods producer  $f \in [0,1]$  purchases goods  $y_t^i$  at a price  $\phi_t$  (expressed in terms of final goods) from intermediate goods producers. He converts these goods one for one into a unique retail good  $y_t^f = y_t^i$ . The fact that retail good f is unique provides retail goods producer f a monopoly for good f. As mentioned above, however, due to the fact that final goods producers purchase retail goods from all retail goods producers, retail goods producer f effectively operates in a market with monopolistic competition. However, monopolistic competition allows retail goods producer f to set the price  $P_t^f$  while taking the demand schedule (36) into account, thereby allowing him to charge a markup over the price  $\phi_t$  of intermediate goods. Retail goods producers, however, are subject to price-stickiness a la?. This implies that there is a probability  $\psi$ , which is constant across time and cross-section, that retail goods producer f will not be able to change its nominal price  $P_t^f$  in the future. Hence retail goods producers do not only maximize current profits, but also expected future profits when setting a new price  $P_t^*$  today. Future expected profits are discounted using the households' stochastic discount factor  $\beta^s \Lambda_{t,t+s}$ , as they are the ultimate owners of the retail goods producers. The optimization problem is given by:

$$\max_{P_t^*} E_t \left\{ \sum_{s=0}^{\infty} \psi^s \beta^s \Lambda_{t,t+s} \left[ \frac{P_t^*}{P_{t+s}} - \phi_{t+s} \right] y_{t+s}^f \right\},\$$

subject to the demand curve (36). Substitution of this demand curve gives the following optimization objective:

$$\max_{P_t^*} E_t \left\{ \sum_{s=0}^{\infty} \psi^s \beta^s \Lambda_{t,t+s} \left[ \left( \frac{P_t^*}{P_{t+s}} \right)^{1-\epsilon} - \phi_{t+s} \left( \frac{P_t^*}{P_{t+s}} \right)^{-\epsilon} \right] y_{t+s} \right\},$$

Differentiation with respect to  $P^{\ast}_t$  gives the following first order condition:

$$(\epsilon - 1) E_t \left[ \sum_{s=0}^{\infty} \psi^s \beta^s \Lambda_{t,t+s} \left( \frac{P_t^*}{P_{t+s}} \right)^{1-\epsilon} \frac{y_{t+s}}{P_t^*} \right] = \epsilon E_t \left[ \sum_{s=0}^{\infty} \psi^s \beta^s \Lambda_{t,t+s} \phi_{t+s} \left( \frac{P_t^*}{P_{t+s}} \right)^{-\epsilon} \frac{y_{t+s}}{P_t^*} \right],$$

Rearranging this expression gives:

$$\frac{P_t^*}{P_t} = \left(\frac{\epsilon}{\epsilon - 1}\right) \frac{E_t \left[\sum_{s=0}^{\infty} \psi^s \beta^s \Lambda_{t,t+s} \phi_{t+s} \left(\frac{P_{t+s}}{P_t}\right)^{\epsilon} y_{t+s}\right]}{E_t \left[\sum_{s=0}^{\infty} \psi^s \beta^s \Lambda_{t,t+s} \left(\frac{P_{t+s}}{P_t}\right)^{\epsilon-1} y_{t+s}\right]},$$

$$= \left(\frac{\epsilon}{\epsilon - 1}\right) \frac{E_t \left[\sum_{s=0}^{\infty} \psi^s \beta^s \Lambda_{t,t+s} \phi_{t+s} \left(\prod_{j=1}^{s} \pi_{t+j}^{\epsilon}\right) y_{t+s}\right]}{E_t \left[\sum_{s=0}^{\infty} \psi^s \beta^s \Lambda_{t,t+s} \left(\prod_{j=1}^{s} \pi_{t+j}^{\epsilon-1}\right) y_{t+s}\right]},$$

$$= \left(\frac{\epsilon}{\epsilon - 1}\right) \frac{\Xi_{1,t}}{\Xi_{2,t}},$$

where  $\Xi_{1,t}$  and  $\Xi_{2,t}$  are given by:

$$\Xi_{1,t} = E_t \left[ \sum_{s=0}^{\infty} \psi^s \beta^s \Lambda_{t,t+s} \phi_{t+s} \left( \prod_{j=1}^{s} \pi_{t+j}^{\epsilon} \right) y_{t+s} \right] = \phi_t y_t + E_t \left[ \psi \beta \Lambda_{t,t+1} \pi_{t+1}^{\epsilon} \Xi_{1,t+1} \beta \right]$$
$$\Xi_{2,t} = E_t \left[ \sum_{s=0}^{\infty} \psi^s \beta^s \Lambda_{t,t+s} \left( \prod_{j=1}^{s} \pi_{t+j}^{\epsilon-1} \right) y_{t+s} \right] = y_t + E_t \left[ \psi \beta \Lambda_{t,t+1} \pi_{t+1}^{\epsilon-1} \Xi_{2,t+1} \right]. \quad (39)$$

The price level  $P_t$  evolves according to the following law of motion, see (37):

$$P_t^{1-\epsilon} = (1-\psi) \left(P_t^*\right)^{1-\epsilon} + \psi \left(1-\psi\right) \left(P_{t-1}^*\right)^{1-\epsilon} + \psi^2 \left(1-\psi\right) \left(P_{t-2}^*\right)^{1-\epsilon} + \dots$$

Lagging by one period, and multiplying by  $\psi$  gives the following expression:

$$\psi P_{t-1}^{1-\epsilon} = \psi \left(1-\psi\right) \left(P_{t-1}^*\right)^{1-\epsilon} + \psi^2 \left(1-\psi\right) \left(P_{t-2}^*\right)^{1-\epsilon} + \psi^3 \left(1-\psi\right) \left(P_{t-3}^*\right)^{1-\epsilon} + \dots$$

Hence I can write the general price level  $P_t$  as:

$$P_t^{1-\epsilon} = (1-\psi) \left(P_t^*\right)^{1-\epsilon} + \psi P_{t-1}^{1-\epsilon}.$$
(40)

Division by  $P_t^{1-\epsilon}$  allows me to express everything in terms of the relative new price  $\pi_t^* \equiv P_t^*/P_t$  and the gross inflation rate  $\pi_t \equiv P_t/P_{t-1}$ :

$$1 = (1 - \psi) (\pi_t^*)^{1 - \epsilon} + \psi \pi_t^{\epsilon - 1}.$$
 (41)

Finally, I calculate price dispersion, which is defined as:

$$\mathcal{D}_{t} \equiv \int_{0}^{1} \left(\frac{P_{t}^{f}}{P_{t}}\right)^{-\epsilon} df = (1-\psi) \left(\frac{P_{t}^{*}}{P_{t}}\right)^{-\epsilon} + \psi (1-\psi) \left(\frac{P_{t-1}^{*}}{P_{t}}\right)^{-\epsilon} + \psi^{2} (1-\psi) \left(\frac{P_{t-2}^{*}}{P_{t}}\right)^{-\epsilon} + \dots$$
$$= P_{t}^{\epsilon} \left[ (1-\psi) \left(P_{t}^{*}\right)^{-\epsilon} + \psi (1-\psi) \left(P_{t-1}^{*}\right)^{-\epsilon} + \psi^{2} (1-\psi) \left(P_{t-2}^{*}\right)^{-\epsilon} + \dots \right]$$

Lagging by one period, and multiplying with  $\psi (P_t/P_{t-1})^{\epsilon}$  gives:

$$\psi\left(\frac{P_{t}}{P_{t-1}}\right)^{\epsilon} \mathcal{D}_{t-1} = P_{t}^{\epsilon} \left[\psi\left(1-\psi\right)\left(P_{t-1}^{*}\right)^{-\epsilon} + \psi^{2}\left(1-\psi\right)\left(P_{t-2}^{*}\right)^{-\epsilon} + \dots\right]$$

Hence I find for dispersion  $\mathcal{D}_t$  the following expression:

$$\mathcal{D}_{t} = P_{t}^{\epsilon} (1 - \psi) (P_{t}^{*})^{-\epsilon} + \psi \left(\frac{P_{t}}{P_{t-1}}\right)^{\epsilon} \mathcal{D}_{t-1}$$
$$= (1 - \psi) (\pi_{t}^{*})^{-\epsilon} + \psi \pi_{t}^{\epsilon} \mathcal{D}_{t-1}.$$
(42)

#### A.2.3 Intermediate goods producers

The production technology of intermediate goods producer  $i \in [0, 1]$  is given by:

$$y_t^i = z_t h_t^i, \tag{43}$$

where  $y_t^i$  is the number of intermediate goods produced,  $z_t$  productivity, and  $h_t^i$  the amount of labor hired by intermediate goods producer *i*. Both the labor market and the market for intermediate goods are perfectly competitive, and intermediate goods producers therefore take the wage rate  $w_t$  and the price of intermediate goods  $\phi_t$  as given. Intermediate goods producers' decision problem is static, and mathematically represented in the following way:

$$\max_{\{h_t^i\}} \quad \phi_t y_t^i - w_t h_t^i,$$

subject to the production technology (43). Taking the derivative with respect to  $h_t^i$  results in the following first order condition:

$$w_t = \phi_t z_t,\tag{44}$$

# A.3 Government

### A.3.1 Fiscal authority

The fiscal authority raises revenues through lump sum taxes  $P_t \tau_t$  on households, and issuing one period nominal government bonds  $B_t$ . Revenues are used to purchase final goods  $P_t g_t$ , where  $g_t$  denotes the number of final goods purchased, and for repayment of principal and interest on debt issued in the previous period  $(1 + R_{t-1}^n) B_{t-1}$ . Hence the nominal government budget constraint is given by:

$$P_t \tau_t + B_t = P_t g_t + \left(1 + R_{t-1}^n\right) B_{t-1}.$$
(45)

Division by the price level  $P_t$  results in the following government budget constraint in terms of final goods:

$$\tau_t + b_t = g_t + \left(\frac{1 + R_{t-1}^n}{\pi_t}\right) b_{t-1},\tag{46}$$

where  $b_t \equiv B_t/P_t$  is the stock of government debt in real terms. Government spending is as specified in the main text. I assume that there is a feedback rule from the stock of government debt on the level of lump sum taxes satisfying the Bohn (1998) principle:

$$\tau_t = \bar{\tau} + \kappa_b \left( b_{t-1} - \bar{b} \right). \tag{47}$$

Therefore, the model satisfies Ricardian equivalence, and as a result the equilibrium allocations for lump sum taxes  $\tau_t$  and government bonds  $b_t$  will not affect the

equilibrium allocation of the other variables. Finally, government spending will be equal to steady state under the monetary regime, while it will endogenously respond to inflation and the output gap under the fiscal regime.

### A.3.2 Central bank

The central bank sets the nominal interest rate  $R_t^n$  on government bonds. I assume that the central bank sets the nominal interest rate according to a standard Taylor rule under the monetary regime, while the interest rate will be equal its steady state value under the fiscal regime:

$$R_t^n = \bar{R}_n. \tag{48}$$

# A.4 Market clearing

Market clearing occurs when the supply of final goods  $y_t$  equals demand for final goods:

$$y_t = c_t + g_t, \tag{49}$$

# A.5 Aggregation

I start by observing that there is a mass of one of households, each of which makes the same decisions for consumption and labor supply. Therefore, we know that  $c_t \equiv \int_0^1 c_t(i) di = c_t(i) \int_0^1 di = c_t(i)$  and  $h_t \equiv \int_0^1 h_t(i) di = h_t(i) \int_0^1 di = h_t(i)$ . Therefore, I can simply replace  $c_t(i)$  by  $c_t$  and  $h_t(i)$  by  $h_t$  in households' first order conditions for consumption and labor supply.

Next, I integrate equation (36) over all retail goods producers:

$$\int_0^1 y_t^f df = \int_0^1 \left(\frac{P_t^f}{P_t}\right)^{-\epsilon} y_t df = y_t \int_0^1 \left(\frac{P_t^f}{P_t}\right)^{-\epsilon} df = \mathcal{D}_t y_t,$$

Integration over the left hand side occurs by remembering that  $y_t^f = y_t^i = z_t h_t^i$ , and then integrating over all intermediate goods producers:

$$\int_0^1 y_t^f df = \int_0^1 y_t^i di = z_t \int_0^1 h_t^i di = z_t h_t.$$

Therefore, the aggregate equivalent of equation (36) is given by:

$$\mathcal{D}_t y_t = z_t h_t. \tag{50}$$

# A.6 Overview first order conditions (RANK)

A competitive equilibrium is a series of quantities  $\{c_t, h_t, y_t, g_t, b_t, \tau_t\}$ , (shadow) prices

 $\{\lambda_t, R_t^n, \phi_t, w_t, \pi_t, \pi_t^*, \mathcal{D}_t, \Xi_{1,t}, \Xi_{2,t}\}$ , and exogenous processes  $\{z_t, \xi_t\}$  satisfying the following equations:

$$\lambda_t = \xi_t c_t^{-\sigma}, \tag{51}$$

$$\chi_h h_t^{\varphi} = w_t c_t^{-\sigma}, \tag{52}$$

$$\begin{bmatrix} \chi_h h_t^{\varphi} &= w_t c_t^{-\sigma}, \\ \left(1 + R_t^n\right) \end{bmatrix} = 1 \tag{52}$$

$$E_t \left[ \beta \Lambda_{t,t+1} \left( \frac{1+1 \phi_t}{\pi_{t+1}} \right) \right] = 1,$$

$$w_t = \phi_t z_t,$$
(53)
(54)

$$w_t = \phi_t z_t, \tag{34}$$

$$\mathcal{D}_t y_t = z_t h_t, \tag{55}$$
$$\pi_t^* = \left(\frac{\epsilon}{1-\epsilon}\right) \frac{\Xi_{1,t}}{1-\epsilon}. \tag{56}$$

$$\pi_{t} = \left(\frac{1}{\epsilon-1}\right) \overline{\Xi_{2,t}}, \tag{30}$$
$$\Xi_{1,t} = \phi_{t} y_{t} + E_{t} \left[\psi \beta \Lambda_{t,t+1} \pi_{t+1}^{\epsilon} \Xi_{1,t+1}\right], \tag{37}$$

$$\Xi_{2,t} = y_t + E_t \left[ \psi \beta \Lambda_{t,t+1} \pi_{t+1}^{\epsilon-1} \Xi_{2,t+1} \right], \qquad (58)$$

$$1 = (1 - \psi) (\pi_t^*)^{1 - \epsilon} + \psi \pi_t^{\epsilon - 1}.$$
 (59)

$$\mathcal{D}_t = (1 - \psi) \left(\pi_t^*\right)^{-\epsilon} + \psi \pi_t^{\epsilon} \mathcal{D}_{t-1}.$$

$$(60)$$

$$\tau_t + b_t = g_t + \left(\frac{1 + 1 \tau_{t-1}}{\pi_t}\right) b_{t-1}, \tag{61}$$

$$\tau_t = \bar{\tau} + \kappa_b \left( b_{t-1} - \bar{b} \right), \tag{62}$$

$$R_t^n = \dots, (63)$$

$$g_t = \dots \tag{64}$$

$$y_t = c_t + g_t, (65)$$

$$\log(z_t) = \rho_z \log(z_{t-1}) + \varepsilon_{z,t}, \tag{66}$$

$$\log\left(\xi_{t}\right) = \rho_{\xi} \log\left(\xi_{t-1}\right) + \varepsilon_{\xi,t},\tag{67}$$

where  $\beta \Lambda_{t,t+s} \equiv \beta \lambda_{t+s}/\lambda_t$  denotes the representative households' stochastic discount factor. In addition, there is a transversality condition for government bonds, and the process for government purchases  $g_t$  is as specified in the main text.

# A.7 Linearized FOCs

$$\varphi \hat{h}_t = -\sigma \hat{c}_t + \hat{w}_t, \tag{68}$$

$$\hat{\xi}_{t} - \sigma \hat{c}_{t} = E_{t} \left[ \hat{\xi}_{t+1} - \sigma \hat{c}_{t+1} \right] + \hat{R}_{t}^{n} - E_{t} \left[ \hat{\pi}_{t+1} \right],$$
(69)

$$\hat{w}_t = \phi_t + \hat{z}_t, \tag{70}$$

$$\hat{\mathcal{D}}_t + \hat{y}_t = \hat{z}_t + \hat{h}_t, \tag{71}$$

$$\hat{\pi}_{t}^{*} = \hat{\Xi}_{1,t} - \hat{\Xi}_{2,t}, \tag{72}$$

$$\Xi_{1,t} = (1 - \psi \beta \overline{\pi}^{\epsilon}) \left( \phi_t + \hat{y}_t \right) + \psi \beta \overline{\pi}^{\epsilon} \left( -\sigma E_t \left[ \hat{c}_{t+1} \right] + \sigma \hat{c}_t + \epsilon E_t \left[ \hat{\pi}_{t+1} \right] + E_t \left[ \hat{\Xi}_{1,t+1} \right] \right), \quad (73)$$

$$\hat{\Xi}_{2,t} = (1 - \psi \beta \bar{\pi}^{\epsilon - 1}) \hat{y}_{t} 
+ \psi \beta \bar{\pi}^{\epsilon - 1} \left( -\sigma E_{t} [\hat{c}_{t+1}] + \sigma \hat{c}_{t} + (\epsilon - 1) E_{t} [\hat{\pi}_{t+1}] + E_{t} [\hat{\Xi}_{2,t+1}] \right),$$
(74)

$$(1 - \psi) (\bar{\pi}^*)^{1 - \epsilon} \hat{\pi}_t^* = \psi (\bar{\pi})^{\epsilon - 1} \hat{\pi}_t,$$
(75)

$$\hat{\mathcal{D}}_{t} = (1 - \psi(\bar{\pi})^{\epsilon}) (-\epsilon \hat{\pi}_{t}^{*}) + \psi(\bar{\pi})^{\epsilon} \left(\epsilon \hat{\pi}_{t} + \hat{\mathcal{D}}_{t-1}\right),$$
(76)

$$\hat{y}_t = (\bar{c}/\bar{y})\,\hat{c}_t + (\bar{g}/\bar{y})\,\hat{g}_t, \tag{77}$$

$$\bar{\tau}\hat{\tau}_t + \bar{b}\hat{b}_t = \bar{g}\hat{g}_t + \left(\frac{1+R_n}{\bar{\pi}}\right)\bar{b}\left(\hat{R}_{t-1}^n - \hat{\pi}_t + \hat{b}_{t-1}\right),\tag{78}$$

$$\bar{\tau}\hat{\tau}_t = \kappa \bar{b}\hat{b}_{t-1},\tag{79}$$

$$\hat{R}_t^n = \dots, \tag{80}$$

$$\hat{g}_t = \dots, \tag{81}$$

$$\hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon_{z,t}, \tag{82}$$

$$\hat{\xi}_t = \rho_{\xi} \hat{\xi}_{t-1} + \varepsilon_{\xi,t}, \tag{83}$$

# A.8 Further derivations

Going forward, I assume that gross steady state inflation is equal to one:  $\bar{\pi} = 1$ . Next, I substitute equations (73) and (74) into (72) to obtain:

$$\hat{\pi}_t^* = (1 - \psi\beta)\,\hat{\phi}_t + \psi\beta E_t\,[\hat{\pi}_{t+1}] + \psi\beta E_t\,[\hat{\pi}_{t+1}^*]\,,\tag{84}$$

Substitution of  $\hat{\pi}_t^* = (\psi/(1-\psi))\hat{\pi}_t$  (75) delivers the traditional New Keynesian Phillips-curve:

$$\hat{\pi}_t = \beta E_t \left[ \hat{\pi}_{t+1} \right] + \zeta \hat{\phi}_t, \tag{85}$$

where  $\zeta \equiv (1 - \psi \beta) (1 - \psi) / \psi$ .

I can rewrite the aggregate resource constraint (77) to obtain an expression for consumption  $\hat{c}$ :

$$\hat{c}_t = \frac{\bar{y}}{\bar{c}} \left[ \hat{y}_t - \left( \frac{\bar{g}}{\bar{y}} \right) \hat{g}_t \right],\tag{86}$$

### A.8.1 The flexible prices equilibrium

Now I aim to derive the flexible prices equilibrium. To do so, I set  $\psi = 0$  in equation (84)

$$\hat{\pi}_t^* = \hat{\phi}_t,$$

Since I know from equation (75) that  $\hat{\pi}_t^* = 0$  when  $\psi = 0$ , I find that  $\hat{\phi}_t = 0$ . Next, I substitute expression (75) into equation (76), and find that  $\hat{\mathcal{D}}_t = 0$ , irrespective of whether  $\psi = 0$  or not.

Now I consider equation (68), and substitute equation (70) for  $\hat{w}_t$ , expression (71) for  $\hat{h}_t$ , and equation (86) to obtain:

$$\varphi\left(\hat{y}_t - \hat{z}_t\right) = -\sigma\left(\bar{y}/\bar{c}\right)\left[\hat{y}_t - \left(\frac{\bar{g}}{\bar{y}}\right)\hat{g}_t\right] + \hat{z}_t,$$

Solving for output delivers the flexible prices level of output, or the natural level of output:

$$\hat{y}_t^n = \left(\frac{1+\varphi}{\sigma\left(\bar{y}/\bar{c}\right)+\varphi}\right)\hat{z}_t + \sigma\left(\bar{y}/\bar{c}\right)\left(\frac{(\bar{g}/\bar{y})}{\sigma\left(\bar{y}/\bar{c}\right)+\varphi}\right)\hat{g}_t,\tag{87}$$

### A.8.2 The sticky prices equilibrium

Agiain I consider equation (174), and substitute equation (184) for  $\hat{w}_t$ , expression (183) for  $\hat{h}_t$ , and equation (197). However, the difference with respect to the flexible prices equilibrium is that  $\hat{\phi}_t$  is no longer zero in equation equation (184). I thus obtain:

$$\varphi\left(\hat{y}_t - \hat{z}_t\right) = -\sigma\left(\bar{y}/\bar{c}\right) \left[\hat{y}_t - \left(\frac{\bar{g}}{\bar{y}}\right)\hat{g}_t\right] + \hat{z}_t + \hat{\phi}_t,$$

Rearranging gives:

$$\hat{\phi}_t = (\sigma \left( \bar{y}/\bar{c} \right) + \varphi) \, \hat{y}_t - (1+\varphi) \, \hat{z}_t - \sigma \left( \bar{y}/\bar{c} \right) \left( \bar{g}/\bar{y} \right) \, \hat{g}_t = (\sigma \left( \bar{y}/\bar{c} \right) + \varphi) \, (\hat{y}_t - \hat{y}_t^n) = (\sigma \left( \bar{y}/\bar{c} \right) + \varphi) \, \tilde{y}_t,$$
(88)

where  $\tilde{y}_t \equiv \hat{y}_t - \hat{y}_t^n$  denotes the output gap, the difference between the level of output under the sticky prices equilibrium and the flexible prices equilibrium.

Substitution of expression (88) into equation (85) delivers the familiar New Keynesian Phillips curve in the output gap  $\tilde{y}_t$ :

$$\hat{\pi}_t = \beta E_t \left[ \hat{\pi}_{t+1} \right] + \kappa \tilde{y}_t, \tag{89}$$

where  $\kappa$  is given by:

$$\kappa = \left(\sigma\left(\bar{y}/\bar{c}\right) + \varphi\right)\zeta,\tag{90}$$

Finally, I substitute expressions (86) into the Euler equation (69):

$$\hat{\xi}_{t} - \sigma\left(\bar{y}/\bar{c}\right) \left[\hat{y}_{t} - \left(\frac{\bar{g}}{\bar{y}}\right)\hat{g}_{t}\right] = E_{t}\left[\hat{\xi}_{t+1}\right] - \sigma\left(\bar{y}/\bar{c}\right)E_{t}\left[\hat{y}_{t+1} - \left(\frac{\bar{g}}{\bar{y}}\right)\hat{g}_{t+1}\right] + \hat{R}_{t}^{n} - E_{t}\left[\hat{\pi}_{t+1}\right],\tag{91}$$

Now I substitute  $\hat{y}_t = \hat{y}_t^n + \tilde{y}_t$ , and substitute expression (87) to get the aggregate demand equation:

$$\sigma\left(\bar{y}/\bar{c}\right)\tilde{y}_{t} = \sigma\left(\bar{y}/\bar{c}\right)E_{t}\left[\tilde{y}_{t+1}\right] - \left(\hat{R}_{t}^{n} - E_{t}\left[\hat{\pi}_{t+1}\right] - \hat{R}_{t}^{*}\right),\tag{92}$$

where  $\hat{R}_t^*$  is given by:

$$\hat{R}_t^* = \hat{R}_t^{z*} + \hat{R}_t^{\xi*} + \hat{R}_t^{g*}, \tag{93}$$

where  $\hat{R}_t^{z*}$ ,  $\hat{R}_t^{\xi*}$ , and  $\hat{R}_t^{g*}$  are given by:

$$\hat{R}_t^{z*} = -\sigma\left(\bar{y}/\bar{c}\right) \left(\frac{1+\varphi}{\sigma\left(\bar{y}/\bar{c}\right)+\varphi}\right) (1-\rho_z) \,\hat{z}_t,\tag{94}$$

$$\hat{R}_{t}^{\xi*} = (1 - \rho_{\xi}) \hat{\xi}_{t}, \tag{95}$$

$$\hat{R}_t^{g*} = \sigma\left(\bar{y}/\bar{c}\right) \left(\frac{(\bar{g}/\bar{y})\varphi}{\sigma\left(\bar{y}/\bar{c}\right) + \varphi}\right) \left(\hat{g}_t - E_t\left[\hat{g}_{t+1}\right]\right),\tag{96}$$

# A.9 The fiscal divine coincidence

A divine coincidence is characterized as an equilibrium in which we have permanent  $\hat{\pi}_t = 0$  and  $\tilde{y}_t = 0$ . The fiscal regime is characterized by  $\hat{R}_t^n = 0$ . Such an equilibrium, however, can only be achieved when the natural rate of interest  $\hat{R}_t^* = 0$  irrespective of the realization of the productivity shock  $\hat{z}_t$  and preference shock  $\hat{\xi}_t$ . To achieve that the divine coincidence equilibrium, government spending must respond to productivity and preference shocks in such a way that its influence on government spending exactly offsets the change in the natural rate arising from the productivity and preference shocks. Therefore, I assume that government spending is given by:

$$\hat{g}_t = A_z \hat{z}_t + A_\xi \hat{\xi}_t. \tag{97}$$

I then immediately find that:

$$\hat{g}_t - E_t \left[ \hat{g}_{t+1} \right] = A_z \left( 1 - \rho_z \right) \hat{z}_t + A_\xi \left( 1 - \rho_\xi \right) \hat{\xi}_t$$

Substitution of the process for government spending into (93) allows me to find the values of  $A_z$  and  $A_{\xi}$  such that the natural rate of interest  $\hat{R}_t^* = 0$  period by period:

$$A_z = \frac{1+\varphi}{(\bar{g}/\bar{y})\,\varphi} > 0,$$
  
$$A_{\xi} = -\frac{\sigma\left(\bar{y}/\bar{c}\right)+\varphi}{\sigma\left(\bar{g}/\bar{c}\right)\varphi} < 0,$$

# A.10 Further derivations

For the RANK-model, I will only investigate the stability conditions for the fiscal regime. To do so, I substitute the government spending rule from the main text (11) into the aggregate demand equation, which together with the New Keynesian Phillips curve gives the following system of two-by-two equations (while keeping  $\hat{R}_t^n = 0$ ):

$$\hat{\pi}_{t} = \beta E_{t} [\hat{\pi}_{t+1}] + \kappa \tilde{y}_{t}, -Bg_{\pi} \hat{\pi}_{t} + (\sigma (\bar{y}/\bar{c}) - Bg_{y}) \tilde{y}_{t} = (1 - Bg_{\pi}) E_{t} [\hat{\pi}_{t+1}] + (\sigma (\bar{y}/\bar{c}) - Bg_{y}) E_{t} [\tilde{y}_{t+1}] + \hat{R}_{t}^{z*} + \hat{R}_{t}^{\xi*},$$

where B is given by:

$$B = \frac{\sigma\left(\bar{g}/\bar{c}\right)\varphi}{\sigma\left(\bar{y}/\bar{c}\right)+\varphi},\tag{98}$$

Now I can write the above system of equations into the following matrix equation:

$$\begin{pmatrix} 1 & -\kappa \\ -Bg_{\pi} & \sigma\left(\bar{y}/\bar{c}\right) - Bg_{y} \end{pmatrix} \begin{pmatrix} \hat{\pi}_{t} \\ \tilde{y}_{t} \end{pmatrix} = \begin{pmatrix} \beta & 0 \\ 1 - Bg_{\pi} & \sigma\left(\bar{y}/\bar{c}\right) - Bg_{y} \end{pmatrix} \begin{pmatrix} E_{t}\left[\hat{\pi}_{t+1}\right] \\ E_{t}\left[\tilde{y}_{t+1}\right] \end{pmatrix} + \begin{pmatrix} 0 \\ \hat{R}_{t}^{z*} + \hat{R}_{t}^{\xi*} \end{pmatrix},$$
(99)

I first establish the inverse of the matrix in front of current inflation and output gap:

$$\begin{pmatrix} 1 & -\kappa \\ -Bg_{\pi} & \sigma \bar{y}/\bar{c} - Bg_{y} \end{pmatrix}^{-1} = \frac{1}{\kappa Bg_{\pi} - (\sigma \bar{y}/\bar{c} - Bg_{y})} \begin{pmatrix} -(\sigma \bar{y}/\bar{c} - Bg_{y}) & -\kappa \\ -Bg_{\pi} & -1 \end{pmatrix}.$$

Now I can write the system of equations (99) in the following way:

$$\begin{pmatrix} \hat{\pi}_t \\ \tilde{y}_t \end{pmatrix} = M \begin{pmatrix} E_t \left[ \hat{\pi}_{t+1} \right] \\ E_t \left[ \tilde{y}_{t+1} \right] \end{pmatrix} + N,$$
(100)

where the matrices M and N are given by:

$$M = \frac{1}{\kappa B g_{\pi} - (\sigma \bar{y}/\bar{c} - B g_{y})} \begin{pmatrix} -\kappa \left(1 - B g_{\pi}\right) - \beta \left(\sigma \bar{y}/\bar{c} - B g_{y}\right) & -\kappa \left(\sigma \bar{y}/\bar{c} - B g_{y}\right) \\ -\beta B g_{\pi} - \left(1 - B g_{\pi}\right) & -\left(\sigma \bar{y}/\bar{c} - B g_{y}\right) \end{pmatrix},$$
(101)

$$N = \frac{1}{\kappa B g_{\pi} - (\sigma \bar{y}/\bar{c} - B g_{y})} \begin{pmatrix} -(\sigma \bar{y}/\bar{c} - B g_{y}) & -\kappa \\ -B g_{\pi} & -1 \end{pmatrix} \begin{pmatrix} 0 \\ \hat{R}_{t}^{z*} + \hat{R}_{t}^{\xi*} \end{pmatrix},$$
(102)

Now I determine trace and determinant of M:

trace 
$$M = \frac{-\kappa \left(1 - Bg_{\pi}\right) - \left(1 + \beta\right) \left(\sigma \bar{y}/\bar{c} - Bg_{y}\right)}{\kappa Bg_{\pi} - \left(\sigma \bar{y}/\bar{c} - Bg_{y}\right)},$$
(103)

$$\det M = \frac{-\beta \left(\sigma \bar{y}/\bar{c} - Bg_y\right)}{\kappa Bg_{\pi} - \left(\sigma \bar{y}/\bar{c} - Bg_y\right)}.$$
(104)

As I have two forward-looking variables, I need two roots of the characteristic equation of matrix M that are inside the unit circle (Bullard and Mitra, 2002). First, I calculate the characteristic equation, and find that it is given by:

$$\lambda^2 - \operatorname{trace} M\lambda + \det M = 0.$$

Bullard and Mitra (2002) take the following characteristic equation:

$$\lambda^2 + a_1\lambda + a_0 = 0.$$

Hence in my case  $a_1$  and  $a_0$  are given by:

$$a_1 = -\operatorname{trace} M = \frac{\kappa \left(1 - Bg_{\pi}\right) + \left(1 + \beta\right) \left(\sigma \bar{y} / \bar{c} - Bg_y\right)}{\kappa Bg_{\pi} - \left(\sigma \bar{y} / \bar{c} - Bg_y\right)},$$
(105)

$$a_0 = \det M = \frac{-\beta \left(\sigma \bar{y}/\bar{c} - Bg_y\right)}{\kappa Bg_\pi - \left(\sigma \bar{y}/\bar{c} - Bg_y\right)}.$$
(106)

A.10.1 The case where  $g_{\pi} \leq 0$  and  $g_{y} \leq 0$ 

When  $g_y \leq 0$ , we see that  $\sigma \bar{y}/\bar{c} - Bg_y > 0$ . With  $g_{\pi} \leq 0$ , it immediately follows that the denominator of (105) and (106) is negative, i.e.  $\kappa Bg_{\pi} - (\sigma \bar{y}/\bar{c} - Bg_y) < 0$ .

I first calculate the absolute value of  $a_0$ :

$$|a_0| = \frac{-\beta \left(\sigma \bar{y}/\bar{c} - Bg_y\right)}{\kappa Bg_{\pi} - \left(\sigma \bar{y}/\bar{c} - Bg_y\right)} = \frac{\beta}{1 + \frac{-\kappa Bg_{\pi}}{\sigma \bar{y}/\bar{c} - Bg_y}} < 1.$$

Hence the first condition of Bullard and Mitra (2002) is satisfied. Now I look at the second condition, i.e.  $|a_1| < 1 + a_0$ . To do that, I need to compute  $|a_1|$ . Since  $g_{\pi} \leq 0$ , we immediately see that the numerator of (105) is always positive. Hence the absolute value of  $a_1$  is given by:

$$|a_1| = \frac{-\kappa \left(1 - Bg_{\pi}\right) - \left(1 + \beta\right) \left(\sigma \bar{y}/\bar{c} - Bg_y\right)}{\kappa Bg_{\pi} - \left(\sigma \bar{y}/\bar{c} - Bg_y\right)}$$

The condition that  $|a_1| < 1 + a_0$  then boils down to:

$$\frac{-\kappa\left(1-Bg_{\pi}\right)-\left(1+\beta\right)\left(\sigma\bar{y}/\bar{c}-Bg_{y}\right)}{\kappa Bg_{\pi}-\left(\sigma\bar{y}/\bar{c}-Bg_{y}\right)} < \frac{\kappa Bg_{\pi}-\left(1+\beta\right)\left(\sigma\bar{y}/\bar{c}-Bg_{y}\right)}{\kappa Bg_{\pi}-\left(\sigma\bar{y}/\bar{c}-Bg_{y}\right)}$$

Multiplication of both sides of the inequality with the negative denominator  $\kappa Bg_{\pi} - (\sigma \bar{y}/\bar{c} - Bg_y) < 0$  transforms the inequality into the following way (where I have to flip the inequality sign):

$$-\kappa \left(1 - Bg_{\pi}\right) - \left(1 + \beta\right) \left(\sigma \bar{y}/\bar{c} - Bg_{y}\right) > \kappa Bg_{\pi} - \left(1 + \beta\right) \left(\sigma \bar{y}/\bar{c} - Bg_{y}\right).$$

After canceling equal terms on the left and right hand side of the equation, I find the condition  $-\kappa > 0$ , which does not hold, since  $\kappa > 0$ . Hence the second condition of Bullard and Mitra (2002), i.e.  $|a_1| < 1 + a_0$ , and hence there are not two roots inside the unit circle. Hence there is no unique stable equilibrium for the case where  $g_{\pi} \leq 0$  and  $g_y \leq 0$ .

# A.11 Analytical expressions for impulse response functions

In this section I calculate analytical expressions for the impulse response functions to the productivity and preference shocks, and show that there exists an isomorphic mapping between the coefficients of the monetary and fiscal policy reactions such that the impulse response functions are identical.

### A.11.1 Monetary regime

I start by writing down the two-system equations for the monetary regime, where I replace  $\hat{R}_t^{x*} = \mathcal{R}^{x*} \hat{x}_t$ , where  $x = \{z, \xi\}$ .

$$\hat{\pi}_{t} = \beta E_{t} [\hat{\pi}_{t+1}] + \kappa \tilde{y}_{t},$$
  
$$\sigma (\bar{y}/\bar{c}) \tilde{y}_{t} = \sigma (\bar{y}/\bar{c}) E_{t} [\hat{y}_{t+1}] - \left(\kappa_{\pi} \hat{\pi}_{t} + \kappa_{y} \tilde{y}_{t} - E_{t} [\hat{\pi}_{t+1}] - \mathcal{R}^{z*} \hat{z}_{t} - \mathcal{R}^{\xi*} \hat{\xi}_{t}\right),$$

Since there are no endogenous backward-looking state variables, I know that the only state variables are  $\hat{z}_t$  and  $\hat{\xi}_t$ . Hence I can employ the method of undetermined coefficients to find the analytical solution to productivity and preference shocks. I assume that  $\hat{\pi}_t$  and  $\tilde{y}_t$  are given by the following solutions:

$$\hat{\pi}_t = \alpha_{\pi,z} \hat{z}_t + \alpha_{\pi,\xi} \hat{\xi}_t, \qquad (107)$$

$$\tilde{y}_t = \alpha_{y,z} \hat{z}_t + \alpha_{y,\xi} \hat{\xi}_t.$$
(108)

Since both shocks are given by exogenous AR(1) shocks, I know that their expected value is given by:

$$E_t \left[ \hat{\pi}_{t+1} \right] = \rho_z \alpha_{\pi,z} \hat{z}_t + \rho_\xi \alpha_{\pi,\xi} \hat{\xi}_t, \qquad (109)$$

$$E_t \left[ \tilde{y}_{t+1} \right] = \rho_z \alpha_{y,z} \hat{z}_t + \rho_\xi \alpha_{y,\xi} \tilde{\xi}_t.$$
(110)

Substitution of the above expressions into the New Keynesian Phillips curve gives the following relations between the inflation and output gap coefficients:

$$\alpha_{y,z} = \left(\frac{1-\beta\rho_z}{\kappa}\right)\alpha_{\pi,z},\tag{111}$$

$$\alpha_{y,\xi} = \left(\frac{1-\beta\rho_{\xi}}{\kappa}\right)\alpha_{\pi,\xi} \tag{112}$$

Substitution of the guessed solutions for the output gap and inflation, and the relation between the output gap coefficients and the inflation coefficients into the aggregate demand equation generate the following expressions for the coefficients:

$$\alpha_{\pi,z}^{m} = \frac{\kappa \mathcal{R}^{z*}}{\kappa \left(\kappa_{\pi} - \rho_{z}\right) + \left[\kappa_{y} + \sigma \left(\bar{y}/\bar{c}\right)\left(1 - \rho_{z}\right)\right]\left(1 - \beta \rho_{z}\right)},\tag{113}$$

$$\alpha_{\pi,\xi}^{m} = \frac{\kappa \mathcal{R}^{\zeta^{*}}}{\kappa (\kappa_{\pi} - \rho_{\xi}) + [\kappa_{y} + \sigma (\bar{y}/\bar{c}) (1 - \rho_{\xi})] (1 - \beta \rho_{\xi})}, \qquad (114)$$
$$(1 - \beta \rho_{z}) \mathcal{R}^{z*}$$

$$\alpha_{y,z}^{m} = \frac{(1-\beta\rho_{z})\kappa}{\kappa(\kappa_{\pi}-\rho_{z}) + [\kappa_{y}+\sigma(\bar{y}/\bar{c})(1-\rho_{z})](1-\beta\rho_{z})},$$
(115)

$$\alpha_{y,\xi}^m = \frac{(1-\beta\rho_{\xi}) \mathcal{K}^s}{\kappa (\kappa_{\pi} - \rho_{\xi}) + [\kappa_y + \sigma (\bar{y}/\bar{c}) (1-\rho_{\xi})] (1-\beta\rho_{\xi})},$$
(116)

where  $\mathcal{R}^{z*}$  and  $\mathcal{R}^{\xi*}$  are given by:

$$\mathcal{R}^{z*} = -\sigma \left( \bar{y}/\bar{c} \right) \left( \frac{1+\varphi}{\sigma \left( \bar{y}/\bar{c} \right) + \varphi} \right) \left( 1-\rho_z \right), \tag{117}$$

$$\mathcal{R}^{\xi*} = (1 - \rho_{\xi}) \tag{118}$$

### A.11.2 Fiscal regime

Next I solve for the impulse response functions under the fiscal regime. The two equation system is again given by:

$$\hat{\pi}_{t} = \beta E_{t} [\hat{\pi}_{t+1}] + \kappa \tilde{y}_{t}, -Bg_{\pi} \hat{\pi}_{t} + \left[\sigma \left(\bar{y}/\bar{c}\right) - Bg_{y}\right] \tilde{y}_{t} = (1 - Bg_{\pi}) E_{t} [\hat{\pi}_{t+1}] + \left[\sigma \left(\bar{y}/\bar{c}\right) - Bg_{y}\right] E_{t} [\hat{y}_{t+1}] + \mathcal{R}^{z*} \hat{z}_{t} + \mathcal{R}^{\xi*} \hat{\xi}_{t},$$

Again employing the method of undetermined coefficients generates the same relationship between the inflation coefficients and the output gap coefficients, and eventually results in the following expressions for the coefficients:

$$\alpha_{\pi,z}^{f} = \frac{\kappa \mathcal{R}^{z*}}{\kappa \left[-Bg_{\pi} \left(1-\rho_{z}\right)-\rho_{z}\right]+\left[\sigma \left(\bar{y}/\bar{c}\right)-Bg_{y}\right]\left(1-\rho_{z}\right)\left(1-\beta\rho_{z}\right)}, \quad (119)$$

$$\alpha_{\pi,\xi}^{J} = \frac{\pi}{\kappa \left[-Bg_{\pi} \left(1-\rho_{\xi}\right)-\rho_{\xi}\right] + \left[\sigma \left(\bar{y}/\bar{c}\right)-Bg_{y}\right] \left(1-\rho_{\xi}\right) \left(1-\beta\rho_{\xi}\right)}, \quad (120)$$

$$\alpha_{y,z}^{J} = \frac{(1 - \mu_{z})}{\kappa \left[-Bg_{\pi} \left(1 - \rho_{z}\right) - \rho_{z}\right] + \left[\sigma \left(\bar{y}/\bar{c}\right) - Bg_{y}\right] \left(1 - \rho_{z}\right) \left(1 - \beta\rho_{z}\right)}, \quad (121)$$

$$\alpha_{y,\xi}^{f} = \frac{(1 - \beta \rho_{\xi}) \kappa}{\kappa \left[-Bg_{\pi} \left(1 - \rho_{\xi}\right) - \rho_{\xi}\right] + \left[\sigma \left(\bar{y}/\bar{c}\right) - Bg_{y}\right] \left(1 - \rho_{\xi}\right) \left(1 - \beta \rho_{\xi}\right)}, \quad (122)$$

Comparing the solutions (113) - (116) under the monetary regime with those under the fiscal regime (119) - (122), we see that there is an isomorphic mapping under which the equilibrium paths for inflation and the output gap are identical under both regimes. This is the case for the productivity shock when:

$$\kappa_{\pi} = -B(1-\rho_z) g_{\pi} \Longrightarrow g_{\pi} = -\frac{\kappa_{\pi}}{B(1-\rho_z)}, \qquad (123)$$

$$\kappa_y = -B(1-\rho_z) g_y \Longrightarrow g_y = -\frac{\kappa_y}{B(1-\rho_z)}, \qquad (124)$$

while we have the following mapping for the preference shock:

$$\kappa_{\pi} = -B\left(1-\rho_{\xi}\right)g_{\pi} \Longrightarrow g_{\pi} = -\frac{\kappa_{\pi}}{B\left(1-\rho_{\xi}\right)},\tag{125}$$

$$\kappa_y = -B(1-\rho_\xi) g_y \Longrightarrow g_y = -\frac{\kappa_y}{B(1-\rho_\xi)}, \qquad (126)$$

# **B** Overlapping Generations Model

# **B.1** Households

A generation lives for two periods. The first period they are young, and in the second period of their existence they are old, after which each generation dies. I assume that each generation has a constant mass of 1 that does not change over time. In the first period, the young earn income  $w_t h_t(i)$  from providing labor, and from ownership of the production firms  $\omega_t(i)$  (in terms of final goods). This income is spent on consumption  $c_t^1(i)$ , lump sum taxes  $\tau_t^1(i)$ , and savings in the form of government bonds  $b_t(i)$ . Their budget constraint is then (in terms of final goods) given by:

$$c_t^1(i) + \tau_t^1(i) + b_t(i) = w_t h_t(i) + \omega_t(i), \qquad (127)$$

When turning from young to old, the old receive income from gross repayment of the government bonds that were purchased when young as well as a pension income  $s_t(i)$  provided by the government. This income is then used for consumption  $c_t^2(i)$ and lump sum taxes  $\tau_t^2(i)$ . The budget constraint for the old (in terms of final goods) is then given by:

$$c_t^2(i) + \tau_t^2(i) = \left(\frac{1 + R_{t-1}^n}{\pi_t}\right) b_{t-1}(i) + s_t(i), \qquad (128)$$

The old's maximization problem is given by maximizing current consumption subject to the budget constraint (128):

$$\max_{\{c_t^2(i)\}} \xi_t^2 \cdot \frac{(c_t(i)^2)^{1-\sigma} - 1}{1 - \sigma},$$

where  $\xi_t^2$  is a preference shock of the old. The Lagrangian for this problem is given by:

$$\mathcal{L} = \xi_t^2 \cdot \frac{(c_t(i)^2)^{1-\sigma} - 1}{1 - \sigma} + \lambda_t^2 \left( \left( \frac{1 + R_{t-1}^n}{\pi_t} \right) b_{t-1}(i) + s_t(i) - c_t^2(i) - \tau_t^2(i) \right)$$

The first order conditions are given by:

$$c_t^2(i)$$
 :  $\lambda_t^2 = \xi_t^2 \left( c_t^2(i) \right)^{-\sigma}$ , (129)

$$\lambda_t^2 : \left( \left( \frac{1 + R_{t-1}^n}{\pi_t} \right) b_{t-1}(i) + s_t(i) - c_t^2(i) - \tau_t^2(i) \right) = 0.$$
(130)

Now I move to the young's optimization problem, which is given by:

$$\max_{\left\{c_{t}^{1}(i),h_{t}(i),b_{t}(i)\right\}} \xi_{t}^{1}\left(\frac{\left(c_{t}^{1}(i)\right)^{1-\sigma}-1}{1-\sigma}-\chi_{h}\frac{\left(h_{t}(i)\right)^{1+\varphi}}{1+\varphi}\right)+E_{t}\left[\beta\xi_{t+1}^{2}\cdot\frac{\left(c_{t+1}^{2}(i)\right)^{1-\sigma}-1}{1-\sigma}\right],$$

subject to the budget constraints (127) and (128). This results in the following Lagrangian:

$$\mathcal{L} = \xi_t^1 \left( \frac{(c_t^1(i))^{1-\sigma} - 1}{1-\sigma} - \chi_h \frac{(h_t(i))^{1+\varphi}}{1+\varphi} \right) + E_t \left[ \beta \xi_{t+1}^2 \cdot \frac{(c_{t+1}^2(i))^{1-\sigma} - 1}{1-\sigma} \right] + \lambda_t^1 \left( w_t h_t(i) + \omega_t(i) - c_t^1(i) - \tau_t^1(i) - b_t(i) \right) + E_t \left[ \beta \lambda_{t+1}^2 \left( \left( \frac{1+R_t^n}{\pi_{t+1}} \right) b_t(i) + s_{t+1}(i) - c_{t+1}^2(i) - \tau_{t+1}^2(i) \right) \right].$$

After taking the derivatives with respect to  $c_t^1(i)$ ,  $h_t(i)$ ,  $b_t(i)$ , and  $\lambda_t^1$ , I obtain the following first order conditions:

$$c_t^1(i) : \lambda_t^1 = \xi_t \left( c_t^1(i) \right)^{-\sigma},$$
 (131)

$$h_t(i) : \chi_h (h_t(i))^{\varphi} = w_t (c_t^1(i))^{-\sigma},$$
 (132)

$$b_t(i) : E_t \left[ \beta \Lambda_{t,t+1}^{1,2} \left( \frac{1+R_t^n}{\pi_{t+1}} \right) \right] = 1,$$
 (133)

$$\lambda_t^1 : \left( w_t h_t(i) + \omega_t(i) - c_t^1(i) - \tau_t^1(i) - b_t(i) \right) = 0,$$
(134)

where  $\beta \Lambda_{t,t+1}^{1,2} \equiv \beta \lambda_{t+1}^2 / \lambda_t^1$  denotes the young generation's stochastic discount factor.

# **B.2** Production firms

In this subsection I only discuss the changes that I make to the production sector, which turn out to be few. The structure with final goods producers, retail goods producers, and intermediate goods producers remains the same as before, as well as all assumptions regarding their production technologies and the type of markets they operate in. The only change that I have to incorporate is the fact that the old generation is now the owner of all the production firms, rather than the infinitelylived household in the RANK model.

### **B.2.1** Final goods producers

As final goods producers face a static optimization problem, and do not make any profits in equilibrium as they operate in a perfectly competitive market for final goods, the optimization problem is exactly the same as in the case of a representative infinitely-lived household.

### B.2.2 Retail goods producers

The production technology of retail goods producers, as well as the fact that they operate under monopolistic competition makes that their optimization problem is the same as under the representative infinitely-lived household. The only difference is that they are owned in period t by the generation that was born in period t, while they will be owned in period t+1 by the generation born in period t+1, etc. Therefore, the stochastic discount factor with which they discount future expected profits will differ. I assume that they will value a cash flow in period t+s with the marginal utility  $\beta \lambda_{t+s}^y$  of the generation that will be young in period t+s, where future profits are discounted with the subjective discount factor  $\beta$  with which they discount next period's utility relative to today's utility. Therefore, the retail goods producers' maximization problem changes into:

$$\max_{P_t^*} E_t \left\{ \sum_{s=0}^{\infty} \psi^s \beta^s \Lambda_{t,t+s}^y \left[ \frac{P_t^*}{P_{t+s}} - \phi_{t+s} \right] y_{t+s}^f \right\},\,$$

subject to the demand curve (36), and where  $\beta^s \Lambda^y_{t,t+s} \equiv \beta \lambda^1_{t+s} / \lambda^1_t$ . Note that this stochastic discount factor differs from the stochastic discount factor that the young employ to discount the future cash flow from the government bond!

Apart from the change in the discount factor, the retail goods producers' optimization problem is identical to the optimization problem when a representative households are infinitely lived. Therefore, all the first order conditions are the same, except for the replacement of the stochastic discount factor in first order conditions (38) and (39), which are now given by:

$$\Xi_{1,t} = \phi_t y_t + E_t \left[ \psi \beta \Lambda^y_{t,t+1} \pi^\epsilon_{t+1} \Xi_{1,t+1} \right], \qquad (135)$$

$$\Xi_{2,t} = y_t + E_t \left[ \psi \beta \Lambda_{t,t+1}^y \pi_{t+1}^{\epsilon-1} \Xi_{2,t+1} \right].$$
(136)

However, it will be relevant to calculate period t profits  $\omega_t^f$  (in terms of the final good) for retail goods producer  $f \in [0, 1]$ :

$$\omega_t^f = \left(\frac{P_t^f}{P_t} - \phi_t\right) y_t^f = \left(\frac{P_t^f}{P_t}\right)^{1-\epsilon} y_t - \left(\frac{P_t^f}{P_t}\right)^{-\epsilon} \phi_t y_t.$$
(137)

where I substituted the demand schedule (36) for retail goods  $f \in [0, 1]$ .

### **B.2.3** Intermediate goods producers

The optimization problem of intermediate goods producers is exactly the same as in the RANK-model.

### **B.3** Government

#### B.3.1 Fiscal authority

The government budget constraint is now extended by a pension payment  $s_t$  to the old. Otherwise the budget constraint is the same as in the RANK-model, and is therefore given by:

$$\tau_t + b_t = g_t + s_t + \left(\frac{1 + R_{t-1}^n}{\pi_t}\right) b_{t-1},$$
(138)

Lump sum taxes are now not raised on a representative household, but on both the young and old generation, i.e.  $\tau_t = \tau_t^1 + \tau_t^2$ , where  $\tau_t^1$  and  $\tau_t^2$  denotes aggregate lump sum taxes on the young and old, respectively. In order to derive at a system with only inflation and the output gap, I need an analytical expression for the output gap, something I cannot achieve in a model which still features endogenous state variables. In order to eliminate these endgenous state variables, I assume that lump sum taxes on the old  $\tau_t^2$  are exactly equal to the gross interest payments on the bonds they purchased when they were young:

$$\tau_t^2 = \left(\frac{1 + R_{t-1}^n}{\pi_t}\right) b_{t-1}.$$
(139)

Substitution of (139) results in the following government budget constraint:

$$\tau_t^1 + b_t = g_t + s_t, \tag{140}$$

For the young, I assume that the level of lump sum taxes  $\tau_t^1$  is linear in last period's stock of government debt  $b_{t-1}$ :

$$\tau_t^1 = \bar{\tau}_1 + \kappa_b \left( b_{t-1} - \bar{b} \right). \tag{141}$$

As in the RANK-model, government purchases will be equal to its steady state value under the monetary regime, while it will endogenously respond to inflation and the output gap under the fiscal regime. Finally, I assume that the pension payment  $s_t$  is linear in output. When the economy is in aboom, pensioners get paid more than when the economy is in recession:

$$s_t = \left(\bar{s}/\bar{y}\right) y_t,\tag{142}$$

### B.3.2 Central bank

Monetary policy is exactly the same as in the RANK-model.

# **B.4** Aggregation

I assume that each member of the young is identical, and chooses the same level of consumption and labor supply in equilibrium. Integrating over all young  $i \in [0, 1]$  gives the following expressions for aggregate consumption and labor supply of the

young.

$$c_t^1 \equiv \int_0^1 c_t^1(i) di = c_t^1(i) \int_0^1 di = c_t^1(i).$$

Similarly, I find aggregate labor supply by the young to be equal to  $h_t^1 = h_t^1(i)$ , as well as government debt holdings  $b_t = b_t(i)$ , aggregate lump sum taxes  $\tau_t^1 = \tau_t^1(i)$ , and aggregate profits from production firms  $\omega_t^1 = \omega_t^1(i)$ . Aggregation over young member  $i \in [0, 1]$  budget constraint (127) then gives the aggregate young's budget constraint:

$$c_t^1 + \tau_t^1 + b_t = w_t h_t + \omega_t, (143)$$

Substitution of the government budget constraint (140) then reads:

$$c_t^1 + g_t + s_t = w_t h_t + \omega_t, (144)$$

Similarly, I can integrate over member  $i \in [0, 1]$  of the old generation to obtain the aggregate old bduget constraint:

$$c_t^2 + \tau_t^2 = \left(\frac{1 + R_{t-1}^n}{\pi_t}\right) b_{t-1} + s_t, \qquad (145)$$

Substitution of the old's lump sum taxes (139) gives:

$$c_t^2 = s_t. (146)$$

In other words, consumpton of the old is equal to the pension payment from the government.

To find aggregate profits of the retail goods producers, I integrate the profits (137) of retail goods producer  $f \in [0, 1]$ 

$$\omega_t = \int_0^1 \omega_t^f df = \int_0^1 \left[ \left( \frac{P_t^f}{P_t} \right)^{1-\epsilon} y_t - \left( \frac{P_t^f}{P_t} \right)^{-\epsilon} \phi_t y_t \right] df = y_t - \mathcal{D}_t \phi_t y_t, \quad (147)$$

where I employed equations (37) and (42). Now I aggregate over the left hand side

of production technology (43) of intermediate goods producer  $i \in [0, 1]$ :

$$\int_0^1 y_t^i di = \int_0^1 y_t^f df = y_t \int_0^1 \left(\frac{P_t^f}{P_t}\right)^{-\epsilon} df = \mathcal{D}_t y_t.$$

Integration over the right hand side of equation (43) gives:

$$\int_0^1 z_t h_t^i di = z_t \int_0^1 h_t^i di = z_t h_t.$$

Combining the aggregated left and right hand side gives:

$$\mathcal{D}_t y_t = z_t h_t. \tag{148}$$

Substitution of this relation into the expression for the profits of retail goods producers gives:

$$\omega_t = y_t - \phi_t z_t h_t = y_t - w_t h_t, \tag{149}$$

where I used first order condition (44). Substitution of equation (149) into the young's aggregate budget constraint (144) gives:

$$c_t^1 + g_t + s_t = y_t, (150)$$

Substitution of the aggregate budget constraint of the old generation (146) gives the aggregate resource constraint of the economy:

$$c_t^1 + c_t^2 + g_t = y_t. (151)$$

# B.5 Overview first order conditions (OLG)

A competitive equilibrium is a series of quantities  $\{c_t^1, c_t^2, h_t, y_t, g_t, b_t, \tau_t^1, \tau_t^2, s_t, \omega_t\}$ , (shadow) prices

 $\{\lambda_t^1, \lambda_t^2, R_t^n, \phi_t, w_t, \pi_t, \pi_t^*, \mathcal{D}_t, \Xi_{1,t}, \Xi_{2,t}\},\ \text{and exogenous processes }\{z_t, \xi_t\}\ \text{satisfying}$ 

the following equations:

$$\lambda_t^1 = \xi_t \left( c_t^1 \right)^{-\sigma}, \qquad (152)$$

$$\lambda_t^2 = \xi_t \left(c_t^2\right)^{-\sigma}, \qquad (153)$$

$$\chi_h h_t^{\varphi} = w_t \left( c_t^1 \right)^{-\sigma}, \qquad (154)$$

$$E_t \left[ \beta \Lambda_{t,t+1}^{1,2} \left( \frac{1+R_t^n}{\pi_{t+1}} \right) \right] = 1, \qquad (155)$$

$$c_t^2 + \tau_t^2 = \left(\frac{1+R_{t-1}^n}{\pi_t}\right)b_{t-1} + s_t, \qquad (156)$$

$$\omega_t = y_t - \phi_t \mathcal{D}_t y_t = y_t - w_t h_t, \qquad (157)$$

$$\pi_t^* = \left(\frac{\epsilon}{\epsilon - 1}\right) \frac{\underline{\Xi}_{1,t}}{\underline{\Xi}_{2,t}},\tag{158}$$

$$\Xi_{1,t} = \phi_t y_t + E_t \left[ \psi \beta \Lambda^y_{t,t+1} \pi^{\epsilon}_{t+1} \Xi_{1,t+1} \right], \qquad (159)$$

$$\Xi_{2,t} = y_t + E_t \left[ \psi \beta \Lambda_{t,t+1}^y \pi_{t+1}^{\epsilon-1} \Xi_{2,t+1} \right], \qquad (160)$$

$$1 = (1 - \psi) (\pi_t^*)^{1-\epsilon} + \psi \pi_t^{\epsilon-1}, \qquad (161)$$

$$\mathcal{D}_t = (1 - \psi) (\pi_t^*)^{-\epsilon} + \psi \pi_t^{\epsilon} \mathcal{D}_{t-1}, \qquad (162)$$

$$\mathcal{D}_t y_t = z_t h_t, \tag{163}$$

$$w_t = \phi_t z_t, \tag{164}$$

$$y_t = c_t^1 + c_t^2 + g_t, (165)$$

$$\tau_t^1 + \tau_t^2 + b_t = g_t + s_t + \left(\frac{1 + R_{t-1}^n}{\pi_t}\right) b_{t-1}, \qquad (166)$$

$$\tau_t^1 = \bar{\tau}_1 + \kappa \left( b_{t-1} - \bar{b} \right), \qquad (167)$$

$$\tau_t^2 = \left(\frac{1+R_{t-1}}{\pi_t}\right) b_{t-1}, \tag{168}$$

$$s_t = (\bar{s}/\bar{y}) y_t, \tag{169}$$

$$R_t^n = \dots, (170)$$

$$g_t = \dots, \tag{171}$$

$$\log(z_t) = \rho_z \log(z_{t-1}) + \varepsilon_{z,t}, \qquad (172)$$

$$\log\left(\xi_{t}\right) = \rho_{\xi} \log\left(\xi_{t-1}\right) + \varepsilon_{\xi,t}, \qquad (173)$$

where  $\beta \Lambda_{t,t+1}^{1,2} = \beta \lambda_{t+1}^2 / \lambda_t^1$  and  $\beta \Lambda_{t,t+1}^y = \beta \lambda_{t+1}^1 / \lambda_t^1$ .

# B.6 Linearized FOCs

$$\varphi \hat{h}_t = -\sigma \hat{c}_t^1 + \hat{w}_t, \tag{174}$$

$$\hat{\xi}_{t} - \sigma \hat{c}_{t}^{1} = E_{t} \left[ \hat{\xi}_{t+1} - \sigma \hat{c}_{t+1}^{2} \right] + \hat{R}_{t}^{n} - E_{t} \left[ \hat{\pi}_{t+1} \right], \qquad (175)$$

$$\bar{c}_2 \hat{c}_t^2 + \bar{\tau}_2 \hat{\tau}_t^2 = \left(\frac{1+R_n}{\bar{\pi}}\right) \bar{b} \left(\hat{R}_{t-1}^n - \hat{\pi}_t + \hat{b}_{t-1}\right) + \bar{s}\hat{s}_t,$$
(176)

$$\bar{\omega}\hat{\omega}_t = \bar{y}\hat{y}_t - \bar{w}\bar{h}\left(\hat{w}_t + \hat{h}_t\right), \qquad (177)$$

$$\hat{\pi}_{t}^{*} = \hat{\Xi}_{1,t} - \hat{\Xi}_{2,t}, \tag{178}$$

$$\hat{\Xi}_{1,t} = (1 - \psi \beta \overline{\pi}^{\epsilon}) \left( \hat{\phi}_t + \hat{y}_t \right) + \psi \beta \overline{\pi}^{\epsilon} \left( -\sigma E_t \left[ \hat{c}_{t+1}^1 \right] + \sigma \hat{c}_t^1 + \epsilon E_t \left[ \hat{\pi}_{t+1} \right] + E_t \left[ \hat{\Xi}_{1,t+1} \right] \right), \quad (179)$$

$$\hat{\Xi}_{2,t} = (1 - \psi \beta \bar{\pi}^{\epsilon - 1}) \hat{y}_{t} 
+ \psi \beta \bar{\pi}^{\epsilon - 1} \left( -\sigma E_{t} \left[ \hat{c}_{t+1}^{1} \right] + \sigma \hat{c}_{t}^{1} + (\epsilon - 1) E_{t} \left[ \hat{\pi}_{t+1} \right] + E_{t} \left[ \hat{\Xi}_{2,t+1} \right] \right),$$
(180)

$$(1-\psi)(\bar{\pi}^{*})^{1-\epsilon}\hat{\pi}_{t}^{*} = \psi(\bar{\pi})^{\epsilon-1}\hat{\pi}_{t},$$
(181)

$$\hat{\mathcal{D}}_{t} = (1 - \psi(\bar{\pi})^{\epsilon}) (-\epsilon \hat{\pi}_{t}^{*}) + \psi(\bar{\pi})^{\epsilon} \left(\epsilon \hat{\pi}_{t} + \hat{\mathcal{D}}_{t-1}\right), \qquad (182)$$

$$\hat{\mathcal{D}}_t + \hat{y}_t = \hat{z}_t + \hat{h}_t, \tag{183}$$

$$\hat{w}_t = \phi_t + \hat{z}_t, \tag{184}$$

$$\hat{y}_t = (\bar{c}_1/\bar{y})\,\hat{c}_t^1 + (\bar{c}_2/\bar{y})\,\hat{c}_t^2 + (\bar{g}/\bar{y})\,\hat{g}_t, \tag{185}$$

$$\bar{\tau}_1 \hat{\tau}_t^1 + \bar{\tau}_2 \hat{\tau}_t^2 + \bar{b}\hat{b}_t = \bar{g}\hat{g}_t + \bar{s}\hat{s}_t + \left(\frac{1+R_n}{\bar{\pi}}\right)\bar{b}\left(\hat{R}_{t-1}^n - \hat{\pi}_t + \hat{b}_{t-1}\right),$$
(186)

$$\bar{\tau}_1 \hat{\tau}_t^1 = \kappa \bar{b} \hat{b}_{t-1}, \tag{187}$$

$$\bar{\tau}_2 \hat{\tau}_t^2 = \left(\frac{1+R_n}{\bar{\pi}}\right) \bar{b} \left(\hat{R}_{t-1}^n - \hat{\pi}_t + \hat{b}_{t-1}\right), \qquad (188)$$

$$\hat{s}_t = \hat{y}_t, \tag{189}$$

$$R_t^n = \dots, (190)$$

$$\hat{g}_t = \dots, \tag{191}$$

$$\hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon_{z,t}, \tag{192}$$

$$\hat{\xi}_t = \rho_{\xi} \hat{\xi}_{t-1} + \varepsilon_{\xi,t}, \tag{193}$$

# **B.7** Further derivations

Going forward, I assume that gross steady state inflation is equal to one:  $\bar{\pi} = 1$ . Next, I substitute equations (179) and (180) into (178) to obtain:

$$\hat{\pi}_t^* = (1 - \psi\beta)\,\hat{\phi}_t + \psi\beta E_t\left[\hat{\pi}_{t+1}\right] + \psi\beta E_t\left[\hat{\pi}_{t+1}^*\right],\tag{194}$$

Substitution of  $\hat{\pi}_t^* = (\psi/(1-\psi))\hat{\pi}_t$  (181) delivers the traditional New Keynesian Phillips-curve:

$$\hat{\pi}_t = \beta E_t \left[ \hat{\pi}_{t+1} \right] + \zeta \hat{\phi}_t, \tag{195}$$

where  $\zeta \equiv (1 - \psi \beta) (1 - \psi) / \psi$ .

Next, I substitute expression (188) into expression (176) and find that:

$$\hat{c}_t^2 = \hat{s}_t = \hat{y}_t, \tag{196}$$

where I employed equation (189) and the knowledge that  $\bar{c}_2 = \bar{s}$ . Substitution of the above expression into equation (185) gives me the following expression for  $\hat{c}_1$ :

$$\hat{c}_t^1 = \frac{\bar{y}}{\bar{c}_1} \left[ \left( 1 - \frac{\bar{c}_2}{\bar{y}} \right) \hat{y}_t - \left( \frac{\bar{g}}{\bar{y}} \right) \hat{g}_t \right],\tag{197}$$

### B.7.1 The flexible prices equilibrium

Now I aim to derive the flexible prices equilibrium. To do so, I set  $\psi = 0$  in equation (194)

$$\hat{\pi}_t^* = \phi_t,$$

Since I know from equation (181) that  $\hat{\pi}_t^* = 0$  when  $\psi = 0$ , I find that  $\hat{\phi}_t = 0$ . Next, I substitute expression (181) into equation (182), and find that  $\hat{\mathcal{D}}_t = 0$ , irrespective of whether  $\psi = 0$  or not.

Now I consider equation (174), and substitute equation (184) for  $\hat{w}_t$ , expression (183) for  $\hat{h}_t$ , and equation (197) to obtain:

$$\varphi\left(\hat{y}_t - \hat{z}_t\right) = -\sigma\left(\bar{y}/\bar{c}_1\right) \left[ \left(1 - \frac{\bar{c}_2}{\bar{y}}\right)\hat{y}_t - \left(\frac{\bar{g}}{\bar{y}}\right)\hat{g}_t \right] + \hat{z}_t,$$

Solving for output delivers the flexible prices level of output, or the natural level

of output:

$$\hat{y}_t^n = \left(\frac{1+\varphi}{\sigma\left(\bar{y}/\bar{c}_1\right)\left(1-\frac{\bar{c}_2}{\bar{y}}\right)+\varphi}\right)\hat{z}_t + \sigma\left(\bar{y}/\bar{c}_1\right)\left(\frac{(\bar{g}/\bar{y})}{\sigma\left(\bar{y}/\bar{c}_1\right)\left(1-\frac{\bar{c}_2}{\bar{y}}\right)+\varphi}\right)\hat{g}_t, \quad (198)$$

### B.7.2 The sticky prices equilibrium

Agiain I consider equation (174), and substitute equation (184) for  $\hat{w}_t$ , expression (183) for  $\hat{h}_t$ , and equation (197). However, the difference with respect to the flexible prices equilibrium is that  $\hat{\phi}_t$  is no longer zero in equation equation (184). I thus obtain:

$$\varphi\left(\hat{y}_t - \hat{z}_t\right) = -\sigma\left(\bar{y}/\bar{c}_1\right) \left[ \left(1 - \frac{\bar{c}_2}{\bar{y}}\right)\hat{y}_t - \left(\frac{\bar{g}}{\bar{y}}\right)\hat{g}_t \right] + \hat{z}_t + \hat{\phi}_t,$$

Rearranging gives:

$$\hat{\phi}_{t} = \left(\sigma\left(\bar{y}/\bar{c}_{1}\right)\left(1-\frac{\bar{c}_{2}}{\bar{y}}\right)+\varphi\right)\hat{y}_{t}-\left(1+\varphi\right)\hat{z}_{t}-\sigma\left(\bar{y}/\bar{c}_{1}\right)\left(\bar{g}/\bar{y}\right)\hat{g}_{t} \\
= \left(\sigma\left(\bar{y}/\bar{c}_{1}\right)\left(1-\frac{\bar{c}_{2}}{\bar{y}}\right)+\varphi\right)\left(\hat{y}_{t}-\hat{y}_{t}^{n}\right)=\left(\sigma\left(\bar{y}/\bar{c}_{1}\right)\left(1-\frac{\bar{c}_{2}}{\bar{y}}\right)+\varphi\right)\tilde{y}_{t},$$
(199)

where  $\tilde{y}_t \equiv \hat{y}_t - \hat{y}_t^n$  denotes the output gap, the difference between the level of output under the sticky prices equilibrium and the flexible prices equilibrium.

Substitution of expression (199) into equation (195) delivers the familiar New Keynesian Phillips curve in the output gap  $\tilde{y}_t$ :

$$\hat{\pi}_t = \beta E_t \left[ \hat{\pi}_{t+1} \right] + \kappa \tilde{y}_t, \tag{200}$$

where  $\kappa$  is given by:

$$\kappa = \left(\sigma\left(\bar{y}/\bar{c}_1\right)\left(1 - \frac{\bar{c}_2}{\bar{y}}\right) + \varphi\right)\zeta,\tag{201}$$

Finally, I substitute expressions (196) and (197) into the Euler equation (175):

$$\hat{\xi}_t - \sigma \left( \bar{y}/\bar{c}_1 \right) \left[ \left( 1 - \frac{\bar{c}_2}{\bar{y}} \right) \hat{y}_t - \left( \frac{\bar{g}}{\bar{y}} \right) \hat{g}_t \right] = E_t \left[ \hat{\xi}_{t+1} - \sigma \hat{y}_{t+1} \right] + \hat{R}_t^n - E_t \left[ \hat{\pi}_{t+1} \right], \quad (202)$$

Now I substitute  $\hat{y}_t = \hat{y}_t^n + \tilde{y}_t$ , and substitute expression (198) to get the aggregate demand equation:

$$\sigma\left(\bar{y}/\bar{c}_{1}\right)\left(1-\frac{\bar{c}_{2}}{\bar{y}}\right)\tilde{y}_{t} = \sigma E_{t}\left[\tilde{y}_{t+1}\right] - \left(\hat{R}_{t}^{n} - E_{t}\left[\hat{\pi}_{t+1}\right] - \hat{R}_{t}^{*}\right),\tag{203}$$

where  $\hat{R}_t^*$  is given by:

$$\hat{R}_t^* = \hat{R}_t^{z*} + \hat{R}_t^{\xi*} + \hat{R}_t^{g*}, \qquad (204)$$

where  $\hat{R}_t^{z*}$ ,  $\hat{R}_t^{\xi*}$ , and  $\hat{R}_t^{g*}$  are given by:

$$\hat{R}_{t}^{z*} = -\left(\frac{\sigma\left(1+\varphi\right)}{\sigma\left(\bar{y}/\bar{c}_{1}\right)\left(1-\frac{\bar{c}_{2}}{\bar{y}}\right)+\varphi}\right)\left[\frac{\bar{y}}{\bar{c}_{1}}\left(1-\frac{\bar{c}_{2}}{\bar{y}}\right)-\rho_{z}\right]\hat{z}_{t},\qquad(205)$$

$$\hat{R}_{t}^{\xi*} = (1 - \rho_{\xi}) \,\hat{\xi}_{t}, \tag{206}$$

$$\hat{R}_{t}^{g*} = \sigma\left(\bar{y}/\bar{c}_{1}\right) \left(\frac{(\bar{g}/\bar{y})}{\sigma\left(\bar{y}/\bar{c}_{1}\right)\left(1-\frac{\bar{c}_{2}}{\bar{y}}\right)+\varphi}\right) \left(\varphi\hat{g}_{t}+\sigma E_{t}\left[\hat{g}_{t+1}\right]\right), \quad (207)$$

### **B.8** The fiscal divine coincidence in the OLG model

As before, a divine coincidence is characterized as an equilibrium in which I have permanent  $\hat{\pi}_t = 0$  and  $\tilde{y}_t = 0$ . The fiscal regime is characterized by  $\hat{R}_t^n = 0$ . Such an equilibrium, however, can only be achieved when the natural rate of interest  $\hat{R}_t^* = 0$  irrespective of the realization of the productivity shock  $\hat{z}_t$  and preference shock  $\hat{\xi}_t$ . To achieve that the divine coincidence equilibrium, government spending must respond to productivity and preference shocks in such a way that its influence on government spending exactly offsets the change in the natural rate arising from the productivity and preference shocks. Similarly to before, government spending is given by:

$$\hat{g}_t = A_z^{OLG} \hat{z}_t + A_{\xi}^{OLG} \hat{\xi}_t.$$
 (208)

I then immediately find that:

$$\varphi \hat{g}_t + \sigma E_t \left[ \hat{g}_{t+1} \right] = A_z^{OLG} \left( \varphi + \sigma \rho_z \right) \hat{z}_t + A_{\xi}^{OLG} \left( \varphi + \sigma \rho_{\xi} \right) \hat{\xi}_t.$$

Substitution of the process for government spending into (204) allows me to find the values of  $A_z^{OLG}$  and  $A_{\xi}^{OLG}$  such that the natural rate of interest  $\hat{R}_t^* = 0$  period by period:

$$\begin{split} A_z^{OLG} &= \frac{\left(1+\varphi\right)\left[\left(\bar{y}/\bar{c}_1\right)\left(1-\frac{\bar{c}_2}{\bar{y}}\right)-\rho_z\right]}{\left(\bar{g}/\bar{c}_1\right)\left(\varphi+\sigma\rho_z\right)} > 0, \\ A_\xi^{OLG} &= -\frac{\left(1-\rho_\xi\right)\left(\sigma\left(\bar{y}/\bar{c}_1\right)\left(1-\frac{\bar{c}_2}{\bar{y}}\right)+\varphi\right)}{\sigma\left(\bar{g}/\bar{c}_1\right)\left(\varphi+\sigma\rho_\xi\right)} < 0, \end{split}$$

# **B.9** Stability conditions under the OLG-model

In this subsction I will investigate the conditions under which unique stable equilibria are possible in the OLG New Keynesian model. I start by inspecting the stability conditions under the monetary regime, after which I investigate the stability conditions for the fiscal regime.

### B.9.1 The monetary regime

As in the main text, I employ a standard Taylor rule (10):

$$\hat{R}_t^n = \kappa_\pi \hat{\pi}_t + \kappa_y \tilde{y}_t,$$

while I set government spending equal to steady state, i.e.  $\hat{g}_t = 0$ . After substitution of the Taylor rule into the aggregate demand equation, and combining this with the New Keynesian Phillips curve (200), I get the following two by two system of equations:

$$\hat{\pi}_{t} = \beta E_{t} \left[ \hat{\pi}_{t+1} \right] + \kappa \tilde{y}_{t},$$

$$\sigma \left( \bar{y} / \bar{c}_{1} \right) \left( 1 - \frac{\bar{c}_{2}}{\bar{y}} \right) \tilde{y}_{t} = \sigma E_{t} \left[ \hat{y}_{t+1} \right] - \left( \kappa_{\pi} \hat{\pi}_{t} + \kappa_{y} \tilde{y}_{t} - E_{t} \left[ \hat{\pi}_{t+1} \right] - \hat{R}_{t}^{z*} - \hat{R}_{t}^{\xi*} \right),$$

I write this in the following way:

$$\begin{pmatrix} 1 & -\kappa \\ \kappa_{\pi} & \kappa_{y} + \sigma \left( \bar{y}/\bar{c}_{1} \right) \left( 1 - \frac{\bar{c}_{2}}{\bar{y}} \right) \end{pmatrix} \begin{pmatrix} \hat{\pi}_{t} \\ \tilde{y}_{t} \end{pmatrix} = \begin{pmatrix} \beta & 0 \\ 1 & \sigma \end{pmatrix} \begin{pmatrix} E_{t} \left[ \hat{\pi}_{t+1} \right] \\ E_{t} \left[ \tilde{y}_{t+1} \right] \end{pmatrix} + \begin{pmatrix} 0 \\ \hat{R}_{t}^{z*} + \hat{R}_{t}^{\xi*} \end{pmatrix},$$

This can be rewritten in the following way:

$$\begin{pmatrix} \hat{\pi}_t \\ \tilde{y}_t \end{pmatrix} = M \begin{pmatrix} E_t \left[ \hat{\pi}_{t+1} \right] \\ E_t \left[ \tilde{y}_{t+1} \right] \end{pmatrix} + N_t$$

where M and N are given by:

$$M = \frac{1}{\kappa_y + \sigma\left(\bar{y}/\bar{c}_1\right)\left(1 - \frac{\bar{c}_2}{\bar{y}}\right) + \kappa\kappa_\pi} \begin{pmatrix} \beta\left(\kappa_y + \sigma\left(\bar{y}/\bar{c}_1\right)\left(1 - \frac{\bar{c}_2}{\bar{y}}\right)\right) + \kappa & \kappa\sigma \\ -\beta\kappa_\pi + 1 & \sigma \end{pmatrix},$$

$$(209)$$

$$N = \frac{1}{\kappa_y + \sigma\left(\bar{y}/\bar{c}_1\right)\left(1 - \frac{\bar{c}_2}{\bar{y}}\right) + \kappa\kappa_\pi} \begin{pmatrix} \kappa_y + \sigma\left(\bar{y}/\bar{c}_1\right)\left(1 - \frac{\bar{c}_2}{\bar{y}}\right) & \kappa \\ -\kappa_\pi & 1 \end{pmatrix} \begin{pmatrix} 0 \\ \hat{R}_t^{z*} + \hat{R}_t^{\xi*} \end{pmatrix},$$

$$(210)$$

and where I note that:

$$\begin{pmatrix} 1 & -\kappa \\ \kappa_{\pi} & \kappa_{y} + \sigma\left(\bar{y}/\bar{c}_{1}\right) \left(1 - \frac{\bar{c}_{2}}{\bar{y}}\right) \end{pmatrix}^{-1} = \frac{1}{\kappa_{y} + \sigma\left(\bar{y}/\bar{c}_{1}\right) \left(1 - \frac{\bar{c}_{2}}{\bar{y}}\right) + \kappa\kappa_{\pi}} \begin{pmatrix} \kappa_{y} + \sigma\left(\bar{y}/\bar{c}_{1}\right) \left(1 - \frac{\bar{c}_{2}}{\bar{y}}\right) & \kappa \\ -\kappa_{\pi} & 1 \end{pmatrix}$$

The trace and the determinant of M are given by:

trace 
$$M = \frac{\beta \left(\kappa_y + \sigma \left(\bar{y}/\bar{c}_1\right) \left(1 - \frac{\bar{c}_2}{\bar{y}}\right)\right) + \kappa + \sigma}{\kappa_y + \sigma \left(\bar{y}/\bar{c}_1\right) \left(1 - \frac{\bar{c}_2}{\bar{y}}\right) + \kappa \kappa_\pi},$$
 (211)

$$\det M = \frac{\beta \sigma}{\kappa_y + \sigma \left( \bar{y}/\bar{c}_1 \right) \left( 1 - \frac{\bar{c}_2}{\bar{y}} \right) + \kappa \kappa_\pi},\tag{212}$$

Now I have to inspect the sign of the two roots of the matrix M to determine under which conditions I have a unique stable equilibrium. As I have two forwardlooking variables, I need two eigenvalues that are smaller in absolute value than one. I start by calculating the characteristic equation of M.

$$\lambda^2 - \operatorname{trace} M\lambda + \det M = 0.$$

I now employ Bullard and Mitra (2002) to determine whether this is the case. They start from the following characteristic equation:  $\lambda^2 + a_1\lambda + a_0 = 0$ . In this case I have:

$$a_1 = -\operatorname{trace} M = -\frac{\beta \left(\kappa_y + \sigma \left(\bar{y}/\bar{c}_1\right) \left(1 - \frac{\bar{c}_2}{\bar{y}}\right)\right) + \kappa + \sigma}{\kappa_y + \sigma \left(\bar{y}/\bar{c}_1\right) \left(1 - \frac{\bar{c}_2}{\bar{y}}\right) + \kappa \kappa_\pi}, \qquad (213)$$

$$a_0 = \det M = \frac{\beta \sigma}{\kappa_y + \sigma \left( \bar{y}/\bar{c}_1 \right) \left( 1 - \frac{\bar{c}_2}{\bar{y}} \right) + \kappa \kappa_\pi},$$
(214)

The first condition that needs to be satisfied according to Bullard and Mitra (2002) is  $|a_0| < 1$ :

$$|a_0| = \frac{\beta\sigma}{\kappa_y + \sigma\left(\bar{y}/\bar{c}_1\right) \left(1 - \frac{\bar{c}_2}{\bar{y}}\right) + \kappa\kappa_\pi} < 1,$$

where I assume in line with the literature that  $\kappa_{\pi} \ge 0$  and  $\kappa_{y} \ge 0$ . This condition can be rewritten as:

$$\kappa \kappa_{\pi} + \kappa_{y} + \sigma \left[ \frac{\bar{y}}{\bar{c}_{1}} \left( 1 - \frac{\bar{c}_{2}}{\bar{y}} \right) - \beta \right] > 0.$$

The second condition is that  $|a_1| < 1 + a_0$ . Again assuming that  $\kappa_{\pi} \ge 0$  and  $\kappa_y \ge 0$ , I find that:

$$|a_1| = \frac{\beta \left(\kappa_y + \sigma \left(\bar{y}/\bar{c}_1\right) \left(1 - \frac{\bar{c}_2}{\bar{y}}\right)\right) + \kappa + \sigma}{\kappa_y + \sigma \left(\bar{y}/\bar{c}_1\right) \left(1 - \frac{\bar{c}_2}{\bar{y}}\right) + \kappa \kappa_\pi},$$

Back to the condition that  $|a_1| < 1 + a_0$ , which boils down to:

$$\frac{\beta\left(\kappa_{y}+\sigma\left(\bar{y}/\bar{c}_{1}\right)\left(1-\frac{\bar{c}_{2}}{\bar{y}}\right)\right)+\kappa+\sigma}{\kappa_{y}+\sigma\left(\bar{y}/\bar{c}_{1}\right)\left(1-\frac{\bar{c}_{2}}{\bar{y}}\right)+\kappa\kappa_{\pi}} < \frac{\kappa_{y}+\sigma\left(\bar{y}/\bar{c}_{1}\right)\left(1-\frac{\bar{c}_{2}}{\bar{y}}\right)+\kappa\kappa_{\pi}+\beta\sigma}{\kappa_{y}+\sigma\left(\bar{y}/\bar{c}_{1}\right)\left(1-\frac{\bar{c}_{2}}{\bar{y}}\right)+\kappa\kappa_{\pi}}$$
(215)

Multiplying by the denominator and rearranging, I can write this condition as:

$$\kappa \left(\kappa_{\pi} - 1\right) + \left(1 - \beta\right) \left[\kappa_{y} + \sigma \left(\bar{y}/\bar{c}_{1}\right) \left(1 - \frac{\bar{c}_{2}}{\bar{y}}\right) - \sigma\right] > 0, \qquad (216)$$

### B.9.2 The fiscal regime

Next, I study stability under the fiscal regime. As in the main text, I employ the government spending rule from the main text (11):

$$\hat{g}_t = g_\pi \hat{\pi}_t + g_y \tilde{y}_t.$$

while I set the nominal interest rate equal to steady state, i.e.  $\hat{R}_t^n = 0$ . After substitution of the government spending rule into the aggregate demand equation, and combining this with the New Keynesian Phillips curve (200), I get the following two by two system of equations:

$$\begin{aligned} \hat{\pi}_t &= \beta E_t \left[ \hat{\pi}_{t+1} \right] + \kappa \tilde{y}_t, \\ -\varphi B^* g_\pi \hat{\pi}_t + \left[ \sigma \left( \bar{y} / \bar{c}_1 \right) \left( 1 - \frac{\bar{c}_2}{\bar{y}} \right) - \varphi B^* g_y \right] \tilde{y}_t &= (1 + B^* \sigma g_\pi) E_t \left[ \hat{\pi}_{t+1} \right] + \sigma \left( 1 + B^* g_y \right) E_t \left[ \hat{y}_{t+1} \right] \\ &+ \hat{R}_t^{z*} + \hat{R}_t^{\xi*}, \end{aligned}$$

I write this in the following way:

$$\begin{pmatrix} 1 & -\kappa \\ -\varphi B^* g_{\pi} & \sigma \left( \bar{y}/\bar{c}_1 \right) \left( 1 - \frac{\bar{c}_2}{\bar{y}} \right) - \varphi B^* g_y \end{pmatrix} \begin{pmatrix} \hat{\pi}_t \\ \tilde{y}_t \end{pmatrix} = \begin{pmatrix} \beta & 0 \\ 1 + B^* \sigma g_{\pi} & \sigma \left( 1 + B^* g_y \right) \end{pmatrix} \begin{pmatrix} E_t \left[ \hat{\pi}_{t+1} \right] \\ E_t \left[ \tilde{y}_{t+1} \right] \end{pmatrix} \\ + \begin{pmatrix} 0 \\ \hat{R}_t^{z*} + \hat{R}_t^{\xi*} \end{pmatrix},$$

This can be rewritten in the following way:

$$\begin{pmatrix} \hat{\pi}_t \\ \tilde{y}_t \end{pmatrix} = M \begin{pmatrix} E_t \left[ \hat{\pi}_{t+1} \right] \\ E_t \left[ \tilde{y}_{t+1} \right] \end{pmatrix} + N_t$$

where M and N are given by:

$$M = \frac{1}{D} \begin{pmatrix} \beta \left[ \sigma \left( \bar{y}/\bar{c}_1 \right) \left( 1 - \frac{\bar{c}_2}{\bar{y}} \right) - \varphi B^* g_y \right] + \kappa \left( 1 + B^* \sigma g_\pi \right) & \kappa \sigma \left( 1 + B^* g_y \right) \\ \beta \varphi B^* g_\pi + 1 + B^* \sigma g_\pi & \sigma \left( 1 + B^* g_y \right) \end{pmatrix},$$
(217)

$$N = \frac{1}{D} \begin{pmatrix} \sigma \left( \bar{y}/\bar{c}_{1} \right) \left( 1 - \frac{\bar{c}_{2}}{\bar{y}} \right) - \varphi B^{*} g_{y} & \kappa \\ \varphi B^{*} g_{\pi} & 1 \end{pmatrix} \begin{pmatrix} 0 \\ \hat{R}_{t}^{z*} + \hat{R}_{t}^{\xi*} \end{pmatrix},$$
(218)

where D is given by:

$$D = \sigma \left( \bar{y}/\bar{c}_1 \right) \left( 1 - \frac{\bar{c}_2}{\bar{y}} \right) - \varphi B^* \left( g_y + \kappa g_\pi \right), \qquad (219)$$

and where I note that:

$$\begin{pmatrix} 1 & -\kappa \\ -\varphi B^* g_\pi & \sigma \left( \bar{y}/\bar{c}_1 \right) \left( 1 - \frac{\bar{c}_2}{\bar{y}} \right) - \varphi B^* g_y \end{pmatrix}^{-1} = \frac{1}{D} \begin{pmatrix} \sigma \left( \bar{y}/\bar{c}_1 \right) \left( 1 - \frac{\bar{c}_2}{\bar{y}} \right) - \varphi B^* g_y & \kappa \\ \varphi B^* g_\pi & 1 \end{pmatrix}.$$

The trace and the determinant of M are given by:

$$\operatorname{trace} M = \frac{\beta \left[ \sigma \left( \bar{y}/\bar{c}_{1} \right) \left( 1 - \frac{\bar{c}_{2}}{\bar{y}} \right) - \varphi B^{*} g_{y} \right] + \kappa \left( 1 + B^{*} \sigma g_{\pi} \right) + \sigma \left( 1 + B^{*} g_{y} \right)}{\sigma \left( \bar{y}/\bar{c}_{1} \right) \left( 1 - \frac{\bar{c}_{2}}{\bar{y}} \right) - \varphi B^{*} \left( g_{y} + \kappa g_{\pi} \right)},$$

$$(220)$$

$$\det M = \frac{\beta \sigma \left(1 + B^* g_y\right)}{\sigma \left(\bar{y}/\bar{c}_1\right) \left(1 - \frac{\bar{c}_2}{\bar{y}}\right) - \varphi B^* \left(g_y + \kappa g_\pi\right)},\tag{221}$$

As such, I now obtain the following Bullard and Mitra (2002) coefficients:

$$a_{1} = -\operatorname{trace} M = -\frac{\beta \left[\sigma \left(\bar{y}/\bar{c}_{1}\right) \left(1 - \frac{\bar{c}_{2}}{\bar{y}}\right) - \varphi B^{*}g_{y}\right] + \kappa \left(1 + B^{*}\sigma g_{\pi}\right) + \sigma \left(1 + B^{*}g_{y}\right)}{\sigma \left(\bar{y}/\bar{c}_{1}\right) \left(1 - \frac{\bar{c}_{2}}{\bar{y}}\right) - \varphi B^{*}\left(g_{y} + \kappa g_{\pi}\right)},$$

$$(222)$$

$$a_0 = \det M = \frac{\beta \sigma \left(1 + B^* g_y\right)}{\sigma \left(\bar{y}/\bar{c}_1\right) \left(1 - \frac{\bar{c}_2}{\bar{y}}\right) - \varphi B^* \left(g_y + \kappa g_\pi\right)},\tag{223}$$

Now assuming that  $g_{\pi} \leq 0$  and  $g_y \leq 0$  implies that the denominator of  $a_1$  and  $a_0$  are positive. If I assume that in addition that the numerator of  $a_1$  is positive, I get the following expression:

$$|a_1| = \frac{\beta \left[ \sigma \left( \bar{y}/\bar{c}_1 \right) \left( 1 - \frac{\bar{c}_2}{\bar{y}} \right) - \varphi B^* g_y \right] + \kappa \left( 1 + B^* \sigma g_\pi \right) + \sigma \left( 1 + B^* g_y \right)}{\sigma \left( \bar{y}/\bar{c}_1 \right) \left( 1 - \frac{\bar{c}_2}{\bar{y}} \right) - \varphi B^* \left( g_y + \kappa g_\pi \right)}.$$

Applying  $|a_1| < 1 + a_0$  gives:

$$\frac{\beta \left[\sigma \left(\bar{y}/\bar{c}_{1}\right)\left(1-\frac{\bar{c}_{2}}{\bar{y}}\right)-\varphi B^{*}g_{y}\right]+\kappa \left(1+B^{*}\sigma g_{\pi}\right)+\sigma \left(1+B^{*}g_{y}\right)}{\sigma \left(\bar{y}/\bar{c}_{1}\right)\left(1-\frac{\bar{c}_{2}}{\bar{y}}\right)-\varphi B^{*}\left(g_{y}+\kappa g_{\pi}\right)}$$

$$\frac{\sigma \left(\bar{y}/\bar{c}_{1}\right)\left(1-\frac{\bar{c}_{2}}{\bar{y}}\right)-\varphi B^{*}\left(g_{y}+\kappa g_{\pi}\right)+\beta \sigma \left(1+B^{*}g_{y}\right)}{\sigma \left(\bar{y}/\bar{c}_{1}\right)\left(1-\frac{\bar{c}_{2}}{\bar{y}}\right)-\varphi B^{*}\left(g_{y}+\kappa g_{\pi}\right)}.$$

Multiplication with the positive denominator results in the following condition:

$$\beta \left[ \sigma \left( \bar{y}/\bar{c}_1 \right) \left( 1 - \frac{\bar{c}_2}{\bar{y}} \right) - \varphi B^* g_y \right] + \kappa \left( 1 + B^* \sigma g_\pi \right) + \sigma \left( 1 + B^* g_y \right) < \sigma \left( \bar{y}/\bar{c}_1 \right) \left( 1 - \frac{\bar{c}_2}{\bar{y}} \right) - \varphi B^* \left( g_y + \kappa g_\pi \right) + \beta \sigma \left( 1 + B^* g_y \right).$$

This condition can be rewritten in the following way:

$$\kappa \left[-B^*\left(\sigma+\varphi\right)g_{\pi}-1\right] + \left(1-\beta\right)\left[-\left(\sigma+\varphi\right)B^*g_y + \sigma\left(\bar{y}/\bar{c}_1\right)\left(1-\frac{\bar{c}_2}{\bar{y}}\right) - \sigma\right] > 0,\tag{224}$$

Comparing inequality (224) with inequality (216), we see that there is a direct mapping from the monetary policy coefficients  $\kappa_{\pi}$  and  $\kappa_{y}$  to the government spending coefficients  $g_{\pi}$  and  $g_{y}$  that makes the two inequalities isomorphic:

$$\kappa_{\pi} = -B^* \left(\sigma + \varphi\right) g_{\pi} \Longrightarrow g_{\pi} = -\frac{\kappa_{\pi}}{B^* \left(\sigma + \varphi\right)}, \qquad (225)$$

$$\kappa_y = -B^* \left(\sigma + \varphi\right) g_y \Longrightarrow g_y = -\frac{\kappa_y}{B^* \left(\sigma + \varphi\right)}, \tag{226}$$

### **B.10** Analytical expressions for impulse response functions

In the previous section I showed that there is an isomorphic mapping from the stability conditions for the monetary regime to the stability conditions for the fiscal regime. In this section I calculate analytical expressions for the impulse response functions to the productivity and preference shocks, and show that there also exists an isomorphic mapping between the coefficients of the monetary and fiscal policy reactions such that the impulse response functions are identical.

### B.10.1 Monetary regime

I start by writing down the two-system equations for the monetary regime, where I replace  $\hat{R}_t^{x*} = \mathcal{R}^{x*} \hat{x}_t$ , where  $x = \{z, \xi\}$ .

$$\hat{\pi}_t = \beta E_t \left[ \hat{\pi}_{t+1} \right] + \kappa \tilde{y}_t,$$
  
$$\sigma \left( \bar{y}/\bar{c}_1 \right) \left( 1 - \frac{\bar{c}_2}{\bar{y}} \right) \tilde{y}_t = \sigma E_t \left[ \hat{y}_{t+1} \right] - \left( \kappa_\pi \hat{\pi}_t + \kappa_y \tilde{y}_t - E_t \left[ \hat{\pi}_{t+1} \right] - \mathcal{R}^{z*} \hat{z}_t - \mathcal{R}^{\xi*} \hat{\xi}_t \right),$$

Since there are no endogenous backward-looking state variables, I know that the only state variables are  $\hat{z}_t$  and  $\hat{\xi}_t$ . Hence I can employ the method of undetermined coefficients to find the analytical solution to productivity and preference shocks. I assume that  $\hat{\pi}_t$  and  $\tilde{y}_t$  are given by the following solutions:

$$\hat{\pi}_t = \alpha_{\pi,z} \hat{z}_t + \alpha_{\pi,\xi} \hat{\xi}_t, \qquad (227)$$

$$\tilde{y}_t = \alpha_{y,z} \hat{z}_t + \alpha_{y,\xi} \hat{\xi}_t.$$
(228)

Since both shocks are given by exogenous AR(1) shocks, I know that their expected value is given by:

$$E_t \left[ \hat{\pi}_{t+1} \right] = \rho_z \alpha_{\pi,z} \hat{z}_t + \rho_\xi \alpha_{\pi,\xi} \hat{\xi}_t, \qquad (229)$$

$$E_t \left[ \tilde{y}_{t+1} \right] = \rho_z \alpha_{y,z} \hat{z}_t + \rho_\xi \alpha_{y,\xi} \xi_t.$$
(230)

Substitution of the above expressions into the New Keynesian Phillips curve gives the following relations between the inflation and output gap coefficients:

$$\alpha_{y,z} = \left(\frac{1-\beta\rho_z}{\kappa}\right)\alpha_{\pi,z},\tag{231}$$

$$\alpha_{y,\xi} = \left(\frac{1-\beta\rho_{\xi}}{\kappa}\right)\alpha_{\pi,\xi} \tag{232}$$

Substitution of the guessed solutions for the output gap and inflation, and the relation between the output gap coefficients and the inflation coefficients into the aggregate demand equation generate the following expressions for the coefficients:

$$\alpha_{\pi,z}^{m} = \frac{\kappa \mathcal{R}^{z*}}{\kappa \left(\kappa_{\pi} - \rho_{z}\right) + \left[\kappa_{y} + \sigma \left(\bar{y}/\bar{c}_{1}\right) \left(1 - \frac{\bar{c}_{2}}{\bar{y}}\right) - \sigma \rho_{z}\right] \left(1 - \beta \rho_{z}\right)}, \quad (233)$$

$$\alpha_{\pi,\xi}^{m} = \frac{\kappa \kappa^{\gamma}}{\kappa \left(\kappa_{\pi} - \rho_{\xi}\right) + \left[\kappa_{y} + \sigma \left(\bar{y}/\bar{c}_{1}\right) \left(1 - \frac{\bar{c}_{2}}{\bar{y}}\right) - \sigma \rho_{\xi}\right] \left(1 - \beta \rho_{\xi}\right)}, \quad (234)$$

$$\alpha_{y,z}^{m} = \frac{(1-\beta\rho_{z}) \mathcal{K}^{*}}{\kappa \left(\kappa_{\pi} - \rho_{z}\right) + \left[\kappa_{y} + \sigma \left(\bar{y}/\bar{c}_{1}\right) \left(1 - \frac{\bar{c}_{2}}{\bar{y}}\right) - \sigma\rho_{z}\right] (1-\beta\rho_{z})}, \quad (235)$$

$$\alpha_{y,\xi}^{m} = \frac{(1-\rho\rho_{\xi}) \mathcal{K}^{s}}{\kappa \left(\kappa_{\pi} - \rho_{\xi}\right) + \left[\kappa_{y} + \sigma \left(\bar{y}/\bar{c}_{1}\right) \left(1 - \frac{\bar{c}_{2}}{\bar{y}}\right) - \sigma\rho_{\xi}\right] \left(1 - \beta\rho_{\xi}\right)}, \quad (236)$$

where  $\mathcal{R}^{z*}$  and  $\mathcal{R}^{\xi*}$  are given by:

$$\mathcal{R}^{z*} = -\left(\frac{\sigma\left(1+\varphi\right)}{\sigma\left(\bar{y}/\bar{c}_{1}\right)\left(1-\frac{\bar{c}_{2}}{\bar{y}}\right)+\varphi}\right)\left(\frac{\bar{y}}{\bar{c}_{1}}\left(1-\frac{\bar{c}_{2}}{\bar{y}}\right)-\rho_{z}\right),\qquad(237)$$

$$\mathcal{R}^{\xi*} = (1 - \rho_{\xi}) \tag{238}$$

#### B.10.2 Fiscal regime

Next I solve for the impulse response functions under the fiscal regime. The two equation system is again given by:

$$\begin{aligned} \hat{\pi}_t &= \beta E_t \left[ \hat{\pi}_{t+1} \right] + \kappa \tilde{y}_t, \\ -\varphi B^* g_\pi \hat{\pi}_t + \left[ \sigma \left( \bar{y} / \bar{c}_1 \right) \left( 1 - \frac{\bar{c}_2}{\bar{y}} \right) - \varphi B^* g_y \right] \tilde{y}_t &= (1 + B^* \sigma g_\pi) E_t \left[ \hat{\pi}_{t+1} \right] + \sigma \left( 1 + B^* g_y \right) E_t \left[ \hat{y}_{t+1} \right] \\ &+ \mathcal{R}^{z*} \hat{z}_t + \mathcal{R}^{\xi*} \hat{\xi}_t, \end{aligned}$$

Again employing the method of undetermined coefficients generates the same relationship between the inflation coefficients and the output gap coefficients, and eventually results in the following expressions for the coefficients:

$$\alpha_{\pi,z}^{f} = \frac{\kappa \mathcal{R}^{z*}}{\kappa \left[-B^{*}g_{\pi}\left(\varphi + \sigma\rho_{z}\right) - \rho_{z}\right] + \left[-B^{*}g_{y}\left(\varphi + \sigma\rho_{z}\right) + \sigma\left(\bar{y}/\bar{c}_{1}\right)\left(1 - \frac{\bar{c}_{2}}{\bar{y}}\right) - \sigma\rho_{z}\right]\left(1 - \beta\rho_{z}\right)},\tag{239}$$

$$\alpha_{\pi,\xi}^{f} = \frac{\kappa \mathcal{R}^{\xi*}}{\kappa \left[-B^{*}g_{\pi}\left(\varphi + \sigma\rho_{\xi}\right) - \rho_{\xi}\right] + \left[-B^{*}g_{y}\left(\varphi + \sigma\rho_{\xi}\right) + \sigma\left(\bar{y}/\bar{c}_{1}\right)\left(1 - \frac{\bar{c}_{2}}{\bar{y}}\right) - \sigma\rho_{\xi}\right]\left(1 - \beta\rho_{\xi}\right)},\tag{240}$$

$$\alpha_{y,z}^{f} = \frac{(1-\beta\rho_{z})\mathcal{R}^{z*}}{\kappa\left[-B^{*}g_{\pi}\left(\varphi+\sigma\rho_{z}\right)-\rho_{z}\right]+\left[-B^{*}g_{y}\left(\varphi+\sigma\rho_{z}\right)+\sigma\left(\bar{y}/\bar{c}_{1}\right)\left(1-\frac{\bar{c}_{2}}{\bar{y}}\right)-\sigma\rho_{z}\right]\left(1-\beta\rho_{z}\right)},$$
(241)

$$\alpha_{y,\xi}^{f} = \frac{(1-\beta\rho_{\xi})\mathcal{R}^{\xi*}}{\kappa\left[-B^{*}g_{\pi}\left(\varphi+\sigma\rho_{\xi}\right)-\rho_{\xi}\right] + \left[-B^{*}g_{y}\left(\varphi+\sigma\rho_{\xi}\right)+\sigma\left(\bar{y}/\bar{c}_{1}\right)\left(1-\frac{\bar{c}_{2}}{\bar{y}}\right)-\sigma\rho_{\xi}\right]\left(1-\beta\rho_{\xi}\right)},\tag{242}$$

Comparing the solutions (233) - (236) under the monetary regime with those under the fiscal regime (239) - (242), I see that there is an isomorphic mapping under which the equilibrium paths for inflation and the output gap are identical under both regimes. This is the case for the productivity shock when:

$$\kappa_{\pi} = -B^* \left( \sigma \rho_z + \varphi \right) g_{\pi} \Longrightarrow g_{\pi} = -\frac{\kappa_{\pi}}{B^* \left( \sigma \rho_z + \varphi \right)}, \tag{243}$$

$$\kappa_y = -B^* \left(\sigma \rho_z + \varphi\right) g_y \Longrightarrow g_y = -\frac{\kappa_y}{B^* \left(\sigma \rho_z + \varphi\right)}, \qquad (244)$$

while I have the following mapping for the preference shock:

$$\kappa_{\pi} = -B^* \left( \sigma \rho_{\xi} + \varphi \right) g_{\pi} \Longrightarrow g_{\pi} = -\frac{\kappa_{\pi}}{B^* \left( \sigma \rho_{\xi} + \varphi \right)}, \tag{245}$$

$$\kappa_y = -B^* \left(\sigma \rho_{\xi} + \varphi\right) g_y \Longrightarrow g_y = -\frac{\kappa_y}{B^* \left(\sigma \rho_{\xi} + \varphi\right)}, \qquad (246)$$

## C Calibration

The numerical values for the relevant parameters can be found in Table 1. The parameter  $\kappa$  that is part of the linearized Phillips-curve can be found from the expression  $\kappa \equiv [\sigma (\bar{y}/\bar{c}) + \varphi] (1 - \psi) (1 - \beta \psi) / \psi$ , an expression which will be derived in the Appendix.

Parameter	Definition	Value
Households		
eta	Subjective discount factor	1
$\sigma$	CRRA coefficient	1
arphi	Inverse Frisch elasticity	1
Production Sector		
$\epsilon$	Elasticity of substit. (goods)	10
$\psi$	Calvo prob. (price stickiness)	0.75
Policy Parameters		
$\bar{\pi}$	Inflation rate target	1
$ar{g}/ar{y}$	Steady state govt expenditures	0.2
Shock processes		
$ ho_z$	Productivity shock	0.95
$ ho_{\xi}$	Preference shock	0.9
$\sigma_z$	Std. dev. productivity shock	0.01
$\sigma_{\xi}$	Std. dev. preference shock	0.01

Table 1: List of calibrated parameter values and source of calibration.

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