## 2020011-EEF

## Unintended Consequences of Central Bank Lending in Financial Crises

July 2020

Christiaan van der Kwaak

SOM is the research institute of the Faculty of Economics \& Business at the University of Groningen. SOM has six programmes:

- Economics, Econometrics and Finance
- Global Economics \& Management
- Innovation \& Organization
- Marketing
- Operations Management \& Operations Research
- Organizational Behaviour

Research Institute SOM
Faculty of Economics \& Business
University of Groningen
Visiting address:
Nettelbosje 2
9747 AE Groningen
The Netherlands
Postal address:
P.O. Box 800

9700 AV Groningen
The Netherlands
T +31 50363 9090/7068/3815
www.rug.nl/feb/research

# Uninteded Consequences of Central Bank Lending in Financial Crises 

Christiaan van der Kwaak<br>University of Groningen, Faculty of Economics and Business, Department of Economics, Econometrics and Finance<br>c.g.f.van.der.kwaak@rug.nl

# Unintended Consequences of Central Bank Lending in Financial Crises* 

Christiaan van der $\mathrm{Kwaak}^{\dagger}$

July 6, 2020


#### Abstract

I investigate the effectiveness of central bank lending to undercapitalized financial intermediaries in mitigating the macroeconomic impact of financial crises. I show that the requirement to pledge collateral has a contractionary effect on private credit everything else equal when central banks provide more funding for one euro of government bonds than for one euro of private credit. I apply the model to the Italian economy during the time of the ECB's three-year Longer-Term Refinancing Operations (LTROs), and show that this collateral effect can explain why Italian banks' private credit grew by only $2 \%$ while their holdings of domestic government bonds grew by $30 \%$. Finally, I find that the three-year LTROs contained an implicit subsidy to the Italian banking system of 140 basis points.


Keywords: 'Financial Intermediation; Macrofinancial Fragility; Unconventional Monetary Policy'

JEL: E32, E44, E52, G21

[^0]
## 1 Introduction

In this paper I investigate the effectiveness of central bank lending to undercapitalized financial intermediaries in mitigating the macroeconomic impact of financial crises. I show that the requirement to pledge collateral gives rise to a collateral effect that has a contractionary impact on the macroeconomy, everything else equal, and thereby reduces the expansionary effect that such lending otherwise has. This effect arises because central banks typically provide more funding for one euro of government bonds than for one euro of private credit. Consequently, the possibility to borrow from the central bank can induce intermediaries to reduce private credit to create additional space for government bonds when they have limited balance sheet capacity in a financial crisis. I apply the framework developed in this paper to the unconventional three-year LTROs of December 2011 and February 2012 under which the Italian banking system borrowed $€ 181.5$ billion from the European Central Bank (ECB). The collateral effect explains why Italian banks' private credit only grew by $2 \%$ (relative to no intervention Carpinelli and Crosignani, 2018) ), while their holdings of domestic sovereign debt increased by $30 \%$. While this paper's focus is on the three-year LTROs, my framework could also be relevant for studying the macroeconomic impact of other central banks' lending programs, such as the Federal Reserve's Term Auction Facility and the Treasury Securities Lending Facility.

I first analyze a two-period general equilibrium model incorporating leverage-constrained financial intermediaries that are partially financed through central bank funding. This model allows me to analytically establish the collateral effect and to identify the deep parameters that determine the relative strength of the collateral effect with respect to the expansionary effect that such lending also has (Gertler and Kiyotaki, 2010, Bocola, 2016, Engler and Große Steffen, 2016; Cahn et al., 2017). To demonstrate the empirical relevance, I construct a New Keynesian DSGE model with financial frictions which I estimate with the help of Bayesian techniques and a moment-matching exercise using Italian data.

I capture the fact that Italian banks were undercapitalized since the Great Financial Crisis (International Monetary Fund, 2011, Hoshi and Kashyap, 2015) by employing the Gertler and Karadi (2011) framework, in which an incentive compatibility constraint limits the size of intermediaries' balance sheet by the amount of net worth. I extend this framework in two directions. First, financial intermediaries have a portfolio choice between government bonds, reserves and corporate securities, the last of which is used by non-financial corporations to finance productive 'physical' capital (Gertler and Karadi, 2013; Van der Kwaak and Van Wijnbergen, 2014 , Kirchner and van Wijnbergen, 2016; Bocola, 2016). Second, I introduce collateralized central bank lending, which represents an alternative form of funding in addition to net worth and deposits. Intermediaries have to pledge collateral to obtain central bank funding, for which both government bonds and corporate securities can be used. However, one euro of government bonds provides more funding than one euro of corporate securities. The central bank supplies any amount of funding as long as sufficient collateral is pledged. I argue in Section 2 that the threeyear LTROs contained an implicit subsidy to Italian banks with respect to the ECB's regular
short-term funding. I capture this implicit subsidy by temporarily decreasing the interest rate on central bank funding with respect to that on reserves (Engler and Große Steffen, 2016), both of which are set by the central bank. I investigate the policy both within a closed economy and a small open economy that is a member of a currency union; these economies capture the two extremes in terms of the influence that Italian macrodevelopments have on the Italian policy rate, which is set by the ECB and based on macrodevelopments in the Eurozone as a whole ${ }_{\square}^{1}$

The main contribution of this paper is the identification of the collateral effect, and the key parameters that determine its strength. I show how this effect reduces or offsets the expansionary effect that central bank lending to intermediaries has on the macroeconomy in other New Keynesian models with financial frictions (Gertler and Kiyotaki, 2010, Bocola, 2016; Engler and Große Steffen, 2016, Cahn et al., 2017). The modeling innovation that gives rise to this effect is the combination of i) balance-sheet-constrained financial intermediaries that are subject to ii) differential collateral requirements when obtaining central bank funding. A second contribution is that the collateral effect explains the accumulation of domestic government bonds by Southern-European commercial banks following the announcement of the three-year LTROs as documented in Crosignani et al. (forthcoming) and Section 2 while simultaneously explaining the limited growth of Italian private credit by $2 \%$ (relative to no intervention, Carpinelli and Crosignani (2018)). A third contribution is to provide an estimate of the implicit subsidy to Italian commercial banks that was contained in the three-year LTROs, which I find to be equal to 140 annual basis points.

A final contribution is that my model provides an explanation for the empirical finding of Carpinelli and Crosignani (2018) that the maturity of the three-year LTROs of December 2011 and February 2012 was a key feature for these operations to have an expansionary effect on credit provision to the real economy: the longer financial intermediaries can profit from lower funding costs, the larger the increase in the expected discounted sum of future profits, and the larger the relaxation of their incentive compatibility constraints. While the collateral effect still induces a relative shift from corporate credit to government bonds, the longer maturity creates sufficient balance sheet space for financial intermediaries to simultaneously expand the level of credit provision to the real economy.

## Related literature

In this literature review I limit myself to the papers closest related to my paper. A more elaborate review can be found in Appendix A.

Drechsler et al. (2016), Carpinelli and Crosignani (2018), Garcia-Posada and Marchetti

[^1](2016), and Andrade et al. (2019) study the ECB's unconventional LTROs at the level of individual banks. Drechsler et al. (2016) focus on the role of the ECB as a Lender of Last Resort (LOLR) during the European sovereign debt crisis. They find that weakly capitalized banks borrowed more from the ECB, pledged riskier collateral, and actively invested the funds borrowed from the ECB in distressed sovereign debt after the start of the European sovereign debt crisis in 2010. Their sample, however, does not include the three-year LTRO of February 2012.

Carpinelli and Crosignani (2018), Garcia-Posada and Marchetti (2016), and Andrade et al. (2019) specifically focus on the three-year LTROs, and find a positive effect on credit provision to the real economy in Italy, Spain, and France, respectively. In addition, Andrade et al. (2019) find that three-year LTROs expand loan supply by more than shorter-maturity LTROs.

Other mechanisms that explain why banks were accumulating government bonds during the European sovereign debt crisis are moral suasion Altavilla et al., 2017, Becker and Ivashina, 2018; Ongena et al., 2019) and risk-shifting (Acharya and Steffen, 2015, Drechsler et al., 2016; Crosignani, 2016; Acharya et al., 2018). These papers also find that such an accumulation of government bonds reduced credit provision to the real economy. A second reason why credit provision to the real economy was reduced during the sovereign debt crisis was capital losses on impaired sovereign bond holdings on bank balance sheets (Popov and Horen, 2015, Altavilla et al., 2017, Acharya et al., 2018).

My paper also relates to Gertler and Kiyotaki (2010); Gertler and Karadi (2011, 2013), who study the transmission to the macroeconomy of shocks to the balance sheets of financial intermediaries. The key property of these papers is that the size of intermediaries' balance sheets is limited by the amount of net worth through an endogenous leverage constraint.

A key result of this paper is that there is crowding out of credit provision to the real economy by government bonds through the collateral effect. Other theoretical papers that feature crowding out are Kirchner and van Wijnbergen (2016) and Crosignani (2016), where it is caused by a debt-financed fiscal expansion increasing commercial banks' bond holdings (Kirchner and van Wijnbergen, 2016), and risk shifting (Crosignani 2016). Other reasons for a reduction in credit provision to the real economy are capital losses on government bonds that reduce intermediaries' net worth through the so-called bank-sovereign nexus (Van der Kwaak and Van Wijnbergen, 2014, Bocola, 2016).

My paper is also related to the Lender of Last Resort (LOLR) literature, of which Bagehot (1873) was the first to argue that central banks should lent freely against good collateral at high rates. In order for banks to take out central bank funding during a financial crisis, LOLR funding must be subsidized in some way relative to funding sources in private markets: otherwise LOLR lending would offer no benefit over the private market, and banks would not borrow from it. I capture this implicit subsidy by temporarily reducing the interest rate on central bank funding relative to that on deposit funding, in line with Engler and Große Steffen (2016).

The more recent literature that investigates the effects from central bank lending within the standard DSGE framework can broadly speaking be distinguished between collateralized and
uncollateralized lending. One of the first papers to explicitly model uncollateralized central bank lending is Gertler and Kiyotaki (2010). Bocola (2016) and Cahn et al. (2017) extend this framework to investigate the impact of the ECB's unconventional LTROs. These papers do not feature a collateral requirement, and therefore miss the contractionary collateral effect. As a result, LTROs only have an expansionary effect on bank lending and output because central bank lending directly relaxes intermediaries' incentive compatibility constraint.

A second strand of literature features a collateral requirement to obtain central bank funding, but the agents who borrow from the central bank are not balance-sheet-constrained (Schabert, 2015, Hörmann and Schabert, 2015, Engler and Große Steffen, 2016). As a result, these agents can perfectly elastically acquire additional collateral in case central bank funding becomes more attractive. This contrasts with my paper, where the combination of collateral requirements and endogenous leverage constraints causes a tradeoff to emerge between acquiring additional government bonds (which provide the most central bank funding per euro) and credit provision to the real economy.

I describe some stylized facts in section 2. The two-period model is analyzed in section 3, while the infinite-horizon model description can be found in section 4 , while section 5 discusses the calibration and estimation procedure. Section 6 presents the results from my simulations, while section 7 discusses the results and evaluates several robustness checks. Finally, section 8 concludes the paper.

## 2 Stylized facts

In this section I present some stylized facts regarding the aggregated balance sheets of Monetary Financial Institutions (MFIs) from Italy, Portugal and Spain at the time of the three-year LTROs. I do so for two reasons. First, I show that the three-year LTROs induced MFIs from these countries to purchase large amounts of domestic government bonds. Second, I argue that the three-year LTROs contained an implicit subsidy for MFIs from the above countries.

Data from the refinancing operations of the ECB were collected from Bruegel (2015), while balance sheet data of MFIs were collected from the ECB's statistical warehouse European Central Bank, 2015, $2^{2}$ The time series have a monthly frequency. Balance sheet data of MFIs, excluding the European System of Central Banks, are available at a country level ${ }^{3}$ The vast majority of euro-area MFIs are credit institutions (i.e., commercial banks, savings banks, postbanks, specialized credit institutions, among others) European Central Bank, 2011b).

Figure 1 shows domestic government bond holdings as a percentage of total assets of Monetary Financial Institutions (MFIs) excluding the European System of Central Banks in Italy, Spain,

[^2]and Portugal. From the figure we see a clear increase in domestic government bondholdings of one to one-and-a-half percentage points of total MFI assets for all three countries during the period in which the three-year LTROs took place. The increase in holdings of domestic government bonds is also large in absolute levels, amounting to a striking $30 \%$ measured in euros see Appendix H. Finally, there is a clear break in the holdings of domestic government bonds around the time of the three-year LTROs, which make it plausible that this increase can be attributed to the three-year LTROs. These results are in line with the findings of Carpinelli and Crosignani (2018) and Crosignani et al. (forthcoming).


Figure 1: Domestic government bond holdings as a percentage of total assets of Monetary Financial Institutions (MFIs) excluding the European System of Central Banks in Italy (IT), Spain (ES), and Portugal (PT) from January 2011 to January 2013. The two dashed vertical lines refer to December 1st, 2011 and March 1st, 2012, respectively, which mark the beginning and the end of the period in which the two LTROs took place, respectively. Source: ECB.

Figure 2 shows the stock of total refinancing operations at the ECB, as well as the country use by MFIs in Italy, Spain and Portugal. Total refinancing operations consist of the sum of main refinancing operations (MROs) and all longer-term refinancing operations (LTROs). MROs are one-week liquidity providing operations in euro, while regular LTROs are three-month liquidity providing operations ${ }^{4}$

Figure 2 suggests three main observations. First, the stock of total refinancing operations increased by more than $40 \%$ from $€ 800$ billion to approximately $€ 1150$ billion during the period in which the three-year LTROs took place.

Second, a disproportionate share of the funding went to MFIs in Italy, Spain and Portugal. By March 1st, 2012, more than $50 \%$ of total ECB funding had been borrowed by MFIs from these countries, while their cumulative share in Eurozone GDP is less than one-third. Apparently, the

[^3]Country use of ECB funding


Figure 2: Country use of the stock of total refinancing operations, consisting of the sum of of outstanding MROs and LTROs, by MFIs in Italy (IT), Spain (ES), Portugal (PT) and the rest of the Eurozone (RE) in € billion from January 2011 to January 2013. The two vertical lines refer to December 1st, 2011 and March 1st, 2012, which mark the beginning of the period in which the three-year LTROs took place and the end, respectively. Source: Bruegel (2015).
three-year LTROs of December 2011 and February 2012 were especially attractive for MFIs in Italy, Spain and Portugal.

Third, the use of ECB funding by MFIs from these three countries amounted to a large share of their respective GDP. On March 1st, 2012, ECB funding accounted for $€ 181.5$ billion and $€ 400$ billion of debt funding for Italian and Spanish MFIs, respectively, amounting to approximately $10 \%$ of Italian GDP and $40 \%$ of Spanish GDP, respectively.

The above observations suggest that the three-year LTROs were an attractive source of funding for MFIs from Italy, Spain and Portugal, as they borrowed significant amounts from the ECB. This raises the question whether there might have been a subsidy element that made these LTROs particularly attractive. At first sight, however, one would argue that this is not the case; the interest rate was 'fixed at the average rate of the main refinancing operations over the life of the respective operation' (European Central Bank, 2011a). As a result, there was no difference in terms of funding costs between a strategy where MFIs borrowed at the ECB at a three-year maturity, and a strategy where they borrowed at a weekly maturity, and roll over for three years (Carpinelli and Crosignani, 2018). Therefore, there was no direct subsidy from the ECB to the MFIs that participated in the three-year LTROs. However, there are two reasons why it can be argued that these unconventional LTROs contained an implicit subsidy.

First, after the bankruptcy of Lehman Brothers in September 2008 the ECB started to provide a so-called 'haircut subsidy' on risky securities such as distressed sovereign debt from the abovementioned countries (Drechsler et al. 2016). A haircut is the difference between the value of the
collateral pledged and the amount of funding received. The haircut subsidy entailed the ECB offering haircuts that were below private-market haircuts, thereby providing more funding for the same amount of collateral. Without any implicit subsidy, MFIs from Italy, Spain and Portugal would have been indifferent between private market funding and ECB funding, and would not have borrowed from the ECB (Drechsler et al., 2016).

Second, there was uncertainty whether MFIs would be able to continue to roll over MRO funding, as the fixed-rate full allotment policy under which this was possible was supposed to be a temporary measure ${ }^{5}$ The three-year LTROs eliminated this uncertainty, and was therefore more attractive than the ECB's regular MROs, despite the fact that the cumulative interest payments would be the same under the two strategies (Carpinelli and Crosignani, 2018).

For these two reasons I argue that the three-year LTROs contained an implicit subsidy to MFIs from Italy, Spain and Portugal. This was different for MFIs from countries such as Germany and the Netherlands, whose domestic bonds were not subject to a haircut subsidy, and who took out relatively little ECB funding over this period, as private market funding offered equally or more attractive sources of funding (Carpinelli and Crosignani, 2018).

## 3 Analytical results within a two period model

In this section I develop a two-period model to analyse the key mechanisms that affect lending by undercapitalized financial intermediaries to the real economy when the central bank provides them with low-interest-rate funding. In particular, I show that such a policy can potentially have a contractionary effect on lending, despite lowering funding costs for financial intermediaries. In addition, I investigate the way in which deep parameters affect lending decisions to prepare for the quantitative analysis in Section 6.

### 3.1 Model setup

The economy contains periods $t=0$ and $t=1$ and is populated by households, production firms, financial intermediaries, and a government. The government consists of a fiscal authority and a central bank, which sets the interest rate on central bank reserves and on loans to financial intermediaries. Financial intermediaries have access to unlimited amounts of central bank funding, provided that they pledge sufficient government bonds as collateral. In addition, they are financed by household deposits and net worth. Assets consist of corporate loans to production firms, government bonds and central bank reserves, presenting intermediaries with a portfolio choice similar to Gertler and Karadi (2013), and Bocola 2016). Intermediaries are subject to an incentive compatibility constraint as in Gertler and Karadi (2011), which prevents them from perfectly elastically expanding the balance sheet in case of arbitrage opportunities. Households

[^4]choose in period $t=0$ between consumption and saving through deposits and government bonds, which are subject to quadratic transaction costs when their bond holdings deviate from the target level. Income in period $t=1$ is consumed after lump sum taxes have been paid to the fiscal authority. Households have a standard utility function that is concave in consumption. Production firms borrow from financial intermediaries in period $t=0$ to purchase physical capital in a perfectly competitive market, and use this capital to produce goods in period $t=1$ using a production function that is concave in physical capital. After paying intermediaries the marginal product of capital in period $t=1$, the remaining profits are transferred to households. The fiscal authority enters period $t=0$ with outstanding long-term bonds that are held by households and financial intermediaries. No revenues or expenditures are raised in period $t=0$, and hence the stock of long-term bonds at the end of period $t=0$ is equal to that at the beginning of period $t=0$. At the beginning of period $t=1$, the fiscal authority receives central bank profits, and raises lump sum taxes on households to repay bondholders.

### 3.1.1 Central bank

Central bank reserves $m_{0}^{R}$ enter the economy through lending $d_{0}^{c b}$ to financial intermediaries. I assume the central bank has zero net worth in period $t=0$. Therefore, the central bank balance sheet is given by $d_{0}^{c b}=m_{0}^{R}$. To obtain central bank funding, intermediaries have to pledge government bonds $q_{0}^{b} s_{0}^{b}$ as collateral:

$$
\begin{equation*}
d_{0}^{c b} \leq \theta^{b} q_{0}^{b} s_{0}^{b} \tag{1}
\end{equation*}
$$

where $0 \leq \theta^{b}<1$ is set by the central bank, and determines how much central bank funding is obtained for one euro of government bonds ${ }^{6}$ Intermediaries remain the legal owner of the bonds, and receive the accompanying cash flows after repayment of the central bank loan in period $t=1$.

The central bank sets the interest rate $r_{0}^{R}$ on central bank reserves $m_{0}^{R}$ and the interest rate $r_{0}^{c b}$ on central bank funding to intermediaries $d_{0}^{c b}$. In line with the ECB's fixed rate full allotment policy, the central bank provides any amount of central bank funding (full allotment) as long as sufficient government bonds are pledged as collateral. 7 Central bank profits in period $t=1$ are transferred to the fiscal authority.

I argued in Section 2 that the three-year LTROs contained an implicit subsidy which I capture by decreasing the interest rate on central bank funding with respect to that on reserves, which will turn out to be equal to the interest rate on deposits in equilibrium. As a result, central

[^5]bank funding becomes a more attractive source of funding than deposit funding (Engler and Große Steffen, 2016.

### 3.1.2 Financial intermediaries

Financial intermediaries enter period $t=0$ with net worth $n_{0}$. They attract deposits $d_{0}$ from households, and obtain funding $d_{0}^{c b}$ from the central bank to purchase government bonds $s_{0}^{b}$ at a price $q_{0}^{b}$, finance loans $s_{0}^{k}$ to production firms, and keep reserves $m_{0}^{R}$ at the central bank:

$$
\begin{equation*}
s_{0}^{k}+q_{0}^{b} s_{0}^{b}+m_{0}^{R}=n_{0}+d_{0}+d_{0}^{c b} . \tag{2}
\end{equation*}
$$

As discussed above, the central bank requires intermediaries to pledge government bonds $q_{0}^{b} s_{0}^{b}$ as collateral. Loans $s_{0}^{k}$ and government bonds $q_{0}^{b} s_{0}^{b}$ pay a net return $r_{0}^{k}$ and $r_{0}^{b}$ in period $t=1$ respectively, while reserves earn an interest rate $r_{0}^{R}$. Intermediaries pay a net interest $r_{0}^{d}$ on deposits and $r_{0}^{c b}$ on central bank funding. Therefore, net worth $n_{1}$ in period $t=1$ is given by:

$$
\begin{equation*}
n_{1}=\left(1+r_{0}^{k}\right) s_{0}^{k}+\left(1+r_{0}^{b}\right) q_{0}^{b} s_{0}^{b}+\left(1+r_{0}^{R}\right) m_{0}^{R}-\left(1+r_{0}^{d}\right) d_{0}-\left(1+r_{0}^{c b}\right) d_{0}^{c b} . \tag{3}
\end{equation*}
$$

Intermediaries are interested in maximizing expected discounted net worth $E_{0}\left[\beta \Lambda_{0,1} n_{1}\right]$, where $\beta \Lambda_{0,1}$ denotes the households' stochastic discount factor, as households are the ultimate owners of financial intermediaries. However, intermediaries face an incentive compatibility constraint as in Gertler and Karadi (2011) that arises from the possibility to costlessly divert a fraction $\lambda_{a}$ of asset $a \in\{a=k, b\}$ at the end of period $t=0 \sqrt{8}^{8}$ Depositors, however, anticipate this possibility, and will in equilibrium only provide deposits up to the point where the continuation value of the intermediary is larger than or equal to the benefits from diverting assets:

$$
\begin{equation*}
E_{0}\left[\beta \Lambda_{0,1} n_{1}\right] \geq \lambda_{k} s_{0}^{k}+\lambda_{b} q_{0}^{b} s_{0}^{b} . \tag{4}
\end{equation*}
$$

The optimization problem of intermediaries is given by maximizing $E_{0}\left[\beta \Lambda_{0,1} n_{1}\right]$ subject to (1) - (4). In Appendix B, I derive the first order conditions, and show that the interest rate on central bank reserves $r_{0}^{R}$ equals the interest rate on deposits $r_{0}^{d}$, as financial intermediaries can perfectly elastically attract additional deposits to increase reserves. Next, I consider the first order condition that pins down the portfolio choice between corporate loans and government bonds:

$$
\begin{equation*}
\frac{\lambda_{b}}{\lambda_{k}} E_{0}\left[\beta \Lambda_{0,1}\left(r_{0}^{k}-r_{0}^{d}\right)\right]=E_{0}\left[\beta \Lambda_{0,1}\left(r_{0}^{b}-r_{0}^{d}\right)\right]+\underbrace{\theta^{b} E_{0}\left[\beta \Lambda_{0,1}\left(r_{0}^{d}-r_{0}^{c b}\right)\right]}_{\substack{\text { Collateral value of } \\ \text { gov't bonds }}}, \tag{5}
\end{equation*}
$$

[^6]Throughout my analysis, I assume that $r_{0}^{d} \geq r_{0}^{c b}$. Otherwise, intermediaries would not use any central bank funding, as deposit funding would have lower costs while not requiring intermediaries to pledge collateral ${ }^{10}$ The first two terms are familiar from Gertler and Karadi (2013). The left hand side denotes the marginal cost from reducing corporate loans by one euro, as expected net worth decreases by $r_{0}^{k}-r_{0}^{d}$ everything else equal. This wedge between the return on corporate loans and deposits exists because of the binding incentive compatibility constraint (4). Similarly, the first term on the right hand side denotes an increase in expected net worth from increasing government bonds by one euro. However, the first order condition contains an additional term relative to Gertler and Karadi (2013) which captures the collateral value that government bonds provide: an additional euro of government bonds provides $\theta_{b}$ euros of central bank funding, which reduces intermediaries' funding costs when $r_{0}^{c b}<r_{0}^{d}$, and thereby raises their expected net worth everything else equal. As such, we see that the possibility to pledge government bonds as collateral shifts intermediaries' portfolio choice from corporate loans to government bonds everything else equal.

In addition, observe that the collateral value increases with the interest rate difference $r_{0}^{d}-r_{0}^{c b}$ : in that case an additional euro of government bonds decreases funding costs by more, and intermediaries will therefore want to increase their stock of government bonds. Finally, central bank lending will not affect intermediaries' portfolio decisions when $r_{0}^{d}=r_{0}^{c b}$. In that case, intermediaries are indifferent between deposit funding and central bank funding. As a result, the collateral value of government bonds is zero, and intermediaries' portfolio choice between corporate loans and government bonds is only determined by the expected return differences between corporate loans and government bonds on the one hand, and deposits on the other.

Next, I employ the intermediaries' first order conditions in Appendix B , together with the law of motion for net worth (3), to rewrite the incentive compatibility constraint (4) in the following way:

$$
\begin{equation*}
\left(1+\mu_{0}\right) n_{0} \geq \lambda_{k} s_{0}^{k}+\lambda_{b} q_{0}^{b} s_{0}^{b} \tag{6}
\end{equation*}
$$

where $\mu_{0}$ denotes the multiplier on the incentive compatibility constraint (4). This (in)equality says that the weighted sum of loans $s_{0}^{k}$ and government bonds $q_{0}^{b} s_{0}^{b}$ is limited by the amount of net worth $n_{0}$ when constraint (6) is binding. In that case, equality (6) can be interpreted as the intermediary being undercapitalized, which is the relevant case in this paper, since European commercial banks have been undercapitalized since the financial crisis of 2007-2009 (International Monetary Fund, 2011, Hoshi and Kashyap, 2015). Finally, I assume that intermediaries carry over bond holdings $s_{-1}^{b}$ that were acquired in period $t=-1$, as commercial banks in SouthernEurope already had large holdings of domestic government bonds before the announcement of the three-year LTROs. As a result, net worth $n_{0}$ depends on the bond price in period $t=0$ :

$$
\begin{equation*}
n_{0}=n_{0}^{e x}+q_{0}^{b} s_{-1}^{b} \tag{7}
\end{equation*}
$$

[^7]where $n_{0}^{e x}$ does not depend upon decisions taken in period $t=0$. Therefore, an increase in the bond price $q_{0}^{b}$ relaxes intermediaries' incentive compatibility constraint (6) everything else equal, and allows them to expand their balance sheet.

### 3.2 Analysis of a decrease in central bank funding costs

The main goal of this section is to investigate the short-run effect on credit provision to the real economy of a policy under which the interest rate on central bank funding $r_{0}^{c b}$ is reduced while keeping the interest rate on reserves $r_{0}^{R}$ constant, a policy which I will refer to as LTRO-policy. I focus on credit provision to the real economy, as this is the key transmission mechanism through which the three-year LTROs should have affected the Eurozone economy. A second goal is to determine which deep parameters are driving the short-run impact of the LTRO-policy to inform my estimation procedure for the full infinite-horizon model in subsequent sections.

To enhance the analysis, I introduce the variable $\Gamma_{0}^{c b}$, which is the difference between the interest rate on reserves $r_{0}^{R}$ and central bank funding $r_{0}^{c b}$. Since the interest rates on reserves and deposits are equal in equilibrium, see Section 3.1.2, a decrease of the interest rate on central bank funding will reduce funding costs relative to deposit funding $\Gamma_{0}^{c b} \equiv r_{0}^{R}-r_{0}^{c b}=r_{0}^{d}-r_{0}^{c b}$ Engler and Große Steffen, 2016). There are no other shocks, therefore my analysis is deterministic.

I start the analysis by differentiating the incentive compatibility constraint (6), the first order condition for intermediaries' portfolio choice between corporate loans and government bond (5), and the market clearing condition for government bonds with respect to $\Gamma_{0}^{c b}$. I subsequently substitute the last two expressions into the first to obtain the following results, the details of which can be found in Appendix B.

Proposition 1. The bond price $q_{0}^{b}$ always increases in response to an increase in $\Gamma_{0}^{c b}$, i.e. $\frac{d q_{0}^{b}}{d \Gamma_{0}^{c b}}>0$.
Proof of Proposition 1. See Appendix B.
The intuition is the following: an increase in $\Gamma_{0}^{c b}$ induces intermediaries to shift from deposit funding to central bank funding. As they need to pledge additional government bonds as collateral, the demand for bonds increases while the supply is unchanged. Therefore, the bond price has to increase to clear the market. This result is in line with the observed drop in SouthernEuropean bond yields around the time of the three-year LTROs, as yields move inversely with bond prices (Crosignani et al., forthcoming, Krishnamurthy et al., 2018).

Having established that bond prices will always increase, we can immediately see that intermediaries' net worth $n_{0}$ will always increase as a result of the LTRO-policy:

Proposition 2. Net worth $n_{0}$ always increases in response to an increase in $\Gamma_{0}^{c b}$, i.e. $\frac{d n_{0}}{d \Gamma_{0}^{b b}}>0$. Proof of Proposition 2. Differentiation of equation (7) gives the following derivative: $\frac{d n_{0}}{d \Gamma_{0}^{c b}}=$ $s_{-1}^{b} \cdot \frac{d q_{0}^{b}}{d \Gamma_{0}^{b}}>0$, where the last inequality follows from Proposition 1 .

Hence the LTRO-policy always increases intermediaries' net worth $n_{0}$ as a result of capital gains on intermediaries' existing bond holdings. Therefore, the policy relaxes intermediaries' incentive compatibility constraints everything else equal, which allows them to expand their balance sheets, for example by expanding credit provision to the real economy. Indeed, such an indirect recapitalization of the financial sector was found to have an empirically relevant effect on credit supply in the context of the ECB's Outright Monetary Transactions (OMT) program (Acharya et al., 2019).

Interestingly, we see in Proposition 3 that such an indirect recapitalization does not necessarily expand credit provision to the real economy, and can even have a contractionary effect on credit provision:

Proposition 3. The impact of a marginal increase in $\Gamma_{0}^{c b}$ on credit provision to the real economy is ambiguous, i.e. $\frac{d s_{0}^{k}}{d \Gamma_{0}^{c b}} \lessgtr 0$.

Proof of Proposition 3. Differentiation of the incentive compatibility constraint (6) with respect to $\Gamma_{0}^{c b}$, and subsequent substition of Proposition 2 and the differentiated market clearing condition for government bonds gives the following expression for lending to the real ecnomy:

$$
\frac{d s_{0}^{k}}{d \Gamma_{0}^{c b}}=\frac{1}{\lambda_{k}-C n_{0}}(\underbrace{\left(1+\mu_{0}\right) s_{-1}^{b}}_{\begin{array}{c}
\text { capital gains }  \tag{8}\\
\text { effect }
\end{array}}-\underbrace{\lambda_{b}\left(s_{0}^{b}+\frac{q_{0}^{b}}{\kappa_{s_{b, h}}}\right)}_{\begin{array}{c}
\text { collateral } \\
\text { effect }
\end{array}}) \cdot \frac{d q_{0}^{b}}{d \Gamma_{0}^{c b}},
$$

where $C<0$, and $\kappa_{s_{b, h}}>0$ the coefficient in front of the quadratic adjustment costs facing households when purchasing government bonds. The sign of $(8)$ is ambiguous, since the collateral effect and the capital gains effect have opposite signs. Details can be found in Appendix B

Besides the above-mentioned capital gains effect, we see the emergence of a collateral effect that reduces credit provision to the real economy everything else equal: the shift from deposit funding to central bank funding forces intermediaries to purchase additional government bonds to be pledged as collateral. As a result, the market value of intermediaries' holdings of government bonds increases because of higher bond prices (first term of the collateral effect) and additional bonds purchased from households (second term of the collateral effect). As the size of their balance sheets is limited by the amount of net worth, lending to the real economy $s_{0}^{k}$ decreases everything else equal. Interestingly, we see from expression (8) that the net effect of the LTROpolicy on credit provision to the real economy can be negative if the collateral effect dominates the capital gains effect. This suggests that the policy could be counterproductive in improving short run macroeconomic conditions.

Note from equation (8) that the collateral effect is eliminated when $\lambda_{b}=0$, in which case intermediaries can expand their holdings of government bonds without tightening the incentive compatibility constraint (6). Alternatively, if existing bond holdings $s_{-1}^{b}$ are zero, intermediaries
will not incur any capital gains on existing bond holdings while there is crowding out of lending to the real economy when acquiring additional government bonds. In that case, the LTRO-policy is always contractionary when $\lambda_{b}>0$.

In addition to disentangling the capital gains effect and the collateral effect, the analysis also shows the crucial role of the coefficient governing households' transaction costs $\kappa_{s_{b, h}}$ in determining the strength of the collateral effect. To see how this parameter affects the collateral effect, I take the partial derivative of (8), which captures the direct effect of a change in $\kappa_{s_{b, h}}$.

Proposition 4. The direct effect from a marginal increase in $\kappa_{s_{b, h}}$ raises lending to the real economy: $\frac{\partial}{\partial \kappa_{s_{b, h}}}\left(\frac{d s_{0}^{k}}{d \Gamma_{0}^{c b}}\right)>0$.
Proof of Proposition 4. See Appendix B.
An increase in $\kappa_{s_{b, h}}$ raises households' marginal cost from changing their holdings of government bonds, which makes them less willing to sell government bonds everything else equal. Therefore, intermediaries will be able to buy fewer bonds in equilibrium, which reduces the strength of the collateral effect. At the same time, bond prices have to increase by more to achieve market clearing, which strengthens the capital gains effect. Therefore, lending to the real economy will increase in equilibrium.

To sum up: I show that the LTRO-policy has an ambiguous effect on lending by financial intermediaries to the real economy, which is the key transmission mechanism through which the LTRO-policy can affect macroeconomic conditions. I disentangle an expansionary capital gains effect and a contractionary collateral effect. This contractionary effect arises because intermediaries need to acquire additional government bonds to pledge as collateral. The possibility that the general equilibrium effect on credit provision to the real economy can be contractionary rather than expansionary sharply contrasts with the existing DSGE literature, in which central bank discount window lending always has an expansionary effect (Gertler and Kiyotaki, 2010; Bocola, 2016; Engler and Große Steffen, 2016, Cahn et al., 2017). In addition, the collateral effect has the potential to explain why Italian banks only invested $€ 22.6$ billion out of $€ 181.5$ billion in three-year LTRO funding in private credit, while they invested almost four times this amount ( $€ 82.7$ billion) in Italian government bonds (Carpinelli and Crosignani, 2018).

To quantitatively investigate whether this is the case, I will extend the current model to an infinite-horizon DSGE model that I estimate on Italian data. From my analysis we see that $\kappa_{s_{b, h}}$ will be an important parameter in the estimation procedure, as it is a key parameter in determining the strength of the collateral effect. In line with the collateral policy of the ECB I will also allow corporate loans to be pledged as collateral. While this will obviously reduce the strength of the collateral effect, it will not eliminate it, as central banks typically provide more funding for one euro of government bonds than for one euro of private loans.

## 4 Infinite-horizon DSGE model

In this section I extend the two-period model to an infinite-horizon DSGE model to quantitatively assess the strength of the collateral effect and the extent to which the three-year LTROs were capable of improving macroeconomic conditions in Italy. Specifically, I employ a standard closed economy New Keynesian model, and check in Appendix E. 2 that the results carry over to a model version of a small open economy that is a member of a currency union. I do so to check that my results do not depend on the way conventional monetary policy is modeled; these model versions capture the two extremes in terms of the influence that Italian macrodevelopments have on the Italian policy rate. One extreme is that they affect the policy rate one-for-one in the closed economy, while the other extreme is that they have zero influence on the policy rate in the small open economy model. In reality, the Italian policy rate is set by the ECB, which bases its policy decisions on macrodevelopments in the Eurozone as a whole. With the Italian economy comprising around $15 \%$ of Eurozone GDP, the influence of Italian macrodevelopments will be somehwere in between that in the closed economy and the small open economy model.

The structure of the financial sector is the same as in the two-period model and again subject to the Gertler and Karadi (2011) incentive compatibility constraint. However, intermediaries can also pledge corporate securities as collateral, but obtain less central bank funding than for one euro of government bonds. The central bank sets the nominal rather than the real interest rate on central bank funding and reserves, the last of which follows an active Taylor rule.

Households maximize the sum of expected discounted utility with habit formation in consumption to more realistically capture consumption dynamics (Christiano et al., 2005). They save through deposits, corporate securities, and government bonds, the last two of which are subject to quadratic adjustment costs (Gertler and Karadi, 2013). Wages are sticky, as households' wage and labor decisions are modeled as in Erceg et al. (2000). Households receive profits from ownership of all firms in the economy, and pay lump sum taxes to the government. The government honors outstanding obligations and purchases final goods. These expenditures are financed from central bank dividends, lump-sum taxes and issuance of (long-term) debt.

Intermediate goods producers borrow from financial intermediaries to purchase physical capital from capital goods producers that are subject to convex adjustment costs. Final labor and physical capital are then used for the production of the intermediate goods, which are sold to retail goods producers who face monopolistic competition and sticky price adjustments as in Calvo (1983). Final goods producers purchase retail goods to produce a final good that is sold in a perfectly competitive market. The final good is used by households for consumption, by capital goods producers for investment, by the government, and for adjustment costs arising from households' transactions in financial markets. A full exposition of the model can be found in Appendix C. 1.

Finally, I do not include sovereign default risk in the main text, despite the fact that Italy was in the middle of a sovereign debt crisis at the time of the three-year LTROs. Instead, I report in Appendix E. 1 a model version which includes endogenous sovereign default risk, and
check that my results from the main text continue to hold.

### 4.1 Government

### 4.1.1 Fiscal authority

The government has three sources of revenue: debt issue $q_{t}^{b} b_{t}$, lump sum taxes $\tau_{t}$, and dividends from the central bank $\Delta_{t}^{c b}$. These revenues are used to pay for (exogenous) government purchases of the final good $g_{t}$ and service outstanding government liabilities $\left(1+r_{t}^{b}\right) q_{t-1}^{b} b_{t-1}$. Therefore, the period $t$ government budget constraint (in terms of the price level of the final good $P_{t}$ ) is given by:

$$
\begin{equation*}
q_{t}^{b} b_{t}+\tau_{t}+\Delta_{t}^{c b}=g_{t}+\left(1+r_{t}^{b}\right) q_{t-1}^{b} b_{t-1} . \tag{9}
\end{equation*}
$$

Government debt is long-term, and its maturity structure follows Woodford (1998, 2001). These bonds pay a cash flow $x_{c}$ that is decaying at a rate $1-\rho$ per period. Hence $\rho$ effectively determines the maturity structure of the bonds. In Appendix C.4.1 I formally show that the real rate of return $r_{t}^{b}$ on a bond issued in period $t-1$ is given by:

$$
\begin{equation*}
1+r_{t}^{b}=\left(x_{c}+(1-\rho) q_{t}^{b}\right) /\left(\pi_{t} q_{t-1}^{b}\right), \tag{10}
\end{equation*}
$$

where $\pi_{t} \equiv P_{t} / P_{t-1}$ denotes the gross inflation rate of the final good. Finally, lump sum taxes $\tau_{t}$ are given by a rule which ensures the intertemporal government budget constraint is satisfied (Bohn, 1998). A more elaborate description of the fiscal authority can be found in Appendix C.4.1.

### 4.1.2 Central Bank

The central bank sets the nominal interest rate $r_{t}^{n, r}$ on reserves $m_{t}^{R}$ by employing a standard Taylor-rule that satisfies the Taylor-principle. Reserves are created when the central bank provides funding $d_{t}^{c b}$ to financial intermediaries. However, unlike Section 3, I assume that part of the central bank's assets are financed by net worth $n_{t}^{c b}{ }^{11}$ In that case, the central bank's balance sheet constraint (in terms of the price level of the final good $P_{t}$ ) is given by:

$$
\begin{equation*}
d_{t}^{c b}=n_{t}^{c b}+m_{t}^{R} \tag{11}
\end{equation*}
$$

Financial intermediaries have to pledge collateral in the form of corporate securities and government bonds to obtain central bank funding. The central bank provides $\theta_{t}^{a}$ eurocents in funding for one euro of collateral from asset class $a=\{k, b\}$. Intermediaries remain the legal owner of the assets they pledge as collateral, and therefore receive the accompanying cash flows after

[^8]repayment of the central bank loan in period $t+1,{ }^{12}$ The collateral constraint has the following functional form:
\[

$$
\begin{equation*}
d_{j, t}^{c b} \leq \theta_{t}^{k} q_{t}^{k} s_{j, t}^{k, p}+\theta_{t}^{b} q_{t}^{b} s_{j, t}^{b, p} . \tag{12}
\end{equation*}
$$

\]

where the central bank is in charge of the haircut parameters $\theta_{t}^{k}$ and $\theta_{t}^{b}$, which I assume to be constant over time. Just as in Section 3 the central bank provides as much funding as demanded by financial intermediaries (full allotment), provided they pledge sufficient collateral. The central bank receives a nominal interest rate $r_{t}^{n, c b}$ on loans $d_{t}^{c b}$ to financial intermediaries. Pre-dividend net worth $n_{t}^{c b *}$ (in terms of the price level $P_{t}$ of the final good) is the difference between the gross return on loans $d_{t-1}^{c b}$ provided to financial intermediaries in period $t-1$, and the gross return on reserves $m_{t-1}^{R}$ issued in period $t-1$ :

$$
\begin{equation*}
n_{t}^{c b *}=\left(\frac{1+r_{t-1}^{n, c b}}{\pi_{t}}\right) d_{t-1}^{c b}-\left(\frac{1+r_{t-1}^{n, r}}{\pi_{t}}\right) m_{t-1}^{R}=\left(1+r_{t}^{c b}\right) d_{t-1}^{c b}-\left(1+r_{t}^{r}\right) m_{t-1}^{R} \tag{13}
\end{equation*}
$$

where $r_{t}^{r}$ and $r_{t}^{c b}$ denote the net real return on reserves and central bank funding, respectively. A fraction $\delta_{t}^{c b}$ of $n_{t}^{c b *}$ is paid out to the fiscal authority, while the remaining funds are retained by the central bank:

$$
\begin{align*}
\Delta_{t}^{c b} & =\delta_{t}^{c b} n_{t}^{c b *}  \tag{14}\\
n_{t}^{c b} & =\left(1-\delta_{t}^{c b}\right) n_{t}^{c b *} \tag{15}
\end{align*}
$$

where I assume $\delta_{t}^{c b}$ to be constant over time ${ }^{13}$ In addition to the nominal interest rate on reserves, the central bank also controls the nominal interest rate $r_{t}^{n, c b}$ on central bank funding by adjusting the spread $\Gamma_{t}^{c b}$ :

$$
\begin{equation*}
r_{t}^{n, c b}=r_{t}^{n, r}-\Gamma_{t}^{c b}, \quad \text { with } \Gamma_{t}^{c b}=\bar{\Gamma}_{c b}+\varkappa_{c b}\left(c b_{t}-\overline{c b}\right)+\varkappa_{\xi}\left(\xi_{t}-\bar{\xi}\right) \tag{16}
\end{equation*}
$$

where $\bar{\Gamma}_{c b}$ is the steady state spread. $c b_{t}$ and $\xi_{t}$ follow lognormal $\operatorname{AR}(1)$ processes, with $\xi_{t}$ representing the quality of capital, a negative shock of which triggers financial crises as in Gertler and Karadi (2011). Just as in Section 3 the LTRO-policy will be captured through an increase in $\Gamma_{t}^{c b}$, and the equilibrium interest rate on reserves will be equal to that on deposits. Therefore, an increase in $\Gamma_{t}^{c b}$ will also decrease the interest rate on central bank funding with respect to that on deposits.

[^9]
### 4.2 Financial intermediaries

Financial intermediaries purchase corporate securities $s_{j, t}^{k, p}$ that are issued by intermediate goods producers at a price $q_{t}^{k}$, and government bonds $s_{j, t}^{b, p}$ at a price $q_{t}^{b}$. In addition, they hold reserves $m_{j, t}^{R}$ at the central bank. They fund their assets through net worth $n_{j, t}$, risk-free household deposits $d_{j, t}$ and central bank funding $d_{j, t}^{c b}$, for which they need to pledge collateral, see equation (12). Total assets $p_{j, t}$ are given by:

$$
\begin{equation*}
p_{j, t}=q_{t}^{k} s_{j, t}^{k, p}+q_{t}^{b} s_{j, t}^{b, p}+m_{j, t}^{R}=n_{j, t}+d_{j, t}+d_{j, t}^{c b}, \tag{17}
\end{equation*}
$$

Net worth in period $t+1$ is the difference between the return on assets and the return on liabilities:
$n_{j, t+1}=\left(1+r_{t+1}^{k}\right) q_{t}^{k} s_{j, t}^{k, p}+\left(1+r_{t+1}^{b}\right) q_{t}^{b} s_{j, t}^{b, p}+\left(1+r_{t+1}^{R}\right) m_{j, t}^{R}-\left(1+r_{t+1}^{d}\right) d_{j, t}-\left(1+r_{t+1}^{c b}\right) d_{j, t}^{c b}$,
where $r_{t}^{k}$ is the net real return on corporate securities in period $t, r_{t}^{b}$ the net real return on government bonds, $r_{t}^{R}$ the net real return on central bank reserves, $r_{t}^{d}$ the net real return on deposits and $r_{t}^{c b}$ the net real return on central bank funding. Following Gertler and Karadi (2011), intermediaries are forced to shut down with probability $\sigma$, which is i.i.d. and exogenous, both in time and the cross-section. Intermediaries that are forced to stop operating pay out all remaining net worth to their respective household, the ultimate owner of the intermediary. As long as they continue operating, intermediaries maximize their continuation value, which is the sum of expected future discounted profits:

$$
\begin{aligned}
& V\left(s_{j, t-1}^{k, p}, s_{j, t-1}^{b, p}, m_{j, t-1}^{R}, d_{j, t-1}, d_{j, t-1}^{c b}\right) \\
= & \max E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma) n_{j, t+1}+\sigma V\left(s_{j, t}^{k, p}, s_{j, t}^{b, p}, m_{j, t}^{R}, d_{j, t}, d_{j, t}^{c b}\right)\right]\right\},
\end{aligned}
$$

where $\beta \Lambda_{t, t+1}$ denotes households' stochastic discount factor. A similar agency problem between depositors and managers of intermediaries arises as in Section 3 (Gertler and Karadi, 2011):

$$
\begin{equation*}
V\left(s_{j, t-1}^{k, p}, s_{j, t-1}^{b, p}, m_{j, t-1}^{R}, d_{j, t-1}, d_{j, t-1}^{c b}\right) \geq \lambda_{k} q_{t}^{k} s_{j, t}^{k, p}+\lambda_{b} q_{t}^{b} s_{j, t}^{b, p} \tag{19}
\end{equation*}
$$

which I assume to be binding throughout my simulations as the Italian banking system was undercapitalized at the time of the three-year LTROs (International Monetary Fund, 2011; Hoshi and Kashyap, 2015). The optimization problem of intermediary $j$ is given by maximizing $V\left(s_{j, t-1}^{k, p}, s_{j, t-1}^{b, p}, m_{j, t-1}^{R}, d_{j, t-1}, d_{j, t-1}^{c b}\right)$ subject to the balance sheet constraint 17 , the collateral constraint (12), the law of motion for net worth (18), and the incentive compatibility constraint (19).

The resulting first order conditions are derived in Appendix C.2 Just as in Section 3. I find that the equilibrium interest rate on reserves and deposits is the same, as intermediaries can perfectly elastically obtain reserves by attracting additional deposits to arbitrage away any (ex-
pected) return difference between reserves and deposits. Intermediaries' portfolio choice between corporate securities and government bonds is given by a very similar expression as equation (5) for the two-period model:
$\frac{\lambda_{b}}{\lambda_{k}} E_{t}\left[\Omega_{t, t+1}\left(r_{t+1}^{k}-r_{t+1}^{d}\right)\right]=E_{t}\left[\Omega_{t, t+1}\left(r_{t+1}^{b}-r_{t+1}^{d}\right)\right]+\left(\theta_{t}^{b}-\frac{\lambda_{b}}{\lambda_{k}} \theta_{t}^{k}\right) E_{t}\left[\Omega_{t, t+1}\left(r_{t+1}^{d}-r_{t+1}^{c b}\right)\right]$,
where $\Omega_{t, t+1}=\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]$ is the intermediary's stochastic discount factor with $\chi_{t}$ the Lagrangian multiplier on the intermediary's balance sheet constraint 17. This discount factor can be interpreted as the household's stochastic discount factor $\beta \Lambda_{t, t+1}$, augmented by an additional term to incorporate the effect of the financial frictions. Again, the second term on the right hand side denotes the collateral effect. Observe, however, that the strength of the collateral effect is reduced, everything else equal, because of the presence of the term $\left(\lambda_{b} / \lambda_{k}\right) \theta_{t}^{k}$ : there is less need to shift from corporate securities to government bonds now that corporate securities also provide intermediaries with central bank funding.

Two other features from the two-period model carry directly over to the infinite-horizon version of the model. First, central bank lending will not affect intermediaries' portfolio decisions when $r_{t}^{d}=r_{t}^{c b}$ as the collateral value of assets becomes zero in that case. Second, the incentive compatibility constraint effectively limits the size of intermediaries' holdings of corporate securities and government bonds by the amount of net worth similar to equation $6 \sqrt{6}: 14$

$$
\begin{equation*}
\chi_{t} n_{j, t}=\lambda_{k} q_{t}^{k} s_{j, t}^{k, p}+\lambda_{b} q_{t}^{b} s_{j, t}^{b, p} \tag{21}
\end{equation*}
$$

Finally, a fraction $1-\sigma$ of intermediaries is forced to stop operating at the beginning of each period. They are replaced by new intermediaries which are provided with a starting net worth equal to a fraction $\chi_{b}$ of previous period net worth of the old intermediary.

### 4.3 Production sector

The production sector is modeled in standard New Keynesian fashion. I will shortly outline the setup below, with a more detailed exposition in Appendix C.3. There is a continuum $i \in[0,1]$ of intermediate goods producers that operate in a perfectly competitive market. They issue corporate securities at the end of period $t-1$ and use the proceeds to purchase physical capital $k_{i, t-1}$ from capital producers at a price $q_{t-1}^{k}$. As in Gertler and Kiyotaki 2010), intermediate goods producers can credibly pledge all after-wage revenues from period $t$ to the buyers of these securities. Shocks are realized at the beginning of period $t$, among which a capital quality shock that transforms capital $k_{i, t-1}$ into $\xi_{t} k_{i, t-1}$ effective units of capital (Gertler and Karadi, 2011). Next, intermediate goods producers hire labor $h_{i, t}$ in a perfectly competitive market from labor

[^10]agencies at a wage rate $w_{t}$, and start producing intermediate goods with a constant-returns-to-scale production function with effective capital $\xi_{t} k_{i, t-1}$ and labor $h_{i, t}$ as inputs, and capital income share $\alpha$. Output $y_{i, t}$ is sold to retail firms at a price $m_{t}$, while the effective capital stock (after depreciation $\delta$ per effective unit of capital) is sold to capital producers at a price $q_{t}^{k}$. As the remaining after-wage revenues are paid out to corporate securities' holders, I get the following expression for the net return $r_{t}^{k}$ on these securities:
\[

$$
\begin{equation*}
1+r_{t}^{k}=\frac{\alpha m_{t} y_{i, t} / k_{i, t-1}+q_{t}^{k}(1-\delta) \xi_{t}}{q_{t-1}^{k}} \tag{22}
\end{equation*}
$$

\]

As in Gertler and Karadi (2011), the capital quality shock $\xi_{t}$ decreases the return on corporate securities $r_{t}^{k}$ for two reasons. First, output $y_{i, t}$ decreases as capital becomes less productive, thereby reducing the first term of 22 . Second, the capital price $q_{t}^{k}$ will fall in the presence of a binding incentive compatibility constraint (21), thereby reducing the second term in 22 .

Capital producers purchase the after-production capital stock $(1-\delta) \xi_{t} k_{t-1}$ from intermediate goods producers at price $q_{t}^{k}$, and convert it one-for-one into new capital. In addition, they purchase final goods to produce additional capital. However, the conversion from final goods to capital goods is subject to quadratic adjustment costs. As a result, one unit of investment typically results in less than one unit of capital goods. The newly produced capital stock is sold to intermediate goods producers at a price $q_{t}^{k}$.

A continuum of retail firms transforms intermediate goods one-for-one into differentiated retail goods. Retail firms operate in a monopolistic competitive market, and are therefore pricesetters that charge a markup over the input price $m_{t}$. Following Calvo (1983), each retail firm faces a probability $\psi_{p}$ that they cannot choose a new price in the current period. In that case, they can partially index with previous period inflation. Final good producers purchase goods from all retail firms to produce a final good with a CES production function, and sell in a perfectly competitive market.

### 4.4 Market clearing \& equilibrium

In equilibrium, the total number of corporate securities $k_{t}$ must equal the total number of securities purchased by households and financial intermediaries. Similarly, the total supply of bonds must equal the number of bonds purchased by households and financial intermediaries:

$$
\begin{align*}
k_{t} & =s_{t}^{k, h}+s_{t}^{k, p}  \tag{23}\\
b_{t} & =s_{t}^{b, h}+s_{t}^{b, p} \tag{24}
\end{align*}
$$

The aggregate resource constraint is given by:

$$
\begin{equation*}
y_{t}=c_{t}+i_{t}+g_{t}+\frac{1}{2} \kappa_{s_{k, h}}\left(s_{t}^{k, h}-\hat{s}_{k, h}\right)^{2}+\frac{1}{2} \kappa_{s_{b, h}}\left(s_{t}^{b, h}-\hat{s}_{b, h}\right)^{2} \tag{25}
\end{equation*}
$$

where the last two terms represent the quadratic adjustment costs paid by households when purchasing corporate securities and government bonds (Gertler and Karadi, 2013). The resulting equilibrium definition can be found in Appendix D, which gives a complete overview of the first order conditions.

## 5 Calibration \& estimation

I employ a mix of calibration and estimation with both Bayesian methods and moment matching to match the Italian economy as close as possible. To do so, I employ data downloaded from Eurostat, the ECB, and the Italian National Institute for Statistics, a description of which can be found in Appendix F

I break the identification of parameter values into four stages. In the first two stages I employ a model version without financial frictions and central bank lending operations, while I employ the full model in the last two stages. Specifically, I partially calibrate my model in the first stage by either taking parameter values that are standard in the macroeconomic literature or targeting first order moments such as the steady state labor supply. I subsequently estimate the remaining parameters in the second stage by employing Bayesian techniques. I calibrate some of the parameters relating to financial frictions in the third stage, and estimate the remaining parameters in the fourth stage through a second-order moment matching exercise on data from the financial crisis period 2008Q1-2011Q4.

Specifically, I employ the following quarterly time series from the period 1999Q1-2007Q4 in my Bayesian estimation: real GDP per capita, real consumption per capita, real investment per capita, hours worked, inflation, and the three-month nominal interest rate. Unlike many papers in the literature (Bocola, 2016, Cahn et al., 2017, Darracq-Pariès and Kühl, 2017, Kühl, 2018), I specifically perform the Bayesian estimation on a model version without financial frictions and central bank lending operations. Such a model version accurately captures the pre-2008 economic environment for several reasons. First, there was no implicit subsidy to the Italian banking system pre-2008 (Drechsler et al. 2016), which is in my model captured by $r_{t}^{n, r}=r_{t}^{n, c b} 15$ In that case the collateral value of intermediaries' assets is equal to zero (equation 20 ), and hence intermediaries' choice between corporate securities and government bonds is not affected by central bank lending operations. Second, financial frictions were likely to be absent or negligible during the 1999Q1-2007Q4 period, which is in my model captured by intermediaries' incentive compatibility constraints 19 being non-binding. Under those conditions the equilibrium allocation coincides exactly with that in a model version without financial frictions. The choice to estimate a model version without financial frictions is further supported by Del Negro et al. (2016), who

[^11]show that estimated models with financial frictions perform worse during normal times than models without those frictions $\sqrt{16}$

To save space, I will only discuss the calibration/estimation of key parameters, and refer the interested reader to Appendix $G$ for more information on how the remaining parameters are chosen, as well as tables with calibrated and estimated parameter values.

Two key parameters that determine the strength of the collateral effect are $\theta_{t}^{j}$ with $j \in\{k, b\}$. While the ECB does not publish the haircut $1-\theta_{t}^{j}$ it applies to different asset types, Bruegel (2018) replicates them using long-term credit ratings. Bruegel (2018) finds that the haircut on Italian government bonds was between 3 and $7 \%$ during the sovereign debt crisis of 2011-2013. I therefore set $\theta_{t}^{b}=0.95$. Next, I set $\theta_{t}^{k}=0.40$, which implies a haircut of $60 \%$ on corporate securities. Although most assets that can be pledged as collateral at the ECB have haircuts that are significantly smaller, my model setup does not distinguish between private assets that are eligible as collateral and those that are not, for which the ECB provides zero euros in liquidity ${ }^{17}$ Therefore, the $60 \%$ haircut can be interpreted as the average haircut on a portfolio that contains both pledgeable and non-pledgeable private assets.

We remember from Section 3 that $\lambda_{b}$ and $\kappa_{s_{b, h}}$ are the key parameters that determine the strength of the collateral effect. I determine the ratio $\lambda_{b} / \lambda_{k}$ by comparing the average spread between the return on corporate securities and deposits with the average spread between the Italian bond yield and the return on deposits ${ }^{18}$ Next, I employ a moment-matching exercise to the full model with binding financial constraints to pin down $\kappa_{s_{b, h}}$ and the quadratic adjustment costs for corporate securities $\kappa_{s_{k, h}}[19$ The moment-matching exercise targets the standard errors of real GDP per capita, real consumption per capita, real investment per capita, and the credit spread between the expected return on corporate securities and deposits over the period 2008Q1-2011Q4. Ideally, I would have estimated these two parameters in the Bayesian estimation procedure of the model version without financial frictions to ensure consistency between the models with and without financial frictions. However, the Bayesian estimation would not converge for this model version. Therefore, I take the point estimates from the momentmatching exercise, and redo the Bayesian estimation on a model version that includes quadratic adjustment costs to check that the parameter estimates from the original Bayesian estimation are not biased. More details on this specific issue can be found in Appendix G.3.

Finally, I pin down the size of the implicit subsidy of the three-year LTROs (captured by the increase in $\Gamma_{t}^{c b}$ in response to a financial crisis) by matching the net uptake of ECB funding by

[^12]the aggregate Italian commercial banking system over the period in which the unconventional LTROs of December 2011 and February 2012 took place. I find the implicit subsidy to be equal to annual 140 basis points, or equivalently 35 quarterly basis points, something I will discuss more elaborately in Sections 6.4 and 7 .

## 6 Results

In this section I show the results from numerical simulations of my model, which I solve by performing a first order perturbation around the deterministic steady state. As mentioned above, I will model the LTRO-policy by (temporarily) reducing the nominal interest rate on central bank funding relative to that on reserves, in line with Engler and Große Steffen (2016).

I set the stage by showing the results to a regular central bank funding shock $c b_{t}$, see equation (16), which allows me to highlight the key mechanisms that are operative under the LTRO-policy. I follow up on the two-period model analysis by investigating how the impulse response functions change when varying key parameters that determine the strength of the collateral effect, such as the collateral requirements $\theta_{t}^{k}$ and $\theta_{t}^{b}$, and the household transaction costs parameter $\kappa_{s_{b, h}}$.

Next I look at a scenario where the LTRO-policy is employed in the middle of a financial crisis to properly capture the fact that the three-year LTROs took place in the middle of a financial crisis. This crisis is initiated through a negative capital quality shock as in Gertler and Karadi (2011). I distinguish between a limited LTRO-policy in which the interest rate spread between the nominal interest rate on reserves and central bank funding $\Gamma_{t}^{c b}$ follows the capital quality shock (and reverts back to the steady state following a regular AR(1)-process), and three-year LTROs in which $\Gamma_{t}^{c b}$ is increased for 12 quarters, after which $\Gamma_{t}^{c b}$ immediately returns to steady state. This allows me to highlight the influence of the maturity of the program on credit provision and the macroeconomy.

### 6.1 A regular central bank funding shock

I start by investigating the regular central bank funding shock $c b_{t}$ in equation (16). This shock reduces the nominal interest rate on central bank funding by 35 basis points with respect to the nominal interest rate on reserves on impact, after which it reverts back to steady state, see panel "Interest rate difference" in Figure 3, Let us first focus on the blue solid line, which denotes the base case, and defer discussion of the red slotted line to the next section. This simulation will allow me to identify the key mechanisms of the model, and set the stage for subsequent sections in which I look at the interaction of this policy with financial crises.

Figure 3 shows that a reduction of the interest rate on central bank funding relative to that on deposit funding has a positive effect on the macroeconomy: financial intermediaries expand lending to the real economy (see panel "Corporate securities (b)"), and investment and output increase.

Reducing intermediaries' funding costs increases profitability and net worth everything else equal, and therefore raises intermediaries' continuation value. This relaxes intermediaries' incentive compatibility constraint 21, which allows them to expand their balance sheet. As a result, both investment and capital accumulation (not shown) increase, which ultimately leads to higher output, a mechansim that is well known from the bank lending channel literature (Bernanke and Gertler, 1995). However, I will refer to this effect as the subsidy effect since I explain in Section 3.1.1 that the policy of reducing central bank funding costs is meant to capture the implicit subsidy that was contained in the three-year LTROs, see Section 2 .

In addition to the subsidy effect, there are two other channels through which the positive effects from the LTRO-policy are amplified in the financial sector. First, the initial relaxation of intermediaries' incentive compatibility constraint allows intermediaries to expand their balance sheets, which provides them with additional collateral that can be pledged at the central bank to obtain additional funding. This, in turn, further reduces intermediaries' funding costs, which then further relaxes their incentive compatibility constraint, and allows for a second round of balance sheet expansion, and so on.

Second, there is the capital gains effect Gertler and Karadi (2011): the price of corporate securities and government bonds increases as a result of the additional demand for these assets, which in turn raises the ex post return on intermediaries' existing holdings of corporate securities and government bonds, see equations (22) and 10 respectively. Intermediaries' net worth increases as a result, which further relaxes their incentive compatibility constraints, thereby allowing for another round of balance sheet expansion. Therefore, we see that the capital gains effect that was identified in Section 3 is also operative within the infinite-horizon model and plays a crucial role in the amplification of the funding shock. Interestingly, this capital gains effect was identified to be empirically relevant in the context of another ECB program, namely the Outright Monetary Transactons (OMT) program that was launched in Spetember 2012 (Acharya et al., 2019). Note, however, that financial intermediaries can now also incur capital gains on corporate securities, which amplifies the capital gains effect relative to the two-period model in Section 3

Finally, we can already identify the collateral effect in Figure 3. while the market value of intermediaries' holdings of corporate securities only increases by $0.5 \%$ after 5 quarters, intermediaries' holdings of government bonds increases by more than $15 \%$ ! As mentioned before, government bonds provide twice as much central bank funding as corporate securities, and thus have a larger collateral value. As a result, most of the additional balance sheet capacity that is created by the reduction of intermediaries' funding costs is taken up by government bonds which allow intermediaries to increase the amount of central bank funding.

Also observe that consumption decreases in equilibrium, as the expansion of intermediaries' balance sheets is not only financed by attracting additional central bank funding, but also through an expansion of deposits. Higher household savings are achieved through higher nominal and real interest rates, which result from higher inflation and output.

Finally, observe that the response is hump-shaped. The reason for this is that consumption
features habit formation, and investment is subject to adjustment costs that are quadratic in the change with respect to previous period investment.

### 6.2 The collateral effect

Next I investigate the collateral effect that was identified in Section 3. I start by shutting down the effect by setting $\theta_{t}^{k}=\left(\lambda_{k} / \lambda_{b}\right) \theta_{t}^{b}$, see equation 20). To have a fair comparison, however, I do not only increase $\theta_{t}^{k}$, which would increase the steady state amount of central bank funding, but simultaneously reduce $\theta_{t}^{b}$ such that the steady state amount of central bank funding is equal to the base case. This is achieved by setting $\theta_{t}^{k}=\theta_{t}^{b}=0.425$, which is represented by the red slotted line in Figure $3{ }^{20}$

We see that shutting down the collateral effect has a strong effect on the composition of intermediaries' balance sheets. The market value of intermediaries' holdings of corporate securities $q_{t}^{k} s_{t}^{k}$ increases from $0.1 \%$ of steady state to $0.7 \%$ on impact, whereas the peak of the market value of intermediaries' holdings of government bonds decreases from more than $15 \%$ to only $3 \%$. Clearly, the collateral effect has a substantial effect on intermediaries' credit provision to the real economy.

However, the substantial change in the balance sheet composition of financial intermediaries does not have large effects on the real economy. Investment, output, and intermediary net worth increase, but not by much, whereas consumption hardly changes at all. The reason for this is the fact that household adjustment costs from changing their holdings of corporate securities and government bonds are relatively small, which allows them to sell corporate securities to satisfy intermediaries' increased demand for these securities (relative to the base case). Meanwhile, intermediaries sell part of their government bond holdings to households to create additional balance sheet capacity for corporate securities. As a result, the total stock of corporate securities and government bonds hardly changes at all. The small expansion of investment and output is caused by the fact that intermediaries do not face adjustment costs, and are therefore more effective in credit intermediation to the real economy.

Finally, note that intermediaries borrow less from the central bank compared with the base case. The reason is that the reduction of $\theta_{t}^{b}$ from 0.95 to 0.425 reduces the collateral value of government bonds (represented by $\left.\theta_{t}^{b} E_{t}\left[\Omega_{t, t+1}\left(r_{t+1}^{d}-r_{t+1}^{c b}\right)\right]\right)$ by more than the increase in the collateral value of corporate securities, see equation 20).

The fact that the collateral effect has a large effect on the composition of intermediaries' balance sheets, but a small effect on investment and capital accumulation is further confirmed in Figure 4, where I decrease $\kappa_{s_{b, h}}$ from 0.0001 (blue, solid line) to 0.00005 (red, slotted line). In line with the prediction from Section 3 , we see that a decrease in $\kappa_{s_{b, h}}$ induces financial intermediaries to increase government bonds holdings from $16 \%$ to $28 \%$ of steady state at the expense of credit

[^13]Central bank funding shock 35 bps.: base case vs. $\theta_{k}=\theta_{b}=0.425$


Figure 3: Impulse response functions for a central bank funding shock of 35 basis points ("Interest rate difference") for the base case (blue, solid) versus a model version in which $\theta_{t}^{k}=\theta_{t}^{b}=0.425$. The panel "Corporate securities (b)" denotes the value of intermediaries' corporate securities holdings $q_{t}^{k} s_{t}^{k}$. Similarly, the panel "Government bonds (b)" denotes the value of intermediaries' government bond holdings $q_{t}^{b} s_{t}^{b}$. Central bank funding is expressed in terms of annual steady state output.
provision to the real economy, which flips from an expansion by $0.1 \%$ of steady state to a contraction by $0.4 \%$. This simulation therefore confirms the theoretical prediction from Section 3 that the net effect on intermediaries' credit provision is contractionary when the collateral effect dominates the capital gains effect. However, just as in the case where $\theta_{t}^{k}=\theta_{t}^{b}=0.425$, the general equilibrium effect on investment and output is small as a result of the relatively small transaction costs for households, who increase their holdings of corporate securities relative to the base case (not shown).

Above I have identified the collateral effect, and show that it has a relatively large effect on intermediaries' balance sheets, but a relatively small effect on investment and output. However, around the time of the three-year LTROs of December 2011 and February 2012 the ECB left the amount of funding for one euro of government bonds unchanged, while it accepted additional asset types as collateral, thereby effectively increasing the fraction of commercial banks' balance sheets that could be pledged at the $\mathrm{ECB}{ }^{21}$ To capture this policy, I investigate in Figure 5 an increase of $\theta_{t}^{k}$ from 0.40 to 0.50 (red, slotted line), while keeping $\theta_{t}^{b}$ fixed at 0.95 .

We see that this policy has a more substantial and persistent effect on the economy. Whereas most macro variables in Figures 3 and 4 return to the baseline case within 20 quarters or less, we see in Figure 5that intermediaries' credit provision and output remain persistently above the base case $\theta_{t}^{k}=0.40$. The reason for this is that, contrary to the previous experiments, there is an effective relaxation of collateral requirements: more central bank funding can be obtained for the same amount of collateral, which decreases effective funding costs by more than in the base case. Therefore, the subsidy effect becomes stronger, incentive compatibility constraints relax, and more credit can persistently be provided to the real economy by financial intermediaries. I therefore conclude that the relaxation of collateral requirements is potentially a powerful tool that the central bank can employ in a financial crisis, a scenario which I will explicitly investigate in the next sections ${ }^{22}$

### 6.3 Financial crises

In the previous sections I investigated the effects from a temporary decrease in central bank funding costs relative to deposit funding costs. I found that such a policy increases credit provision to the real economy, investment and output. In addition, I identified the collateral effect, and showed that a relaxation of collateral requirements in the form of an expansion of the amount of central bank funding per euro of corporate securities had an expansionary effect on the macroeconomy. The unconventional three-year LTROs of December 2011 and February 2012, however, occurred against the backdrop of a financial crisis, whereas I have thus far looked at a central bank funding shock in isolation. Therefore, I investigate in this section the effectiveness

[^14]Central bank funding shock $35 \mathrm{bps} .: \kappa_{s_{b, h}}=0.0001$ vs. $\kappa_{s_{b, h}}=0.00005$


Figure 4: Impulse response functions for a central bank funding shock of 35 basis points ("Interest rate difference") for the base case $\kappa_{s_{b, h}}=0.0001$ (blue, solid) versus the case where $\kappa_{s_{b, h}}=$ 0.00005 (red, slotted). The panel "Corporate securities (b)" denotes the value of intermediaries' corporate securities holdings $q_{t}^{k} s_{t}^{k}$. Similarly, the panel "Government bonds (b)" denotes the value of intermediaries' government bond holdings $q_{t}^{b} s_{t}^{b}$. Central bank funding is expressed in terms of annual steady state output.

Central bank funding shock 35 bps.: $\theta_{t}^{k}=0.40$ vs. $\theta_{t}^{k}=0.50$


Figure 5: Impulse response functions for a central bank funding shock of 35 basis points ("Interest rate difference") for the base case $\theta_{t}^{k}=0.40$ (blue, solid) versus the case where $\theta_{t}^{k}=0.50$ (red, slotted). The panel "Corporate securities (b)" denotes the value of intermediaries' corporate securities holdings $q_{t}^{k} s_{t}^{k}$. Similarly, the panel "Government bonds (b)" denotes the value of intermediaries' government bond holdings $q_{t}^{b} s_{t}^{b}$. Central bank funding is expressed in terms of annual steady state output.
of a temporary decrease in central bank funding costs in times of financial crises.
In Figure 6I initiate such a crisis through a capital quality shock as in Gertler and Karadi (2011), the size of which is calibrated to match the cumulative drop in quarterly Italian GDP from 2011Q4 to 2013Q2 and amounts to $3.8 \%$. Specifically, the blue solid line depicts the case where the central bank does not engage in the LTRO-policy $\left(\Gamma_{t}^{c b}=0\right)$. The other two simulations feature the LTRO-policy where $\Gamma_{t}^{c b}$ increases by 35 basis points on impact by setting $\varkappa_{\xi}<0$, see equation (16), and afterwards returns to its steady state along with capital quality $\xi_{t}$ after approximately 12 quarters. I distinguish between the base case with $\theta_{t}^{k}=0.40$ (red, slotted line), and a policy where collateral requirements are relaxed by setting $\theta_{t}^{k}=0.50$ (black, dashed line).

I start by focusing on the blue solid impulse response functions to understand the economy's response to the financial crisis shock in the absence of the LTRO-policy. The resulting simulations are very similar to those in Gertler and Karadi (2011): the capital quality shock reduces the return on corporate securities which imposes significant losses on financial intermediaries. Net worth falls, and intermediaries' incentive compatibility constraints tighten as a result, which in turn reduces the amount of funding that intermediaries can provide to intermediate goods producers for purchasing physical capital. As a result, the price of capital drops, which further decreases the ex post return on corporate securities, as can be seen from the second term of equation $\sqrt[22]{ }$. Intermediaries' net worth falls further, leading to a second round of balance sheet tightening. The fact that intermediaries have to shrink the balance sheet does not only affect the price of corporate securities, but also the price of government bonds, which further reduces net worth. The shrinking of intermediaries' balance sheets reduces central bank funding by $20 \%$, as intermediaries have less collateral to pledge at the central bank.

The results for the real economy are pronounced: credit provision to the real economy drops by almost $20 \%$, as net worth drops by more than $30 \%$. As a result, investment drops by $10 \%$ with respect to steady state, and a significantly lower capital stock leads to lower wages and reduced household income (not shown). The wealth effect causes consumption to fall by $4 \%$. Together with the fall in investment, this results in a drop in output of approximately $3.8 \%$, in line with the observed drop in quarterly Italian GDP between 2011Q4 and 2013Q2.

Next I investigate the red, slotted simulations in Figure 6, in which the central bank responds to the financial crisis by initiating the LTRO-policy. We see that this policy has a positive but almost negligible effect on the economy. Just as in the case of the LTRO-policy without financial crisis, the collateral effect causes intermediaries to use most of the additional balance sheet capacity that is created by lower funding costs to be taken up by additional government bonds, which increase by more than $15 \%$ of steady state, whereas the expansion in corporate securities is negligible. Closer inspection learns that the same positive effects on net worth, investment, and output exist (relative to no LTRO-policy) as in the case of an isolated LTRO-policy, but that these positive effects are completely dwarfed by the negative effects from the financial crisis. Therefore, the LTRO-policy is relatively ineffective in ameliorating the negative macroconomic effects of the financial crisis.

Financial crisis: no policy vs limited LTRO vs limited LTRO $\theta_{k}=0.5$


Figure 6: Impulse response functions for a financial crisis, initiated by a capital quality shock of $3.5 \%$. The figure compares a scenario with no additional policy (blue, solid) with a scenario in which the central bank lowers the nominal interest rate on central bank funding with respect to the nominal interest rate on deposits with 35 quarterly basis points in line with the capital quality shock (red, slotted), and with the same intervention with $\theta_{k}=0.5$ (black, dashed). The panel "Corporate securities (b)" denotes the value of intermediaries' corporate securities holdings $q_{t}^{k} s_{t}^{k}$. Similarly, the panel "Government bonds (b)" denotes the value of intermediaries' government bond holdings $q_{t}^{b} s_{t}^{b}$. Central bank funding is expressed in terms of annual steady state output.

Next, I investigate the degree to which the relaxation of collateral requirements makes the LTRO-policy more effective in a financial crisis (black, dashed line). We see that net worth increases by $3 \%$ of steady state, intermediaries' holdings of corporate securities by $1 \%$ of steady state, and investment by more than $1 \%$ with respect to no LTRO-policy. Just as in the previous section, financial intermediaries obtain more central bank funding for one euro of corporate securities, and the increase in $\theta_{t}^{k}$ reduces the distortionary effect from the collateral effect. However, output increases only marginally as consumption falls with respect to the case without LTRO-policy, just as it did in Figure 5. I therefore conclude that the LTRO-policy is relatively ineffective in mitigating the negative effects that the financial crisis has on the macroeconomy.

Finally, observe in Figure 6 that it looks as if central bank funding decreases for $\theta_{t}^{k}=0.50$ relative to $\theta_{t}^{k}=0.40$. Note, however, that the figures displays the deviation from the steady state, and that steady state central bank funding is higher for the case with $\theta_{t}^{k}=0.50$. Therefore, to see whether the LTRO-policy expands central bank funding for $\theta_{t}^{k}=0.50$, I check that the amount of central bank funding falls by even more in the absence of the LTRO-policy.

### 6.4 Three-year LTROs

In the previous section we looked at what one could call a 'limited' LTRO-policy, as $\Gamma_{t}^{c b}$ immediately started to revert back to steady state after the initial crisis response. However, the unconventional LTROs of December 2011 and February 2012 had a maturity of three years. Therefore I investigate in Figure 7 a policy in which $\Gamma_{t}^{c b}$ is increased by 35 basis points for 12 quarters, after which it reverts back to its steady state value. Just as in Figure 6 I compare the baseline case of a financial crisis without LTRO-policy (blue, solid line) with the three-year LTROs (red, slotted), and the same LTRO-policy with $\theta_{t}^{k}=0.50$ (black, dashed). Setting $\Gamma_{t}^{c b}$ at this particular value allows me to match the net uptake of ECB funding by the aggregate Italian commercial banking system over the period in which the three-year LTROs of December 2011 and February 2012 took place, which amounted to $3.7 \%$ of annual Italian GDP (Bruegel, 2015) ${ }^{23}$ Interestingly, by pinning down $\Gamma_{t}^{c b}$ in this way I am able to obtain an estimate of $140(=4 \times 35)$ annual basis points for the implicit subsidy that was contained in the three-year LTROs, something I will further discuss in the next section.

We see from Figure 7 that lengthening the maturity of the LTRO-policy significantly increases its effectiveness in expanding credit provision to the real economy, investment and output relative to no LTRO-policy. The trough in investment decreases by approximately a quarter, while output is persistently $0.3 \%$ of steady state above the scenario in which the central bank does not embark upon the three-year LTROs. However, we see that the collateral effect is still present. As before, we see that most of the additional balance sheet capacity created by a reduction of intermediaries' funding costs is used to increase their holdings of government bonds. Whereas the market value

[^15]of their corporate securities grows by almost $2 \%$ in the base case three-year LTROs, the market value of intermediaries' holdings of government bonds grows by almost $30 \%$. Interestingly, these numbers are very close to the growth rates reported by Carpinelli and Crosignani (2018) ${ }^{24}$ As such it seems that my model is well capable of matching the data, which is remarkable given the fact that I did not target these two growth rates in my estimation procedure, see Appendix G.4. Also observe that a further manifestation of the collateral effect can be seen from the immediate drop in intermediaries' bondholdings after the three-year LTROs have ended, as the collateral value of government bonds suddenly decreases at that point in time.

The key take-away from Figure 7 however, is the fact that the three-year LTROs have an expansionary effect on the real economy (relative to no LTROs), which contrasts with the negligible expansion under the limited LTROs in Figure 6. This result is driven by the fact that lengthening the maturity increases the strength of the subsidy effect: the longer financial intermediaries can borrow at an interest rate below the policy rate, the longer the period in which their funding costs decrease, and the larger the relaxation of intermediaries' incentive compatibility constraints. This, in turn, allows for a larger balance sheet expansion compared with the limited LTROs in Figure 6. Therefore, despite the fact that the collateral effect still induces intermediaries to use most of the additional balance sheet capacity to expand their bond holdings, the increase in balance sheet capacity is now large enough to also allow a substantial expansion of credit provision to the real economy.

The result that the maturity of the LTROs is a key determinant for the effectiveness of the unconventional LTROs is in line with Carpinelli and Crosignani (2018), who empirically establish that the longer maturity of the three-year LTROs was key in reducing the credit contraction by the Italian banking system. In addition, Cahn et al. (2017), who estimate a DSGE model with financial frictions a la Gertler and Karadi (2011) on Eurozone data, also find that lengthening the maturity improves the macroeconomic effectiveness of the unconventional LTRO policy. However, their data do not contain the period of the three-year LTROs.

Finally, I look at the effect of the three-year LTROs when $\theta_{t}^{k}=0.50$ (black, dashed line), as the three-year LTROs also featured a relaxation of collateral requirements. We see from Figure 7 that the relaxation further increases credit provision, investment, and output. However, quantitatively, we see that the expansionary effect from increasing the maturity of the unconventional LTROs dominates the effects from providing more central bank funding for one euro of corporate securities. I therefore conclude that the ECB's lengthening of the maturity of the LTROs was the key feature that made the three-year LTROs effective in expanding credit provision to the real economy.

[^16]Financial crisis: no policy vs LTRO vs LTRO with $\theta_{k}=0.5$


Figure 7: Impulse response functions for a financial crisis, initiated by a capital quality shock of $3.5 \%$. The figure compares a scenario with no additional policy (blue, solid) with a scenario in which the central bank lowers the nominal interest rate on central bank funding with respect to the nominal interest rate on deposits with 35 quarterly basis points for 12 quarters, capturing the three-year LTROs (red, slotted), and the same central bank intervention with $\theta_{k}=0.5$ (black, dashed). The panel "Corporate securities (b)" denotes the value of intermediaries' corporate securities holdings $q_{t}^{k} s_{t}^{k}$. Similarly, the panel "Government bonds (b)" denotes the value of intermediaries' government bond holdings $q_{t}^{b} s_{t}^{b}$. Central bank funding is expressed in terms of annual steady state output.

## 7 Discussion \& Robustness

In this section I discuss some of the results of the previous sections, and check the influence of some of the assumptions that I have made.

First I discuss the size of the implicit subsidy of the three-year LTROs. 140 annual basis points sounds like a large (implicit) subsidy for the Italian banking system. However, Italy was in the middle of a financial and sovereign debt crisis at the time of the three-year LTROs. One feature of this crisis was that Italian banks experienced a sharp reduction of their foreign wholesale funding, and were therefore having difficulties financing their balance sheets. As a result, the provision by the ECB of unlimited funding at an attractive interest rate and relatively long maturity made the funding very attractive to Italian banks (Carpinelli and Crosignani, 2018). It is conceivable that Italian banks would have been forced to offer an interest rate that would have been 140 basis points above the ECB interest rate to induce foreign investors not to withdraw their funds. Therefore, the implicit subsidy of 140 basis points might be a reasonable estimate.

The previous paragraph touches upon another issue that could potentially play a big role in my results, namely the presence of sovereign default risk. The Eurozone was in the middle of a sovereign debt crisis at the time of the three-year LTROs, and the default risk of the Italian government as indicated by the spread on credit default swaps (CDS) had increased by approximately 300 basis points since the first half of 2011 . Therefore, I check the extent to which the inclusion of endogenous sovereign default risk affects my results in Appendix $H$, and find that my results are qualitatively not affected. The underlying reason is that including sovereign default risk does not affect the collateral effect, as government bonds continue to provide more central bank funding per euro than corporate securities.

A third modeling issue is that I study central bank lending in a closed economy model. Within such a model, the nominal interest rate is determined by a standard Taylor rule through which macrodevelopments affect the policy rate one for one. However, Italy being part of the Eurozone implies that the Italian nominal interest rate is determined by the ECB, which adjusts its monetary policy in response to macrodvelopments in the Eurozone as a whole, rather than to Italian macrodevelopments alone. As a result, the monetary policy response in my closed economy model will overstate the extent to which the ECB responds to Italian macrodevelopments. To check whether this choice influences my results, I construct a small open economy that is a member of a currency union in which the nominal interest rate on reserves is permanently fixed at its steady state value. The resulting simulations can be found in Appendix Here I show that the results are hardly affected by the switch from a closed to a small open economy model. The reason why the way I model conventional monetary policy does not affect my main results is the fact that my results are driven by a decrease of the interest rate on central bank funding relative to that on reserves, and because the focus of my paper is on what happens to my economy relative to the baseline case of not engaging in the LTRO-policy. Therefore, the specific modeling of the interest rate on reserves does not affect my results.

Finally, I check the extent to which the relative diversion rate $\bar{\lambda}_{b} / \bar{\lambda}_{k}$ affects the results in

Appendix H I do so as Section 3 showed that $\lambda_{b}$ is a key parameter in determining the strength of the collateral effect. I find that my results are not qualitatively affected by a change in $\lambda_{b}$.

## 8 Conclusion

In this paper I investigate the effectiveness of central bank lending to balance-sheet-constrained financial intermediaries in mitigating the macroeconomic impact of a financial crisis. A key feature of such lending operations is the requirement to pledge collateral. I find that this requirement gives rise to a collateral effect that reduces credit provision to the real economy everything else equal when central banks provide more funding for one euro of government bonds than for one euro of corporate credit. As a result, intermediaries with limited balance sheet capacity will shift from corporate credit to government bonds, so as to increase the amount of central bank funding they can obtain. Therefore the collateral effect can reduce or offset in equilibrium the expansionary effects of central bank lending to intermediaries that occur in other DSGE models (Gertler and Kiyotaki, 2010, Bocola, 2016; Engler and Große Steffen, 2016; Cahn et al., 2017).

My formal analysis can be disentangled into two strands. First, I investigate a two-period general equilibrium model with balance-sheet-constrained financial intermediaries that are partially financed through central bank funding for which they have to pledge government bonds as collateral. This model allows me to disentangle the short-run effect on credit provision into a capital gains effect on intermediaries' existing holdings of government bonds, which everything else equal has an expansionary effect on credit provision, and a collateral effect that has a contractionary effect everything else equal. The two-period model analysis also allows me to identify the key parameters that determine the strength of the collateral effect.

I then extend a standard New Keynesian model with price and wage stickiness and financial frictions a la Gertler and Karadi (2011) to include central bank lending to financial intermediaries. In addition to government bonds, intermediaries can now also pledge corporate securities as collateral, although one euro of corporate securities provides less central bank funding than one euro of government bonds. I argue that the three-year LTROs can be considered to have contained an implicit subsidy for Italian banks, which I model as a reduction of the nominal interest rate on central bank funding relative to that on reserves Engler and Große Steffen, 2016). I employ a Bayesian estimation procedure as well as a moment-matching exercise using Italian data to match the Italian economy as close as possible. The estimation procedure allows me to pin down the values of the key parameters that determine the strength of the collateral effect.

The quantitative exercise shows that the collateral effect can explain the empirical findings that Italian banks increased their holdings of domestic government bonds by $30 \%$ during the period of the three-year LTROs, while credit provision to the real economy expanded by a mere $2 \%$ (relative to no LTROs (Carpinelli and Crosignani, 2018)). The exercise also allows me to obtain an estimate of the implicit subsidy that the three-year LTROs contained for Italian banks
by matching the net uptake of ECB funding by Italian banks. I find this implicit subsidy to equal 140 annual basis points. Finally, my model confirms that the maturity of the three-year LTROs was key for the policy to have an expansionary effect on credit provision to the real economy (Carpinelli and Crosignani, 2018): the longer intermediaries can profit from lower funding costs, the larger the increase in the sum of expected discounted future profits, and the larger the relaxation of intermediaries' incentive compatibility constraints. While the collateral effect remains operative for longer-maturity LTROs and still induces a relative shift from corporate securities to government bonds, the additional balance sheet capacity that is now created is large enough to also allow for an expansion of the level of credit provision to the real economy.

The presence of the collateral effect could also explain why the three-year LTROs of December 2011 and February 2012 were adjusted in subsequent ECB lending operations; Under the so-called Targeted Longer-Term Refinancing Operations (TLTROs) commercial banks can still borrow long-term, but the amount they can borrow is linked to their loans to non-financial corporations and households. ${ }^{25}$ While such a requirement could (partially) eliminate the collateral effect, it could also prevent banks from acquiring as much TLTRO-funding as they otherwise would have, as a smaller fraction of the initial funds taken out can be used to purchase additional government bonds that can then be pledged to obtain a second round of TLTRO-funding. However, I leave a study of this particular policy for future research.

Finally, although I focus on the three-year LTROs, the framework developed in this paper could easily be adjusted to investigate other central banks' lending programs at the height of financial crisis, such as the Federal Reserve's Term Auction Facility and the Treasury Securities Lending Facility.

## References

Acharya, V.V., Eisert, T., Eufinger, C., Hirsch, C., 2018. Real Effects of the Sovereign Debt Crisis in Europe: Evidence from Syndicated Loans. Review of Financial Studies 31, 2855-2896.

Acharya, V.V., Eisert, T., Eufinger, C., Hirsch, C., 2019. Whatever it takes: The Real Effects of Unconventional Monetary Policy. Review of Financial Studies 32, 3366-3411.

Acharya, V.V., Steffen, S., 2015. The "greatest" carry trade ever? Understanding eurozone bank risks. Journal of Financial Economics 115, 215-236. doi $10.1016 / \mathrm{j}$. jfineco.2014.10.

Altavilla, C., Pagano, M., Simonelli, S., 2017. Bank Exposures and Sovereign Stress Transmission. Review of Finance 21, 2103-2139. URL: https://ideas.repec.org/a/oup/revfin/ v21y2017i6p2103-2139..html.

Andrade, P., Cahn, C., Fraisse, H., Mésonnier, J.S., 2019. Can the Provision of Long-Term Liquidity Help to Avoid a Credit Crunch? Evidence from the Eurosystem's LTRO. Journal

[^17]of the European Economic Association 17, 1070-1106. URL: https://ideas.repec.org/a/ oup/jeurec/v17y2019i4p1070-1106..html.

Bagehot, W., 1873. Lombard Street: A Description of the Money Market.
Becker, B., Ivashina, V., 2018. Financial repression in the european sovereign debt crisis. Review of Finance 22, 83-115.

Bernanke, B.S., Gertler, M., 1995. Inside the Black Box: The Credit Channel of Monetary Policy Transmission. Journal of Economic Perspectives 9, 27-48. URL: https://ideas.repec.org/ a/aea/jecper/v9y1995i4p27-48.html.

Bocola, L., 2016. The pass-through of sovereign risk. Journal of Political Economy 124, 879-926.
Bohn, H., 1998. The behavior of U.S. public debt and deficits. The Quarterly Journal of Economics 113, 949-963.

Brooks, S.P., Gelman, A., 1998. General methods for Monitoring Convergence of Iterative Simulations. Journal of Computational and Graphical Statistics , 434-455.

Bruegel, 2015. Bruegel database on Eurosystem liquidity. http://bruegel.org/publications/ datasets/eurosystem-liquidity/.

Bruegel, 2018. Is the ECB collateral framework compromising the safe-asset status of euro-area sovereign bonds? https://www.bruegel.org/2018/06/ is-the-ecb-collateral-framework-compromising-the-safe-asset-status-of-euro-area-sovereign-bonc

Brutti, F., 2011. Sovereign defaults and liquidity crises. Journal of International Economics 84, 65-72. URL: https://ideas.repec.org/a/eee/inecon/v84y2011i1p65-72.html.

Burriel, P., Fernández-Villaverde, J., Rubio-Ramírez, J., 2010. MEDEA: a DSGE model for the Spanish economy. SERIEs: Journal of the Spanish Economic Association 1, 175-243. doi $10.1007 /$ s13209-009-0011-x.

Cahn, C., Matheron, J., Sahuc, J., 2017. Assessing the Macroeconomic Effects of LTROs during the Great Recession. Journal of Money, Credit and Banking 49, 1443-1482. URL: https: //ideas.repec.org/a/wly/jmoncb/v49y2017i7p1443-1482.html

Calvo, G.A., 1983. Staggered prices in a utility-maximizing framework. Journal of Monetary Economics 12, 383-398.

Carpinelli, L., Crosignani, M., 2018. The design and transmission of central bank liquidity provisions. Working Paper .

Choi, D.B., Santos, J.A., Yorulmazer, T., 2019. A Theory of Collateral for the Lender of Last Resort. Technical Report.

Christiano, L., Motto, R., Rostagno, M., 2010. Financial factors in economic fluctuations. Working Paper Series 1192. European Central Bank. URL: https://ideas.repec.org/p/ecb/ ecbwps/20101192.html.

Christiano, L.J., Eichenbaum, M., Evans, C.L., 2005. Nominal rigidities and the dynamic effects of a shock to monetary policy. Journal of Political Economy 113, 1-45.

Corsetti, G., Kuester, K., Meier, A., Müller, G.J., 2013. Sovereign Risk, Fiscal Policy, and Macroeconomic Stability. Economic Journal 0, F99-F132. URL: https://ideas.repec.org/ a/ecj/econjl/vy2013ipf99-f132.html.

Crosignani, M., 2016. Why Are Banks Not Recapitalized During Crises? Technical Report.
Crosignani, M., Faria-e Castro, M., Fonseca, L., forthcoming. The (unintended?) consequences of the largest liquidity injection ever. Journal of Monetary Economics .

Darracq-Paries, M., De Santis, R.A., 2015. A non-standard monetary policy shock: The ECB's 3 -year LTROs and the shift in credit supply. Journal of International Money and Finance 54, 1-34. URL: https://ideas.repec.org/a/eee/jimfin/v54y2015icp1-34.html, doi 10 . 1016/j.jimonfin.2015.0.

Darracq-Pariès, M., Kühl, M., 2017. The optimal conduct of central bank asset purchases. Technical Report.

Del Negro, M., Eggertsson, G., Ferrero, A., Kiyotaki, N., 2017. The Great Escape? A Quantitative Evaluation of the Fed's Liquidity Facilities. American Economic Review 107, 824-857.

Del Negro, M., Hasegawa, R.B., Schorfheide, F., 2016. Dynamic prediction pools: An investigation of financial frictions and forecasting performance. Journal of Econometrics 192, 391405. URL: https://ideas.repec.org/a/eee/econom/v192y2016i2p391-405.html, doi 10 . 1016/j.jeconom.2016.02.

Drechsler, I., Drechsel, T., Marques-Ibanez, D., Schnabl, P., 2016. Who Borrows from the Lender of Last Resort? Journal of Finance 71, 1933-1974.

Engler, P., Große Steffen, C., 2016. Sovereign risk, interbank freezes, and aggregate fluctuations. European Economic Review 87, 34-61. doi 10.1016/j.euroecorev. 2016

Erceg, C.J., Henderson, D.W., Levin, A.T., 2000. Optimal monetary policy with staggered wage and price contracts. Journal of Monetary Economics 46, 281-313. URL: https://ideas. repec.org/a/eee/moneco/v46y2000i2p281-313.html.

European Central Bank, 2011a. Press release: 8 December 2011 - ECB announces measures to support bank lending and money market activity URL: http://www.ecb.europa.eu/press/ pr/date/2011/html/pr111208_1.en.html

European Central Bank, 2011b. The Monetary policy of the ECB URL: http://www.ecb.int,
European Central Bank, 2015 URL: http://www.ecb.int.
Fahr, S., Motto, R., Rostagno, M., Smets, F., Tristani, O., 2013. A monetary policy strategy in good and bad times: lessons from the recent past. Economic Policy 28, 243-288. URL: https: //ideas.repec.org/a/bla/ecpoli/v28y2013i74p243-288.html, doi 10.1111/1468-0327. 12008 .

Ferrero, G., Loberto, M., Miccoli, M., 2017. The collateral channel of unconventional monetary policy. Temi di discussione (Economic working papers) 1119. Bank of Italy, Economic Research and International Relations Area. URL: https://ideas.repec.org/p/bdi/wptemi/ td_1119_17.html.

Garcia-Posada, M., Marchetti, M., 2016. The Bank Lending Channel of Unconventional Monetary Policy: the Impact of the VLTROs on Credit Supply in Spain. Economic Modelling 58, 427-441.

Gerali, A., Neri, S., Sessa, L., Signoretti, F.M., 2010. Credit and Banking in a DSGE Model of the Euro Area. Journal of Money, Credit and Banking 42, 107-141. URL: https://ideas. repec.org/a/mcb/jmoncb/v42y2010is1p107-141.html.

Gertler, M., Karadi, P., 2011. A model of unconventional monetary policy. Journal of Monetary Economics 58, 17-34.

Gertler, M., Karadi, P., 2013. QE 1 vs. 2 vs. 3. . . : a framework for analyzing large-scale asset purchases as a monetary policy tool. International Journal of Central Banking 9, 5-53.

Gertler, M., Kiyotaki, N., 2010. Financial intermediation and credit policy in business cycle analysis, in: Friedman, B.M., Woodford, M. (Eds.), Handbook of Monetary Economics. Elsevier. volume 3. chapter 11, pp. 547-599.

Gertler, M., Kiyotaki, N., 2015. Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy. American Economic Review 105, 2011-2043. URL: https://ideas.repec.org/a/ aea/aecrev/v105y2015i7p2011-43.html.

Giannone, D., Lenza, M., Pill, H., Reichlin, L., 2012. The ECB and the Interbank Market. Economic Journal 122, F467-F486.

Güntner, J.H., 2015. The federal funds market, excess reserves, and unconventional monetary policy. Journal of Economic Dynamics and Control 53, 225-250. URL: https://ideas. repec.org/a/eee/dyncon/v53y2015icp225-250.html, doi 10.1016/j.jedc.2015.02.01.

Hörmann, M., Schabert, A., 2015. A Monetary Analysis of Balance Sheet Policies. Economic Journal 125, 1888-1917.

Hoshi, T., Kashyap, A.K., 2015. Will the U.S. and Europe Avoid a Lost Decade? Lessons from Japan's Postcrisis Experience. IMF Economic Review 63, 110-163.

International Monetary Fund, 2011. "Global Financial Stability Report", Washington D.C.
Kirchner, M., van Wijnbergen, S., 2016. Fiscal deficits, financial fragility, and the effectiveness of government policies. Journal of Monetary Economics 80, 51-68. doi 10.1016/j.jmoneco. 2016.04.

Kiyotaki, N., Moore, J., 1997. Credit cycles. Journal of Political Economy 105, 211-48.
Koulischer, F., Struyven, D., 2014. Central bank liquidity provision and collateral quality. Journal of Banking \& Finance 49, 113-130. doi 10.1016/j.jbankfin. 2014.0

Krishnamurthy, A., Nagel, S., Vissing-Jorgensen, A., 2018. ECB Policies Involving Government Bond Purchases: Impact and Channels. Review of Finance 22, 1-44.

Kuttner, K.N., 2018. Outside the Box: Unconventional Monetary Policy in the Great Recession and Beyond. Journal of Economic Perspectives 32, 121-146. URL: https://ideas.repec. org/p/wil/wileco/2018-04.html.

Van der Kwaak, C., Van Wijnbergen, S., 2014. Financial fragility, sovereign default risk and the limits to commercial bank bail-outs. Journal of Economic Dynamics and Control 43, 218-240.
van der Kwaak, C., van Wijnbergen, S., 2017. Financial Fragility and the Keynesian Multiplier. CEPR Discussion Papers 12394. C.E.P.R. Discussion Papers. URL: https://ideas.repec. org/p/cpr/ceprdp/12394.html.

Kühl, M., 2018. The Effects of Government Bond Purchases on Leverage Constraints of Banks and Non-Financial Firms. International Journal of Central Banking 14, 93-161.

Lakdawala, A., Minetti, R., Olivero, M.P., 2018. Interbank markets and bank bailout policies amid a sovereign debt crisis. Journal of Economic Dynamics and Control 93, 131-153. URL: https://ideas.repec.org/a/eee/dyncon/v93y2018icp131-153.html, doi 10.1016/ j-jedc.2018.01.03
Ongena, S., Popov, A., Horen, N.V., 2019. The Invisible Hand of the Government: Moral Suasion during the European Sovereign Debt Crisis. American Economic Journal: Macroeconomics 11, 346-379. URL: https://ideas.repec.org/a/aea/aejmac/v11y2019i4p346-79.html.

Pfeifer, J., 2018. A Guide to Specifying Observation Equations for the Estimation of DSGE Models. Technical Report. University of Cologne. URL: https://sites.google.com/site/ pfeiferecon/dynare.

Popov, A., Horen, N.V., 2015. Exporting Sovereign Stress: Evidence from Syndicated Bank Lending during the Euro Area Sovereign Debt Crisis. Review of Finance 19, 1825-1866. URL: https://ideas.repec.org/a/oup/revfin/v19y2015i5p1825-1866..html.

Schabert, A., 2015. Optimal Central Bank Lending. Journal of Economic Theory , 485-516.
Schabert, A., van Wijnbergen, S.J.G., 2014. Sovereign Default and the Stability of InflationTargeting Regimes. IMF Economic Review 62, 261-287.

Uhlig, H., 2014. Sovereign Default Risk and Banks in a Monetary Union. German Economic Review 15, 23-41. URL: https://ideas.repec.org/a/bla/germec/v15y2014i1p23-41.html.

Woodford, M., 1990. Public Debt as Private Liquidity. American Economic Review 80, 382-388. URL: https://ideas.repec.org/a/aea/aecrev/v80y1990i2p382-88.html.

Woodford, M., 1998. Public debt and the price level. unpublished manuscript Columbia University.

Woodford, M., 2001. Fiscal requirements for price stability. Journal of Money, Credit and Banking 33, 669-728.

## A Extensive literature review

Early papers that investigate the effects of longer-maturity LTROs do so using estimations based on aggregated data. For example, Giannone et al. (2012) perform a Bayesian estimation of a VAR model using a sample up to April 2011, while Darracq-Paries and De Santis (2015) estimate a panel-VAR for euro area countries, and focus their analysis on the three-year LTROs. Both these papers find an expansion of bank lending to non-financial corporations and real activity.

While these papers perform their analysis at the level of the aggregated banking system, Drechsler et al. (2016), Carpinelli and Crosignani (2018), Garcia-Posada and Marchetti (2016), and Andrade et al. (2019) study the ECB's unconventional LTROs at the level of individual banks. Drechsler et al. (2016) focus on the role of the ECB as a Lender of Last Resort (LOLR) during the European sovereign debt crisis. They find that weakly capitalized banks borrowed more from the ECB, pledged riskier collateral, and actively invested the funds borrowed from the ECB in distressed sovereign debt after the start of the European sovereign debt crisis in 2010. Their sample, however, ends in December 2011, and does not include the three-year LTRO of February 2012.

Carpinelli and Crosignani (2018), Garcia-Posada and Marchetti (2016), and Andrade et al. (2019) specifically focus on the three-year LTROs, and find a positive effect on credit provision to the real economy in Italy, Spain, and France, respectively. In addition, Andrade et al. (2019) find that three-year LTROs expand loan supply by more than shorter-maturity LTROs.

LTROs also induced banks to expand their holdings of government bonds (Drechsler et al., 2016, Carpinelli and Crosignani, 2018; Crosignani et al. forthcoming). Carpinelli and Crosignani (2018) report that Italian banks used $€ 82.7$ billion of $€ 181.5$ billion in ECB funding to purchase
additional government bonds, while only $€ 22.6$ billion was invested in private credit. This number increased to $€ 0.83$ in Italian government bonds per euro of LTRO-funding for banks that were not exposed to the wholesale funding dry-up that took place in the second half of 2011. Similarly, Crosignani et al. (forthcoming) report that the three-year LTROs induced Portuguese banks to purchase $10.6 \%$ of the total stock of outstanding short-term domestic government bonds, and pledge them as collateral to obtain funding from the ECB. Crosignani et al. (forthcoming) also find that the yields on short-term Portuguese government securities collapsed in the aftermath of the announcement of the three-year LTROs, a result in line with Krishnamurthy et al. (2018), whose analysis covers all GIIPS sovereign bond yields and multiple ECB programs ${ }^{26}$

Other mechanisms that explain why banks were accumulating government bonds during the European sovereign debt crisis are moral suasion (Altavilla et al., 2017, Becker and Ivashina, 2018; Ongena et al., 2019) and risk-shifting (Acharya and Steffen, 2015, Drechsler et al., 2016; Crosignani, 2016; Acharya et al., 2018). These papers also find that such an accumulation of government bonds reduced credit provision to the real economy. A second reason why credit provision to the real economy was reduced during the sovereign debt crisis was capital losses on impaired sovereign bond holdings on bank balance sheets (Popov and Horen, 2015, Altavilla et al., 2017, Acharya et al., 2018).

My paper also relates to Gertler and Kiyotaki (2010); Gertler and Karadi (2011, 2013), who study the transmission to the macroeconomy of shocks to the balance sheets of financial intermediaries. The key property of these papers is that the size of intermediaries' balance sheets is limited by the amount of net worth through an endogenous leverage constraint. Gertler and Karadi (2013); Kirchner and van Wijnbergen (2016) and Bocola (2016) extend this framework by introducing a portfolio choice between corporate securities and government debt.

A key result of my paper is that there is crowding out of credit provision to the real economy by government bonds through the collateral effect. Other theoretical papers that feature crowding out are Kirchner and van Wijnbergen (2016) and Crosignani (2016), where it is caused by a debt-financed fiscal expansion increasing commercial banks' bond holdings Kirchner and van Wijnbergen, 2016), and risk shifting (Crosignani, 2016). When banks are undercapitalized and will anyhow default in case of a sovereign default, they have an incentive to increase their holdings of domestic government bonds: these bonds have the highest payoff in the good state of the world, while limited liability protects the banks against the bad state of the world (i.e sovereign default). Other reasons for a reduction in credit provision to the real economy are capital losses on government bonds that reduce intermediaries' net worth, the so-called bank-sovereign nexus (Bocola, 2016, Van der Kwaak and Van Wijnbergen, 2014).

My paper is also related to the Lender of Last Resort (LOLR) literature, of which Bagehot (1873) was the first to argue that central banks should lent freely against good collateral at high rates. In order for banks to take out central bank funding during a financial crisis, LOLR funding must be subsidized in some way relative to funding sources in private markets: otherwise LOLR

[^18]lending would offer no benefit over the private market, and banks would not borrow from it. I capture this implicit subsidy by temporarily reducing the interest rate on central bank funding relative to that on deposit funding, in line with Engler and Große Steffen (2016).

The more recent literature that investigates the effects from central bank lending within the standard DSGE framework can broadly speaking be distinguished between collateralized and uncollateralized lending. One of the first papers to explicitly model uncollateralized central bank lending is Gertler and Kiyotaki (2010). Bocola (2016) and Cahn et al. (2017) extend this framework to investigate the impact of the ECB's unconventional LTROs. Both papers find that LTROs have an expansionary effect on bank lending and output because central bank lending directly relaxes intermediaries' incentive compatibility constraint. My paper differs in three dimensions: first, central bank funding does not directly relax intermediaries' incentive compatibility constraints, but only does so indirectly by decreasing intermediaries' funding costs. Second, the absence of a collateral requirement in Bocola (2016) implies that the provision of central bank funding does not directly affect intermediaries' portfolio choice between government bonds and credit provision to the real economy, while such a portfolio choice does not even feature in Cahn et al. (2017). Therefore, these papers do not feature the collateral effect, and are always expansionary. Third, the central bank determines the volume of lending in Gertler and Kiyotaki (2010) and Cahn et al. (2017), with the interest rate endogenously determined in equilibrium. This contrasts with the ECB's Fixed Rate Full Alotment (FRFA) policy, under which the nominal interest rate and collateral requirements are set by the ECB, and any amount of funding is supplied provided sufficient collateral is pledged (full allotment), a modeling strategy followed in this paper.

Other papers that look at uncollateralized liquidity provision by the central bank also find such operations to be expansionary (Güntner, 2015, Fahr et al., 2013). In Güntner (2015), such operations provide additional funds to intermediaries when they loose access to the unsecured interbank market (thereby preventing a cutback on lending to the real economy), while additional central bank liquidity helps financial intermediaries perform liquidity services for households and firms in Fahr et al. (2013), whose setup is based on Christiano et al. (2010). After a Bayesian estimation on Eurozone data, Fahr et al. (2013) conclude that the LTROs helped mitigate the contractionary impact of the impairment of money and interbank markets.

One of the first papers to explicitly introduce collateralized borrowing within a macroeconomic model is Kiyotaki and Moore (1997), in which land serves as collateral. However, as government bonds are considered to be the safest type of collateral, they typically serve as collateral in papers in which the secured interbank market is explicitly modeled (Engler and Große Steffen, 2016; Ferrero et al. 2017, Lakdawala et al., 2018).

A natural extension from such models is to allow the central bank to provide collateralized funding (Schabert, 2015, Hörmann and Schabert, 2015; Engler and Große Steffen, 2016). The key difference with my paper is that the agents with recourse to the central bank balance sheet are not subject to an endogenous leverage constraint, and can therefore perfectly elastically acquire
additional collateral in case central bank funding becomes more attractive. This contrasts with my paper, where the combination of collateral requirements and endogenous leverage constraints causes a tradeoff to emerge between acquiring additional government bonds (which provide the most central bank funding per euro) and credit provision to the real economy. Whereas central bank funding relaxes households' cash-in-advance constraint in Schabert (2015) and Hörmann and Schabert 2015), it finances credit provision to the real economy in Engler and Große Steffen (2016). Like in my paper, Engler and Große Steffen (2016) model the unconventional LTROs of December 2011 and February 2012 as an intervention in which the interest rate on central bank funding is temporarily reduced with respect to that on private sector funding.

My paper is also related to the literature in which government bonds provide liquidity services (Del Negro et al. 2017, Woodford, 1990, Brutti, 2011). Liquidity, however, is obtained by selling government bonds rather than by pledging them as collateral, and is not necessary to satisfy a cash-in-advance constraint, as in Schabert (2015) and Hörmann and Schabert (2015), but to finance new investment. In Del Negro et al. (2017), central bank liquidity operations consist of swapping less liquid assets for government bonds, which therefore has an expansionary effect on investment.

Central bank lending is also investigated within corporate finance type of models, in which moral hazard and risk shifting issues are more easily incorporated. Uhlig (2014) finds that countries with higher sovereign default risk have an incentive to set looser regulation, as this induces banks to purchase more government bonds, thereby reducing the sovereign's funding costs. Despite creating counterparty risk for the central bank, both Koulischer and Struyven (2014) and Choi et al. (2019) find that it might be optimal for the central bank to relax its collateral requirements. Doing so can mitigate the contraction in lending to the real economy during a credit crunch (Koulischer and Struyven, 2014), and improve liquidity creation in private markets (Choi et al. 2019).

Finally, my paper connects to the broader strand of literature that investigates unconventional monetary policies, which can broadly speaking be separated into asset purchase programs, central bank lending, negative interest rate policies, and forward guidance. Both the theoretical and empirical literature typically finds these measures to be expansionary, an overview of which can be found in Kuttner (2018).

## B Two period model

## B. 1 Model description

## B.1. 1 Households

There is a representative household that cares about consumption $c_{t}$ in period $t=0$ and $t=1$ because consumption generates utility $u(c)$ with the standard assumption that $u^{\prime}(c)>0$ and $u^{\prime \prime}(c)<0$. Households discount the expected future cashflow in period $t=1$ by the subjective
discount factor $\beta$. In period $t=0$, the household obtains an endowment $\mathcal{W}_{0}$ and income $\left(x_{0}^{c}+q_{0}^{b}\right)$ per government bond $s_{-1}^{b, h}$ purchased in period $t=-1$. Income in period $t=0$ is divided between consumption $c_{0}$, deposits $d_{0}$ at financial intermediaries on which households receive a net of principal interest rate $r_{0}^{d}$ in period $t=1$, and government bonds $s_{0}^{b, h}$, which are purchased at price $q_{0}^{b}$. Households pay a transaction cost upon purchasing bonds, which is quadratic in the deviation from a level $\hat{s}_{b, h}$. Upon arrival in period $t=1$, households receive income from repayment of savings $d_{0}$ including interest $r_{0}^{d} d_{0}$, repayment of principal and coupon $\left(1+x_{1}^{c}\right)$ of government bonds, the payout of net worth $n_{1}$ of the financial intermediary that it owns, and the profit $\Pi_{1}$ of the production firm it owns. Although the household owns the financial intermediary, it is not capable of influencing its investment decisions. The household therefore regards the net worth $n_{1}$ as a lump sum payment from the financial intermediary. The household uses the funds for consumption $c_{1}$ and lump sum taxes $\tau_{1}$ that are paid to the government. The household's optimization problem is now given by:

$$
\begin{aligned}
& \max _{\left\{c_{0}, c_{1}, d_{0}, s_{0}^{b, h}\right\}} \\
& u\left(c_{0}\right)+\beta E_{0}\left[u\left(c_{1}\right)\right] \\
& \text { s.t. }
\end{aligned}
$$

where $\Pi_{1}$ are profits from production firms in period $t=1$, which are given by:

$$
\begin{equation*}
\Pi_{1}=y_{1}-\left(1+r_{0}^{k}\right) k_{0}=k_{0}^{\alpha}-\left(1+r_{0}^{k}\right) k_{0} \tag{26}
\end{equation*}
$$

I set up the accompanying Lagrangian for the household's optimization problem:

$$
\begin{aligned}
\mathcal{L} & =u\left(c_{0}\right)+\beta E_{0}\left[u\left(c_{1}\right)\right]+\lambda_{0}\left[\mathcal{W}_{0}+\left(x_{0}^{c}+q_{0}^{b}\right) s_{-1}^{b, h}-c_{0}-d_{0}-q_{0}^{b} s_{0}^{b, h}-\frac{1}{2} \kappa_{s_{b, h}}\left(s_{0}^{b, h}-\hat{s}_{b, h}\right)^{2}\right] \\
& +\beta E_{0}\left\{\lambda_{1}\left[\left(1+r_{0}^{d}\right) d_{0}+\left(1+x_{1}^{c}\right) s_{0}^{b, h}+n_{1}+\Pi_{1}-c_{1}-\tau_{1}\right]\right\}
\end{aligned}
$$

Differentiation with respect to $c_{0}, d_{0}, s_{0}^{b, h}$ and $c_{1}$ gives the following first order conditions:

$$
\begin{align*}
c_{0}: & u^{\prime}\left(c_{0}\right)-\lambda_{0}=0 \Rightarrow \lambda_{0}=u^{\prime}\left(c_{0}\right),  \tag{27}\\
c_{1}: & \beta u^{\prime}\left(c_{1}\right)-\beta \lambda_{1}=0 \Rightarrow \lambda_{1}=u^{\prime}\left(c_{1}\right),  \tag{28}\\
d_{0}: & -\lambda_{0}+\beta E_{0}\left[\lambda_{1}\left(1+r_{0}^{d}\right)\right]=0 \Rightarrow E_{0}\left[\beta \Lambda_{0,1}\left(1+r_{0}^{d}\right)\right]=1 .  \tag{29}\\
s_{0}^{b, h}: & -\lambda_{0}\left[q_{0}^{b}+\kappa_{s_{b, h}}\left(s_{0}^{b, h}-\hat{s}_{b, h}\right)\right]+\beta E_{0}\left[\lambda_{1}\left(1+x_{1}^{c}\right)\right]=0 \Rightarrow \\
& E_{0}\left\{\beta \Lambda_{0,1}\left[\frac{1+x_{1}^{c}}{q_{0}^{b}+\kappa_{s_{b, h}}\left(s_{0}^{b, h}-\hat{s}_{b, h}\right)}\right]\right\}=1, \tag{30}
\end{align*}
$$

where $\beta \Lambda_{0,1}=\beta \lambda_{1} / \lambda_{0}$ is the households' stochastic discount factor.

## B.1.2 Production firms

Production firms borrow funds from financial intermediaries to purchase physical capital $k_{0}$ in period $t=0$. They employ this capital to produce goods $y_{1}$ in period $t=1$, with a concave production function:

$$
y_{1}=k_{0}^{\alpha} .
$$

Intermediaries are repaid in period $t=1$, together with a net rate of return $r_{0}^{k}$. The production firms' optimization problem is subsequently given by:

$$
\max _{\left\{k_{0}\right\}} E_{0}\left\{\beta \Lambda_{0,1}\left[y_{1}-\left(1+r_{0}^{k}\right) k_{0}\right]\right\}
$$

where $\beta \Lambda_{0,1}$ is the households' stochastic discount factor. The production firms' first order condition is given by:

$$
\begin{equation*}
r_{0}^{k}=\alpha k_{0}^{\alpha-1}-1 \tag{31}
\end{equation*}
$$

## B.1.3 Government

Period $t=0$

The government enters period $t=0$ with outstanding long-term government bonds $b_{-1}$ which are traded at a price $q_{0}^{b}$. At the beginning of period $t=0$, these bonds pay a coupon $x_{0}^{c}$. A second coupon $x_{1}^{c}$ is paid at the beginning of period $t=1$, at which moment the principal $b_{-1}$ is repaid.

To meet outstanding liabilities, the government issues new bonds $b_{0}^{\text {new }}$ at the end of period $t=0$. Bonds $b_{0}^{\text {new }}$ pay a coupon $x_{1}^{c}$ at the beginning of period $t=1$, at which moment the principal is repaid. The cash flows from bonds $b_{-1}$ and $b_{0}^{\text {new }}$ are therefore exactly the same from the end of period $t=0$ going forward, which implies that the price of both bonds equals $q_{0}^{b}$. Revenue $q_{0}^{b} b_{0}^{\text {new }}$ from issuance of new bonds $b_{0}^{\text {new }}$ is used to pay the coupon $x_{0}^{c} b_{-1}$ on outstanding
government bonds:

$$
q_{0}^{b} b_{0}^{\text {new }}=x_{0}^{c} b_{-1} .
$$

Define the total number of bonds outstanding at the end of period $t=0$ by $b_{0}=b_{0}^{\text {new }}+b_{-1}$. Then the government budget constraint in period $t=0$ is given by:

$$
\begin{equation*}
q_{0}^{b} b_{0}=q_{0}^{b}\left(b_{0}^{n e w}+b_{-1}\right)=\left(x_{0}^{c}+q_{0}^{b}\right) b_{-1} . \tag{32}
\end{equation*}
$$

## Period $t=1$

The government enters period $t=1$ with a stock of government bonds $b_{0}$ on which it has to pay a coupon $x_{1}^{c}$ per bond issued, and it has to repay the principal. Government liabilities at the beginning of period $t=1$ are therefore equal to $\left(1+x_{1}^{c}\right) b_{0}$. The government meets these obligations by raising lump sum taxes $\tau_{1}$ from households, which leads to the following government budget constraint in period $t=1$ :

$$
\begin{equation*}
\tau_{1}=\left(1+x_{1}^{c}\right) b_{0} \tag{33}
\end{equation*}
$$

## B.1.4 Central Bank

The central bank description can to a large extent be found in Section 3.1.1 of the main text. However, central bank profits in period $t=1$ were not described at that place.

Remember that the central bank receives an interest rate $r_{0}^{c b}$ in period $t=1$ on loans $d_{0}^{c b}$ issued in period $t=0$, while the central bank pays an interest rate $r_{0}^{R}$ in period $t=1$ on reserves $m_{0}^{R}$ issued in period $t=0$. Hence central bank profits $\Pi_{1}^{c b}$ in period $t=1$ are given by:

$$
\begin{equation*}
\Pi_{1}^{c b}=\left(1+r_{0}^{c b}\right) d_{0}^{c b}-\left(1+r_{0}^{R}\right) m_{0}^{R} \tag{34}
\end{equation*}
$$

## B.1.5 Financial intermediaries

Financial intermediaries enter period $t=0$ with net worth $n_{0}$. They raise deposits $d_{0}$ from households and obtain central bank funding $d_{0}^{c b}$. To obtain central bank funding $d_{0}^{c b}$, financial intermediaries need to pledge government bonds as collateral:

$$
\begin{equation*}
d_{0}^{c b} \leq \theta^{b} q_{0}^{b} s_{0}^{b} \tag{35}
\end{equation*}
$$

These funds are used to finance loans $s_{0}^{k}$ to production firms, purchase government bonds $s_{0}^{b}$ at a price $q_{0}^{b}$, and central bank reserves $m_{0}^{R}$. Therefore, the balance sheet of the representative intermediary is given by:

$$
\begin{equation*}
s_{0}^{k}+q_{0}^{b} s_{0}^{b}+m_{0}^{R}=n_{0}+d_{0}+d_{0}^{c b} \tag{36}
\end{equation*}
$$

Lending to production firms earns a net return $r_{0}^{k}$ at the beginning of period $t=1$. The government repays the principal and pays a coupon $x_{1}^{c}$ on bonds $s_{0}^{b}$ at the beginning of period $t=1$, while central bank reserves $m_{0}^{R}$ earn interest $r_{0}^{R}$. Intermediaries repay principal and interest $r_{0}^{d}$ and $r_{0}^{c b}$, respectively, on depsoits $d_{0}$ and central bank funding $d_{0}^{c b}$, respectively. Net worth in period $t=1$ is given by:

$$
\begin{equation*}
n_{1}=\left(1+r_{0}^{k}\right) s_{0}^{k}+\left(1+r_{0}^{b}\right) q_{0}^{b} s_{0}^{b}+\left(1+r_{0}^{R}\right) m_{0}^{R}-\left(1+r_{0}^{d}\right) d_{0}-\left(1+r_{0}^{c b}\right) d_{0}^{c b} \tag{37}
\end{equation*}
$$

with $r_{0}^{b}$ the net return on government bonds:

$$
\begin{equation*}
r_{0}^{b}=\frac{1+x_{1}^{c}}{q_{0}^{b}}-1 \tag{38}
\end{equation*}
$$

Financial intermediaries maximize expected discounted net worth $E_{0}\left[\beta \Lambda_{0,1} n_{1}\right]$, with $\beta \Lambda_{0,1}$ the households' stochastic discount factor, as households are the ultimate owners of financial intermediaries. Financial intermediaries face an incentive compatibility constraint as in Gertler and Karadi (2011): after purchasing assets in period $t=0$, financial intermediaries have the opportunity to divert assets at the end of period $t=0$. Depositors anticipate this possibility, and will in equilibrium provide deposits up to the point where the continuation value is larger than or equal to the value gained by diverting assets:

$$
\begin{equation*}
E_{0}\left[\beta \Lambda_{0,1} n_{1}\right] \geq \lambda_{k} s_{0}^{k}+\lambda_{b} q_{0}^{b} s_{0}^{b} \tag{39}
\end{equation*}
$$

where I assume that central bank reserves cannot be diverted by the managers of the intermediary. The optimization problem of intermediaries is then given by:

$$
\begin{aligned}
\underset{\left\{s_{0}^{k}, s_{0}^{b}, m_{0}^{R}, d_{0}, d_{0}^{c b}\right\}}{\max } \quad & E_{0}\left[\beta \Lambda_{0,1} n_{1}\right] \\
& \text { s.t. } \\
E_{0}\left[\beta \Lambda_{0,1} n_{1}\right] & \geq \lambda_{k} s_{0}^{k}+\lambda_{b} q_{0}^{b} s_{0}^{b} \\
s_{0}^{k}+q_{0}^{b} s_{0}^{b}+m_{0}^{R} & =n_{0}+d_{0}+d_{0}^{c b} \\
\theta^{b} q_{0}^{b} s_{0}^{b} & \geq d_{0}^{c b} \\
n_{1} & =\left(1+r_{0}^{k}\right) s_{0}^{k}+\left(1+r_{0}^{b}\right) q_{0}^{b} s_{0}^{b}+\left(1+r_{0}^{R}\right) m_{0}^{R}-\left(1+r_{0}^{d}\right) d_{0}-\left(1+r_{0}^{c b}\right) d_{0}^{c b}
\end{aligned}
$$

After substituting out net worth in period $t=1$ with the help of the law of motion for net worth (37), I set up the Lagrangian belonging to the intermediary's optimization problem:

$$
\begin{aligned}
\mathcal{L} & =\left(1+\mu_{0}\right) E_{0}\left\{\beta \Lambda_{0,1}\left[\left(1+r_{0}^{k}\right) s_{0}^{k}+\left(1+r_{0}^{b}\right) q_{0}^{b} s_{0}^{b}+\left(1+r_{0}^{R}\right) m_{0}^{R}-\left(1+r_{0}^{d}\right) d_{0}-\left(1+r_{0}^{c b}\right) d_{0}^{c b}\right]\right\} \\
& -\mu_{0}\left(\lambda_{k} s_{0}^{k}+\lambda_{b} q_{0}^{b} s_{0}^{b}\right)+\chi_{0}\left(n_{0}+d_{0}+d_{0}^{c b}-s_{0}^{k}-q_{0}^{b} s_{0}^{b}-m_{0}^{R}\right)+\psi_{0}^{c b}\left(\theta^{b} q_{0}^{b} s_{0}^{b}-d_{0}^{c b}\right),
\end{aligned}
$$

where $\mu_{0}$ is the Lagrangian multiplier on the incentive compatibility constraint of the commercial bank, $\chi_{0}$ the Lagrangian multiplier on the balance sheet constraint of the commercial bank, and $\psi_{0}^{c b}$ the Lagrangian multiplier on the collateral constraint. Differentiation with respect to loans, bonds, deposits and central bank funding results in the following first order conditions:

$$
\begin{align*}
s_{0}^{k} & :\left(1+\mu_{0}\right) E_{0}\left[\beta \Lambda_{0,1}\left(1+r_{0}^{k}\right)\right]-\lambda_{k} \mu_{0}-\chi_{0}=0  \tag{40}\\
s_{0}^{b} & :\left(1+\mu_{0}\right) E_{0}\left[\beta \Lambda_{0,1}\left(1+r_{0}^{b}\right)\right]-\lambda_{b} \mu_{0}-\chi_{0}+\psi_{0}^{c b} \theta^{b}=0  \tag{41}\\
m_{0}^{R} & :\left(1+\mu_{0}\right) E_{0}\left[\beta \Lambda_{0,1}\left(1+r_{0}^{R}\right)\right]-\chi_{0}=0,  \tag{42}\\
d_{0} & :-\left(1+\mu_{0}\right) E_{0}\left[\beta \Lambda_{0,1}\left(1+r_{0}^{d}\right)\right]+\chi_{0}=0 .  \tag{43}\\
d_{0}^{c b} & :-\left(1+\mu_{0}\right) E_{0}\left[\beta \Lambda_{0,1}\left(1+r_{0}^{c b}\right)\right]+\chi_{0}-\psi_{0}^{c b}=0 . \tag{44}
\end{align*}
$$

I can rewrite the first order condition for deposits as $\chi_{0}=\left(1+\mu_{0}\right) E_{0}\left[\beta \Lambda_{0,1}\left(1+r_{0}^{d}\right)\right]$, and substitute in the first order conditions for loans, bond, and reserves to get:

$$
\begin{align*}
s_{0}^{k} & : \quad E_{0}\left[\beta \Lambda_{0,1}\left(r_{0}^{k}-r_{0}^{d}\right)\right]=\frac{\lambda_{k} \mu_{0}}{1+\mu_{0}},  \tag{45}\\
s_{0}^{b} & : \quad E_{0}\left[\beta \Lambda_{0,1}\left(r_{0}^{b}-r_{0}^{d}\right)\right]=\frac{\lambda_{b} \mu_{0}}{1+\mu_{0}}-\theta^{b}\left(\frac{\psi_{0}^{c b}}{1+\mu_{0}}\right),  \tag{46}\\
m_{0}^{R} & : \quad E_{0}\left[\beta \Lambda_{0,1}\left(r_{0}^{R}-r_{0}^{d}\right)\right]=0,  \tag{47}\\
d_{0}^{c b} & : \quad E_{0}\left[\beta \Lambda_{0,1}\left(r_{0}^{d}-r_{0}^{c b}\right)\right]=\frac{\psi_{0}^{c b}}{1+\mu_{0}} . \tag{48}
\end{align*}
$$

We observe from equation (47) that $r_{0}^{R}=r_{0}^{d}$ : in equilibrium, the interest rate on deposits equals the interest rate on central bank reserves. Continuing, I solve for $\mu_{0} /\left(1+\mu_{0}\right)$ from equation (45), and substitute the resulting expression into equation 46) delivers the following first order condition for the portfolio choice between corporate loans and government bonds:

$$
\begin{equation*}
\frac{\lambda_{b}}{\lambda_{k}} E_{0}\left[\beta \Lambda_{0,1}\left(r_{0}^{k}-r_{0}^{d}\right)\right]=E_{0}\left[\beta \Lambda_{0,1}\left(r_{0}^{b}-r_{0}^{d}\right)\right]+\theta^{b}\left(\frac{\psi_{0}^{c b}}{1+\mu_{0}}\right) . \tag{49}
\end{equation*}
$$

Substitution of equation (48) into this expression gives the following expression:

$$
\begin{equation*}
\frac{\lambda_{b}}{\lambda_{k}} E_{0}\left[\beta \Lambda_{0,1}\left(r_{0}^{k}-r_{0}^{d}\right)\right]=E_{0}\left[\beta \Lambda_{0,1}\left(r_{0}^{b}-r_{0}^{d}\right)\right]+\theta^{b} E_{0}\left[\beta \Lambda_{0,1}\left(r_{0}^{d}-r_{0}^{c b}\right)\right] . \tag{50}
\end{equation*}
$$

I continue by looking at the law of motion for net worth $n_{1}$ :

$$
\begin{align*}
n_{1} & =\left(1+r_{0}^{k}\right) s_{0}^{k}+\left(1+r_{0}^{b}\right) q_{0}^{b} s_{0}^{b}+\left(1+r_{0}^{R}\right) m_{0}^{R}-\left(1+r_{0}^{d}\right) d_{0}-\left(1+r_{0}^{c b}\right) d_{0}^{c b} \\
& =\left(1+r_{0}^{k}\right) s_{0}^{k}+\left(1+r_{0}^{b}\right) q_{0}^{b} s_{0}^{b}+\left(1+r_{0}^{R}\right) m_{0}^{R}-\left(1+r_{0}^{d}\right)\left(s_{0}^{k}+q_{0}^{b} s_{0}^{b}+m_{0}^{R}-n_{0}-d_{0}^{c b}\right)-\left(1+r_{0}^{c b}\right) d_{0}^{c b} \\
& =\left(r_{0}^{k}-r_{0}^{d}\right) s_{0}^{k}+\left(r_{0}^{b}-r_{0}^{d}\right) q_{0}^{b} s_{0}^{b}+\left(r_{0}^{R}-r_{0}^{d}\right) m_{0}^{R}+\left(1+r_{0}^{d}\right) n_{0}+\left(r_{0}^{d}-r_{0}^{c b}\right) d_{0}^{c b} \tag{51}
\end{align*}
$$

Now I take a look at the incentive compatibility constraint of the commercial bank 39. I start by substituting (51) for $n_{1}$ :

$$
\begin{align*}
& E_{0}\left[\beta \Lambda_{0,1}\left(r_{0}^{k}-r_{0}^{d}\right)\right] s_{0}^{k}+E_{0}\left[\beta \Lambda_{0,1}\left(r_{0}^{b}-r_{0}^{d}\right)\right] q_{0}^{b} s_{0}^{b}+E_{0}\left[\beta \Lambda_{0,1}\left(r_{0}^{R}-r_{0}^{d}\right)\right] m_{0}^{R} \\
+ & E_{0}\left[\beta \Lambda_{0,1}\left(1+r_{0}^{d}\right)\right] n_{0}+E_{0}\left[\beta \Lambda_{0,1}\left(r_{0}^{d}-r_{0}^{c b}\right)\right] d_{0}^{c b} \geq \lambda_{k} s_{0}^{k}+\lambda_{b} q_{0}^{b} s_{0}^{b} \tag{52}
\end{align*}
$$

Substitution of the first order conditions for loan, bonds, reserves, and central bank funding (45) - (48) allows me to rewrite 52 in the following way:
$\frac{\mu_{0}}{1+\mu_{0}}\left(\lambda_{k} s_{0}^{k}+\lambda_{b} q_{0}^{b} s_{0}^{b}\right)-\theta^{b}\left(\frac{\psi_{0}^{c b}}{1+\mu_{0}}\right) q_{0}^{b} s_{0}^{b}+E_{0}\left[\beta \Lambda_{0,1}\left(1+r_{0}^{d}\right)\right] n_{0}+\left(\frac{\psi_{0}^{c b}}{1+\mu_{0}}\right) d_{0}^{c b} \geq \lambda_{k} s_{0}^{k}+\lambda_{b} q_{0}^{b} s_{0}^{b}$
I can rewrite this constraint into the following form:

$$
\begin{align*}
n_{0}-\left(\frac{\psi_{0}^{c b}}{1+\mu_{0}}\right)\left(\theta^{b} q_{0}^{b} s_{0}^{b}-d_{0}^{c b}\right) & \geq\left(1-\frac{\mu_{0}}{1+\mu_{0}}\right)\left(\lambda_{k} s_{0}^{k}+\lambda_{b} q_{0}^{b} s_{0}^{b}\right) \Rightarrow \\
\left(1+\mu_{0}\right) n_{0} & \geq \lambda_{k} s_{0}^{k}+\lambda_{b} q_{0}^{b} s_{0}^{b} \tag{53}
\end{align*}
$$

since $E_{0}\left[\beta \Lambda_{0,1}\left(1+r_{0}^{d}\right)\right]=1$ and $\psi_{0}^{c b}\left(\theta^{b} q_{0}^{b} s_{0}^{b}-d_{0}^{c b}\right)=0$ because of the Kuhn-Tucker conditions: either the collateral constraint is not binding, in which case $\psi_{0}^{c b}=0$, or the collateral constraint is binding, in which case $\theta^{b} q_{0}^{b} s_{0}^{b}-d_{0}^{c b}=0$.

I assume that net worth $n_{0}$ depends upon the bond price $q_{0}^{b}$ :

$$
\begin{align*}
n_{0} & =\left(1+r_{-1}^{k}\right) s_{-1}^{k}+\left(x_{0}^{c}+q_{0}^{b}\right) s_{-1}^{b}+\left(1+r_{-1}^{R}\right) m_{-1}^{R}-\left(1+r_{-1}^{d}\right) d_{-1}-\left(1+r_{-1}^{c b}\right) d_{-1}^{c b} \\
& =n_{0}^{e x}+\left(x_{0}^{c}+q_{0}^{b}\right) s_{-1}^{b} \tag{54}
\end{align*}
$$

where $n_{0}^{e x}=\left(1+r_{-1}^{k}\right) s_{-1}^{k}+\left(1+r_{-1}^{R}\right) m_{-1}^{R}-\left(1+r_{-1}^{d}\right) d_{-1}-\left(1+r_{-1}^{c b}\right) d_{-1}^{c b}$. Note that I have set $x_{0}^{c}=0$ in the main text.

## B.1.6 Market clearing

Market clearing for corporate loans requires that loans held by intermediaries $s_{0}^{k}$ equal totals physical capital $k_{0}$ held by production firms:

$$
\begin{equation*}
k_{0}=s_{0}^{k} \tag{55}
\end{equation*}
$$

Market clearing for bonds requires that bonds issued by the government equal bonds purchased by households and financial intermediaries:

$$
\begin{equation*}
b_{0}=s_{0}^{b}+s_{0}^{b, h} \tag{56}
\end{equation*}
$$

## B.1.7 Aggregate resource constraints

To derive the aggregate resource constraint in period $t=0$ and $t=1$, I start from the households' budget constraints in period $t=0$ and $t=1$. I start with period $t=0$, and substitute the intermediaries' balance sheet constraint (36), the market clearing condition for government bonds (56), and the government budget constraint (32):

$$
\begin{align*}
\mathcal{W}_{0}+\left(x_{0}^{c}+q_{0}^{b}\right) s_{-1}^{b, h} & =c_{0}+d_{0}+q_{0}^{b} s_{0}^{b, h}+\frac{1}{2} \kappa_{s_{b, h}}\left(s_{0}^{b, h}-\hat{s}_{b, h}\right)^{2} \\
& =c_{0}+d_{0}+q_{0}^{b}\left(b_{0}-s_{0}^{b}\right)+\frac{1}{2} \kappa_{s_{b, h}}\left(s_{0}^{b, h}-\hat{s}_{b, h}\right)^{2} \\
& =c_{0}+s_{0}^{k}+q_{0}^{b} s_{0}^{b}+m_{0}^{R}-n_{0}-d_{0}^{c b}+q_{0}^{b} b_{0}-q_{0}^{b} s_{0}^{b}+\frac{1}{2} \kappa_{s_{b, h}}\left(s_{0}^{b, h}-\hat{s}_{b, h}\right)^{2} \\
& =c_{0}+s_{0}^{k}-n_{0}+q_{0}^{b} b_{0}+\frac{1}{2} \kappa_{s_{b, h}}\left(s_{0}^{b, h}-\hat{s}_{b, h}\right)^{2} \\
& =c_{0}+s_{0}^{k}-n_{0}+x_{0}^{c} b_{-1}+q_{0}^{b} b_{-1}+\frac{1}{2} \kappa_{s_{b, h}}\left(s_{0}^{b, h}-\hat{s}_{b, h}\right)^{2} . \tag{57}
\end{align*}
$$

where I employed the central bank's balance sheet constraint $d_{0}^{c b}=m_{0}^{R}$. Now we remember from equation (55) that $s_{0}^{k}=k_{0}$. In addition, intermediaries' net worth in period $t=0$ is given by $n_{0}=n_{0}^{e x}+\left(x_{0}^{c}+q_{0}^{b}\right) s_{-1}^{b}$. Substituting this expression into the households' budget constraint (57) gives the following expression:

$$
\begin{align*}
\mathcal{W}_{0}+\left(x_{0}^{c}+q_{0}^{b}\right) s_{-1}^{b, h} & =c_{0}+k_{0}-n_{0}^{e x}-\left(x_{0}^{c}+q_{0}^{b}\right) s_{-1}^{b}+\left(x_{0}^{c}+q_{0}^{b}\right) b_{-1}+\frac{1}{2} \kappa_{s_{b, h}}\left(s_{0}^{b, h}-\hat{s}_{b, h}\right)^{2} \Rightarrow \\
\mathcal{W}_{0}+\left(x_{0}^{c}+q_{0}^{b}\right)\left(s_{-1}^{b}+s_{-1}^{b, h}\right) & =c_{0}+k_{0}-n_{0}^{e x}+\left(x_{0}^{c}+q_{0}^{b}\right) b_{-1}+\frac{1}{2} \kappa_{s_{b, h}}\left(s_{0}^{b, h}-\hat{s}_{b, h}\right)^{2} \Rightarrow \\
\mathcal{W}_{0}+n_{0}^{e x} & =c_{0}+k_{0}+\frac{1}{2} \kappa_{s_{b, h}}\left(s_{0}^{b, h}-\hat{s}_{b, h}\right)^{2} \tag{58}
\end{align*}
$$

Next I start from the households' budget constraint in period $t=1$, and substitute the equation for lump sum taxes in period $t=1$ (33), central bank profits (34), the market clearing conditions (55) - (56) and the law of motion for net worth $n_{1}$ in period $t=1$, equation (37), and
production firms' profits 26):

$$
\begin{aligned}
c_{1}+\tau_{1} & =\left(1+r_{0}^{d}\right) d_{0}+\left(1+x_{1}^{c}\right) s_{0}^{b, h}+n_{1}+\Pi_{1} \Rightarrow \\
c_{1} & =\left(1+r_{0}^{d}\right) d_{0}+\left(1+x_{1}^{c}\right) s_{0}^{b, h}+n_{1}+\Pi_{1}-\tau_{1} \\
& =\left(1+r_{0}^{d}\right) d_{0}+\left(1+x_{1}^{c}\right)\left(b_{0}-s_{0}^{b}\right) \\
& +\left(1+r_{0}^{k}\right) s_{0}^{k}+\left(1+x_{1}^{c}\right) s_{0}^{b}+\left(1+r_{0}^{R}\right) m_{0}^{R}-\left(1+r_{0}^{d}\right) d_{0}-\left(1+r_{0}^{c b}\right) d_{0}^{c b} \\
& +k_{0}^{\alpha}-\left(1+r_{0}^{k}\right) k_{0}-\left(1+x_{1}^{c}\right) b_{0}+\Pi_{1}^{c b} \\
& =\left(1+r_{0}^{k}\right) s_{0}^{k}+\left(1+r_{0}^{R}\right) m_{0}^{R}-\left(1+r_{0}^{c b}\right) d_{0}^{c b} \\
& +k_{0}^{\alpha}-\left(1+r_{0}^{k}\right) k_{0}+\left(1+r_{0}^{c b}\right) d_{0}^{c b}-\left(1+r_{0}^{R}\right) m_{0}^{R} \\
& =\left(1+r_{0}^{k}\right) k_{0}+k_{0}^{\alpha}-\left(1+r_{0}^{k}\right) k_{0} \\
& =k_{0}^{\alpha}
\end{aligned}
$$

Hence we find as the aggregate resource constraint in period $t=1$ :

$$
\begin{equation*}
c_{1}=k_{0}^{\alpha} . \tag{59}
\end{equation*}
$$

## B.1.8 Some further derivations

I start by remembering from equation (47) that $r_{0}^{R}=r_{0}^{d}$. We remember from the main text that the central bank is in charge of both the interest rate on reserves $r_{0}^{R}$ and central bank loans $r_{0}^{c b}$, and that I assume that in my analysis the central bank only changes $r_{0}^{c b}$ without changing $r_{0}^{R}$. Therefore, I know from equation (47) that the interest rate on deposits $r_{0}^{d}$ also remains constant. Consider now a change in the interest rate difference $\Gamma_{0}^{c b}$ between the return on deposits and the return on central bank funding, and assume that no further shocks occur. Therefore, I can drop the expectations operator $E_{0}$ in my analysis. I start by inspecting the households' Euler equation for deposits:

$$
\begin{aligned}
0 & =\frac{d}{d \Gamma_{0}^{c b}}\left\{E_{0}\left[\beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot\left(1+r_{0}^{d}\right)\right]\right\}=\frac{d}{d \Gamma_{0}^{c b}}\left[\beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot\left(1+r_{0}^{d}\right)\right] \\
& =\beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot\left(1+r_{0}^{d}\right)\left[\frac{u^{\prime \prime}\left(c_{1}\right)}{u^{\prime}\left(c_{1}\right)} \cdot \frac{d c_{1}}{d \Gamma_{0}^{c b}}-\frac{u^{\prime \prime}\left(c_{0}\right)}{u^{\prime}\left(c_{0}\right)} \cdot \frac{d c_{0}}{d \Gamma_{0}^{c b}}\right] .
\end{aligned}
$$

From this equation I find:

$$
\begin{equation*}
\frac{u^{\prime \prime}\left(c_{1}\right)}{u^{\prime}\left(c_{1}\right)} \cdot \frac{d c_{1}}{d \Gamma_{0}^{c b}}-\frac{u^{\prime \prime}\left(c_{0}\right)}{u^{\prime}\left(c_{0}\right)} \cdot \frac{d c_{0}}{d \Gamma_{0}^{c b}}=0 \tag{60}
\end{equation*}
$$

It will be useful to have a direct relationship between the tightness incentive compatibility constraint of the financial intermediary, captured by the Lagrangian multiplier $\mu_{0}$, and the interest rate shock $\Gamma_{0}^{c b}$. Such a direct relationship, however, is not directly available, but an indirect one is in the form of equation 45 . I can rewrite this relationship between the return on capital and
the Lagrangian multiplier on the incentive compatibility constraint in the following way:

$$
\begin{equation*}
\mu_{0}=\frac{\beta \Lambda_{0,1}\left(r_{0}^{k}-r_{0}^{d}\right)}{\lambda_{k}-\beta \Lambda_{0,1}\left(r_{0}^{k}-r_{0}^{d}\right)} \tag{61}
\end{equation*}
$$

I show that $\lambda_{k}>\beta \Lambda_{0,1}\left(r_{0}^{k}-r_{0}^{d}\right)$ in the following way:

$$
\lambda_{k}-\beta \Lambda_{0,1}\left(r_{0}^{k}-r_{0}^{d}\right)=\lambda_{k}-\lambda_{k}\left(\frac{\mu_{0}}{1+\mu_{0}}\right)=\frac{\lambda_{k}}{1+\mu_{0}}>0
$$

Now I differentiate $\mu_{0}$ with respect to the interest rate shock $\Gamma_{0}^{c b}$ :

$$
\begin{aligned}
\frac{d \mu_{0}}{d \Gamma_{0}^{c b}} & =\frac{\left[\beta \Lambda_{0,1}\left(r_{0}^{k}-r_{0}^{d}\right)\right]^{\prime}\left[\lambda_{k}-\beta \Lambda_{0,1}\left(r_{0}^{k}-r_{0}^{d}\right)\right]-\left[\beta \Lambda_{0,1}\left(r_{0}^{k}-r_{0}^{d}\right)\right]\left[\lambda_{k}-\beta \Lambda_{0,1}\left(r_{0}^{k}-r_{0}^{d}\right)\right]^{\prime}}{\left[\lambda_{k}-\beta \Lambda_{0,1}\left(r_{0}^{k}-r_{0}^{d}\right)\right]^{2}} \\
& =\frac{\left[\beta \Lambda_{0,1}\left(r_{0}^{k}-r_{0}^{d}\right)\right]^{\prime}\left[\lambda_{k}-\beta \Lambda_{0,1}\left(r_{0}^{k}-r_{0}^{d}\right)\right]+\left[\beta \Lambda_{0,1}\left(r_{0}^{k}-r_{0}^{d}\right)\right]\left[\beta \Lambda_{0,1}\left(r_{0}^{k}-r_{0}^{d}\right)\right]^{\prime}}{\left[\lambda_{k}-\beta \Lambda_{0,1}\left(r_{0}^{k}-r_{0}^{d}\right)\right]^{2}} \\
& =\frac{\lambda_{k}\left[\beta \Lambda_{0,1}\left(r_{0}^{k}-r_{0}^{d}\right)\right]^{\prime}}{\left[\lambda_{k}-\beta \Lambda_{0,1}\left(r_{0}^{k}-r_{0}^{d}\right)\right]^{2}}=\frac{\lambda_{k}\left[\beta \Lambda_{0,1}\left(r_{0}^{k}-r_{0}^{d}\right)\right]^{\prime}}{\left[\lambda_{k} /\left(1+\mu_{0}\right)\right]^{2}}=\frac{\left(1+\mu_{0}\right)^{2}\left[\beta \Lambda_{0,1}\left(r_{0}^{k}-r_{0}^{d}\right)\right]^{\prime}}{\lambda_{k}},
\end{aligned}
$$

where $[. . .]^{\prime}$ denotes the derivative of the object between brackets with respect to the interest rate shock $\Gamma_{0}^{c b}$. Now I continue to calculate $\left[\beta \Lambda_{0,1}\left(r_{0}^{k}-r_{0}^{d}\right)\right]^{\prime}$ :

$$
\begin{aligned}
{\left[\beta \Lambda_{0,1}\left(r_{0}^{k}-r_{0}^{d}\right)\right]^{\prime} } & =\left[\beta\left(\frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)}\right)\left(\alpha k_{0}^{\alpha-1}-1-r_{0}^{d}\right)\right]^{\prime}=\left[\beta\left(\frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)}\right) \alpha k_{0}^{\alpha-1}-1\right]^{\prime} \\
& =\beta\left(\frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)}\right) \alpha k_{0}^{\alpha-1}\left[\frac{u^{\prime \prime}\left(c_{1}\right)}{u^{\prime}\left(c_{1}\right)} \cdot \frac{d c_{1}}{d \Gamma_{0}^{c b}}-\frac{u^{\prime \prime}\left(c_{0}\right)}{u^{\prime}\left(c_{0}\right)} \cdot \frac{d c_{0}}{d \Gamma_{0}^{c b}}+\frac{\alpha-1}{k_{0}} \cdot \frac{d k_{0}}{d \Gamma_{0}^{c b}}\right] \\
& =\beta\left(\frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)}\right) \alpha k_{0}^{\alpha-1}\left(\frac{\alpha-1}{k_{0}}\right) \frac{d k_{0}}{d \Gamma_{0}^{c b}},
\end{aligned}
$$

where I used equation $\sqrt[29]{ }$ and 60 . Hence I end up with the following expression for $\frac{d \mu_{0}}{d \Gamma_{0}^{c b}}$ :

$$
\begin{equation*}
\frac{d \mu_{0}}{d \Gamma_{0}^{c b}}=\frac{\left(1+\mu_{0}\right)^{2}}{\lambda_{k}} \beta\left(\frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)}\right) \alpha k_{0}^{\alpha-1}\left(\frac{\alpha-1}{k_{0}}\right) \frac{d k_{0}}{d \Gamma_{0}^{c b}}=C \cdot \frac{d k_{0}}{d \Gamma_{0}^{c b}}, \tag{62}
\end{equation*}
$$

where $C$ is given by:

$$
\begin{equation*}
C=\frac{\left(1+\mu_{0}\right)^{2}}{\lambda_{k}} \beta\left(\frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)}\right) \alpha k_{0}^{\alpha-1}\left(\frac{\alpha-1}{k_{0}}\right)<0 . \tag{63}
\end{equation*}
$$

Now I implicitly differentiate the households' first order condition for government bonds:

$$
\begin{aligned}
0 & =\beta\left(\frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)}\right)\left[\frac{1+x_{1}^{c}}{q_{0}^{b}+\kappa_{s_{b, h}}\left(s_{0}^{b, h}-\hat{s}_{b, h}\right)}\right] \\
& \times\left[\frac{u^{\prime \prime}\left(c_{1}\right)}{u^{\prime}\left(c_{1}\right)} \cdot \frac{d c_{1}}{d \Gamma_{0}^{c b}}-\frac{u^{\prime \prime}\left(c_{0}\right)}{u^{\prime}\left(c_{0}\right)} \cdot \frac{d c_{0}}{d \Gamma_{0}^{c b}}-\left(\frac{1}{q_{0}^{b}+\kappa_{s_{b, h}}\left(s_{0}^{b, h}-\hat{s}_{b, h}\right)}\right)\left(\frac{d q_{0}^{b}}{d \Gamma_{0}^{c b}}+\kappa_{s_{b, h}} \cdot \frac{d s^{b, h}}{d \Gamma_{0}^{c b}}\right)\right] \Rightarrow \\
0 & =\frac{u^{\prime \prime}\left(c_{1}\right)}{u^{\prime}\left(c_{1}\right)} \cdot \frac{d c_{1}}{d \Gamma_{0}^{c b}}-\frac{u^{\prime \prime}\left(c_{0}\right)}{u^{\prime}\left(c_{0}\right)} \cdot \frac{d c_{0}}{d \Gamma_{0}^{c b}}-\left(\frac{1}{q_{0}^{b}+\kappa_{s_{b, h}}\left(s_{0}^{b, h}-\hat{s}_{b, h}\right)}\right)\left(\frac{d q_{0}^{b}}{d \Gamma_{0}^{c b}}+\kappa_{s_{b, h}} \cdot \frac{d s^{b, h}}{d \Gamma_{0}^{c b}}\right) \Rightarrow \\
0 & =\frac{d q_{0}^{b}}{d \Gamma_{0}^{c b}}+\kappa_{s_{b, h}} \cdot \frac{d s^{b, h}}{d \Gamma_{0}^{c b}}
\end{aligned}
$$

where I used first order condition (60) in the last step. I then obtain the following expression for the change in the number of government bonds held by the households:

$$
\begin{equation*}
\frac{d s_{0}^{b, h}}{d \Gamma_{0}^{c b}}=-B \cdot \frac{d q_{0}^{b}}{d \Gamma_{0}^{c b}} \tag{64}
\end{equation*}
$$

where $B$ is given by:

$$
\begin{equation*}
B=\frac{1}{\kappa_{s_{b, h}}} \tag{65}
\end{equation*}
$$

I can also derive a direct relation between the price of government bonds $q_{0}^{b}$ and the amount of physical capital $k_{0}$. I start by combining the first order conditions for physical capital (45) and government bonds 46):

$$
\begin{equation*}
\beta\left(\frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)}\right)\left(r_{0}^{b}-r_{0}^{d}\right)=\frac{\lambda_{b}}{\lambda_{k}} \beta\left(\frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)}\right)\left(r_{0}^{k}-r_{0}^{d}\right)-\theta^{b} \beta\left(\frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)}\right)\left(r_{0}^{d}-r_{0}^{c b}\right) \tag{66}
\end{equation*}
$$

Now I substitute the expressions for the return on capital and the return on government bonds to get:

$$
\begin{equation*}
\beta\left(\frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)}\right)\left(\frac{1+x_{1}^{c}}{q_{0}^{b}}-1-r_{0}^{d}\right)=\frac{\lambda_{b}}{\lambda_{k}} \beta\left(\frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)}\right)\left(\alpha k_{0}^{\alpha-1}-1-r_{0}^{d}\right)-\theta^{b} \beta\left(\frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)}\right) \Gamma_{0}^{c b}, \tag{67}
\end{equation*}
$$

where $\Gamma_{0}^{c b}=r_{0}^{d}-r_{0}^{c b}$. Using the households' first order condition for savings 29), I can rewrite this first order condition as:

$$
\begin{equation*}
\beta\left(\frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)}\right)\left(\frac{1+x_{1}^{c}}{q_{0}^{b}}\right)-1=\frac{\lambda_{b}}{\lambda_{k}} \beta\left(\frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)}\right) \alpha k_{0}^{\alpha-1}-\frac{\lambda_{b}}{\lambda_{k}}-\theta^{b} \beta\left(\frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)}\right) \Gamma_{0}^{c b} . \tag{68}
\end{equation*}
$$

Now I implicitly differentiate expression with respect to the interest rate spread between
deposits and central bank funding $\Gamma_{0}^{c b}$. I do so first for the left hand side (L.H.S.):

$$
\begin{align*}
{\left[\beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot\left(\frac{1+x_{1}^{c}}{q_{0}^{b}}\right)-1\right]^{\prime} } & =\beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot\left(\frac{1+x_{1}^{c}}{q_{0}^{b}}\right)\left[\frac{u^{\prime \prime}\left(c_{1}\right)}{u^{\prime}\left(c_{1}\right)} \cdot \frac{d c_{1}}{d \Gamma_{0}^{c b}}-\frac{u^{\prime \prime}\left(c_{0}\right)}{u^{\prime}\left(c_{0}\right)} \cdot \frac{d c_{0}}{d \Gamma_{0}^{c b}}-\frac{1}{q_{0}^{b}} \cdot \frac{d q_{0}^{b}}{d \Gamma_{0}^{c b}}\right] \\
& =-\beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot\left(\frac{1+x_{1}^{c}}{q_{0}^{b}}\right) \cdot \frac{1}{q_{0}^{b}} \cdot \frac{d q_{0}^{b}}{d \Gamma_{0}^{c b}} \tag{69}
\end{align*}
$$

where $[. . .]^{\prime}$ denotes differentiation of $[. .$.$] with respect to \Gamma_{0}^{c b}$. Implicit differentiation of the first term on the right hand side (R.H.S.) of (68) results in the following expression:

$$
\begin{align*}
{\left[\frac{\lambda_{b}}{\lambda_{k}} \beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot \alpha k_{0}^{\alpha-1}-\frac{\lambda_{b}}{\lambda_{k}}-\theta^{b} \beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot \Gamma_{0}^{c b}\right]^{\prime} } & =\frac{\lambda_{b}}{\lambda_{k}} \beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot \alpha k_{0}^{\alpha-1} \\
& \times\left[\frac{u^{\prime \prime}\left(c_{1}\right)}{u^{\prime}\left(c_{1}\right)} \cdot \frac{d c_{1}}{d \Gamma_{0}^{c b}}-\frac{u^{\prime \prime}\left(c_{0}\right)}{u^{\prime}\left(c_{0}\right)} \cdot \frac{d c_{0}}{d \Gamma_{0}^{c b}}+\frac{\alpha-1}{k_{0}} \cdot \frac{d k_{0}}{d \Gamma_{0}^{c b}}\right] \\
& -\theta^{b} \beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot \Gamma_{0}^{c b} \\
& \times\left[\frac{u^{\prime \prime}\left(c_{1}\right)}{u^{\prime}\left(c_{1}\right)} \cdot \frac{d c_{1}}{d \Gamma_{0}^{c b}}-\frac{u^{\prime \prime}\left(c_{0}\right)}{u^{\prime}\left(c_{0}\right)} \cdot \frac{d c_{0}}{d \Gamma_{0}^{c b}}+\frac{1}{\Gamma_{0}^{c b}}\right] \\
& =\frac{\lambda_{b}}{\lambda_{k}} \beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot \alpha k_{0}^{\alpha-1}\left(\frac{\alpha-1}{k_{0}}\right) \frac{d k_{0}}{d \Gamma_{0}^{c b}} \\
& -\theta^{b} \beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \tag{70}
\end{align*}
$$

where I again used 60 . Now I combine 69 and 70 to find the following relation:

$$
-\beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot\left(\frac{1+x_{1}^{c}}{q_{0}^{b}}\right) \cdot \frac{1}{q_{0}^{b}} \cdot \frac{d q_{0}^{b}}{d \Gamma_{0}^{c b}}=\frac{\lambda_{b}}{\lambda_{k}} \beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot \alpha k_{0}^{\alpha-1}\left(\frac{\alpha-1}{k_{0}}\right) \frac{d k_{0}}{d \Gamma_{0}^{c b}}-\theta^{b} \beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)}
$$

This can be rewritten in the following form:

$$
\begin{equation*}
\frac{d q_{0}^{b}}{d \Gamma_{0}^{c b}}=E+F \cdot \frac{d k_{0}}{d \Gamma_{0}^{c b}} \tag{71}
\end{equation*}
$$

where $E$ and $F$ are given by:

$$
\begin{align*}
E & =\frac{\theta^{b} \beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)}}{\beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot\left(\frac{1+x_{1}^{c}}{q_{0}^{b}}\right) \cdot \frac{1}{q_{0}^{b}}>0}  \tag{72}\\
F & =\frac{-\frac{\lambda_{b}}{\lambda_{k}} \beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot \alpha k_{0}^{\alpha-1}\left(\frac{\alpha-1}{k_{0}}\right)}{\beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot\left(\frac{1+x_{1}^{c}}{q_{0}^{b}}\right) \cdot \frac{1}{q_{0}^{b}}}>0 \tag{73}
\end{align*}
$$

Implicit differentiation of the market clearing condition for corporate loans (55) with respect
to the central bank interest $\Gamma_{0}^{c b}$ gives the following relation:

$$
\begin{equation*}
\frac{d k_{0}}{d \Gamma_{0}^{c b}}=\frac{d s_{0}^{k}}{d \Gamma_{0}^{c b}} \tag{74}
\end{equation*}
$$

## B.1.9 Government budget constraint

First I look at the government budget constraint (32), which I implicitly differentiate with respect to $\Gamma_{0}^{c b}$ :

$$
\begin{align*}
\frac{d\left(q_{0}^{b} b_{0}\right)}{d \Gamma_{0}^{c b}} & \equiv q_{0}^{b} \frac{d b_{0}}{d \Gamma_{0}^{c b}}+b_{0} \frac{d q_{0}^{b}}{d \Gamma_{0}^{c b}}=b_{-1} \frac{d q_{0}^{b}}{d \Gamma_{0}^{c b}} \Rightarrow \\
\frac{d b_{0}}{d \Gamma_{0}^{c b}} & =-\left(\frac{b_{0}-b_{-1}}{q_{0}^{b}}\right) \frac{d q_{0}^{b}}{d \Gamma_{0}^{c b}} . \tag{75}
\end{align*}
$$

## B.1.10 Market clearing

Implicitly differentiating (56) results in the following expression:

$$
\begin{equation*}
\frac{d b_{0}}{d \Gamma_{0}^{c b}}=\frac{d s_{0}^{b}}{d \Gamma_{0}^{c b}}+\frac{d s_{0}^{b, h}}{d \Gamma_{0}^{c b}} \tag{76}
\end{equation*}
$$

I use (76), together with the government budget constraint (75), and the households' first order condition for bondholdings (64) to express $\frac{d s_{0}^{b}}{d \Gamma_{0}^{c b}}$ in terms of $\frac{d q_{0}^{b}}{d \Gamma_{0}^{c b}}$ :

$$
\begin{align*}
\frac{d s_{0}^{b}}{d \Gamma_{0}^{c b}} & =\frac{d b_{0}}{d \Gamma_{0}^{c b}}-\frac{d s_{0}^{b, h}}{d \Gamma_{0}^{c b}}=-\left(\frac{b_{0}-b_{-1}}{q_{0}^{b}}\right) \frac{d q_{0}^{b}}{d \Gamma_{0}^{c b}}+B \cdot \frac{d q_{0}^{b}}{d \Gamma_{0}^{c b}} \\
& =\left(B-\frac{b_{0}-b_{-1}}{q_{0}^{b}}\right) \frac{d q_{0}^{b}}{d \Gamma_{0}^{c b}} \tag{77}
\end{align*}
$$

## B.1.11 Incentive compatibility constraint

Now I implicitly differentiate the incentive compatibiltiy constraint (53):

$$
\begin{aligned}
n_{0} \cdot \frac{d \mu_{0}}{d \Gamma_{0}^{c b}}+\left(1+\mu_{0}\right) \frac{d n_{0}}{d \Gamma_{0}^{c b}} & =\lambda_{k} \frac{d s_{0}^{k}}{d \Gamma_{0}^{c b}}+\lambda_{b} \cdot \frac{d\left(q_{0}^{b} s_{0}^{b}\right)}{d \Gamma_{0}^{c b}} \Rightarrow \\
n_{0} \cdot \frac{d \mu_{0}}{d \Gamma_{0}^{c b}}+\left(1+\mu_{0}\right) s_{-1}^{b} \frac{d q_{0}^{b}}{d \Gamma_{0}^{c b}} & =\lambda_{k} \frac{d s_{0}^{k}}{d \Gamma_{0}^{c b}}+\lambda_{b}\left(q_{0}^{b} \frac{d s_{0}^{b}}{d \Gamma_{0}^{c b}}+s_{0}^{b} \cdot \frac{d q_{0}^{b}}{d \Gamma_{0}^{c b}}\right)
\end{aligned}
$$

## B. 2 Analysis of a central bank interest rate shock

Consider the impact of a shock to the central bank spread $\Gamma_{0}^{c b}$.

$$
n_{0} C \cdot \frac{d k_{0}}{d \Gamma_{0}^{c b}}+\left(1+\mu_{0}\right) s_{-1}^{b} \cdot \frac{d q_{0}^{b}}{d \Gamma_{0}^{c b}}=\lambda_{k} \frac{d s_{0}^{k}}{d \Gamma_{0}^{c b}}+\lambda_{b}\left[q_{0}^{b}\left(B-\frac{b_{0}-b_{-1}}{q_{0}^{b}}\right) \frac{d q_{0}^{b}}{d \Gamma_{0}^{c b}}+s_{0}^{b} \cdot \frac{d q_{0}^{b}}{d \Gamma_{0}^{c b}}\right]
$$

This can be rewritten in the following way:

$$
\left(C n_{0}-\lambda_{k}\right) \frac{d s_{0}^{k}}{d \Gamma_{0}^{c b}}=\left\{\lambda_{b}\left[s_{0}^{b}-\left(b_{0}-b_{-1}\right)+B q_{0}^{b}\right]-\left(1+\mu_{0}\right) s_{-1}^{b}\right\} \frac{d q_{0}^{b}}{d \Gamma_{0}^{c b}}
$$

where I used $\frac{d s_{0}^{k}}{d \Gamma_{0}^{c b}}=\frac{d k_{0}}{d \Gamma_{0}^{c b}}$. I rewrite this in the following way:

$$
\begin{equation*}
\left(C n_{0}-\lambda_{k}\right) \frac{d s_{0}^{k}}{d \Gamma_{0}^{c b}}=G \cdot \frac{d q_{0}^{b}}{d \Gamma_{0}^{c b}} \tag{78}
\end{equation*}
$$

Since $C<0$, we see that $C n_{0}-\lambda_{k}<0$. Meanwhile, $G$ is given by:

$$
\begin{equation*}
G=\lambda_{b}\left[s_{0}^{b}-\left(b_{0}-b_{-1}\right)+B q_{0}^{b}\right]-\left(1+\mu_{0}\right) s_{-1}^{b} . \tag{79}
\end{equation*}
$$

Substitution of (71) gives the following expression:

$$
\begin{align*}
\left(C n_{0}-\lambda_{k}\right) \frac{d s_{0}^{k}}{d \Gamma_{0}^{c b}} & =G\left(E+F \cdot \frac{d k_{0}}{d \Gamma_{0}^{c b}}\right) \Rightarrow \\
\frac{d s_{0}^{k}}{d \Gamma_{0}^{c b}} & =\frac{G E}{\left(C n_{0}-\lambda_{k}-G F\right)} \tag{80}
\end{align*}
$$

I end by substituting 80 into 71 to get the change in the bond price as a result of a change in the interest rate spread between deposits and central bank funding:

$$
\begin{align*}
\frac{d q_{0}^{b}}{d \Gamma_{0}^{c b}} & =E+F \cdot \frac{d k_{0}}{d \Gamma_{0}^{c b}}=E+F \cdot \frac{d s_{0}^{k}}{d \Gamma_{0}^{c b}} \\
& =E+F \cdot\left[\frac{G E}{\left(C n_{0}-\lambda_{k}-G F\right)}\right] \\
& =\frac{E\left(C n_{0}-\lambda_{k}\right)}{\left(C n_{0}-\lambda_{k}-G F\right)} . \tag{81}
\end{align*}
$$

We know that $E>0$ and $C n_{0}-\lambda_{k}<0$. If I can prove that $C n_{0}-\lambda_{k}-G F<0$, we know that $\frac{d q_{0}^{b}}{d \Gamma_{0}^{c}}>0$. Let me start by writing $C n_{0}-\lambda_{k}-G F$ in the following way:

$$
\begin{equation*}
C n_{0}-\lambda_{k}-G F=C n_{0}-\lambda_{k}-G_{1} F+G_{2} F, \tag{82}
\end{equation*}
$$

where $G_{1}$ and $G_{2}$ are given by:

$$
\begin{align*}
G_{1} & =\lambda_{b}\left[s_{0}^{b}-\left(b_{0}-b_{-1}\right)+B q_{0}^{b}\right]>0  \tag{83}\\
G_{2} & =\left(1+\mu_{0}\right) s_{-1}^{b}>0 \tag{84}
\end{align*}
$$

where $G_{1}>0$ since $b_{0}=b_{-1}$. Since $F>0$, we see that $G_{2} F$ is the only term in 82 that is positive. Now I show that the sum $C n_{0}+G_{2} F$ is smaller than zero. In that case, $C n_{0}-\lambda_{k}-G F<$
0. I start by subsituting expressions (63), 73) and 79):

$$
\begin{aligned}
C n_{0}+G_{2} F & =n_{0} \cdot \frac{\left(1+\mu_{0}\right)^{2}}{\lambda_{k}} \beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot \alpha k_{0}^{\alpha-1}\left(\frac{\alpha-1}{k_{0}}\right) \\
& +\left(1+\mu_{0}\right) s_{-1}^{b}\left(\frac{-\frac{\lambda_{b}}{\lambda_{k}} \beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot \alpha k_{0}^{\alpha-1}\left(\frac{\alpha-1}{k_{0}}\right)}{\beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot\left(\frac{1+x_{1}^{c}}{q_{0}^{b}}\right) \cdot \frac{1}{q_{0}^{b}}}\right) \\
& =\frac{\left(1+\mu_{0}\right) \beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot \alpha k_{0}^{\alpha-1}\left(\frac{\alpha-1}{k_{0}}\right)}{\beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot\left(\frac{1+x_{1}^{c}}{q_{0}^{b}}\right)}\left[\beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot\left(\frac{1+x_{1}^{c}}{q_{0}^{b}}\right)\left(\frac{1+\mu_{0}}{\lambda_{k}}\right) n_{0}-\frac{\lambda_{b}}{\lambda_{k}} \cdot q_{0}^{b} s_{-1}^{b}\right] .
\end{aligned}
$$

We can see that $C n_{0}+G_{2} F<0$ if the term inside the brackets is larger than zero:

$$
\begin{aligned}
\beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot\left(\frac{1+x_{1}^{c}}{q_{0}^{b}}\right)\left(\frac{1+\mu_{0}}{\lambda_{k}}\right) n_{0}-\frac{\lambda_{b}}{\lambda_{k}} \cdot q_{0}^{b} s_{-1}^{b} & =\beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot\left(\frac{1+x_{1}^{c}}{q_{0}^{b}}\right) \frac{\left(1+\mu_{0}\right) n_{0}}{\lambda_{k}}-\frac{\lambda_{b}}{\lambda_{k}} \cdot q_{0}^{b} s_{-1}^{b} \\
& \geq \beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot\left(\frac{1+x_{1}^{c}}{q_{0}^{b}}\right)\left(\frac{\lambda_{k} k_{0}+\lambda_{b} q_{0}^{b} s_{0}^{b}}{\lambda_{k}}\right)-\frac{\lambda_{b}}{\lambda_{k}} \cdot q_{0}^{b} s_{-1}^{b} \\
& =\beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot\left(\frac{1+x_{1}^{c}}{q_{0}^{b}}\right)\left(k_{0}+\frac{\lambda_{b}}{\lambda_{k}} \cdot q_{0}^{b} s_{0}^{b}\right)-\frac{\lambda_{b}}{\lambda_{k}} \cdot q_{0}^{b} s_{-1}^{b} \\
& =\beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot\left(\frac{1+x_{1}^{c}}{q_{0}^{b}}\right) k_{0} \\
& +\frac{\lambda_{b}}{\lambda_{k}}\left[\beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot\left(\frac{1+x_{1}^{c}}{q_{0}^{b}}\right) q_{0}^{b} s_{0}^{b}-q_{0}^{b} s_{-1}^{b}\right] \\
& \geq \beta \cdot \frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{0}\right)} \cdot\left(\frac{1+x_{1}^{c}}{q_{0}^{b}}\right) k_{0}+\frac{\lambda_{b}}{\lambda_{k}} q_{0}^{b}\left(s_{0}^{b}-s_{-1}^{b}\right)>0 .
\end{aligned}
$$

Hence $C n_{0}-\lambda_{k}-G F<0$, and therefore

$$
\frac{d q_{0}^{b}}{d \Gamma_{0}^{c b}}=\frac{E\left(C n_{0}-\lambda_{k}\right)}{\left(C n_{0}-\lambda_{k}-G F\right)}>0
$$

for all $0 \leq \frac{\lambda_{b}}{\lambda_{k}} \leq 1$. Now we remember that the change in lending is given by 80 . Since $E>0$ and $C n_{0}-\lambda_{k}-G F<0$, I know that $\frac{d k_{0}}{d \Gamma_{0}^{c b}}>0$ when $G<0$. This happens when:

$$
\lambda_{b}\left[s_{0}^{b}-\left(b_{0}-b_{-1}\right)+B q_{0}^{b}\right]<\left(1+\mu_{0}\right) s_{-1}^{b}
$$

The left hand side says that intermediaries' incentive compatibility constraints tighten because i) the existing bond holdings $s_{0}^{b}$ increase in value through a higher bond price and ii) because intermediaries buy additional bondholdings, captured by $B q_{0}^{b}$. The right hand side says that intermediaries incentive compatibility constraints become less binding, as previous period bondholdings $s_{-1}^{b}$ increase as the bond price increases as a result of the central bank interest rate shock. An increase in the monetary value of previous period bondholdings $s_{-1}^{b}$ increase intermediaries' net worth. Hence intermediaries are indirectly recapitalized as the central bank policy
increases their net worth. When the above inequality holds, it means that the indirect recapitalization through capital gains on government bonds is larger than the tightening of intermediaries incentive compatibility constraints due to an expansion of bondholdings.

$$
\begin{aligned}
\frac{\partial}{\partial \kappa_{s_{b, h}}}\left(\frac{d s_{0}^{k}}{d \Gamma_{0}^{c b}}\right) & =\frac{\partial}{\partial \kappa_{s_{b, h}}}\left(\frac{G E}{C n_{0}-\lambda_{k}-G F}\right) \\
& =\frac{1}{\left(C n_{0}-\lambda_{k}-G F\right)^{2}}\left\{\frac{\partial(G E)}{\partial \kappa_{s_{b, h}}} \cdot\left(C n_{0}-\lambda_{k}-G F\right)-G E \cdot \frac{\partial}{\partial \kappa_{s_{b, h}}}\left[C n_{0}-\lambda_{k}-G F\right]\right\} \\
& =\frac{1}{\left(C n_{0}-\lambda_{k}-G F\right)^{2}}\left\{E \cdot \frac{\partial G}{\partial \kappa_{s_{b, h}}} \cdot\left(C n_{0}-\lambda_{k}-G F\right)+G E F \cdot \frac{\partial G}{\partial \kappa_{s_{b, h}}}\right\} \\
& =\frac{1}{\left(C n_{0}-\lambda_{k}-G F\right)^{2}}\left\{E \cdot \frac{\partial G}{\partial \kappa_{s_{b, h}}} \cdot\left(C n_{0}-\lambda_{k}\right)-E G F \cdot \frac{\partial G}{\partial \kappa_{s_{b, h}}}+G E F \cdot \frac{\partial G}{\partial \kappa_{s_{b, h}}}\right\} \\
& =\frac{E\left(C n_{0}-\lambda_{k}\right)}{\left(C n_{0}-\lambda_{k}-G F\right)^{2}} \cdot \frac{\partial G}{\partial \kappa_{s_{b, h}}}>0
\end{aligned}
$$

since $\frac{1}{\left(C n_{0}-\lambda_{k}-G F\right)^{2}}>0, E>0$ and $C n_{0}-\lambda_{k}<0$, while the derivative of $G$ is given by:

$$
\begin{equation*}
\frac{\partial G}{\partial \kappa_{s_{b, h}}}=\lambda_{b} q_{0}^{b} \frac{\partial B}{\partial \kappa_{s_{b, h}}}=-\frac{\lambda_{b} q_{0}^{b}}{\kappa_{s_{b, h}}^{2}}<0 \tag{85}
\end{equation*}
$$

We can interpret this result in the following way: an increase in $\kappa_{s_{b, h}}$ implies that households will be less willing to sell government bonds to financial intermediaries. Less space of intermediaries' balance sheet is used for government bond holdings, which implies that there is more space on the balance sheet to expand lending to the real economy.

I now calculate the partial derivative of $\frac{d s_{0}^{k}}{d \Gamma_{0}^{c b}}$ with respect to previous period bondholdings $s_{-1}^{b}$. Similar to the partial derivative with respect to $\kappa_{s_{b, h}}$, the only term that is directly affected by $s_{-1}^{b}$ is $G$. Hence I can write down the following expression:

$$
\frac{\partial}{\partial s_{-1}^{b}}\left(\frac{d s_{0}^{k}}{d \Gamma_{0}^{c b}}\right)=\frac{E\left(C n_{0}-\lambda_{k}\right)}{\left(C n_{0}-\lambda_{k}-G F\right)^{2}} \cdot \frac{\partial G}{\partial s_{-1}^{b}}>0
$$

since the derivative of $G$ with respect to $s_{-1}^{b}$ is given by:

$$
\begin{equation*}
\frac{\partial G}{\partial s_{-1}^{b}}=-\left(1+\mu_{0}\right)<0 \tag{86}
\end{equation*}
$$

The reason why an increase in previous period bondholdings has an expansionary effect everything else equal is that more bondholdings increase net worth $n_{0}$ since the bond price $q_{0}^{b}$ increases. More net worth relaxes the incentive compatibility constraint, which allows financial intermediaries to expand lending to the real economy.

The partial derivative of $\frac{d s_{0}^{k}}{d \Gamma_{0}^{c b}}$ with respect to the haircut parameter $\theta^{b}$ is given by:

$$
\begin{aligned}
\frac{\partial}{\partial \theta^{b}}\left(\frac{d s_{0}^{k}}{d \Gamma_{0}^{c b}}\right) & =\frac{\partial}{\partial \theta^{b}}\left(\frac{G E}{C n_{0}-\lambda_{k}-G F}\right)=\frac{G}{C n_{0}-\lambda_{k}-G F} \cdot \frac{\partial E}{\partial \theta^{b}} \\
& =\frac{G E}{C n_{0}-\lambda_{k}-G F} \cdot \frac{1}{E} \frac{\partial E}{\partial \theta^{b}}=\frac{d s_{0}^{k}}{d \Gamma_{0}^{c b}} \cdot \frac{1}{E} \frac{\partial E}{\partial \theta^{b}}=\frac{1}{\theta^{b}} \cdot \frac{d s_{0}^{k}}{d \Gamma_{0}^{c b}}
\end{aligned}
$$

Hence we see that the sign of the partial derivative with respect to the haircut parameter $\theta^{b}$ depends on the sign of $\frac{d s_{0}^{k}}{d \Gamma_{0}^{c_{0}^{b b}}}$. Remember that the sign of $\frac{d s_{0}^{k}}{d \Gamma_{0}^{c b}}$ depends on two effects that work in opposite direction. The positive effect is that increased demand for government bonds, resulting from central bank funding becoming more attractive for which government bonds need to be pledged as collateral, indirectly recapitalizes financial intermediaries. The negative effect arises from additional government bond purchases crowding out lending to the real economy. Increasing central bank funding obtained for one euro of government bonds amplifies the effects from the policy. If the initial effect is positive, i.e. $\frac{d s_{0}^{k}}{d \Gamma_{0}^{c b}}>0$, providing more central bank funding for one euro of government bonds as collateral amplifies the expansion in lending. A similar conclusion is arrived at when the initial effect is negative, i.e. $\frac{d s_{0}^{k}}{d \Gamma_{0}^{c b}}<0$.

## C Infinite-horizon model

## C. 1 Households

A continuum of households with measure one are infinitely lived, and exhibit identical preferences and asset endowments. Each household consists of bankers and workers. There is perfect consumption insurance within the household. Each period, households provide a unique type of labor that gives them the power to set the nominal wage rate at which they perfectly elastically provide their labor services. However, each period they face a probability $\psi_{w}$ that they will not be able to change the nominal wage rate as in Erceg et al. (2000), based on Calvo (1983). I explain the households' wage decision in detail in Appendix C.3.5. Households can save through deposits at financial intermediaries $d_{t}$, which yield repayment of principal $d_{t}$ and interest $r_{t+1}^{d} d_{t}$ in period $t+1$. Households can also invest in corporate securities and government bonds with net return $r_{t+1}^{k}$ and $r_{t+1}^{b}$ respectively, on their holdings $q_{t}^{k} s_{t}^{k, h}$ and $q_{t}^{b} s_{t}^{b, h}$ respectively. $q_{t}^{a}$ and $s_{t}^{a}$ denote the price and volume, respectively, of asset $a \in\{k, b\}$ in period $t$. In addition, households receive profits $\Pi_{t}$ from the production sector and the financial sector. Income is used for consumption $c_{t}$, deposits $d_{t}$, investment in corporate securities $q_{t}^{k} s_{t}^{k, h}$ and government bonds $q_{t}^{b} s_{t}^{b, h}$. However, households pay a cost when purchasing corporate securities and government bonds, which is quadratic in the deviation of the number of securities and bonds from the level $\hat{s}_{k, h}$ and $\hat{s}_{b, h}$ respectively. Such costs capture in a simple way the limited participation in asset markets by households (Gertler and Karadi 2013). The government levies lump sum taxes $\tau_{t}$. Household members derive utility from consumption and leisure, with habit formation in consumption to
capture consumption dynamics in a more realistical way, as in Christiano et al. (2005). Utility from consumption and leisure is subject to a preference shock $\epsilon_{t}^{c}$, the $\log$ of which is a regular AR(1) process. Households maximize expected life-time utility subject to the budget constraint:
$\max _{\left\{c_{t+i}, d_{t+i}, s_{t+i}^{k, h}, s_{t+i}^{b, h}\right\}_{i=0}^{\infty}} E_{t} \quad\left\{\sum_{i=0}^{\infty} \beta^{i}\left[\epsilon_{t+i}^{c} \log \left(c_{t+i}-v c_{t-1+i}\right)-\chi \frac{h_{t+i}^{1+\varphi}}{1+\varphi}\right]\right\}$
s.t.

$$
\begin{aligned}
c_{t}+\tau_{t}+d_{t} & +q_{t}^{k} s_{t}^{k, h}+q_{t}^{b} s_{t}^{b, h}+\frac{1}{2} \kappa_{s_{k, h}}\left(s_{t}^{k, h}-\hat{s}_{k, h}\right)^{2}+\frac{1}{2} \kappa_{s_{b, h}}\left(s_{t}^{b, h}-\hat{s}_{b, h}\right)^{2} \\
& =w_{t} h_{t}+\left(1+r_{t}^{d}\right) d_{t-1}+\left(1+r_{t}^{k}\right) q_{t-1}^{k} s_{t-1}^{k, h}+\left(1+r_{t}^{b}\right) q_{t-1}^{b} s_{t-1}^{b, h}+\Pi_{t}
\end{aligned}
$$

After setting up the Lagrangian belonging to the household's optimization problem, I arrive at the following first order conditions:

$$
\begin{align*}
& c_{t}:  \tag{87}\\
& d_{t}=\epsilon_{t}^{c}\left(c_{t}-v c_{t-1}\right)^{-1}-\beta v E_{t}\left[\epsilon_{t+1}^{c}\left(c_{t+1}-v c_{t}\right)^{-1}\right],  \tag{88}\\
& d_{t} E_{t}\left[\beta \Lambda_{t, t+1}\left(1+r_{t+1}^{d}\right)\right]=1,  \tag{89}\\
& s_{t}^{k, h}:  \tag{90}\\
& s_{t}^{b, h}\left\{\beta \Lambda_{t, t+1}\left[\frac{\left(1+r_{t+1}^{k}\right) q_{t}^{k}}{q_{t}^{k}+\kappa_{s_{k, h}}\left(s_{t}^{k, h}-\hat{s}_{k, h}\right)}\right]\right\}=1 \\
&: \\
& E_{t}\left\{\beta \Lambda_{t, t+1}\left[\frac{\left(1+r_{t+1}^{b}\right) q_{t}^{b}}{q_{t}^{b}+\kappa_{s_{b, h}}\left(s_{t}^{b, h}-\hat{s}_{b, h}\right)}\right]\right\}=1
\end{align*}
$$

where $\lambda_{t}$ is the marginal utility of consumption. The household's stochastic discount factor is $\beta \Lambda_{t, t+1}=\beta \lambda_{t+1} / \lambda_{t}$. Equation (87) denotes the marginal utility from an additional unit of consumption. Equation - 88 weigh the benefit from an additional unit of consumption tomorrow from investing in deposits, corporate securities, and government bonds respectively, with the cost of lower consumption today.

## C. 2 Financial intermediaries

In the main text, the collateral constraint is given by $d_{j, t}^{c b} \leq \theta_{t}^{k} q_{t}^{k} s_{j, t}^{k, p}+\theta_{t}^{b} q_{t}^{b} s_{j, t}^{b, p}$, where in the main text I assumed $\theta_{t}^{k}=\theta^{k}$ and $\theta_{t}^{b}=\theta^{b}$. In this appendix I will apply a more general formulation, namely $d_{j, t}^{c b} \leq \theta_{t}^{k} \kappa_{t}^{k} s_{j, t}^{k, p}+\theta_{t}^{b} \kappa_{t}^{b} s_{j, t}^{b, p}$, where $\kappa_{t}^{j}$, with $j \in\{k, b\}$, can be equal to:

$$
\kappa_{t}^{j}= \begin{cases}q_{t}^{j} & \text { "Regular collateral constraint" } \\ 1 & \text { "No risk-adjustment collateral constraint" }\end{cases}
$$

In addition to the more general formulation of the collateral requirement, I also include a reserve requirement for financial intermediaries in the derivations. The reason for doing so, is that such
a requirement exists for Eurozone banks. However, after the ECB' switch to the Fixed Rate Full Allotment policy, in which banks could obtain as much liquidity as required from the ECB as long as sufficient collateral was put up, it is unlikely that the reserve requirement continued to be binding, especially since the ECB lowered the requirement from $2 \%$ to $1 \%$ in January 2012. I will do the full derivations including the reserve requirement, but will assume it is not binding anymore in subsection C.2.2, where I combine and simplify the first order conditions for presentation in the main text. As stated above, financial intermediaries are also subject to reserve requirements, which require them to carry central bank reserves $m_{j, t}^{R}$ that are equal to or larger than a fraction $\vartheta_{t}$ of household deposits $d_{j, t}$ :

$$
\begin{equation*}
m_{j, t}^{R} \geq \vartheta_{t} d_{j, t} \tag{91}
\end{equation*}
$$

Finally, for generality, I will assume that central bank reserves $m_{j, t}^{R}$ can be diverted by the managers of the intermediaries, which gives a slightly more general incentive compatibility constraint than in the main text:

$$
\begin{equation*}
V_{j, t} \geq \lambda_{k} q_{t}^{k} j_{j, t}^{k, p}+\lambda_{b} q_{t}^{b} s_{j, t}^{b, p}+\lambda_{R} m_{j, t}^{R} \tag{92}
\end{equation*}
$$

Just as for the reserve requirements (91), I will set $\lambda_{R}=0$ in subsection C.2.2; in reality, central bank reserves are electronic accounts that are at full control of the central bank, and can thus never be diverted by financial intermediaries.

The law of motion for net worth, which includes recapitalizations by the government and financial sector repayments is given by:

$$
\begin{aligned}
n_{j, t+1} & =\left(1+r_{t+1}^{k}\right) q_{t}^{k} s_{j, t}^{k, p}+\left(1+r_{t+1}^{b}\right) q_{t}^{b} s_{j, t}^{b, p}+\left(1+r_{t+1}^{R}\right) m_{j, t}^{R} \\
& -\left(1+r_{t+1}^{d}\right) d_{j, t}-\left(1+r_{t+1}^{c b}\right) d_{j, t}^{c b}+n_{j, t+1}^{g}-\tilde{n}_{j, t+1}^{g} \\
& =\left(1+r_{t+1}^{k}\right) q_{t}^{k} s_{j, t}^{k, p}+\left(1+r_{t+1}^{b}\right) q_{t}^{b} s_{j, t}^{b, p}+\left(1+r_{t+1}^{R}\right) m_{j, t}^{R} \\
& -\left(1+r_{t+1}^{d}\right) d_{j, t}-\left(1+r_{t+1}^{c b}\right) d_{j, t}^{c b}+\tau_{t+1}^{n} n_{j, t}-\tilde{\tau}_{t+1}^{n} n_{j, t} \\
& =\left(1+r_{t+1}^{k}\right) q_{t}^{k} s_{j, t}^{k, p}+\left(1+r_{t+1}^{b}\right) q_{t}^{b} s_{j, t}^{b, p}+\left(1+r_{t+1}^{R}\right) m_{j, t}^{R} \\
& -\left(1+r_{t+1}^{d}\right) d_{j, t}-\left(1+r_{t+1}^{c b}\right) d_{j, t}^{c b} \\
& +\left(\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)\left(q_{t}^{k} s_{j, t}^{k, p}+q_{t}^{b} s_{j, t}^{b, p}+m_{j, t}^{R}-d_{j, t}-d_{j, t}^{c b}\right) \\
& =\left(1+r_{t+1}^{k}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right) q_{t}^{k} s_{j, t}^{k, p}+\left(1+r_{t+1}^{b}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right) q_{t}^{b} s_{j, t}^{b, p} \\
& +\left(1+r_{t+1}^{R}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right) m_{j, t}^{R} \\
& -\left(1+r_{t+1}^{d}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right) d_{j, t}-\left(1+r_{t+1}^{c b}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right) d_{j, t}^{c b},
\end{aligned}
$$

where $n_{j, t}^{g}=\tau_{t}^{n} n_{j, t-1}$ denotes a net worth injection by the government that is proportional to previous period net worth $n_{j, t-1}$, while $\tilde{n}_{j, t}^{g}=\tilde{\tau}_{t}^{n} n_{j, t-1}$ denotes the repayment of earlier provided government support, which is also proportional to previous period net worth $n_{j, t-1}$. Now we remember the optimization problem of the financial intermediary, which is mathematically
described in the following way, with $V_{j, t} \equiv V\left(s_{j, t-1}^{k, p}, s_{j, t-1}^{b, p}, m_{j, t-1}^{R}, d_{j, t-1}, d_{j, t-1}^{c b}\right)$ :

$$
\begin{aligned}
& V_{j, t}= \underset{\left\{s_{j, t}^{k, p}, s_{j, t}^{b, p}, m_{j, t}^{R}, d_{j, t}, d_{j, t}^{c b}\right\}}{ } \max _{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma) n_{j, t+1}+\sigma V_{j, t+1}\right]\right\}, \\
& \text { s.t. } \\
& V_{j, t} \geq \lambda_{k} q_{t}^{k} s_{j, t}^{k, p}+\lambda_{b} q_{t}^{b} s_{j, t}^{b, p}+\lambda_{R} m_{j, t}^{R}, \\
& n_{j, t}+d_{j, t}+d_{j, t}^{c b} \geq q_{t}^{k} s_{j, t}^{k, p}+q_{t}^{b} s_{j, t}^{b, p}+m_{j, t}^{R}, \\
& n_{j, t}=\left(1+r_{t}^{k}+\tau_{t}^{n}-\tilde{\tau}_{t}^{n}\right) q_{t-1}^{k} s_{j, t-1}^{k, p}+\left(1+r_{t}^{b}+\tau_{t}^{n}-\tilde{\tau}_{t}^{n}\right) q_{t-1}^{b} s_{j, t-1}^{b, p} \\
&+\left(1+r_{t}^{R}+\tau_{t}^{n}-\tilde{\tau}_{t}^{n}\right) m_{j, t-1}^{R} \\
&-\left(1+r_{t}^{d}+\tau_{t}^{n}-\tilde{\tau}_{t}^{n}\right) d_{j, t-1}-\left(1+r_{t}^{c b}+\tau_{t}^{n}-\tilde{\tau}_{t}^{n}\right) d_{j, t-1}^{c b}, \\
& m_{j, t}^{R} \geq \vartheta_{t} d_{j, t}, \\
& \theta_{t}^{k} \kappa_{t}^{k} s_{j, t}^{k, p}+\theta_{t}^{b} \kappa_{t}^{b} s_{j, t}^{b, p} \geq d_{j, t}^{c b} .
\end{aligned}
$$

Now I set up the accompanying Lagrangian of the problem to find the optimal allocation:

$$
\begin{aligned}
\mathcal{L} & =\left(1+\mu_{t}\right) E_{t}\left(\beta \Lambda _ { t , t + 1 } \left\{( 1 - \sigma ) \left[\left(1+r_{t+1}^{k}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right) q_{t}^{k} s_{j, t}^{k, p}+\left(1+r_{t+1}^{b}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right) q_{t}^{b} s_{j, t}^{b, p}\right.\right.\right. \\
& \left.+\left(1+r_{t+1}^{R}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right) m_{j, t}^{R}-\left(1+r_{t+1}^{d}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right) d_{j, t}-\left(1+r_{t+1}^{c b}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right) d_{j, t}^{c b}\right] \\
& \left.\left.+\sigma V\left(s_{j, t}^{k, p}, s_{j, t}^{b, p}, m_{j, t}^{R}, d_{j, t}, d_{j, t}^{c b}\right)\right\}\right) \\
& -\mu_{t} \lambda_{k} q_{t}^{k} s_{j, t}^{k, p}-\mu_{t} \lambda_{b} q_{t}^{b} s_{j, t}^{b, p}-\mu_{t} \lambda_{R} m_{j, t}^{R} \\
& +\chi_{t}\left[\left(1+r_{t}^{k}+\tau_{t}^{n}-\tilde{\tau}_{t}^{n}\right) q_{t-1}^{k} s_{j, t-1}^{k, p}+\left(1+r_{t}^{b}+\tau_{t}^{n}-\tilde{\tau}_{t}^{n}\right) q_{t-1}^{b} s_{j, t-1}^{b, p}+\left(1+r_{t}^{R}+\tau_{t}^{n}-\tilde{\tau}_{t}^{n}\right) m_{j, t-1}^{R}\right. \\
& \left.-\left(1+r_{t}^{d}+\tau_{t}^{n}-\tilde{\tau}_{t}^{n}\right) d_{j, t-1}-\left(1+r_{t}^{c b}+\tau_{t}^{n}-\tilde{\tau}_{t}^{n}\right) d_{j, t-1}^{c b}-q_{t}^{k} s_{j, t}^{k, p}-q_{t}^{b} s_{j, t}^{b, p}-m_{j, t}^{R}+d_{j, t}+d_{j, t}^{c b}\right] \\
& +\psi_{t}^{R}\left(m_{j, t}^{R}-\vartheta_{t} d_{j, t}\right) \\
& +\psi_{t}^{c b}\left(\theta_{t}^{k} \kappa_{t}^{k} s_{j, t}^{k, p}+\theta_{t}^{b} \kappa_{t}^{b} s_{j, t}^{b, p}-d_{j, t}^{c b}\right) .
\end{aligned}
$$

This gives rise to the following first order conditions:

$$
\begin{align*}
s_{j, t}^{k, p} & :\left(1+\mu_{t}\right) E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)\left(1+r_{t+1}^{k}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right) q_{t}^{k}+\sigma \frac{\partial V\left(s_{j, t}^{k, p}, s_{j, t}^{b, p}, m_{j, t}^{R}, d_{j, t}, d_{j, t}^{c b}\right)}{\partial s_{j, t}^{k, p}}\right]\right\} \\
& -\mu_{t} \lambda_{k} q_{t}^{k}-\chi_{t} q_{t}^{k}+\psi_{t}^{c b} \theta_{t}^{k} \kappa_{t}^{k}=0,  \tag{93}\\
s_{j, t}^{b, p} & :\left(1+\mu_{t}\right) E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)\left(1+r_{t+1}^{b}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right) q_{t}^{b}+\sigma \frac{\partial V\left(s_{j, t}^{k, p}, s_{j, t}^{b, p}, m_{j, t}^{R}, d_{j, t}, d_{j, t}^{c b}\right)}{\partial s_{j, t}^{b, p}}\right]\right\} \\
& -\mu_{t} \lambda_{b} q_{t}^{b}-\chi_{t} q_{t}^{b}+\psi_{t}^{c b} \theta_{t}^{b} \kappa_{t}^{b}=0,  \tag{94}\\
m_{j, t}^{R} & :\left(1+\mu_{t}\right) E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)\left(1+r_{t+1}^{R}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)+\sigma \frac{\partial V\left(s_{j, t}^{k, p}, s_{j, t}^{b, p}, m_{j, t}^{R}, d_{j, t}, d_{j, t}^{c b}\right)}{\partial m_{j, t}^{R}}\right]\right\} \\
& -\mu_{t} \lambda_{R}-\chi_{t}+\psi_{t}^{R}=0,  \tag{95}\\
d_{j, t} & : \quad\left(1+\mu_{t}\right) E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)\left(1+r_{t+1}^{d}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)(-1)+\sigma \frac{\partial V\left(s_{j, t}^{k, p}, s_{j, t}^{b, p}, m_{j, t}^{R}, d_{j, t}, d_{j, t}^{c b}\right)}{\partial d_{j, t}}\right]\right\} \\
& +\chi_{t}-\psi_{t}^{R} \vartheta_{t}=0,  \tag{96}\\
& :\left(1+\mu_{t}\right) E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)\left(1+r_{t+1}^{c b}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)(-1)+\sigma \frac{\partial V\left(s_{j, t}^{k, p}, s_{j, t}^{b, p}, m_{j, t}^{R}, d_{j, t}, d_{j, t}^{c b}\right)}{\partial d_{j, t}^{c b}}\right]\right\} \\
d_{j, t}^{c b} & :  \tag{97}\\
& \chi_{t}-\psi_{t}^{c b}=0,
\end{align*}
$$

with complementary slackness conditions:

$$
\begin{align*}
\mu_{t} & :\left[V\left(s_{j, t-1}^{k, p}, s_{j, t-1}^{b, p}, m_{j, t-1}^{R}, d_{j, t-1}, d_{j, t-1}^{c b}\right)-\lambda_{k} q_{t}^{k} s_{j, t}^{k, p}-\lambda_{b} q_{t}^{b} s_{j, t}^{b, p}-\lambda_{R} m_{j, t}^{R}\right] \mu_{t}=0,  \tag{98}\\
\chi_{t} & :\left[\left(1+r_{t}^{k}+\tau_{t}^{n}-\tilde{\tau}_{t}^{n}\right) q_{t-1}^{k} s_{j, t-1}^{k, p}+\left(1+r_{t}^{b}+\tau_{t}^{n}-\tilde{\tau}_{t}^{n}\right) q_{t-1}^{b} s_{j, t-1}^{b, p}+\left(1+r_{t}^{R}+\tau_{t}^{n}-\tilde{\tau}_{t}^{n}\right) m_{j, t-1}^{R}\right. \\
& \left.-\left(1+r_{t}^{d}+\tau_{t}^{n}-\tilde{\tau}_{t}^{n}\right) d_{j, t-1}-\left(1+r_{t}^{c b}+\tau_{t}^{n}-\tilde{\tau}_{t}^{n}\right) d_{j, t-1}^{c b}-q_{t}^{k} s_{j, t}^{k, p}-q_{t}^{b} s_{j, t}^{b, p}-m_{j, t}^{R}+d_{j, t}+d_{j, t}^{c b}\right] \chi_{t}=0, \tag{99}
\end{align*}
$$

$\psi_{t}^{R} \quad: \quad\left(m_{j, t}^{R}-\vartheta_{t} d_{j, t}\right) \psi_{t}^{R}=0$,
$\psi_{t}^{c b} \quad: \quad\left(\theta_{t}^{k} \kappa_{t}^{k} s_{j, t}^{k}+\theta_{t}^{b} \kappa_{t}^{b} s_{j, t}^{b}-d_{j, t}^{c b}\right) \psi_{t}^{c b}=0$.

Now I apply the envelope theorem to find the derivatives:

$$
\begin{align*}
& \frac{\partial V\left(s_{j, t-1}^{k, p}, s_{j, t-1}^{b, p}, m_{j, t-1}^{R}, d_{j, t-1}, d_{j, t-1}^{c b}\right)}{\partial s_{j, t-1}^{k, p}}=\chi_{t}\left(1+r_{t}^{k}+\tau_{t}^{n}-\tilde{\tau}_{t}^{n}\right) q_{t-1}^{k},  \tag{102}\\
& \frac{\partial V\left(s_{j, t-1}^{k, p}, s_{j, t-1}^{b, p}, m_{j, t-1}^{R}, d_{j, t-1}, d_{j, t-1}^{c b}\right)}{\partial s_{j, t-1}^{b, p}}=\chi_{t}\left(1+r_{t}^{b}+\tau_{t}^{n}-\tilde{\tau}_{t}^{n}\right) q_{t-1}^{b},  \tag{103}\\
& \frac{\partial V\left(s_{j, t-1}^{k, p}, s_{j, t-1}^{b, p}, m_{j, t-1}^{R}, d_{j, t-1}, d_{j, t-1}^{c b}\right)}{\partial m_{j, t-1}^{R}}=\chi_{t}\left(1+r_{t}^{R}+\tau_{t}^{n}-\tilde{\tau}_{t}^{n}\right),  \tag{104}\\
& \frac{\partial V\left(s_{j, t-1}^{k, p}, s_{j, t-1}^{b, p}, m_{j, t-1}^{R}, d_{j, t-1}, d_{j, t-1}^{c b}\right)}{\partial d_{j, t-1}}=-\chi_{t}\left(1+r_{t}^{d}+\tau_{t}^{n}-\tilde{\tau}_{t}^{n}\right),  \tag{105}\\
& \frac{\partial V\left(s_{j, t-1}^{k, p}, s_{j, t-1}^{b, p}, m_{j, t-1}^{R}, d_{j, t-1}, d_{j, t-1}^{c b}\right)}{\partial d_{j, t-1}^{c b}}=-\chi_{t}\left(1+r_{t}^{c b}+\tau_{t}^{n}-\tilde{\tau}_{t}^{n}\right) . \tag{106}
\end{align*}
$$

Subsititution of the envelope conditions (102) - 106) into (93) - (97), I find the following relation between the different assets:

$$
\begin{align*}
s_{j, t}^{k, p} & : \lambda_{k}\left(\frac{\mu_{t}}{1+\mu_{t}}\right)+\frac{\chi_{t}}{1+\mu_{t}}-\left(\frac{\kappa_{t}^{k}}{q_{t}^{k}}\right) \theta_{t}^{k}\left(\frac{\psi_{t}^{c b}}{1+\mu_{t}}\right) \\
& =E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]\left(1+r_{t+1}^{k}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)\right\}  \tag{107}\\
s_{j, t}^{b, p} & : \lambda_{b}\left(\frac{\mu_{t}}{1+\mu_{t}}\right)+\frac{\chi_{t}}{1+\mu_{t}}-\left(\frac{\kappa_{t}^{b}}{q_{t}^{b}}\right) \theta_{t}^{b}\left(\frac{\psi_{t}^{c b}}{1+\mu_{t}}\right) \\
& =E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]\left(1+r_{t+1}^{b}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)\right\}  \tag{108}\\
m_{j, t}^{R} & : \lambda_{R}\left(\frac{\mu_{t}}{1+\mu_{t}}\right)+\frac{\chi_{t}}{1+\mu_{t}}-\left(\frac{\psi_{t}^{R}}{1+\mu_{t}}\right) \\
& =E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]\left(1+r_{t+1}^{R}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)\right\}  \tag{109}\\
d_{j, t} & : \frac{\chi_{t}}{1+\mu_{t}}-\vartheta_{t}\left(\frac{\psi_{t}^{R}}{1+\mu_{t}}\right)=E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]\left(1+r_{t+1}^{d}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)\right\} \\
d_{j, t}^{c b} & : \frac{\chi_{t}}{1+\mu_{t}}-\frac{\psi_{t}^{c b}}{1+\mu_{t}}=E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]\left(1+r_{t+1}^{c b}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)\right\} \tag{110}
\end{align*}
$$

Now I define the following variables:

$$
\begin{align*}
\eta_{t}^{k} & \equiv \lambda_{k}\left(\frac{\mu_{t}}{1+\mu_{t}}\right)+\frac{\chi_{t}}{1+\mu_{t}}-\left(\frac{\kappa_{t}^{k}}{q_{t}^{k}}\right) \theta_{t}^{k}\left(\frac{\psi_{t}^{c b}}{1+\mu_{t}}\right) \\
\eta_{t}^{b} & \equiv \lambda_{b}\left(\frac{\mu_{t}}{1+\mu_{t}}\right)+\frac{\chi_{t}}{1+\mu_{t}}-\left(\frac{\kappa_{t}^{b}}{q_{t}^{b}}\right) \theta_{t}^{b}\left(\frac{\psi_{t}^{c b}}{1+\mu_{t}}\right)  \tag{112}\\
\eta_{t}^{R} & \equiv \lambda_{R}\left(\frac{\mu_{t}}{1+\mu_{t}}\right)+\frac{\chi_{t}}{1+\mu_{t}}-\left(\frac{\psi_{t}^{R}}{1+\mu_{t}}\right)  \tag{114}\\
\eta_{t} & \equiv \frac{\chi_{t}}{1+\mu_{t}}-\vartheta_{t}\left(\frac{\psi_{t}^{R}}{1+\mu_{t}}\right)  \tag{115}\\
\eta_{t}^{c b} & \equiv \frac{\chi_{t}}{1+\mu_{t}}-\frac{\psi_{t}^{c b}}{1+\mu_{t}} \tag{116}
\end{align*}
$$

Hence I can write the first order conditions (107) - (111) in the following way. This gives rise to the following first order conditions:

$$
\begin{align*}
\eta_{t}^{k} & =E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]\left(1+r_{t+1}^{k}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)\right\}  \tag{117}\\
\eta_{t}^{b} & =E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]\left(1+r_{t+1}^{b}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)\right\}  \tag{118}\\
\eta_{t}^{R} & =E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]\left(1+r_{t+1}^{R}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)\right\}  \tag{119}\\
\eta_{t} & =E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]\left(1+r_{t+1}^{d}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)\right\},  \tag{120}\\
\eta_{t}^{c b} & =E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]\left(1+r_{t+1}^{c b}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)\right\} \tag{121}
\end{align*}
$$

Now I assume a particular function for the value function, and will later check whether the first order conditions are consistent with it:

$$
V_{j, t}=V\left(s_{j, t-1}^{k, p}, s_{j, t-1}^{b, p}, m_{j, t-1}^{R}, d_{j, t-1}, d_{j, t-1}^{c b}\right)=\eta_{t}^{k} q_{t}^{k} s_{j, t}^{k, p}+\eta_{t}^{b} q_{t}^{b} s_{j, t}^{b, p}+\eta_{t}^{R} m_{j, t}^{R}-\eta_{t} d_{j, t}-\eta_{t}^{c b} d_{j, t}^{c b}
$$

Substitution of the first order conditions $\sqrt{112)}$ - 116 in the value function of the typical financial
intermediary gives the following expression:

$$
\begin{aligned}
V_{j, t} & =\eta_{t}^{k} q_{t}^{k} s_{j, t}^{k, p}+\eta_{t}^{b} q_{t}^{b} s_{j, t}^{b, p}+\eta_{t}^{R} m_{j, t}^{R}-\eta_{t} d_{j, t}-\eta_{t}^{c b} d_{j, t}^{c b} \\
& =\left[\lambda_{k}\left(\frac{\mu_{t}}{1+\mu_{t}}\right)+\frac{\chi_{t}}{1+\mu_{t}}-\left(\frac{\kappa_{t}^{k}}{q_{t}^{k}}\right) \theta_{t}^{k}\left(\frac{\psi_{t}^{c b}}{1+\mu_{t}}\right)\right] q_{t}^{k} s_{j, t}^{k, p} \\
& +\left[\lambda_{b}\left(\frac{\mu_{t}}{1+\mu_{t}}\right)+\frac{\chi_{t}}{1+\mu_{t}}-\left(\frac{\kappa_{t}^{b}}{q_{t}^{b}}\right) \theta_{t}^{b}\left(\frac{\psi_{t}^{c b}}{1+\mu_{t}}\right)\right] q_{t}^{b} s_{j, t}^{b, p} \\
& +\left[\lambda_{R}\left(\frac{\mu_{t}}{1+\mu_{t}}\right)+\frac{\chi_{t}}{1+\mu_{t}}-\left(\frac{\psi_{t}^{R}}{1+\mu_{t}}\right)\right] m_{j, t}^{R} \\
& -\left[\frac{\chi_{t}}{1+\mu_{t}}-\vartheta_{t}\left(\frac{\psi_{t}^{R}}{1+\mu_{t}}\right)\right] d_{j, t}-\left(\frac{\chi_{t}}{1+\mu_{t}}-\frac{\psi_{t}^{c b}}{1+\mu_{t}}\right) d_{j, t}^{c b} \\
& =\frac{\chi_{t}}{1+\mu_{t}}\left(q_{t}^{k} s_{j, t}^{k, p}+q_{t}^{b} s_{j, t}^{b, p}+m_{j, t}^{R}-d_{j, t}-d_{j, t}^{c b}\right) \\
& +\frac{\mu_{t}}{1+\mu_{t}}\left(\lambda_{k} q_{t}^{k} s_{j, t}^{k, p}+\lambda_{b} q_{t}^{b} s_{j, t}^{b, p}+\lambda_{R} m_{j, t}^{R}\right) \\
& -\frac{\psi_{t}^{R}}{1+\mu_{t}}\left(m_{j, t}^{R}-\vartheta_{t} d_{j, t}\right)-\frac{\psi_{t}^{c b}}{1+\mu_{t}}\left(\theta_{t}^{k} \kappa_{t}^{k} s_{j, t}^{k, p}+\theta_{t}^{b} \kappa_{t}^{b} s_{j, t}^{b, p}-d_{j, t}^{c b}\right) \\
& =\left(\frac{\chi_{t}}{1+\mu_{t}}\right) n_{j, t}+\frac{\mu_{t}}{1+\mu_{t}}\left(\lambda_{k} q_{t}^{k} s_{j, t}^{k, p}+\lambda_{b} q_{t}^{b} s_{j, t}^{b, p}+\lambda_{R} m_{j, t}^{R}\right),
\end{aligned}
$$

where the terms with $\psi_{t}^{R}$ and $\psi_{t}^{c b}$ drop out because of the slackness conditions 100 and (101). Using this expression for the continuation value of intermediary $j$, I can rewrite the incentive compatibility constraint in the following way:

$$
\begin{aligned}
V_{j, t} & \geq \lambda_{k} q_{t}^{k} s_{j, t}^{k, p}+\lambda_{b} q_{t}^{b} s_{j, t}^{b, p}+\lambda_{R} m_{j, t}^{R} \Longrightarrow \\
\left(\frac{\chi_{t}}{1+\mu_{t}}\right) n_{j, t}+\frac{\mu_{t}}{1+\mu_{t}}\left(\lambda_{k} q_{t}^{k} s_{j, t}^{k, p}+\lambda_{b} q_{t}^{b, p} s_{j, t}^{b}+\lambda_{R} m_{j, t}^{R}\right) & \geq \lambda_{k} q_{t}^{k} s_{j, t}^{k, p}+\lambda_{b} q_{t}^{b} s_{j, t}^{b, p}+\lambda_{R} m_{j, t}^{R} \Longrightarrow \\
\left(1-\frac{\mu_{t}}{1+\mu_{t}}\right)\left(\lambda_{k} q_{t}^{k} s_{j, t}^{k, p}+\lambda_{b} q_{t}^{b} s_{j, t}^{b, p}+\lambda_{R} m_{j, t}^{R}\right) & \leq\left(\frac{\chi_{t}}{1+\mu_{t}}\right) n_{j, t} \Longrightarrow \\
\lambda_{k} q_{t}^{k} s_{j, t}^{k, p}+\lambda_{b} q_{t}^{b} s_{j, t}^{b, p}+\lambda_{R} m_{j, t}^{R} & \leq \chi_{t} n_{j, t} .
\end{aligned}
$$

Now I substitute the expressions for the shadow values of the different asset classes in the expression for the expected discounted profits of the financial intermediary to obtain the following
expression:

$$
\begin{aligned}
V_{j, t} & =\max E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma) n_{j, t+1}+\sigma V_{j, t+1}\right]\right\} \\
& =E_{t}\left(\beta \Lambda _ { t , t + 1 } \left\{(1-\sigma) n_{j, t+1}\right.\right. \\
& \left.\left.+\sigma\left[\left(\frac{\chi_{t+1}}{1+\mu_{t+1}}\right) n_{j, t+1}+\frac{\mu_{t+1}}{1+\mu_{t+1}}\left(\lambda_{k} q_{t+1}^{k} s_{j, t+1}^{k, p}+\lambda_{b} q_{t+1}^{b} s_{j, t+1}^{b, p}+\lambda_{R} m_{j, t+1}^{R}\right)\right]\right\}\right) \\
& =E_{t}\left(\beta \Lambda_{t, t+1}\left\{(1-\sigma) n_{j, t+1}+\sigma\left[\left(\frac{\chi_{t+1}}{1+\mu_{t+1}}\right) n_{j, t+1}+\frac{\mu_{t+1}}{1+\mu_{t+1}} \chi_{t+1} n_{j, t+1}\right]\right\}\right) \\
& =E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma) n_{j, t+1}+\sigma \chi_{t+1} n_{j, t+1}\right]\right\} \\
& =E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right] n_{j, t+1}\right\} \\
& =E_{t}\left\{\beta \Lambda _ { t , t + 1 } [ ( 1 - \sigma ) + \sigma \chi _ { t + 1 } ] \left[\left(1+r_{t+1}^{k}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right) q_{t}^{k} s_{j, t}^{k, p}\right.\right. \\
& +\left(1+r_{t+1}^{b}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right) q_{t}^{b} s_{j, t}^{b, p}+\left(1+r_{t+1}^{R}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right) m_{j, t}^{R} \\
& \left.\left.-\left(1+r_{t+1}^{d}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right) d_{j, t}-\left(1+r_{t+1}^{c b}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right) d_{j, t}^{c b}\right]\right\}
\end{aligned}
$$

Comparing with the initial guess for the solution, I obtain the following first order conditions:

$$
\begin{align*}
\eta_{t}^{k} & =E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]\left(1+r_{t+1}^{k}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)\right\}  \tag{122}\\
\eta_{t}^{b} & =E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]\left(1+r_{t+1}^{b}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)\right\}  \tag{123}\\
\eta_{t}^{R} & =E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]\left(1+r_{t+1}^{R}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)\right\},  \tag{124}\\
\eta_{t} & =E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]\left(1+r_{t+1}^{d}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)\right\},  \tag{125}\\
\eta_{t}^{c b} & =E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]\left(1+r_{t+1}^{c b}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)\right\} \tag{126}
\end{align*}
$$

We see that the solutions (117) - 121) and (122) - 126 coincide, and hence that my initial guess for the value function is correct. Finally, the law of motion for aggregate net worth consists of the net worth of the bankers that are allowed to continue operating, together with the aggregate net worth given to new bankers, which is equal to a fraction $\chi_{b}$ of previous period net worth $n_{t-1}$. Together with net government support $n_{t}^{g}-\tilde{n}_{t}^{g}$, I obtain the following law of motion:

$$
\begin{align*}
n_{t} & =\sigma\left[\left(1+r_{t}^{k}\right) q_{t-1}^{k} s_{t-1}^{k, p}+\left(1+r_{t}^{b}\right) q_{t-1}^{b} s_{t-1}^{b, p}+\left(1+r_{t}^{R}\right) m_{t-1}^{R}\right. \\
& \left.-\left(1+r_{t}^{d}\right) d_{t-1}-\left(1+r_{t}^{c b}\right) d_{t-1}^{c b}\right]+\chi n_{t-1}+n_{t}^{g}-\tilde{n}_{t}^{g} \tag{127}
\end{align*}
$$

## C.2.1 Financial Sector First Order Conditions

The resulting first order conditions for the financial sector are now given by:

$$
\begin{align*}
\eta_{t}^{k} & =\lambda_{k}\left(\frac{\mu_{t}}{1+\mu_{t}}\right)+\frac{\chi_{t}}{1+\mu_{t}}-\left(\frac{\kappa_{t}^{k}}{q_{t}^{k}}\right) \theta_{t}^{k}\left(\frac{\psi_{t}^{c b}}{1+\mu_{t}}\right)  \tag{128}\\
\eta_{t}^{b} & =\lambda_{b}\left(\frac{\mu_{t}}{1+\mu_{t}}\right)+\frac{\chi_{t}}{1+\mu_{t}}-\left(\frac{\kappa_{t}^{b}}{q_{t}^{b}}\right) \theta_{t}^{b}\left(\frac{\psi_{t}^{c b}}{1+\mu_{t}}\right)  \tag{129}\\
\eta_{t}^{R} & =\lambda_{R}\left(\frac{\mu_{t}}{1+\mu_{t}}\right)+\frac{\chi_{t}}{1+\mu_{t}}-\left(\frac{\psi_{t}^{R}}{1+\mu_{t}}\right)  \tag{130}\\
\eta_{t} & =\frac{\chi_{t}}{1+\mu_{t}}-\vartheta_{t}\left(\frac{\psi_{t}^{R}}{1+\mu_{t}}\right),  \tag{131}\\
\eta_{t}^{c b} & =\frac{\chi_{t}}{1+\mu_{t}}-\frac{\psi_{t}^{c b}}{1+\mu_{t}}  \tag{132}\\
\eta_{t}^{k} & =E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]\left(1+r_{t+1}^{k}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)\right\},  \tag{133}\\
\eta_{t}^{b} & =E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]\left(1+r_{t+1}^{b}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)\right\},  \tag{134}\\
\eta_{t}^{R} & =E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]\left(1+r_{t+1}^{R}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)\right\},  \tag{135}\\
\eta_{t} & =E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]\left(1+r_{t+1}^{d}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)\right\},  \tag{136}\\
\eta_{t}^{c b} & =E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]\left(1+r_{t+1}^{c b}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)\right\} .  \tag{137}\\
\chi_{t} n_{t} & \geq \lambda_{k} q_{t}^{k} s_{t}^{k, p}+\lambda_{b} q_{t}^{b} s_{t}^{b, p}+\lambda_{R} m_{t}^{R}  \tag{138}\\
n_{t} & =\sigma\left[\left(1+r_{t}^{k}\right) q_{t-1}^{k} s_{t-1}^{k, p}+\left(1+r_{t}^{b}\right) q_{t-1}^{b} s_{t-1}^{b, p}+\left(1+r_{t}^{R}\right) m_{t-1}^{R}\right. \\
& \left.-\left(1+r_{t}^{d}\right) d_{t-1}-\left(1+r_{t}^{c b}\right) d_{t-1}^{c b}\right]+\chi n_{t-1}+n_{t}^{g}-\tilde{n}_{t}^{g},  \tag{139}\\
0 & =\left(m_{t}^{R}-\vartheta_{t} d_{t}\right) \psi_{t}^{R},  \tag{140}\\
d_{t}^{c b} & =\theta_{t}^{k} \kappa_{t}^{k} s_{t}^{k, p}+\theta_{t}^{b} \kappa_{t}^{b} s_{t}^{b, p} . \tag{141}
\end{align*}
$$

## C.2.2 Further simplification of the F.O.C.'s for mathematical proofs

Now I combine some of the F.O.C.'s found in section C.2.1 to obtain a better economic understanding and more intuition. As mentioned above, I assume throughout the simulations that there are so many central bank reserves in the system that the reserve requirement (91) is not binding, and hence that $\psi_{t}^{R}=0$. I start by substituting (131) for $\chi_{t} /\left(1+\mu_{t}\right)$ into the first order conditions for corporate securities (128) and government bonds 129):

$$
\begin{aligned}
\eta_{t}^{k}-\eta_{t} & =\lambda_{k}\left(\frac{\mu_{t}}{1+\mu_{t}}\right)-\left(\frac{\kappa_{t}^{k}}{q_{t}^{k}}\right) \theta_{t}^{k}\left(\frac{\psi_{t}^{c b}}{1+\mu_{t}}\right) \\
\eta_{t}^{b}-\eta_{t} & =\lambda_{b}\left(\frac{\mu_{t}}{1+\mu_{t}}\right)-\left(\frac{\kappa_{t}^{b}}{q_{t}^{b}}\right) \theta_{t}^{b}\left(\frac{\psi_{t}^{c b}}{1+\mu_{t}}\right)
\end{aligned}
$$

Substitution of equations (133), 134 and (136) results in the following expressions:

$$
\begin{align*}
& E_{t}\left[\Omega_{t, t+1}\left(r_{t+1}^{k}-r_{t+1}^{d}\right)\right]=\lambda_{k}\left(\frac{\mu_{t}}{1+\mu_{t}}\right)-\left(\frac{\kappa_{t}^{k}}{q_{t}^{k}}\right) \theta_{t}^{k}\left(\frac{\psi_{t}^{c b}}{1+\mu_{t}}\right)  \tag{142}\\
& E_{t}\left[\Omega_{t, t+1}\left(r_{t+1}^{b}-r_{t+1}^{d}\right)\right]=\lambda_{b}\left(\frac{\mu_{t}}{1+\mu_{t}}\right)-\left(\frac{\kappa_{t}^{b}}{q_{t}^{b}}\right) \theta_{t}^{b}\left(\frac{\psi_{t}^{c b}}{1+\mu_{t}}\right) \tag{143}
\end{align*}
$$

where $\Omega_{t, t+1}=\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]$ refers to the stochastic discount factor of the financial intermediaries, which is equal to the household's stochastic discount factor, augmented to incorporate the financial frictions.

Finally, cmbining equations 142 and 143 results in the following condition for intermediaries' portfolio choice between corporate securities and government bonds::
$\frac{\lambda_{b}}{\lambda_{k}} E_{t}\left[\Omega_{t, t+1}\left(r_{t+1}^{k}-r_{t+1}^{d}\right)\right]=E_{t}\left[\Omega_{t, t+1}\left(r_{t+1}^{b}-r_{t+1}^{d}\right)\right]+\left[\left(\frac{\kappa_{t}^{b}}{q_{t}^{b}}\right) \theta_{t}^{b}-\left(\frac{\lambda_{b}}{\lambda_{k}}\right)\left(\frac{\kappa_{t}^{k}}{q_{t}^{k}}\right) \theta_{t}^{k}\right]\left(\frac{\psi_{t}^{c b}}{1+\mu_{t}}\right)$,
Next, I combine 130 and 131 (again setting $\psi_{t}^{R}=0$ ) to obtain the following relation between the shadow value on central bank reserves and deposit funding:

$$
\eta_{t}^{R}-\eta_{t}=\lambda_{R}\left(\frac{\mu_{t}}{1+\mu_{t}}\right)
$$

Now I substitute equation (128) to eliminate $\mu_{t} /\left(1+\mu_{t}\right)$ :

$$
\eta_{t}^{R}-\eta_{t}=\frac{\lambda_{R}}{\lambda_{k}}\left[\eta_{t}^{k}-\eta_{t}+\left(\frac{\kappa_{t}^{k}}{q_{t}^{k}}\right) \theta_{t}^{k}\left(\frac{\psi_{t}^{c b}}{1+\mu_{t}}\right)\right]
$$

Substitution of the expressions $(133), 134$ and 136 results in the following equation:

$$
\begin{equation*}
E_{t}\left[\Omega_{t, t+1}\left(r_{t+1}^{R}-r_{t+1}^{d}\right)\right]=\frac{\lambda_{R}}{\lambda_{k}} E_{t}\left[\Omega_{t, t+1}\left(r_{t+1}^{k}-r_{t+1}^{d}\right)\right]+\frac{\lambda_{R}}{\lambda_{k}}\left(\frac{\kappa_{t}^{k}}{q_{t}^{k}}\right) \theta_{t}^{k}\left(\frac{\psi_{t}^{c b}}{1+\mu_{t}}\right) \tag{145}
\end{equation*}
$$

Any realistic calibration will feature $\lambda_{R}=0$, because the central bank reserves are electronic accounts controlled by the central bank. In that case, the first order condition shrinks to:

$$
E_{t}\left[\Omega_{t, t+1}\left(r_{t+1}^{R}-r_{t+1}^{d}\right)\right]=0
$$

which implies that the interest rates on reserves and deposits must be equal in equilibrium.
Setting $\psi_{t}^{R}=0$ in equation 131, and combining with equation 136 gives the following expression:

$$
\frac{\chi_{t}}{1+\mu_{t}}=E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]\left(1+r_{t+1}^{d}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)\right\}
$$

Now I combine (131) and (again setting $\psi_{t}^{R}=0$ ) to obtain the following relation between
the shadow value on deposit funding and central bank funding:

$$
\frac{\psi_{t}^{c b}}{1+\mu_{t}}=\eta_{t}-\eta_{t}^{c b}
$$

Substitution of (136) and gives rise to the following relation:

$$
\begin{equation*}
\frac{\psi_{t}^{c b}}{1+\mu_{t}}=E_{t}\left[\Omega_{t, t+1}\left(r_{t+1}^{d}-r_{t+1}^{c b}\right)\right] \tag{146}
\end{equation*}
$$

Finally, I assume that the ECB applies the regular collateral requirements in C.2), and therefore set $\kappa_{t}^{k}=q_{t}^{k}$ and $\kappa_{t}^{b}=q_{t}^{b}$. Now, I can summarize the first order conditions for corporate securities, government bonds, central bank reserves, deposits, central bank funding, and the incentive compatibility constraint (which I assume to be binding in a financial crisis), as they are presented in the main text:

$$
\begin{align*}
\frac{\lambda_{b}}{\lambda_{k}} E_{t}\left[\Omega_{t, t+1}\left(r_{t+1}^{k}-r_{t+1}^{d}\right)\right] & =E_{t}\left[\Omega_{t, t+1}\left(r_{t+1}^{b}-r_{t+1}^{d}\right)\right]+\left(\theta_{t}^{b}-\frac{\lambda_{b}}{\lambda_{k}} \theta_{t}^{k}\right)\left(\frac{\psi_{t}^{c b}}{1+\mu_{t}}\right)  \tag{147}\\
E_{t}\left[\Omega_{t, t+1}\left(r_{t+1}^{b}-r_{t+1}^{d}\right)\right] & =\lambda_{b}\left(\frac{\mu_{t}}{1+\mu_{t}}\right)-\theta_{t}^{b}\left(\frac{\psi_{t}^{c b}}{1+\mu_{t}}\right)  \tag{148}\\
E_{t}\left[\Omega_{t, t+1}\left(r_{t+1}^{R}-r_{t+1}^{d}\right)\right] & =0  \tag{149}\\
\frac{\chi_{t}}{1+\mu_{t}} & =E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]\left(1+r_{t+1}^{d}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right) k(1\right.  \tag{150}\\
\frac{\psi_{t}^{c b}}{1+\mu_{t}} & =E_{t}\left[\Omega_{t, t+1}\left(r_{t+1}^{d}-r_{t+1}^{c b}\right)\right]  \tag{151}\\
\chi_{t} n_{t} & =\lambda_{k} q_{t}^{k} s_{t}^{k, p}+\lambda_{b} q_{t}^{b} s_{t}^{b, p} \tag{152}
\end{align*}
$$

where I have set $\lambda_{R}=0$ in 152 . Finally, note that I set $\tau_{t+1}^{n}=\tilde{\tau}_{t+1}^{n}=0$ in equation 150 in the main text, as I do not discuss recapitalizations of intermediaries.

## C. 3 Production Process

## C.3.1 Capital Producers

At the end of period $t$, when the intermediate goods firms have produced, the capital producers buy the remaining stock of capital $(1-\delta) \xi_{t} k_{t-1}$ from the intermediate goods producers at a price $q_{t}^{k}$. They combine this capital with goods bought from the final goods producers (investment $i_{t}$ ) to produce next period's beginning of period capital stock $k_{t}$. This capital is being sold to the intermediate goods producers at a price $q_{t}^{k}$. I assume that the capital producers face convex adjustment costs when transforming the final goods bought into capital goods, set up such that changing the level of gross investment is costly. Hence I get:

$$
\begin{equation*}
k_{t}=(1-\delta) \xi_{t} k_{t-1}+\left(1-\Psi\left(\iota_{t}\right)\right) \epsilon_{t}^{i} i_{t}, \quad \Psi(x)=\frac{\gamma}{2}(x-1)^{2}, \quad \iota_{t}=i_{t} / i_{t-1} \tag{153}
\end{equation*}
$$

where $\xi_{t}$ represents a capital quality shock Gertler and Karadi, 2011), and $\epsilon_{t}^{i}$ a shock to investment adjustment costs. Profits are passed on to households, who are the ultimate owners of the capital producers. Profits at the end of period $t$ equal:

$$
\Pi_{t}^{c}=q_{t}^{k} k_{t}-q_{t}^{k}(1-\delta) \xi_{t} k_{t-1}-i_{t}=q_{t}^{k}\left(1-\Psi\left(\iota_{t}\right)\right) \epsilon_{t}^{i} i_{t}-i_{t}
$$

where I substituted equation 153 . The capital producers maximize the sum of expected current and (discounted) future profits:

$$
\max _{\left\{i_{t+s}\right\}_{s=0}^{\infty}} E_{t}\left\{\sum_{s=0}^{\infty} \beta^{s} \Lambda_{t, t+s}\left[q_{t+s}^{k}\left(1-\Psi\left(\iota_{t+s}\right)\right) \epsilon_{t+s}^{i} i_{t+s}-i_{t+s}\right]\right\}
$$

Differentiation with respect to investment gives the first order condition for the capital producers:

$$
q_{t}^{k}\left(1-\Psi\left(\iota_{t}\right)\right) \epsilon_{t}^{i}-1-q_{t}^{k} \epsilon_{t}^{i} \iota_{t} \Psi^{\prime}\left(\iota_{t}\right)+E_{t}\left[\beta \Lambda_{t, t+1} q_{t+1}^{k} \epsilon_{t+1}^{i} \iota_{t+1}^{2} \Psi^{\prime}\left(\iota_{t+1}\right)\right]=0
$$

which gives the following expression for the price of capital:
$\frac{1}{q_{t}^{k}}=\left[1-\frac{\gamma}{2}\left(\frac{i_{t}}{i_{t-1}}-1\right)^{2}\right] \epsilon_{t}^{i}-\gamma\left(\frac{i_{t}}{i_{t-1}}-1\right)\left(\frac{i_{t}}{i_{t-1}}\right) \epsilon_{t}^{i}+E_{t}\left[\beta \Lambda_{t, t+1} \frac{q_{t+1}^{k}}{q_{t}^{k}}\left(\frac{i_{t+1}}{i_{t}}\right)^{2} \gamma\left(\frac{i_{t+1}}{i_{t}}-1\right) \epsilon_{t+1}^{i}\right]$

## C.3.2 Intermediate Goods Producers

I remember that period $t$ profits are given by:

$$
\Pi_{i, t}=m_{t} z_{t}\left(\xi_{t} k_{i, t-1}\right)^{\alpha} h_{i, t}^{1-\alpha}+q_{t}^{k}(1-\delta) \xi_{t} k_{i, t-1}-\left(1+r_{t}^{k}\right) q_{t-1}^{k} k_{i, t-1}-w_{t} h_{i, t}
$$

The intermediate goods producing firms hire labor in a perfectly competitive labor market, such that the marginal benefit from an additional unit of labor equals the marginal cost $w_{t}$ from an additional unit of labor:

$$
w_{t}=(1-\alpha) m_{t} y_{i, t} / h_{i, t}
$$

Intermediate goods producers credibly pledge all after-wage profits to financial intermediaries. Hence, in equilibrium profits will be zero. By substituting the first order condition for the wage rate into the zero-profit condition $\Pi_{i, t}=0$, I can find an expression for the ex-post return on capital:

$$
r_{t}^{k}=\left(q_{t-1}^{k}\right)^{-1}\left(\alpha m_{t} y_{i, t} / k_{i, t-1}+q_{t}^{k}(1-\delta) \xi_{t}\right)-1
$$

Now I rewrite the first order condition for labor and the expression for the ex-post return on capital to find the factor demands:

$$
\begin{align*}
k_{i, t-1} & =\alpha m_{t} y_{i, t} /\left[q_{t-1}^{k}\left(1+r_{t}^{k}\right)-q_{t}^{k}(1-\delta) \xi_{t}\right]  \tag{155}\\
h_{i, t} & =(1-\alpha) m_{t} y_{i, t} / w_{t} \tag{156}
\end{align*}
$$

By substituting the factor demands into the production technology function, I get for the relative intermediate output price $m_{t}$ :

$$
\begin{equation*}
m_{t}=\alpha^{-\alpha}(1-\alpha)^{\alpha-1} z_{t}^{-1}\left(w_{t}^{1-\alpha}\left[q_{t-1}^{k}\left(1+r_{t}^{k}\right) \xi_{t}^{-1}-q_{t}^{k}(1-\delta)\right]^{\alpha}\right) \tag{157}
\end{equation*}
$$

## C.3.3 Final Goods Producers

Final goods firms purchase retail goods $y_{t}^{f}$ at a price $P_{t}^{f}$ from a continuum of retail goods firms $f \in[0,1]$. They employ the following technology to produce the final good using retalil goods as input

$$
\begin{equation*}
y_{t}=\left[\int_{0}^{1}\left(y_{t}^{f}\right)^{\left(\epsilon_{p}-1\right) / \epsilon_{p}} d f\right]^{\frac{\epsilon_{p}}{\epsilon_{p}-1}} \tag{158}
\end{equation*}
$$

where $\epsilon_{p}$ is the elasticity of substitution between two different retail goods. Final goods firms operate in a perfectly competitive market, and face a period-by-period optimization problem in which they maximize the difference between the revenue from selling final goods $y_{t}$ at price $P_{t}$ and input costs which are the sum over all retail goods firms of the volume of retail goods $y_{t}^{f}$ purchased from retailer $f$ at price $P_{t}^{f}$. Final goods producers take all prices and the demand for final goods $y_{t}$ as given, and only choose the volume $y_{t}^{f}$ to buy from retail firm $f$. Hence the period $t$ optimization problem is given by:

$$
\max _{y_{t}^{f}} P_{t} y_{t}-\int_{0}^{1} P_{t}^{f} y_{t}^{f} d f
$$

subject to the final goods firm's production technology 158). After substitution of 158, I differentiate with respect to $y_{t}^{f}$, and obtain the following first order condition:

$$
\begin{aligned}
P_{t}^{f} & =P_{t}\left(\frac{\epsilon_{p}}{\epsilon_{p}-1}\right)\left[\int_{0}^{1}\left(y_{t}^{f}\right)^{\left(\epsilon_{p}-1\right) / \epsilon_{p}} d f\right]^{\frac{\epsilon_{p}}{\epsilon_{p}-1}-1}\left(\frac{\epsilon_{p}-1}{\epsilon_{p}}\right)\left(y_{t}^{f}\right)^{\frac{\frac{\epsilon_{p}-1}{\epsilon_{p}}-1}{\epsilon^{\prime}}-} \\
& =P_{t}\left[\int_{0}^{1}\left(y_{t}^{f}\right)^{\left(\epsilon_{p}-1\right) / \epsilon_{p}} d f\right]^{\frac{1}{\epsilon_{p}-1}}\left(y_{t}^{f}\right)^{\frac{-1}{\epsilon_{p}}}=P_{t} y_{t}^{\frac{1}{\epsilon_{p}}}\left(y_{t}^{f}\right)^{\frac{-1}{\epsilon_{p}}}
\end{aligned}
$$

This last expression can be rewritten as:

$$
\begin{equation*}
y_{t}^{f}=\left(\frac{P_{t}^{f}}{P_{t}}\right)^{-\epsilon_{p}} y_{t} \tag{159}
\end{equation*}
$$

Finally, I substitute the demand curve 159 into the production technology 158 of the final goods producers to get:

$$
y_{t}=\left[\int_{0}^{1}\left(\frac{P_{t}^{f}}{P_{t}}\right)^{1-\epsilon_{p}} y_{t}^{\left(\epsilon_{p}-1\right) / \epsilon_{p}} d f\right]^{\frac{\epsilon_{p}}{\epsilon_{p}-1}}=\left[\int_{0}^{1}\left(\frac{P_{t}^{f}}{P_{t}}\right)^{1-\epsilon_{p}} d f\right]^{\frac{\epsilon_{p}}{\epsilon_{p}-1}} y_{t}
$$

Division by $y_{t}$, and raising both sides of the equation to the power $\left(\epsilon_{p}-1\right) / \epsilon_{p}$ results in the following equation:

$$
1=\int_{0}^{1}\left(\frac{P_{t}^{f}}{P_{t}}\right)^{1-\epsilon_{p}} d f
$$

As $P_{t}$ does not depend on $f$, I can take it outside the integral and move it to the left hand side of the equation to obtain:

$$
\begin{equation*}
P_{t}^{1-\epsilon_{p}}=\int_{0}^{1}\left(P_{t}^{f}\right)^{1-\epsilon_{p}} d f \tag{160}
\end{equation*}
$$

## C.3.4 Retail firms

Retail firms purchase goods $y_{t}^{i}$ from intermediate goods proders for a price $m_{t}$ in terms of the general price level $P_{t}$. They convert intermediate goods into retail goods $y_{t}^{f}$, which are sold for a real price $P_{t}^{f} / P_{t}$ to final goods producers. It takes one intermediate good to produce one retail good $\left(y_{t}^{f}=y_{t}^{i}\right)$. All retail firms produce a differentiated retail good, and therfore operate in a monopolistically competitive market. Hence they are capable of charging a markup over the input price earning them profits $\left(P_{t}^{f} / P_{t}-m_{t}\right) y_{t}^{f}$. Retail firms face the following demand curve for retail goods 159). Each period, retail goods firms face price-stickiness as in Calvo (1983). This implies that retail firms face probability $\psi_{p}$ that they cannot adjust their prices in the current period. The probability $\psi_{p}$ is i.i.d. and constant across time and the cross-section of retail goods firms. Retail goods producers that cannot choose a new price are forced to multiply their existing price by $\pi_{t}^{a d j}$. Hence retail goods firms face the following optimization problem:

$$
\max _{\left\{P_{t}^{f}\right\}} E_{t}\left\{\sum_{j=0}^{\infty} \beta^{j} \psi_{p}^{j} \frac{\lambda_{t+j}}{\lambda_{t}}\left[\frac{P_{t}^{f} \Pi_{k=1}^{k=j} \pi_{t+k}^{a d j}}{P_{t+j}} y_{t+j}^{f}-m_{t+j} y_{t+j}^{f}\right]\right\}
$$

Substitution of the demand curve for retail goods (159) gives the following optimization probelm:

$$
\max _{\left\{P_{t}^{f}\right\}} E_{t}\left\{\sum_{j=0}^{\infty} \beta^{j} \psi_{p}^{j} \frac{\lambda_{t+j}}{\lambda_{t}}\left[\left(\frac{P_{t}^{f} \Pi_{k=1}^{k=j} \pi_{t+k}^{a d j}}{P_{t+j}}\right)^{1-\epsilon_{p}} y_{t+j}-m_{t+j}\left(\frac{P_{t}^{f} \Pi_{k=1}^{k=j} \pi_{t+k}^{a d j}}{P_{t+j}}\right)^{-\epsilon_{p}} y_{t+j}\right]\right\}
$$

Now I am in the position to take the first order condition with respect to $P_{t}^{f}$, where I write $P_{t}^{*}$ to denote the optimally chosen price for $P_{t}^{f}$ :

$$
\begin{aligned}
& \left(\epsilon_{p}-1\right) E_{t}\left[\sum_{j=0}^{\infty} \beta^{j} \psi_{p}^{j} \frac{\lambda_{t+j}}{\lambda_{t}}\left(\frac{P_{t}^{*} \Pi_{k=1}^{k=j} \pi_{t+k}^{a d j}}{P_{t+j}}\right)^{1-\epsilon_{p}} \frac{y_{t+j}}{P_{t}^{*}}\right] \\
= & \epsilon_{p} E_{t}\left[\sum_{j=0}^{\infty} \beta^{j} \psi_{p}^{j} \frac{\lambda_{t+j}}{\lambda_{t}} m_{t+j}\left(\frac{P_{t}^{*} \Pi_{k=1}^{k=j} \pi_{t+k}^{a d j}}{P_{t+j}}\right)^{-\epsilon_{p}} \frac{y_{t+j}}{P_{t}^{*}}\right],
\end{aligned}
$$

Taking the price $P_{t}^{*}$ outside of the brackets gives:

$$
\begin{aligned}
& \frac{\left(P_{t}^{*}\right)^{-\epsilon_{p}}}{P_{t}^{1-\epsilon_{p}}}\left(\epsilon_{p}-1\right) E_{t}\left[\sum_{j=0}^{\infty} \beta^{j} \psi_{p}^{j} \lambda_{t+j}\left(\frac{P_{t} \Pi_{k=1}^{k=j} \pi_{t+k}^{a d j}}{P_{t+j}}\right)^{1-\epsilon_{p}} y_{t+j}\right] \\
= & \frac{\left(P_{t}^{*}\right)^{-\epsilon_{p}-1}}{P_{t}^{-\epsilon_{p}}} \epsilon_{p} E_{t}\left[\sum_{j=0}^{\infty} \beta^{j} \psi_{p}^{j} \lambda_{t+j} m_{t+j}\left(\frac{P_{t} \Pi_{k=1}^{k=j} \pi_{t+k}^{a d j}}{P_{t+j}}\right)^{-\epsilon_{p}} y_{t+j}\right],
\end{aligned}
$$

This expression can be further simplified:

$$
\begin{aligned}
& \frac{P_{t}^{*}}{P_{t}}\left(\epsilon_{p}-1\right) E_{t}\left[\sum_{j=0}^{\infty} \beta^{j} \psi_{p}^{j} \lambda_{t+j}\left(\frac{\Pi_{k=1}^{k=j} \pi_{t+k}}{\Pi_{k=1}^{k=j} \pi_{t+k}^{a d j}}\right)^{\epsilon_{p}-1} y_{t+j}\right] \\
= & \epsilon_{p} E_{t}\left[\sum_{j=0}^{\infty} \beta^{j} \psi_{p}^{j} \lambda_{t+j} m_{t+j}\left(\frac{\Pi_{k=1}^{k=j} \pi_{t+k}}{\Pi_{k=1}^{k=j} \pi_{t+k}^{a d j}}\right)^{\epsilon_{p}} y_{t+j}\right],
\end{aligned}
$$

Finally, I can rewrite this as:

$$
\frac{P_{t}^{*}}{P_{t}}=\left(\frac{\epsilon_{p}}{\epsilon_{p}-1}\right) \frac{E_{t}\left[\sum_{j=0}^{\infty} \beta^{j} \psi_{p}^{j} \lambda_{t+j} m_{t+j}\left(\frac{\Pi_{k=1}^{k=j} \pi_{t+k}}{\Pi_{k=1}^{k=1} 1_{t+k}^{a d j}}\right)^{\epsilon_{p}} y_{t+j}\right]}{E_{t}\left[\sum_{j=0}^{\infty} \beta^{j} \psi_{p}^{j} \lambda_{t+j}\left(\frac{\Pi_{k}^{k=j} \pi_{k+k}}{\Pi_{k=1}^{k=j} \pi_{t+k}^{a d j}}\right)^{\epsilon_{p}-1} y_{t+j}\right]} .
$$

Defining $\pi_{t}^{*} \equiv P_{t}^{*} / P_{t}$, I can write the above expression in the following way:

$$
\begin{equation*}
\pi_{t}^{*}=\left(\frac{\epsilon_{p}}{\epsilon_{p}-1}\right) \frac{\Xi_{1, t}}{\Xi_{2, t}} \tag{161}
\end{equation*}
$$

where $\Xi_{1, t}$ and $\Xi_{2, t}$ are given by:

$$
\begin{align*}
& \Xi_{1, t}=\lambda_{t} m_{t} y_{t}+\beta \psi_{p} E_{t}\left[\left(\frac{\pi_{t+1}}{\pi_{t+1}^{a d j}}\right)^{\epsilon_{p}} \Xi_{1, t+1}\right]  \tag{162}\\
& \Xi_{1, t}=\lambda_{t} y_{t}+\beta \psi_{p} E_{t}\left[\left(\frac{\pi_{t+1}}{\pi_{t+1}^{a d j}}\right)^{\epsilon_{p}-1} \Xi_{2, t+1}\right], \tag{163}
\end{align*}
$$

Now I take the law of motion for the aggregate price level 160 , which I can write as:

$$
\begin{equation*}
P_{t}^{1-\epsilon_{p}}=\left(1-\psi_{p}\right)\left(P_{t}^{*}\right)^{1-\epsilon_{p}}+\left(1-\psi_{p}\right) \psi_{p}\left(\pi_{t}^{a d j} P_{t-1}^{*}\right)^{1-\epsilon_{p}}+\left(1-\psi_{p}\right) \psi_{p}^{2}\left(\pi_{t}^{a d j} \pi_{t-1}^{a d j} P_{t-2}^{*}\right)^{1-\epsilon_{p}}+\ldots \ldots \tag{164}
\end{equation*}
$$

Iterating one period backward, and multiplying by $\psi_{p}\left(\pi_{t}^{a d j}\right)^{1-\epsilon_{p}}$ gives:

$$
\psi_{p}\left(\pi_{t}^{a d j}\right)^{1-\epsilon_{p}} P_{t-1}^{1-\epsilon_{p}}=\left(1-\psi_{p}\right) \psi_{p}\left(\pi_{t}^{a d j} P_{t-1}^{*}\right)^{1-\epsilon_{p}}+\left(1-\psi_{p}\right) \psi_{p}^{2}\left(\pi_{t}^{a d j} \pi_{t-1}^{a d j} P_{t-2}^{*}\right)^{1-\epsilon_{p}}+\ldots \ldots
$$

Hence I can rewrite (164) in the following way:

$$
\begin{equation*}
P_{t}^{1-\epsilon_{p}}=\left(1-\psi_{p}\right)\left(P_{t}^{*}\right)^{1-\epsilon_{p}}+\psi_{p}\left(\pi_{t}^{a d j}\right)^{1-\epsilon_{p}} P_{t-1}^{1-\epsilon_{p}} \tag{165}
\end{equation*}
$$

Division of the left and right hand side by $P_{t}^{1-\epsilon_{p}}$ gives the following expression:

$$
\begin{equation*}
1=\left(1-\psi_{p}\right)\left(\pi_{t}^{*}\right)^{1-\epsilon_{p}}+\psi_{p}\left(\frac{\pi_{t}}{\pi_{t}^{a d j}}\right)^{\epsilon_{p}-1} \tag{166}
\end{equation*}
$$

Now I move on to price dispersion $\mathcal{D}_{t}^{p} \equiv \int_{0}^{1}\left(\frac{P_{t}^{f}}{P_{t}}\right)^{-\epsilon_{p}} d f$ :
$\mathcal{D}_{t}^{p}=\left(1-\psi_{p}\right)\left(\frac{P_{t}^{*}}{P_{t}}\right)^{-\epsilon_{p}}+\left(1-\psi_{p}\right) \psi_{p}\left(\frac{\pi_{t}^{a d j} P_{t-1}^{*}}{P_{t}}\right)^{-\epsilon_{p}}+\left(1-\psi_{p}\right) \psi_{p}^{2}\left(\frac{\pi_{t}^{a d j} \pi_{t-1}^{a d j} P_{t-2}^{*}}{P_{t}}\right)^{-\epsilon_{p}}+\ldots$.
Iterating one period backwards, and multiplying by $\psi_{p}\left(\frac{\pi_{t}^{a d j} P_{t-1}}{P_{t}}\right)^{-\epsilon_{p}}$ gives the following expression:
$\psi_{p}\left(\frac{\pi_{t}^{a d j} P_{t-1}}{P_{t}}\right)^{-\epsilon_{p}} \mathcal{D}_{t-1}^{p}=\left(1-\psi_{p}\right) \psi_{p}\left(\frac{\pi_{t}^{a d j} P_{t-1}^{*}}{P_{t}}\right)^{-\epsilon_{p}}+\left(1-\psi_{p}\right) \psi_{p}^{2}\left(\frac{\pi_{t}^{a d j} \pi_{t-1}^{a d j} P_{t-2}^{*}}{P_{t}}\right)^{-\epsilon_{p}}+\ldots$.

Hence I can write 167) in the following way:

$$
\begin{align*}
\mathcal{D}_{t}^{p} & =\left(1-\psi_{p}\right)\left(\frac{P_{t}^{*}}{P_{t}}\right)^{-\epsilon_{p}}+\psi_{p}\left(\frac{\pi_{t}^{a d j} P_{t-1}}{P_{t}}\right)^{-\epsilon_{p}} \mathcal{D}_{t-1}^{p} \\
& =\left(1-\psi_{p}\right)\left(\pi_{t}^{*}\right)^{-\epsilon_{p}}+\psi_{p}\left(\frac{\pi_{t}}{\pi_{t}^{a d j}}\right)^{\epsilon_{p}} \mathcal{D}_{t-1}^{p} \tag{168}
\end{align*}
$$

## C.3.5 Labor market

As mentioned in the main text, the labor market features staggered wage-setting by households, which is similar in spirit to Calvo (1983), and was first introduced by Erceg et al. (2000). I start by investigating labor agencies optimization problem. They have to assemble labor $h_{t}(i)$ from each household $i \in[0,1]$, and combine this into final labor $h_{t}$ using a constant elasticity of substitution aggregation funtcion with elasticity $\epsilon_{w}$ :

$$
\begin{equation*}
h_{t}=\left[\int_{0}^{1}\left(h_{t}(i)\right)^{\frac{\epsilon_{w}-1}{\epsilon_{w}}} d i\right]^{\frac{\epsilon_{w}}{\epsilon_{w}-1}} \tag{169}
\end{equation*}
$$

Labor agencies sell the final labor $h_{t}$ to intermediate goods producers in a perfectly competitive market. Hence labor agencies take the wage rate $W_{t}$ and demand for final labor $h_{t}$ as given. To produce final labor, the agencies hire labor from every household $i \in[0,1]$. Each of these households provide a unique type of labor, and hence they have the power to set the wage rate $W_{t}(i)$ at which they sell their labor. Therefore, the only choice that labor agencies have is to determine how much labor to hire from each household $i$. Therefore, labor agencies' optimization problem is given by:

$$
\max _{h_{t}(i)} W_{t} h_{t}-\int_{0}^{1} W_{t}(i) h_{t}(i) d i
$$

subject to 169). Taking the first order condition with respect to $h_{t}(i)$, I arrive at the following first order condition:

$$
\begin{aligned}
W_{t}(i) & =W_{t}\left[\int_{0}^{1}\left(h_{t}(i)\right)^{\frac{\epsilon_{w}-1}{\epsilon_{w}}} d i\right]^{\frac{\epsilon_{w}}{\epsilon_{w}-1}-1}\left(h_{t}(i)\right)^{\frac{\epsilon_{w}-1}{\epsilon_{w}}-1} \\
& =W_{t}\left[\int_{0}^{1}\left(h_{t}(i)\right)^{\frac{\epsilon_{w}-1}{\epsilon_{w}}} d i\right]^{\frac{1}{\epsilon_{w}-1}}\left(h_{t}(i)\right)^{\frac{-1}{\epsilon_{w}}} \\
& =W_{t} h_{t}^{\frac{1}{\epsilon_{w}}}\left(h_{t}(i)\right)^{\frac{-1}{\epsilon_{w}}}=W_{t}\left(\frac{h_{t}(i)}{h_{t}}\right)^{\frac{-1}{\epsilon_{w}}}
\end{aligned}
$$

This allows me to rewrite the first order condition for the demand for household $i$ 's labor $h_{t}(i)$ :

$$
\begin{equation*}
h_{t}(i)=\left(\frac{W_{t}(i)}{W_{t}}\right)^{-\epsilon_{w}} h_{t} \tag{170}
\end{equation*}
$$

where $h_{t}$ is aggregate labor, $W_{t}(i)$ is the nominal wage rate of household $i$, and $W_{t}$ is the aggregate wage rate. Now I substitute 170 into 169 :

$$
h_{t}=\left[\int_{0}^{1}\left(\frac{W_{t}(i)}{W_{t}}\right)^{1-\epsilon_{w}} h_{t}^{\frac{\epsilon_{w}-1}{\epsilon_{w}}} d i\right]^{\frac{\epsilon_{w}}{\epsilon_{w}-1}}=\left[\int_{0}^{1}\left(\frac{W_{t}(i)}{W_{t}}\right)^{1-\epsilon_{w}} d i\right]^{\frac{\epsilon_{w}}{\epsilon_{w}-1}} h_{t}
$$

Dividing the left and right hand side by $h_{t}$ and raising to the power $\left(\epsilon_{w}-1\right) / \epsilon_{w}$ gives me the following expression:

$$
1=\int_{0}^{1}\left(\frac{W_{t}(i)}{W_{t}}\right)^{1-\epsilon_{w}} d i=\frac{1}{W_{t}^{1-\epsilon_{w}}} \int_{0}^{1} W_{t}(i)^{1-\epsilon_{w}} d i
$$

Multiplying left and right hand side by $W_{t}^{1-\epsilon_{w}}$ gives me the following expression for the aggrgeate final wage $W_{t}$ :

$$
\begin{equation*}
W_{t}^{1-\epsilon_{w}}=\int_{0}^{1} W_{t}(i)^{1-\epsilon_{w}} d i \tag{171}
\end{equation*}
$$

Now I move on to the household $i$ 's optimization problem, which is given by:

$$
\max _{\left\{W_{t}(i)\right\}} E_{t}\left\{\sum_{s=0}^{\infty}\left(\beta \psi_{w}\right)^{s}\left[\lambda_{t+s} \frac{W_{t}(i) \Pi_{j=1}^{j=s} \omega_{t+j}^{a d j}}{P_{t+s}} h_{t+s}(i)-\chi \frac{h_{t+s}(i)^{1+\varphi}}{1+\varphi}\right]\right\},
$$

Substitution of the labor demand curve (170) results in the following optimization problem:

$$
\max _{\left\{W_{t}(i)\right\}} E_{t}\left\{\sum_{s=0}^{\infty}\left(\beta \psi_{w}\right)^{s}\left[\lambda_{t+s} \frac{W_{t+s}}{P_{t+s}}\left(\frac{W_{t}(i) \Pi_{j=1}^{j=s} \omega_{t+j}^{a d j}}{W_{t+s}}\right)^{1-\epsilon_{w}} h_{t+s}-\chi\left(\frac{W_{t}(i) \Pi_{j=1}^{j=s} \omega_{t+j}^{a d j}}{W_{t+s}}\right)^{-\epsilon_{w}(1+\varphi)} \frac{h_{t+s}^{1+\varphi}}{1+\varphi}\right]\right\}
$$

Taking the first order condition with respect to the price $W_{t}(i)$ results in the following first order condition:

$$
\begin{aligned}
& \left(\epsilon_{w}-1\right) E_{t}\left[\sum_{s=0}^{\infty}\left(\beta \psi_{w}\right)^{s} \lambda_{t+s} w_{t+s} \frac{1}{W_{t}(i)}\left(\frac{W_{t}(i) \Pi_{j=1}^{j=s} \omega_{t+j}^{a d j}}{W_{t+s}}\right)^{1-\epsilon_{w}} h_{t+s}\right] \\
= & \epsilon_{w}(1+\varphi) E_{t}\left[\sum_{s=0}^{\infty}\left(\beta \psi_{w}\right)^{s} \chi \frac{1}{W_{t}(i)}\left(\frac{W_{t}(i) \Pi_{j=1}^{j=s} \omega_{t+j}^{a d j}}{W_{t+s}}\right)^{-\epsilon_{w}(1+\varphi)} \frac{h_{t+s}^{1+\varphi}}{1+\varphi}\right],
\end{aligned}
$$

where $w_{t+s}=W_{t+s} / P_{t+s}$. This equation can be rewritten in the following way:

$$
\begin{aligned}
& \left(\epsilon_{w}-1\right) \frac{W_{t}(i)^{-\epsilon_{w}}}{W_{t}^{1-\epsilon_{w}}} E_{t}\left[\sum_{s=0}^{\infty}\left(\beta \psi_{w}\right)^{s}\left(\frac{W_{t} \Pi_{j=1}^{j=s} \omega_{t+j}^{a d j}}{W_{t+s}}\right)^{1-\epsilon_{w}} \lambda_{t+s} w_{t+s} h_{t+s}\right] \\
= & \chi \epsilon_{w} \frac{W_{t}(i)^{-\epsilon_{w}(1+\varphi)-1}}{W_{t}^{-\epsilon_{w}(1+\varphi)}} E_{t}\left[\sum_{s=0}^{\infty}\left(\beta \psi_{w}\right)^{s}\left(\frac{W_{t} \Pi_{j=1}^{j=s} \omega_{t+j}^{a d j}}{W_{t+s}}\right)^{-\epsilon_{w}(1+\varphi)} h_{t+s}^{1+\varphi}\right],
\end{aligned}
$$

Further rewriting gives:

$$
\left(\frac{W_{t}^{*}}{W_{t}}\right)^{1+\epsilon_{w} \varphi}=\chi\left(\frac{\epsilon_{w}}{\epsilon_{w}-1}\right) \frac{E_{t}\left[\sum_{s=0}^{\infty}\left(\beta \psi_{w}\right)^{s}\left(\Pi_{j=1}^{j=s} \frac{\pi_{t+j}^{w}}{\omega_{t+j}^{a d j}}\right)^{\epsilon_{w}(1+\varphi)} h_{t+s}^{1+\varphi}\right]}{E_{t}\left[\sum_{s=0}^{\infty}\left(\beta \psi_{w}\right)^{s} \lambda_{t+s} w_{t+s}\left(\Pi_{j=1}^{j=s} \frac{\pi_{t+j}^{w}}{\omega_{t+j}^{a d j}}\right)^{\epsilon_{w}-1} h_{t+s}\right]},
$$

where I have replaced $W_{t}(i)$ by $W_{t}^{*}$, and where $\pi_{t}^{w} \equiv W_{t} / W_{t-1}=\left(w_{t} / w_{t-1}\right) \pi_{t}$. Defining the relative new wage $\omega_{t}^{*} \equiv W_{t}^{*} / W_{t}$, I can rewrite the first order condition in the following way:

$$
\begin{equation*}
\left(\omega_{t}^{*}\right)^{1+\epsilon_{w} \varphi}=\chi\left(\frac{\epsilon_{w}}{\epsilon_{w}-1}\right) \frac{\Xi_{1, t}^{w}}{\Xi_{2, t}^{w}} \tag{172}
\end{equation*}
$$

where $\Xi_{1, t}^{w}$ and $\Xi_{2, t}^{w}$ are given by:

$$
\begin{align*}
& \Xi_{1, t}^{w}=h_{t}^{1+\varphi}+\beta \psi_{w} E_{t}\left[\left(\frac{\pi_{t+1}^{w}}{\omega_{t+1}^{a d j}}\right)^{\epsilon_{w}(1+\varphi)} \Xi_{1, t+1}^{w}\right],  \tag{173}\\
& \Xi_{2, t}^{w}=\lambda_{t} w_{t} h_{t}+\beta \psi_{w} E_{t}\left[\left(\frac{\pi_{t+1}^{w}}{\omega_{t+1}^{a d j}}\right)^{\epsilon_{w}-1} \Xi_{2, t+1}^{w}\right], \tag{174}
\end{align*}
$$

Now that I have found the first order condition for the households that allowed to set a new wage rate $W_{t}(i)$, I seek to derive an expression for the final wage rate 171, which I can write in the following way:
$W_{t}^{1-\epsilon_{w}}=\left(1-\psi_{w}\right)\left(W_{t}^{*}\right)^{1-\epsilon_{w}}+\left(1-\psi_{w}\right) \psi_{w}\left(\omega_{t}^{a d j} W_{t-1}^{*}\right)^{1-\epsilon_{w}}+\left(1-\psi_{w}\right) \psi_{w}^{2}\left(\omega_{t}^{a d j} \omega_{t-1}^{a d j} W_{t-2}^{*}\right)^{1-\epsilon_{w}} \ldots$
Shifitng the time index one period back, and multiplying left and right hand side by $\psi_{w}\left(\omega_{t}^{a d j}\right)^{1-\epsilon_{w}}$ gives me the following expression:

$$
\psi_{w}\left(\omega_{t}^{a d j} W_{t-1}\right)^{1-\epsilon_{w}}=\left(1-\psi_{w}\right) \psi_{w}\left(\omega_{t}^{a d j} W_{t-1}^{*}\right)^{1-\epsilon_{w}}+\left(1-\psi_{w}\right) \psi_{w}^{2}\left(\omega_{t}^{a d j} \omega_{t-1}^{a d j} W_{t-2}^{*}\right)^{1-\epsilon_{w}} \ldots
$$

This allows me to rewrite equation 175 in the following way:

$$
\begin{equation*}
W_{t}^{1-\epsilon_{w}}=\left(1-\psi_{w}\right)\left(W_{t}^{*}\right)^{1-\epsilon_{w}}+\psi_{w}\left(\omega_{t}^{a d j} W_{t-1}\right)^{1-\epsilon_{w}} \tag{176}
\end{equation*}
$$

Dividing left and right hand side by $W_{t}^{1-\epsilon_{w}}$ allows me to rewrite equation 176 in the following way:

$$
\begin{equation*}
1=\left(1-\psi_{w}\right)\left(\omega_{t}^{*}\right)^{1-\epsilon_{w}}+\psi_{w}\left(\frac{\pi_{t}^{w}}{\omega_{t}^{a d j}}\right)^{\epsilon_{w}-1} \tag{177}
\end{equation*}
$$

I employ a similar technique to find the wage dispersion $\mathcal{D}_{t}^{w} \equiv \int_{0}^{1}\left(\frac{W_{t}(i)}{W_{t}}\right)^{-\epsilon_{w}} d i$ :
$\mathcal{D}_{t}^{w}=\left(1-\psi_{w}\right)\left(\frac{W_{t}^{*}}{W_{t}}\right)^{-\epsilon_{w}}+\left(1-\psi_{w}\right) \psi_{w}\left(\frac{\omega_{t}^{a d j} W_{t-1}^{*}}{W_{t}}\right)^{-\epsilon_{w}}+\left(1-\psi_{w}\right) \psi_{w}^{2}\left(\frac{\omega_{t}^{a d j} \omega_{t-1}^{a d j} W_{t-2}^{*}}{W_{t}}\right)^{-\epsilon_{w}}+\ldots .$.
Iterating one period back, and multiplying with $\psi_{w}\left(\omega_{t}^{a d j} W_{t-1} / W_{t}\right)^{-\epsilon_{w}}$ provides me with the following equation:
$\psi_{w}\left(\frac{\omega_{t}^{a d j} W_{t-1}}{W_{t}}\right)^{-\epsilon_{w}} \mathcal{D}_{t-1}^{w}=\left(1-\psi_{w}\right) \psi_{w}\left(\frac{\omega_{t}^{a d j} W_{t-1}^{*}}{W_{t}}\right)^{-\epsilon_{w}}+\left(1-\psi_{w}\right) \psi_{w}^{2}\left(\frac{\omega_{t}^{a d j} \omega_{t-1}^{a d j} W_{t-2}^{*}}{W_{t}}\right)^{-\epsilon_{w}}+\ldots .$.
This expression allows me to rewrite equation 178 in the following way:

$$
\begin{align*}
\mathcal{D}_{t}^{w} & =\left(1-\psi_{w}\right)\left(\frac{W_{t}^{*}}{W_{t}}\right)^{-\epsilon_{w}}+\psi_{w}\left(\frac{\omega_{t}^{a d j} W_{t-1}}{W_{t}}\right)^{-\epsilon_{w}} \mathcal{D}_{t-1}^{w} \\
& =\left(1-\psi_{w}\right)\left(\omega_{t}^{*}\right)^{-\epsilon_{w}}+\psi_{w}\left(\frac{\pi_{t}^{w}}{\omega_{t}^{a d j}}\right)^{\epsilon_{w}} \mathcal{D}_{t-1}^{w} . \tag{179}
\end{align*}
$$

## C. 4 Government

## C.4.1 Fiscal Authority

The government issues debt that is long-term, and its maturity structure follows Woodford (1998, 2001). Let $q_{t}^{b}$ be the price expressed in terms of the final good of outstanding nominal bonds $B_{t}$. Therefore, the government raises nominal revenue $q_{t}^{b} B_{t}$ from debt issue in period $t$. A bond $B_{t-1}$ issued in period $t-1$ delivers a nominal cash flow $x_{c}$ in period $t,(1-\rho) x_{c}$ in period $t+1$, $(1-\rho)^{2} x_{c}$ in period $t+2$, etc ${ }^{27}$ Therefore, the cash flow from a bond issued in period $t-1$ equals a fraction $1-\rho$ of the cash flow from a bond issued in period $t$. As such, its price equals $(1-\rho) q_{t}^{b}$, where $q_{t}^{b}$ is the price of a bond issued in period $t$.

To determine the nominal and the real return on government bonds, I start by considering a government bond $B_{t-1}$ issued in period $t-1$ : purchasing such a bond requires investors to pay a nominal amount $q_{t-1}^{b} B_{t-1}$ in period $t-1$, and delivers a nominal cash flow $x_{c} B_{t-1}+(1-\rho) q_{t}^{b} B_{t-1}$ in period $t$. Hence the nominal return on bonds $r_{t}^{n, b}$ is given by:

$$
\begin{equation*}
1+r_{t}^{n, b}=\frac{x_{c}+(1-\rho) q_{t}^{b}}{q_{t-1}^{b}} \tag{180}
\end{equation*}
$$

The same bond requires investors to pay an amount (in terms of the final good) equal to $q_{t-1}^{b} B_{t-1} / P_{t-1}=q_{t-1}^{b} b_{t-1}$, where $b_{t} \equiv B_{t} / P_{t}$. In period $t$, such a bond delivers a cash flow

[^19] determined by $\rho$.
(in terms of the final good) equal to $x_{c} B_{t-1} / P_{t}+(1-\rho) q_{t}^{b} B_{t-1} / P_{t}=\frac{x_{c}+(1-\rho) q_{t}^{b}}{\pi_{t}} b_{t-1}$. Hence the real return on bonds $r_{t}^{b}$ (in terms of the final good) is given by:
\[

$$
\begin{equation*}
1+r_{t}^{b}=\frac{x_{c}+(1-\rho) q_{t}^{b}}{\pi_{t} q_{t-1}^{b}} \tag{181}
\end{equation*}
$$

\]

where $\pi_{t}$ denotes the gross inflation rate of the final good. The government also raises revenue by levying lump sum taxes $\tau_{t}$ on households and central bank profits $\Pi_{t}^{c b}$, both in terms of the final good. Finally, the government also receives the funds $\tilde{n}_{t}^{g}$ from repayment of previously administered financial sector support. Government revenues are used to pay for government purchases of the final good $g_{t}$, which is given by an $\mathrm{AR}(1)$ process, for servicing existing liabilities $\left(1+r_{t}^{b}\right) q_{t-1}^{b} b_{t-1}$, and for assistance to the financial sector by injecting new net worth $n_{t}^{g}$. The government budget constraint (in terms of the final good) is therefore given by:

$$
\begin{equation*}
q_{t}^{b} b_{t}+\tau_{t}+\Pi_{t}^{c b}+\tilde{n}_{t}^{g}=g_{t}+n_{t}^{g}+\left(1+r_{t}^{b}\right) q_{t-1}^{b} b_{t-1} . \tag{182}
\end{equation*}
$$

The tax rule of the government is given by a rule which makes sure the intertemporal government budget constraint is satisfied (Bohn, 1998):

$$
\begin{equation*}
\tau_{t}=\bar{\tau}+\kappa_{b}\left(b_{t-1}-\bar{b}\right)+\kappa_{n} n_{t}^{g}, \quad \kappa_{b} \in(0,1], \quad \kappa_{n} \in[0,1] . \tag{183}
\end{equation*}
$$

where $\bar{b}$ is the steady state level of debt. $\kappa_{n}$ controls the way government transfers to the financial sector are financed. If $\kappa_{n}=0$, support is financed by new debt. $\kappa_{n}=1$ implies that the additional spending is completely financed by increasing lump sum taxes. I parametrize government support as follows:

$$
\begin{align*}
n_{t}^{g} & =\tau_{t}^{n} n_{t-1}  \tag{184}\\
\tau_{t}^{n} & =\zeta \varepsilon_{\xi, t-l}, \quad \zeta \leq 0, \quad l \geq 0
\end{align*}
$$

Thus the government provides funds to the financial sector if $\zeta<0$ (a negative shock $\varepsilon_{\xi, t-l}$ to the quality of capital). Depending on the value of $l$, the government can provide support instantaneously $(l=0)$, or with a lag $(l>0)$. Furthermore, $\vartheta$ indicates the extent to which the government support needs to be repaid:

$$
\begin{equation*}
\tilde{n}_{t}^{g}=\vartheta n_{t-e}^{g}, \quad \vartheta \geq 0, \quad e \geq 1 . \tag{185}
\end{equation*}
$$

$\vartheta=0$ means the support is a gift from the government. In case $\vartheta=1$, the government aid is a zero interest loan, while a $\vartheta>1$ implies that financial intermediaries have to pay interest over the support received earlier ${ }^{28}$ The parameter $e$ denotes the amount of time after which the

[^20]government support has to be repaid.

## C. 5 Aggregation

I start by dividing intermediate goods producers' factor demand for capital 155 by the factor demand for labor (156):

$$
\frac{k_{i, t-1}}{h_{i, t}}=\left(\frac{\alpha}{1-\alpha}\right) \frac{w_{t}}{\left[q_{t-1}^{k}\left(1+r_{t}^{k}\right)-q_{t}^{k}(1-\delta) \xi_{t}\right]}
$$

Since the individual intermediate goods producer's capital-labor ratio does not depend on firmspecific variables, I know that every intermediate goods producer will choose the same capitallabor ratio in equilibrium. Therefore, I can write down the aggregate capital-labor ratio:

$$
\frac{k_{t-1}}{h_{t}}=\left(\frac{\alpha}{1-\alpha}\right) \frac{w_{t}}{\left[q_{t-1}^{k}\left(1+r_{t}^{k}\right)-q_{t}^{k}(1-\delta) \xi_{t}\right]}
$$

Now I can calculate aggregate supply by integrating the left and right hand side of the individual intermediate goods producer's production function $y_{i, t}=z_{t}\left(\xi_{t} k_{i, t-1}\right)^{\alpha} h_{i, t}^{1-\alpha}$ :

$$
\int_{0}^{1} z_{t}\left(\xi_{t} k_{i, t-1}\right)^{\alpha} h_{i, t}^{1-\alpha} d i=z_{t} \xi_{t}^{\alpha}\left(\frac{k_{t-1}}{h_{t}}\right)^{\alpha} \int_{0}^{1} h_{i, t} d i=z_{t}\left(\xi_{t} k_{t-1}\right)^{\alpha} h_{t}^{1-\alpha}
$$

while integration over $y_{i, t}$ gives:

$$
\int_{0}^{1} y_{i, t} d f=y_{t} \int_{0}^{1}\left(P_{t}^{f} / P_{t}\right)^{-\epsilon_{p, t}} d f=\mathcal{D}_{t}^{p} y_{t}
$$

So I get the following relation for aggregate supply $y_{t}$ :

$$
\begin{equation*}
\mathcal{D}_{t}^{p} y_{t}=z_{t}\left(\xi_{t} k_{t-1}\right)^{\alpha} h_{t}^{1-\alpha} . \tag{186}
\end{equation*}
$$

aid with a penalty rate of 50 percent. EU state support rules usually require financial intermediaries to repay previously received state support with a penalty rate.

## D First Order Conditions \& Equilibrium

## D. 1 First Order Conditions

The household's first order conditions are given by:

$$
\begin{align*}
\lambda_{t} & =\epsilon_{t}^{c}\left(c_{t}-v c_{t-1}\right)^{-1}-\beta v E_{t}\left[\epsilon_{t+1}^{c}\left(c_{t+1}-v c_{t}\right)^{-1}\right]  \tag{187}\\
1 & =E_{t}\left[\beta \Lambda_{t, t+1}\left(1+r_{t+1}^{d}\right)\right]  \tag{188}\\
1 & =E_{t}\left[\beta \Lambda_{t, t+1}\left(\frac{\left(1+r_{t+1}^{k}\right) q_{t}^{k}}{q_{t}^{k}+\kappa_{s_{k, h}}\left(s_{t}^{k, h}-\hat{s}_{k, h}\right)}\right)\right]  \tag{189}\\
1 & =E_{t}\left[\beta \Lambda_{t, t+1}\left(\frac{\left(1+r_{t+1}^{b}\right) q_{t}^{b}}{q_{t}^{b}+\kappa_{s_{b, h}}\left(s_{t}^{b, h}-\hat{s}_{b, h}\right)}\right)\right] \tag{190}
\end{align*}
$$

where $\Lambda_{t, t+1}=\lambda_{t+1} / \lambda_{t}$. The first order conditions for financial intermediaries are given by:

$$
\begin{align*}
\eta_{t}^{k} & =\lambda_{k}\left(\frac{\mu_{t}}{1+\mu_{t}}\right)+\frac{\chi_{t}}{1+\mu_{t}}-\theta_{t}^{k}\left(\frac{\psi_{t}^{c b}}{1+\mu_{t}}\right)  \tag{191}\\
\eta_{t}^{b} & =\lambda_{b}\left(\frac{\mu_{t}}{1+\mu_{t}}\right)+\frac{\chi_{t}}{1+\mu_{t}}-\theta_{t}^{b}\left(\frac{\psi_{t}^{c b}}{1+\mu_{t}}\right)  \tag{192}\\
\eta_{t}^{R} & =\frac{\chi_{t}}{1+\mu_{t}}-\left(\frac{\psi_{t}^{R}}{1+\mu_{t}}\right)  \tag{193}\\
\eta_{t} & =\frac{\chi_{t}}{1+\mu_{t}}-\vartheta_{t}\left(\frac{\psi_{t}^{R}}{1+\mu_{t}}\right),  \tag{194}\\
\eta_{t}^{c b} & =\frac{\chi_{t}}{1+\mu_{t}}-\frac{\psi_{t}^{c b}}{1+\mu_{t}}  \tag{195}\\
\eta_{t}^{k} & =E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]\left(1+r_{t+1}^{k}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)\right\},  \tag{196}\\
\eta_{t}^{b} & =E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]\left(1+r_{t+1}^{b}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)\right\},  \tag{197}\\
\eta_{t}^{R} & =E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]\left(1+r_{t+1}^{R}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)\right\},  \tag{198}\\
\eta_{t} & =E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]\left(1+r_{t+1}^{d}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)\right\},  \tag{199}\\
\eta_{t}^{c b} & =E_{t}\left\{\beta \Lambda_{t, t+1}\left[(1-\sigma)+\sigma \chi_{t+1}\right]\left(1+r_{t+1}^{c b}+\tau_{t+1}^{n}-\tilde{\tau}_{t+1}^{n}\right)\right\},  \tag{200}\\
\chi_{t} n_{t} & =\lambda_{k} q_{t}^{k} s_{t}^{k, p}+\lambda_{b} q_{t}^{b} s_{t}^{b, p},  \tag{201}\\
n_{t} & =\sigma\left[\left(1+r_{t}^{k}\right) q_{t-1}^{k} s_{t-1}^{k, p}+\left(1+r_{t}^{b}\right) q_{t-1}^{b} s_{t-1}^{b, p}+\left(1+r_{t}^{R}\right) m_{t-1}^{R}\right. \\
& \left.-\left(1+r_{t}^{d}\right) d_{t-1}-\left(1+r_{t}^{c b}\right) d_{t-1}^{c b}\right]+\chi_{b} n_{t-1}+n_{t}^{g}-\tilde{n}_{t}^{g},  \tag{202}\\
\psi_{t}^{R} & =0,  \tag{203}\\
p_{t} & =q_{t}^{k} s_{t}^{k, p}+q_{t}^{b} s_{t}^{b, p}+m_{t}^{R},  \tag{204}\\
p_{t} & =n_{t}+d_{t}+d_{t}^{c b},  \tag{205}\\
\omega_{t}^{k} & =q_{t}^{k} s_{t}^{k, p} / p_{t},  \tag{206}\\
p_{t} & =\phi_{t} n_{t},  \tag{207}\\
d_{t}^{c b} & =\theta_{t}^{k} q_{t}^{k} s_{t}^{k, p}+\theta_{t}^{b} q_{t}^{b} s_{t}^{b, p}, \tag{208}
\end{align*}
$$

where I have taken $\kappa_{t}^{k}=q_{t}^{k}$ and $\kappa_{t}^{b}=q_{t}^{b}$, and $\lambda_{R}=0$. The first order conditions for price setting are given by:

$$
\begin{align*}
\pi_{t}^{*} & =\left(\frac{\epsilon_{t}^{p}}{\epsilon_{t}^{p}-1}\right) \frac{\Xi_{1, t}}{\Xi_{2, t}},  \tag{209}\\
\pi_{t}^{a d j} & =\pi_{t-1}^{\gamma_{P}},  \tag{210}\\
\Xi_{1, t} & =\lambda_{t} m_{t} y_{t}+\beta \psi_{p} E_{t}\left[\left(\frac{\pi_{t+1}}{\pi_{t+1}^{a d j}}\right)^{\epsilon_{t}^{p}} \Xi_{1, t+1}\right]  \tag{211}\\
\Xi_{1, t} & =\lambda_{t} y_{t}+\beta \psi_{p} E_{t}\left[\left(\frac{\pi_{t+1}}{\pi_{t+1}^{a d j}}\right)^{\epsilon_{t}^{p}-1} \Xi_{2, t+1}\right]  \tag{212}\\
1 & =\left(1-\psi_{p}\right)\left(\pi_{t}^{*}\right)^{1-\epsilon_{t}^{p}}+\psi_{p}\left(\frac{\pi_{t}}{\pi_{t}^{a d j}}\right)^{\epsilon_{t}^{p}-1}  \tag{213}\\
\mathcal{D}_{t}^{p} & =\left(1-\psi_{p}\right)\left(\pi_{t}^{*}\right)^{-\epsilon_{t}^{p}}+\psi_{p}\left(\frac{\pi_{t}}{\pi_{t}^{a d j}}\right)^{\epsilon_{t}^{p}} \mathcal{D}_{t-1}^{p} . \tag{214}
\end{align*}
$$

The first order conditions for wage setting are given by:

$$
\begin{align*}
\pi_{t}^{w} & =\left(\frac{w_{t}}{w_{t-1}}\right) \pi_{t},  \tag{215}\\
\left(\omega_{t}^{*}\right)^{1+\epsilon_{t}^{w} \varphi} & =\chi\left(\frac{\epsilon_{t}^{w}}{\epsilon_{t}^{w}-1}\right) \frac{\Xi_{1, t}^{w}}{\Xi_{2, t}^{w}},  \tag{216}\\
\omega_{t}^{a d j} & =\left(\pi_{t-1}^{w}\right)^{\gamma_{W}},  \tag{217}\\
\Xi_{1, t}^{w} & =h_{t}^{1+\varphi}+\beta \psi_{w} E_{t}\left[\left(\frac{\pi_{t+1}^{w}}{\omega_{t+1}^{a d j}}\right)^{\epsilon_{t}^{w}(1+\varphi)} \Xi_{1, t+1}^{w}\right],  \tag{218}\\
\Xi_{2, t}^{w} & =\lambda_{t} w_{t} h_{t}+\beta \psi_{w} E_{t}\left[\left(\frac{\pi_{t+1}^{w}}{\left.\left.\omega_{t+1}^{\text {adj }}\right)^{\epsilon_{t}^{w}-1} \Xi_{2, t+1}^{w}\right],}\right.\right.  \tag{219}\\
1 & \left.=\left(1-\psi_{w}\right)\left(\omega_{t}^{*}\right)^{1-\epsilon_{t}^{w}}+\psi_{w}\left(\frac{\pi_{t}^{w}}{\omega_{t}^{\text {adj }}}\right)^{\epsilon_{t}^{w}-1}\right]  \tag{220}\\
\mathcal{D}_{t}^{w} & =\left(1-\psi_{w}\right)\left(\omega_{t}^{*}\right)^{-\epsilon_{t}^{w}}+\psi_{w}\left(\frac{\pi_{t}^{w}}{\omega_{t}^{a d j}}\right)^{\epsilon_{t}^{w}} \mathcal{D}_{t-1}^{w} . \tag{221}
\end{align*}
$$

Production equations are given by:

$$
\begin{align*}
m_{t} & =\alpha^{-\alpha}(1-\alpha)^{\alpha-1} z_{t}^{-1}\left(w_{t}^{1-\alpha}\left[q_{t-1}^{k}\left(1+r_{t}^{k}\right) \xi_{t}^{-1}-q_{t}^{k}(1-\delta)\right]^{\alpha}\right)  \tag{222}\\
\frac{k_{t-1}}{h_{t}} & =\left(\frac{\alpha}{1-\alpha}\right) \frac{w_{t}}{\left[q_{t-1}^{k}\left(1+r_{t}^{k}\right)-q_{t}^{k}(1-\delta) \xi_{t}\right]},  \tag{223}\\
k_{t} & =(1-\delta) \xi_{t} k_{t-1}+\left[1-\frac{\gamma}{2}\left(\frac{i_{t}}{i_{t-1}}-1\right)^{2}\right] \epsilon_{t}^{i} i_{t}  \tag{224}\\
\frac{1}{q_{t}^{k}} & =\left[1-\frac{\gamma}{2}\left(\frac{i_{t}}{i_{t-1}}-1\right)^{2}\right] \epsilon_{t}^{i}-\gamma\left(\frac{i_{t}}{i_{t-1}}-1\right)\left(\frac{i_{t}}{i_{t-1}}\right) \epsilon_{t}^{i} \\
& +E_{t}\left[\beta \Lambda_{t, t+1} \frac{q_{t+1}^{k}}{q_{t}^{k}}\left(\frac{i_{t+1}}{i_{t}}\right)^{2} \gamma\left(\frac{i_{t+1}}{i_{t}}-1\right) \epsilon_{t+1}^{i}\right]  \tag{225}\\
\mathcal{D}_{t}^{p} y_{t} & =z_{t}\left(\xi_{t} k_{t-1}\right)^{\alpha} h_{t}^{1-\alpha} . \tag{226}
\end{align*}
$$

The first order conditions for the fiscal authority are given by:

$$
\begin{align*}
q_{t}^{b} b_{t}+\tau_{t}+\Delta_{t}^{c b}+\tilde{n}_{t}^{g} & =g_{t}+n_{t}^{g}+\left(1+r_{t}^{b}\right) q_{t-1}^{b} b_{t-1},  \tag{227}\\
\log \left(g_{t} / \bar{g}\right) & =\rho_{g} \log \left(g_{t-1} / \bar{g}\right)+\varepsilon_{g, t},  \tag{228}\\
\tau_{t} & =\bar{\tau}+\kappa_{b}\left(b_{t-1}-\bar{b}\right)+\kappa_{n} n_{t}^{g},  \tag{229}\\
n_{t}^{g} & =\tau_{t}^{n} n_{t-1},  \tag{230}\\
\tau_{t}^{n} & =\zeta \varepsilon_{\xi, t-l},  \tag{231}\\
\tilde{n}_{t}^{g} & =\vartheta n_{t-e}^{g},  \tag{232}\\
\tilde{\tau}_{t}^{n} & =n_{t}^{g} / n_{t-1} . \tag{233}
\end{align*}
$$

The first order conditions for the central bank are given by:

$$
\begin{align*}
d_{t}^{c b} & =n_{t}^{c b}+m_{t}^{R}  \tag{234}\\
n_{t}^{c b *} & =\left(1+r_{t}^{c b}\right) d_{t-1}^{c b}-\left(1+r_{t}^{r}\right) m_{t-1}^{R}  \tag{235}\\
\Delta_{t}^{c b} & =\delta_{t}^{c b} n_{t}^{c b *}  \tag{236}\\
n_{t}^{c b} & =\left(1-\delta_{t}^{c b}\right) n_{t}^{c b *}  \tag{237}\\
\delta_{t}^{c b} & =\bar{\delta}_{c b},  \tag{238}\\
r_{t}^{n, r *} & =\bar{r}_{n, r}+\kappa_{\pi}\left(\pi_{t}-\bar{\pi}\right)+\kappa \log \left(y_{t} / y_{t-1}\right),  \tag{239}\\
r_{t}^{n, r} & =\left(1-\rho_{r}\right) r_{t}^{n, r *}+\rho_{r} r_{t-1}^{n, r}+\varepsilon_{r, t},  \tag{240}\\
r_{t}^{n, c b} & =r_{t}^{n, r}-\Gamma_{t}^{c b}  \tag{241}\\
\Gamma_{t}^{c b} & =\bar{\Gamma}_{c b}+\varkappa_{c b}\left(c b_{t}-\overline{c b}\right)+\varkappa_{\xi}\left(\xi_{t}-\bar{\xi}\right),  \tag{242}\\
\theta_{t}^{k} & =\bar{\theta}_{k},  \tag{243}\\
\theta_{t}^{b} & =\bar{\theta}_{b},  \tag{244}\\
\vartheta_{t} & =\bar{\vartheta}, \tag{245}
\end{align*}
$$

The relation to the nominal return and real return on assets are given by:

$$
\begin{align*}
1+r_{t}^{b} & =\frac{x_{c}+(1-\rho) q_{t}^{b}}{\pi_{t} q_{t-1}^{b}}  \tag{246}\\
1+r_{t}^{R} & =\frac{1+r_{t-1}^{n, r}}{\pi_{t}}  \tag{247}\\
1+r_{t}^{c b} & =\frac{1+r_{t-1}^{n, c b}}{\pi_{t}}  \tag{248}\\
1+r_{t}^{d} & =\frac{1+r_{t-1}^{n}}{\pi_{t}} \tag{249}
\end{align*}
$$

Market clearing conditions are given by:

$$
\begin{align*}
k_{t} & =s_{t}^{k, p}+s_{t}^{k, h}  \tag{250}\\
b_{t} & =s_{t}^{b, p}+s_{t}^{b, h}  \tag{251}\\
y_{t} & =c_{t}+i_{t}+g_{t}+\frac{1}{2} \kappa_{s_{k, h}}\left(s_{t}^{k, h}-\hat{s}_{k, h}\right)^{2}+\frac{1}{2} \kappa_{s_{b, h}}\left(s_{t}^{b, h}-\hat{s}_{b, h}\right)^{2} \tag{252}
\end{align*}
$$

And finally, exogenous processes are given by:

$$
\begin{align*}
\log \left(z_{t}\right) & =\rho_{z} \log \left(z_{t-1}\right)+\varepsilon_{z, t}  \tag{253}\\
\log \left(\xi_{t}\right) & =\rho_{a} \log \left(\xi_{t-1}\right)+\varepsilon_{\xi, t}  \tag{254}\\
\log \left(\epsilon_{t}^{c}\right) & =\rho_{c} \log \left(\epsilon_{t-1}^{c}\right)+\varepsilon_{c, t}  \tag{255}\\
\log \left(\epsilon_{t}^{i}\right) & =\rho_{i} \log \left(\epsilon_{t-1}^{i}\right)+\varepsilon_{i, t}  \tag{256}\\
\log \left(\frac{\epsilon_{t}^{p}}{\bar{\epsilon}_{p}}\right) & =\rho_{p} \log \left(\frac{\epsilon_{t-1}^{p}}{\bar{\epsilon}_{p}}\right)+\varepsilon_{p, t},  \tag{257}\\
\log \left(\frac{\epsilon_{t}^{w}}{\bar{\epsilon}_{w}}\right) & =\rho_{w} \log \left(\frac{\epsilon_{t-1}^{w}}{\bar{\epsilon}_{w}}\right)+\varepsilon_{w, t} . \tag{258}
\end{align*}
$$

## D. 2 Equilibrium Conditions

Let $c_{t-1}, s_{t-1}^{k, h}, s_{t-1}^{b, h}, d_{t-1}, s_{t-1}^{k, p}, s_{t-1}^{b, p}, m_{t-1}^{R}, n_{t-1}, d_{t-1}^{c b}, k_{t-1}, i_{t-1}, b_{t-1}, y_{t-1}, r_{t-1}^{n}, r_{t-1}^{n, r}, r_{t-1}^{n, c b}, \pi_{t-1}, \pi_{t-1}^{w}$ be the endogenous state-variables, while $z_{t}, \xi_{t}, \epsilon_{t}^{c}, \epsilon_{t}^{i}, \epsilon_{t}^{p}, \epsilon_{t}^{w}$ be the exogenous state-variables. A recursive competitive equilibrium is a sequence of quantities and prices $c_{t}, \lambda_{t}, h_{t}, s_{t}^{k, h}, s_{t}^{b, h}$, $\eta_{t}^{k}, \eta_{t}^{b}, \eta_{t}^{R}, \eta_{t}, \eta_{t}^{c b}, \chi_{t}, \mu_{t}, \psi_{t}^{R}, \psi_{t}^{c b}, \phi_{t}, n_{t,} s_{t}^{k, p}, s_{t}^{b, p}, m_{t}^{R}, p_{t}, d_{t}, d_{t}^{c b}, \omega_{t}^{k}, q_{t}^{k}, q_{t}^{b}, r_{t}^{k}, r_{t}^{b}, r_{t}^{R}, r_{t}^{d}, r_{t}^{c b}$, $w_{t}, m_{t}, \pi_{t}, \pi_{t}^{*}, \pi_{t}^{a d j}, \Xi_{1, t}, \Xi_{2, t}, \mathcal{D}_{t}^{p}, \pi_{t}^{w}, \omega_{t}^{*}, \omega_{t}^{\text {adj }}, \Xi_{1, t}^{w}, \Xi_{2, t}^{w}, \mathcal{D}_{t}^{w}, i_{t}, k_{t}, y_{t}$, $b_{t}, g_{t}, n_{t}^{g}, \tilde{n}_{t}^{g}, \tau_{t}, \tau_{t}^{n}, \tilde{\tau}_{t}^{n}$, and $r_{t}^{n}, r_{t}^{n, r}, r_{t}^{n, r *}, r_{t}^{n, c b}, \Gamma_{t}^{c b}, \Delta_{t}^{c b}, n_{t}^{c b}, n_{t}^{c b *}, \delta_{t}^{c b}, \theta_{t}^{k}, \theta_{t}^{b}, \vartheta_{t}$, and exogenous shocks $z_{t}, \xi_{t}, \epsilon_{t}^{c}, \epsilon_{t}^{i}, \epsilon_{t}^{p}, \epsilon_{t}^{w}$ such that:
(i) Households optimize taking prices as given: (187) - 190, together with the wage setting equations 215 - 221.
(ii) Financial intermediaries optimize taking prices as given: 191 - 208.
(iii) Capital producers optimize taking prices as given: 224) - 225 .
(iv) Intermediate goods producers optimize taking prices as given: 222- 223
(v) Retail goods producers that are allowed to change prices optimize taking input prices $m_{t}$ as given: 209 - 214.
(vi) Final goods producers optimize taking prices as given: 226 .
(vii) Asset markets clear: 250 - 251.
(viii) The goods market clears: 252 .
(ix) The fiscal variables evolve according to: 227) - 233.
(x) The monetary variables evolve according to: (??) - 245, and 246-249).
(xi) Exogenous processes evolve according to: 253) and 258.

## E Model variants

In this Appendix I describe two model variants to check that my results in the main text are robust under alternative specifications. The first feature for which I check is sovereign default risk, while the second robustness check consists of checking whether my results continue to hold in a small open economy that is a member of a currency union.

## E. 1 Introducing sovereign default risk

I introduce sovereign default risk by following Corsetti et al. (2013) and Schabert and van Wijnbergen (2014). The result is a setup that is very similar to that in van der Kwaak and van Wijnbergen (2017).

Households and financial intermediaries determine in period $t-1$ how many government bonds to buy. Just as before, government bonds pay a gross return $1+r_{t}^{b}$ (equation 10p) at the beginning of period $t$ in case the government does not default. However, with probability $p_{t}^{\text {def }}$ the fiscal authority defaults at the beginning of period $t$, and imposes a haircut $\vartheta$ on both the cash flow $x_{c}$, as well as on the remaining stock of outstanding bonds. This default is caused by the fact that the level of taxes that needs to be raised according to the tax rule exceeds a stochastic maximum level of taxation (Corsetti et al., 2013; Schabert and van Wijnbergen, 2014). So in that case the return is equal to $(1-\vartheta)\left(1+r_{t}^{b}\right)$. As a result, the expected return on government bonds $1+r_{t}^{b *}$ at the beginning of period $t$ (before the default decision is taken) is given by:

$$
\begin{equation*}
1+r_{t}^{b *}=\left(1-p_{t}^{d e f}\right)\left(1+r_{t}^{b}\right)+p_{t}^{d e f}(1-\vartheta)\left(1+r_{t}^{b}\right)=\left(1-p_{t}^{\text {def }} \vartheta\right)\left(1+r_{t}^{b}\right) \tag{259}
\end{equation*}
$$

The probability of default $p_{t}^{\text {def }}$ depends on the stock of government debt $b_{t}$ using a generalized beta-distribution as in Corsetti et al. (2013):

$$
\begin{equation*}
p_{t}^{d e f}=F_{\beta}\left(\frac{b_{t}}{4 \bar{y}} \frac{1}{\bar{b}_{\max }} ; \alpha_{b}, \beta_{b}\right) \tag{260}
\end{equation*}
$$

where $\alpha_{b}, \beta_{b}$ and $\bar{b}_{\text {max }}$ are parameters of the beta-dsitribution 29
In case of default the government does not have to refinance outstanding liabilities, and thus saves an amount $\left(1+r_{t}^{b}\right) q_{t-1}^{b} b_{t-1}$. Just as in Corsetti et al. (2013) and Schabert and van Wijnbergen (2014), however, I assume that these savings are effectively transferred to households by reducing their lump sum taxes from $\tau_{t}$ in the case of no default (equation (9)) to $\tilde{\tau}_{t}$ :

$$
\begin{equation*}
\tilde{\tau}_{t}=\tau_{t}-\left(1+r_{t}^{b}\right) q_{t-1}^{b} b_{t-1} \tag{261}
\end{equation*}
$$

As a result, the government budget constraint ex post default is given by the following expression

[^21](compare with expression(9)):
\[

$$
\begin{equation*}
q_{t}^{b} b_{t}+\tilde{\tau}_{t}+\Delta_{t}^{c b}=g_{t} \tag{262}
\end{equation*}
$$

\]

Substitution of the ex post default level of lump sum taxes 261 into the ex post default government budget constraint 262 shows that the government budget constraint is not directly affected by the default, as the budget constraint is identical to the budget constraint (9) in the main text:

$$
\begin{equation*}
q_{t}^{b} b_{t}+\tau_{t}+\Delta_{t}^{c b}=g_{t}+\left(1+r_{t}^{b}\right) q_{t-1}^{b} b_{t-1} \tag{263}
\end{equation*}
$$

Although the default will not directly affect the government budget constraint, there will be an indirect effect: households and intermediaries do not anticipate that the default gains will be transferred to households in the form of lower lump sum taxes. In addition, households' default gains will be used to compensate financial intermediaries under their ownership for the default losses incurred by these intermediaries. As a result of the fact that both households and intermediaries do not anticipate the lower lump sum taxes and the recapitalization, respectively, their first order conditions for bond holdings will feature $r_{t}^{b *}$ rather than $r_{t}^{b}$. As a result, the probability that the government might default will be priced in by households and intermediaries, and affect the equilibrium bond price and expected return. However, intermediaries' aggregate law of motion still features $r_{t}^{b}$, as they do not incur any default losses ex post.

Finally, I adjust the rule for lump sum taxes $\tau_{t}$, and include a feedback from output to taxes:

$$
\begin{equation*}
\tau_{t}=\bar{\tau}+\kappa_{b}\left(b_{t-1}-\bar{b}\right)+\kappa_{n} n_{t}^{g}+\kappa_{\tau, y}\left(y_{t}-\bar{y}\right) \tag{264}
\end{equation*}
$$

I introduce the output term for two reasons. First, such a feedback captures the fact that tax revenues deteriorate significantly in a financial crisis. As my model does not feature distortionary taxes through which such a reduction would automatically occur, introducing a feedback on the level of lump sum taxes achieves the same goal.

Second, sovereign default risk will only increase in my model when the level of debt $b_{t}$ increases. In my model, however, there is only a small increase in debt during a financial crisis, as the government does not engage in countercyclical fiscal policy. Reducing the level of lump sum taxes then forces the government to issue more debt, which then subsequently increases sovereign default risk.

To sum up: the introduction of sovereign default risk changes the first order conditions for households' and intermediaries' bond holdings, in which $r_{t}^{b}$ is replaced by $r_{t}^{b *}$. In addition, I see the introduction of two new variables, namely $r_{t}^{b *}$ and $p_{t}^{\text {def }}$. Consequently, I introduce two new equations, namely 259) and 260.

## E. 2 Small open economy member of a currency union

In this subsection I adjust my closed-economy setup to a small open economy member of a currency union by largely following ?. Specifically, the following adjustments are made. First, the nominal interest rate is permanently set equal to the steady state nominal interest rate. Second, the Italian consumption and investment goods are constructed by combining domestically produced goods and foreign produced goods with a constant elasticity of substitution between them. Third, Italy is a small economy relative to the rest of the Eurozone. Fourth, Italian banks only have domestic deposits. Fifth, there is a net foreign asset that domestic households can purchase. Households are borrowing from the rest of the Eurozone when net foreign asset holdings are negative. Sixth, the government only purchases domestically produced goods. Below I discuss the model parts that are different from the closed-economy setup.

## E.2.1 Households

The households' maximization objective is the same as in the closed-economy version. However, their budget constraint is extended by providing households the opportunity to hold foreign assets $F_{t}$ which pay a nominal interest rate $r_{t}^{n, f}$ in period $t+1$. However, there quadratic adjustment costs in the deviation of households' holdings of foreign assets (in terms of the consumption price index $P_{t}$ ) from a target level $\bar{f}$. Households' nominal budget constraint is therefore given by:

$$
P_{t} C_{t}+D_{t}+F_{t}+\frac{1}{2} \kappa_{f}\left(\frac{F_{t}}{P_{t}}-\bar{f}\right)^{2}+P_{t} \tau_{t}=W_{t} h_{t}+\left(1+r_{t-1}^{n}\right) D_{t-1}+\left(1+r_{t-1}^{n, f}\right) F_{t-1}+\Pi_{t} .
$$

This budget constraint can be divided by the consumer price index (CPI) $P_{t}$ to obtain the budget constraint in terms of the CPI:

$$
\begin{equation*}
c_{t}+d_{t}+f_{t}+\frac{1}{2} \kappa_{f}\left(f_{t}-\bar{f}\right)^{2}+\tau_{t}=w_{t} h_{t}+\left(1+r_{t}^{d}\right) d_{t-1}+\left(1+r_{t}^{f}\right) f_{t-1}+\mathcal{P}_{t} \tag{265}
\end{equation*}
$$

where $x_{t} \equiv X_{t} / P_{t}$. The real interest rate $r_{t}^{f}$ is given by:

$$
\begin{equation*}
1+r_{t}^{f}=\frac{1+r_{t-1}^{n, f}}{\pi_{t}} \tag{266}
\end{equation*}
$$

The resulting first order condition for net foreign assets is then given by:

$$
\begin{equation*}
f_{t}: \quad E_{t}\left[\beta \frac{\lambda_{t+1}}{\lambda_{t}}\left(\frac{1+r_{t+1}^{f}}{1+\kappa_{f}\left(f_{t}-\bar{f}\right)}\right)\right]=1 \tag{267}
\end{equation*}
$$

whereas all the other first order conditions are identical to the one in the closed economy.
Households' consumption bundle $c_{t}$ is an aggregate of domestically produced goods $c_{t}^{h}$ and
foreign goods $c_{t}^{f}$ :

$$
\begin{equation*}
c_{t}=\left[\left(1-v_{c}\right)^{\frac{1}{\eta_{c}}}\left(c_{t}^{h}\right)^{\frac{\eta_{c}-1}{\eta_{c}}}+v_{c}^{\frac{1}{\eta_{c}}}\left(c_{t}^{f}\right)^{\frac{\eta_{c}-1}{\eta_{c}}}\right]^{\frac{\eta_{c}}{\eta_{c}-1}} \tag{268}
\end{equation*}
$$

Households purchase $c_{t}^{h}$ and $c_{t}^{f}$ at a nominal price $P_{t}^{h}$ and $P_{t}^{f}$ respectively. They choose $c_{t}^{h}$ and $c_{t}^{f}$ in such a way that their expenditures $P_{t}^{h} c_{t}^{h}+P_{t}^{f} c_{t}^{f}$ to obtain a consumption level $c_{t}$ are minimized. This results in the following first order conditions:

$$
\begin{align*}
& c_{t}^{h}: \quad c_{t}^{h}=\left(1-v_{c}\right)\left(\frac{P_{t}^{h}}{P_{t}}\right)^{-\eta_{c}} c_{t}  \tag{269}\\
& c_{t}^{f} \quad: \quad c_{t}^{f}=v_{c}\left(\frac{P_{t}^{f}}{P_{t}}\right)^{-\eta_{c}} c_{t} \tag{270}
\end{align*}
$$

The domestic Consumer Price Index (CPI) $P_{t}$ is obtained through substitution of 269) and 270 into the aggregate domestic consumption bundle 268):

$$
\begin{equation*}
P_{t}^{1-\eta_{c}}=\left(1-v_{c}\right)\left(P_{t}^{h}\right)^{1-\eta_{c}}+v_{c}\left(P_{t}^{f}\right)^{1-\eta_{c}} \tag{271}
\end{equation*}
$$

Division of equation (271) by $P_{t}^{1-\eta_{c}}$ results in the following expression:

$$
\begin{equation*}
1=\left(1-v_{c}\right)\left(p_{t}^{h}\right)^{1-\eta_{c}}+v_{c}\left(p_{t}^{f}\right)^{1-\eta_{c}} \tag{272}
\end{equation*}
$$

with the relative prices $p_{t}^{h} \equiv P_{t}^{h} / P_{t}$ and $p_{t}^{f} \equiv P_{t}^{f} / P_{t}$ of domestically and foreign produced goods, respectively, expressed in terms of the consumper price index.

## E.2.2 Importers

Importers purchase consumption bundles $c_{t}^{m}$ from every country $m \in[0,1]$ to construct an aggregate $c_{t}^{f}$ of imported goods:

$$
\begin{equation*}
c_{t}^{f}=\left[\int_{0}^{1}\left(c_{t}^{m}\right)^{\frac{\gamma-1}{\gamma}} d m\right]^{\frac{\gamma}{\gamma-1}}, \tag{273}
\end{equation*}
$$

where $c_{t}^{m}$ is the final good that is produced in country $m$. Importers take the price $P_{t}^{f}$ at which the aggregate imported good $c_{t}^{f}$ is sold in the domestic economy as given, as well as the price $P_{t}^{m}$ of good $c_{t}^{m}$ from country $m$. Importers decide how many consumption goods $c_{t}^{m}$ to purchase from each country $m \in[0,1]$ with the goal of maximizing profits $P_{t}^{f} c_{t}^{f}-\int_{0}^{1} P_{t}^{m} c_{t}^{m} d m$ subject to the bundling technology (273). The first order condition for the amount of goods $c_{t}^{m}(h)$ purchased from country $m$ by domestic importers is given by:

$$
\begin{equation*}
c_{t}^{m}(h)=\left(\frac{P_{t}^{m}}{P_{t}^{f}}\right)^{-\gamma} c_{t}^{f} \tag{274}
\end{equation*}
$$

By argument of symmetry, the demand $c_{t}^{h}(m)$ from country $m \in[0,1]$ for goods from the domestic economy $h$ is then given by:

$$
\begin{equation*}
c_{t}^{h}(m)=\left(\frac{P_{t}^{h}}{P_{t}^{f *}(m)}\right)^{-\gamma_{c}} c_{t}^{f *}(m) \tag{275}
\end{equation*}
$$

where $c_{t}^{f *}(m)$ denotes the index of final goods imported from other countries in the monetary union to country $m$ for consumption, and $P_{t}^{f *}(m)$ the price index of this imported consumption good. Total foreign demand $c_{t}^{h, *}$ for domestically produced goods can be found by aggregating over all countries $m \in[0,1]$ :

$$
\begin{equation*}
c_{t}^{h, *}=\int_{0}^{1} c_{t}^{h}(m) d m=\int_{0}^{1}\left(\frac{P_{t}^{h}}{P_{t}^{f *}(m)}\right)^{-\gamma_{c}^{*}} c_{t}^{f *}(m) d m \tag{276}
\end{equation*}
$$

The demand for foreign goods $c_{t}^{f *}(m)$ in country $m$ is qualitatively the same as for the domestic economy:

$$
\begin{equation*}
c_{t}^{f *}(m)=v_{c}^{*}\left(\frac{P_{t}^{f *}(m)}{P_{t}^{*}(m)}\right)^{-\eta_{c}^{*}} c_{t}^{*}(m) \tag{277}
\end{equation*}
$$

where $P_{t}^{*}(m)$ is the consumer price index in country $m$, and $c_{t}^{*}(m)$ the aggregated consumption good obtained from combining domestic and foreign goods. I assume that all the other countries of the monetary union are the same, and face the same shocks with correlation one. Therefore, the consumer price index $P_{t}^{*}(m)$ will be the same in every country $m$ other than the domestic economy: $P_{t}^{*}(m)=P_{t}^{*}$. Since the domestic economy is a small member of the monetary union, the price $P_{t}^{h}$ at which domestically produced goods are sold to other countries in the monetary union will have a negligible influence on foreign consumer price indices. Therefore, I can write $P_{t}^{f *}(m)=P_{t}^{*}(m)=P_{t}^{*}$, and equation (277) can be rewritten as:

$$
\begin{equation*}
c_{t}^{f *}(m)=v_{c}^{*} c_{t}^{*}(m) \tag{278}
\end{equation*}
$$

Substitution into equation (276), together with $P_{t}^{f *}(m)=P_{t}^{*}$ gives:

$$
\begin{equation*}
c_{t}^{h, *}=\int_{0}^{1} c_{t}^{h}(m) d m=\int_{0}^{1}\left(\frac{P_{t}^{h}}{P_{t}^{*}}\right)^{-\gamma_{c}^{*}} v_{c}^{*} c_{t}^{*}(m) d m=v_{c}^{*}\left(\frac{P_{t}^{h}}{P_{t}^{*}}\right)^{-\gamma_{c}^{*}} c_{t}^{*} \tag{279}
\end{equation*}
$$

Now I substitute $P_{t}^{*}=P_{t}^{f}$ since the domestic economy is a small member of the monetary union. As a result, the foreign consumption demand for domestically produced goods $c_{t}^{h, *}$ can be written as:

$$
\begin{equation*}
c_{t}^{h, *}=v_{c}^{*}\left(\frac{P_{t}^{h}}{P_{t}^{f}}\right)^{-\gamma_{c}^{*}} c_{t}^{*}=v_{c}^{*} S_{t}^{\gamma_{c}^{*}} c_{t}^{*}, \tag{280}
\end{equation*}
$$

where $S_{t}$ denotes the terms of trade, which is defined as $S_{t} \equiv P_{t}^{f} / P_{t}^{h}$.

## E.2.3 Investment

Aggregate domestic investment consists of an investment bundle $i_{t}$ which is an aggregate of domestically produced goods $i_{t}^{h}$ and foreign goods $i_{t}^{f}$ :

$$
\begin{equation*}
i_{t}=\left[\left(1-v_{i}\right)^{\frac{1}{\eta_{i}}}\left(i_{t}^{h}\right)^{\frac{\eta_{i}-1}{\eta_{i}}}+v_{i}^{\frac{1}{\eta_{i}}}\left(i_{t}^{f}\right)^{\frac{\eta_{i}-1}{\eta_{i}}}\right]^{\frac{\eta_{i}}{\eta_{i}-1}} \tag{281}
\end{equation*}
$$

Capital goods producers minimize expenditures $P_{t}^{h} i_{t}^{h}+P_{t}^{f} i_{t}^{f}$ on domestic and foreign goods while taking the aggregation technology 281 into account, resulting in the following first order conditions:

$$
\begin{align*}
& i_{t}^{h}: \quad i_{t}^{h}=\left(1-v_{i}\right)\left(\frac{P_{t}^{h}}{P_{t}^{i}}\right)^{-\eta_{i}} i_{t},  \tag{282}\\
& i_{t}^{f}: \quad i_{t}^{f}=v_{i}\left(\frac{P_{t}^{f}}{P_{t}^{i}}\right)^{-\eta_{i}} i_{t} \tag{283}
\end{align*}
$$

where $P_{t}^{i}$ is the price of the domestic Investment Price Index (IPI), which can be found by substitution of 282 and 283) into the aggregate domestic investment bundle 281):

$$
\begin{equation*}
\left(P_{t}^{i}\right)^{1-\eta_{i}}=\left(1-v_{i}\right)\left(P_{t}^{h}\right)^{1-\eta_{i}}+v_{i}\left(P_{t}^{f}\right)^{1-\eta_{i}} \tag{284}
\end{equation*}
$$

Division by $P_{t}^{1-\eta_{i}}$ allows me to write 284 in the following way:

$$
\begin{equation*}
\left(p_{t}^{i}\right)^{1-\eta_{i}}=\left(1-v_{i}\right)\left(p_{t}^{h}\right)^{1-\eta_{i}}+v_{i}\left(p_{t}^{f}\right)^{1-\eta_{i}} \tag{285}
\end{equation*}
$$

with the relative price $p_{t}^{i} \equiv P_{t}^{i} / P_{t}$. The expression for the period $t$ profits of the capital goods producers becomes the following:

$$
\begin{equation*}
\Pi_{t}^{i}=q_{t}^{k} k_{t}-q_{t}^{k}(1-\delta) k_{t-1}-p_{t}^{i} i_{t}=q_{t}^{k}\left[1-\frac{1}{2} \gamma_{k}\left(\frac{i_{t}}{i_{t-1}}-1\right)^{2}\right] i_{t}-p_{t}^{i} i_{t} \tag{286}
\end{equation*}
$$

As a result, the first order condition for investment changes into the following expression:

$$
\frac{p_{t}^{i}}{q_{t}^{k}}=\left[1-\frac{\gamma}{2}\left(\frac{i_{t}}{i_{t-1}}-1\right)^{2}\right] \iota_{t}-\frac{\gamma i_{t}}{i_{t-1}}\left(\frac{i_{t}}{i_{t-1}}-1\right) \iota_{t}+\beta E_{t}\left[\Lambda_{t, t+1} \iota_{t+1} \frac{q_{t+1}^{k}}{q_{t}^{k}}\left(\frac{i_{t+1}}{i_{t}}\right)^{2} \gamma\left(\frac{i_{t+1}}{i_{t}}-1\right)\right](287)
$$

where as before $\iota_{t}$ denotes the investment adjustment costs shock.
Finally, similar to the foreign demand for domestic goods for consumption purposes (equation
(280), I write down the foreign demand for domestic goods for foreign investment:

$$
\begin{equation*}
i_{t}^{h, *}=v_{i}^{*} S_{t}^{\gamma_{i}^{*}} i_{t}^{*} \tag{288}
\end{equation*}
$$

where $i_{t}^{*}$ is aggregate investment in the rest of the monetary union.

## E.2.4 Domestic production

Domestic final goods are now sold at the price $P_{t}^{h}$ instead of the consumer price index $P_{t}$, while domestic retail goods producers sell their retail goods at price $P_{f, t}^{h}$ instead of $P_{f, t}$. Therefore, the demand of domestic final goods producers for retail good $y_{f, t}$ is now given by:

$$
\begin{equation*}
y_{f, t}=\left(\frac{P_{f, t}^{h}}{P_{t}^{h}}\right)^{-\epsilon_{p, t}} y_{t} \tag{289}
\end{equation*}
$$

Next, I replace consumer price inflation $\pi_{t}$ by domestic producer price inflation $\pi_{t}^{h} \equiv P_{t}^{h} / P_{t-1}^{h}$ in the first order conditions for the production sector, which are then given by:

$$
\begin{align*}
\pi_{t}^{*} & =\frac{\epsilon_{t}^{p}}{\epsilon_{t}^{p}-1} \frac{\Xi_{1, t}}{\Xi_{2, t}},  \tag{290}\\
\pi_{t}^{h, a d j} & =\left(\pi_{t-1}^{h}\right)^{\gamma_{p}},  \tag{291}\\
\Xi_{1, t} & =\lambda_{t} m_{t} y_{t}+\beta \psi_{p} E_{t}\left[\left(\frac{\pi_{t+1}^{h}}{\pi_{t+1}^{h, a d j}}\right)^{\epsilon_{t}^{p}} \Xi_{1, t+1}\right],  \tag{292}\\
\Xi_{2, t} & =\lambda_{t} y_{t}+\beta \psi_{p} E_{t}\left[\left(\frac{\pi_{t+1}^{h}}{\pi_{t+1}^{h, a d j}}\right)^{\epsilon_{t}^{p}-1} \Xi_{2, t+1}\right]  \tag{293}\\
1 & =\left(1-\psi_{p}\right)\left(\pi_{t}^{*}\right)^{1-\epsilon_{t}^{p}}+\psi_{p}\left(\frac{\pi_{t}^{h}}{\pi_{t}^{h, a d j}}\right)^{\epsilon_{t}^{p}-1}  \tag{294}\\
\mathcal{D}_{t} & =\left(1-\psi_{p}\right)\left(\pi_{t}^{*}\right)^{-\epsilon_{t}^{p}}+\psi_{p}\left(\frac{\pi_{t}^{h}}{\pi_{t}^{h, a d j}}\right)^{\epsilon_{t}^{p}} \mathcal{D}_{t-1} . \tag{295}
\end{align*}
$$

## E.2.5 Government

The fiscal authority only purchases domestic goods. As a result, the government budget constraint becomes:

$$
\begin{equation*}
q_{t}^{b} b_{t}+\tau_{t}+\Delta_{t}^{c b}=p_{t}^{h} g_{t}+\left(1+r_{t}^{b}\right) q_{t-1}^{b} b_{t-1} \tag{296}
\end{equation*}
$$

with $p_{t}^{h}=P_{t}^{h} / P_{t}$ the relative price of domestically produced goods in terms of the consumer price index.

The nominal interest rate does not respond to domestic economic developments, and is equal
to the steady state interest rate plus an exogenous shock:

$$
\begin{equation*}
r_{t}^{n}=\bar{r}_{n}+\varepsilon_{r, t} \tag{297}
\end{equation*}
$$

## E.2.6 Domestic output \& net international asset position

The market clearing condition for domestic output $y_{t}$ is given by:

$$
\begin{equation*}
y_{t}=c_{t}^{h}+c_{t}^{h, *}+i_{t}^{h}+i_{t}^{h, *}+g_{t} . \tag{298}
\end{equation*}
$$

The nominal trade balance $T_{t}^{b}$ is given by:

$$
\begin{equation*}
T_{t}^{b}=P_{t}^{h}\left(c_{t}^{h, *}+i_{t}^{h, *}\right)-P_{t}^{f}\left(c_{t}^{f}+i_{t}^{f}\right) \tag{299}
\end{equation*}
$$

I get the real trade balance $\tau_{t}^{b}$ through division by the domestic CPI $P_{t}$ :

$$
\begin{equation*}
\tau_{t}^{b}=p_{t}^{h}\left(c_{t}^{h, *}+i_{t}^{h, *}\right)-p_{t}^{f}\left(c_{t}^{f}+i_{t}^{f}\right) \tag{300}
\end{equation*}
$$

This expression can be rewritten in the following way:

$$
c_{t}^{h, *}+i_{t}^{h, *}=\frac{\tau_{t}^{b}+p_{t}^{f}\left(c_{t}^{f}+i_{t}^{f}\right)}{p_{t}^{h}}
$$

Substitution into the equation for domestic output 298) gives the following expression:

$$
\begin{equation*}
p_{t}^{h} y_{t}=p_{t}^{h} c_{t}^{h}+p_{t}^{h} i_{t}^{h}+\tau_{t}^{b}+p_{t}^{f}\left(c_{t}^{f}+i_{t}^{f}\right)+p_{t}^{h} g_{t} \tag{301}
\end{equation*}
$$

I rewrite the above expression into the following relation using the identities $c_{t}=p_{t}^{h} c_{t}^{h}+p_{t}^{f} c_{t}^{f}$ and $p_{t}^{i}=p_{t}^{h} i_{t}^{h}+p_{t}^{f} i_{t}^{f}$ :

$$
\begin{equation*}
p_{t}^{h} y_{t}=c_{t}+p_{t}^{i} i_{t}+p_{t}^{h} g_{t}+\tau_{t}^{b} \tag{302}
\end{equation*}
$$

The nominal current account $C A_{t}$ consists of the nominal trade balance $T_{t}^{b}$ plus the interest payments on the internationally traded asset $F_{t}$ :

$$
\begin{equation*}
C A_{t}=T_{t}^{b}+r_{t-1}^{n, f} F_{t-1} \tag{303}
\end{equation*}
$$

The current account $\mathcal{C}_{t}^{a}$ in terms of the domestic consumer price index is obtained through division of the above expression by $P_{t}$ :

$$
\begin{equation*}
\mathcal{C}_{t}^{a}=\tau_{t}^{b}+\frac{r_{t-1}^{n, f}}{\pi_{t}} f_{t-1} \tag{304}
\end{equation*}
$$

The nominal capital account $C P_{t}$ is equal to the difference between domestic households' current period holdings of the internationally traded asset and their previous period holdings:

$$
\begin{equation*}
C P_{t}=F_{t}-F_{t-1} . \tag{305}
\end{equation*}
$$

I obtain the real capital account $\mathcal{C}_{t}^{p}$ in terms of the domestic consumer price index through division by $P_{t}$ :

$$
\begin{equation*}
\mathcal{C}_{t}^{p}=f_{t}-\frac{f_{t-1}}{\pi_{t}} \tag{306}
\end{equation*}
$$

Next, the current account and the capital account must be equal in equilibrium:

$$
\begin{equation*}
\mathcal{C}_{t}^{a}=\mathcal{C}_{t}^{p} . \tag{307}
\end{equation*}
$$

Finally, the law of motion for the net international asset position of households $f_{t}$ is derived through substitution of (304) and 306) into 307):

$$
\begin{equation*}
f_{t}=\tau_{t}^{b}+\left(\frac{1+r_{t-1}^{n, f}}{\pi_{t}}\right) f_{t-1} \tag{308}
\end{equation*}
$$

## E.2.7 Relations between prices and inflation rates

I define the terms of trade $S_{t}$ as the nominal price of foreign goods over the nominal price of domestically produced goods:

$$
\begin{equation*}
S_{t} \equiv \frac{P_{t}^{f}}{P_{t}^{f}} \tag{309}
\end{equation*}
$$

I define the real exchange rate $\mathcal{Q}_{t}$ as the aggregate nominal foreign price level $P_{t}^{*}$ over the aggregate domestic consumer price index $P_{t}$ :

$$
\begin{equation*}
\mathcal{Q}_{t} \equiv \frac{P_{t}^{*}}{P_{t}} \tag{310}
\end{equation*}
$$

I assume that the price of the foreign produced good is equal to the aggregate foreign price level $P_{t}^{f}=P_{t}^{*}$ because of my assumption that the domestic economy is a small country within the monetary union. As a result, I can write $p_{t}^{f}$ in the following way:

$$
\begin{equation*}
p_{t}^{f} \equiv \frac{P_{t}^{f}}{P_{t}}=\frac{P_{t}^{*}}{P_{t}}=\mathcal{Q}_{t} . \tag{311}
\end{equation*}
$$

Similarly, I can write the relative price $p_{t}^{h}$ in the following way:

$$
\begin{equation*}
p_{t}^{h} \equiv \frac{P_{t}^{h}}{P_{t}}=\frac{P_{t}^{h}}{P_{t}^{f}} \frac{P_{t}^{f}}{P_{t}}=\frac{\mathcal{Q}_{t}}{S_{t}} \tag{312}
\end{equation*}
$$

Next, consider the change in the real exchange rate $\mathcal{Q}_{t}$, which I can write in the following way:

$$
\begin{equation*}
\frac{\mathcal{Q}_{t}}{\mathcal{Q}_{t-1}}=\frac{P_{t}^{*} / P_{t}}{P_{t-1}^{*} / P_{t-1}}=\frac{\pi_{t}^{*}}{\pi_{t}} \tag{313}
\end{equation*}
$$

Finally, I rewrite the expression for the inflation rate $\pi_{t}^{h}$ of the domestic producer price index $P_{t}^{h}$ :

$$
\begin{equation*}
\pi_{t}^{h}=\frac{P_{t}^{h}}{P_{t-1}^{h}}=\frac{P_{t}^{h}}{P_{t}} \cdot \frac{P_{t}}{P_{t-1}} \cdot \frac{P_{t-1}}{P_{t-1}^{h}}=\left(\frac{p_{t}^{h}}{p_{t-1}^{h}}\right) \pi_{t} \tag{314}
\end{equation*}
$$

## F Data sources \& preparation for estimation

In this section I describe the data sources and how I prepare the raw data for estimation of the model. Data were collected from Eurostat, the European Central Bank, and Istat, the Italian statistical agency. For the Bayesian estimation the time series are from 1998Q1-2007Q4, while the time series for the moment estimation run from 2008Q1-2011Q4.

I download gross domestic product, household and NPISH final consumption expenditure, gross fixed capital formation, and final consumption expenditure of general government, all at current prices, from the Eurostat website. These time series represent nominal GDP, consumption, investment, and government spending within my model respectively. In addition, I download gross domestic product in chain-linked volumes (2010) million euros, which represents real GDP. All time series are seasonally and calendar adjusted.

I calculate the GDP-deflator by dividing nominal GDP by real GDP. Next, I divide the time series for nominal consumption, investment, and government spending by the GDP-deflator to obtain a time series for real consumption, investment and government spending. Alternatively, I could have downloaded chain-linked volumes time series for consumption, investment, and government spending. However, output, consumption, investment and government spending have the same price index within my model. As such, I should also have a common price index when converting nominal empirical data to real data, which cannot be achieved by taking chainlinked volumes for each individual time series, see also Pfeifer (2018).

To convert these real time series into per capita time series, I download working population from the website of Istat. Specifically, I download the time series for the total labour force 15 years and more in thousands of persons. To obtain per capita time series for real output, consumption, investment, and government spending, I divide the real time series by the total labour force time series and multiply by 1000 to correct for the fact that the labour force is expressed in thousands of persons while the real time series are measured in (2010) millions of euros.

Inflation, measured as the quarter-to-quarter change in the consumer price index, coincides with the producer price index within my closed-economy model. Therefore, my measure for inflation is the quarter-to-quarter change in the GDP-deflator, rather than the empirical time
series for the Italian consumer price index, which also includes foreign consumption goods.
I take "average number of actual weekly hours of work in main job for all employed persons" as the time series for hours worked. This time series is downloaded from Eurostat.

I employ the time series for the Euribor 3-month money market interest rate as a proxy for the policy rate ("Euro area (changing composition) - Money Market - Euribor 3-month - Historical close, average of observations through period - Euro, provided by Reuters"). This time series is downloaded from the ECB's statistical warehouse. It is measured in percent per annum, so I divide it by 400 to convert into a quarterly interest rate in decimals.

After having constructed time series in the above described way, I take the log of all time series (including inflation and 1 plus the Euribor 3-month), and employ the one-sided HP-filter with smoothing parameter 1600 to obtain the business cycle component of all time series (Pfeifer, 2018). Subsequently I throw away the first four observations as a 'burn-in', and demean the resulting time series. Hence I end up with a time series with 36 observations running from 1999Q1 till 2007Q4, which is then employed in the Bayesian estimation.

Finally, I also obtain time series that are only used for the calibration and the momentmatching exercise. These include time series for the interest rate on loans and the yield on long-term government debt. Both time series are downloaded from the ECB's statistical warehouse. For the interest rate on loans I take the time series "Italy, Annualised agreed rate (AAR) / Narrowly defined effective rate (NDER), Credit and other institutions (MFI except MMFs and central banks) reporting sector - Loans other than revolving loans and overdrafts, convenience and extended credit card debt, Total initial rate fixation, Total amount, New business coverage, Non-Financial corporations (S.11) sector, denominated in Euro" which has the code MIR.M.IT.B.A2A.A.R.A.2240.EUR.N. For the interest rate on long-term government bonds I take the time series "Italy, Long-term interest rate for convergence purposes - Unspecified rate type, Debt security issued, 10 years maturity, New business coverage, denominated in Euro - Unspecified counterpart sector" with code IRS.M.IT.L.L40.CI.0000.EUR.N.Z. Although I explain in Appendix G.2.1 why the maturity of government bonds in my model is only 6 quarters, for which one would like to employ bond yields with a shorter maturity, the above time series is the only one available to the best of my knowledge. Both time series are measured in percent per annum, so I divide by 400 to convert into a quarterly interest rate in decimals.

## G Calibration \& Estimation

I start this section by describing some modifications that I make to the standard New Keynesian model without financial frictions, which is the model I am going to employ for the Bayesian estimation procedure. Next, I describe the calibration of the parameters of this model that I do not estimate, after which I describe how I calibrate the parameters relating to financial intermediaries and the central bank within the full model. Subsequently, I discuss the priors and posteriors of the Bayesian estimation procedure, as well as a robustness check that I perform. Finally, I
describe the moment-matching exercise with which I pin down among others the coefficients in front of households' quadratic adjustment costs for corporate securities and government bonds.

## G. 1 Modifications to the standard New Keynesian model

I start by discussing two modifications to the standard closed economy New Keynesian model without financial frictions. First, in this model there are no financial intermediaries, and therefore there are no reserves on which the central bank can set the nominal interest rate. Therefore, I need another asset on which the central bank can set the nominal interest. Because I want government bond yields to be endogenously determined, just as in the model with financial frictions, I introduce an asset which is in zero net supply and pays the nominal interest rate set by the central bank. This way, the steady state return on this asset will be equal to the steady state return on deposits in the model version with financial intermediaries and central bank lending operations. The second modification consists of introducing a tax on the gross return on corporate securities $\tau_{k}$ and government bonds $\tau_{b}$. I do so to ensure that the pre-tax steady state gross return on corporate securities and government bonds is equal to that in the model version with financial frictions. Therefore, there will be a spread between the gross return on corporate securities and government bonds before taxes on the one hand, and the return on the asset in zero net supply on the other, just as there is a spread in the model version with financial frictions, see also the online Appendix of Kirchner and van Wijnbergen (2016).

At this point, it is important to explain why my model version without financial frictions and central bank lending operations features such a tax rather than the quadratic adjustment costs that households are subject to in the model version with financial frictions. The reason is the following. I saw in Section 3 that the coefficient $\kappa_{s_{b, h}}$ in front of the quadratic adjustment costs for government bonds is a crucial parameter in determining the strength of the collateral effect. And even though the coefficient $\kappa_{s_{k, h}}$ did not feature in the two-period model, it is likely to be important for the strength of the collateral effect as well. As such, it is key for my quantitative exercise to estimate these parameters using empirical data. However, I found in preliminary Bayesian estimations that a model version in which these two parameters were estimated did not converge. The reason for this failure to converge is the fact that households finance the entire stock of government debt, and as such need to be able to hold any amount of government debt in equilibrium. However, when households are subject to adjustment costs, they might not be able to do so when the stock of government debt they need to hold in equilibrium is far away from the reference level $\hat{s}_{b, h}$, as these adjustment costs increase quadratically in the deviation from this reference level.

Therefore, I estimate a model version in which households are subject to a tax on the gross return on corporate securities and government bonds to match the gross returns within the model version with financial intermediaries and central bank lending operations. I subsequently estimate the adjustment costs parameters $\kappa_{s_{b, h}}$ and $\kappa_{s_{k, h}}$ in a moment-matching exercise, see Appendix G.4. I check whether the error I thus introduce into my parameter estimates is large
by employing the point estimates for $\kappa_{s_{k, h}}$ and $\kappa_{s_{b, h}}$ in a Bayesian estimation of the model version with quadratic adjustment costs (instead of the tax on the gross returns), see Appendix G.3.2

Specifically, the introduction of a tax on the gross returns on corporate securities $\tau_{k}$ and government bonds $\tau_{b}$ gives me the following first order conditions for households optimal choices for corporate securities $s_{j, t}^{k, h}$, government bonds $s_{j, t}^{b, h}$, and the asset that is in zero net supply $d_{j, t}$ :

$$
\begin{align*}
& s_{j, t}^{k, h}:  \tag{315}\\
& s_{j, t}^{b, h}:  \tag{316}\\
& d_{j, t}: \\
& d_{t}\left[\beta \Lambda_{t, t+1}\left(1-\tau_{k}\right)\left(1+r_{t, t+1}^{k}\left(1-\tau_{b}\right)\left(1+r_{t+1}^{b}\right)\right]=1,\right. \\
& l_{t}\left[\beta \Lambda_{t, t+1}\left(1+r_{t+1}^{d}\right)\right]=1,
\end{align*}
$$

As the purpose of these taxes is to have a steady state return on corporate securities and government bonds equal to that in the model version with financial frictions, I assume that the proceeds from these taxes are lump sum rebated to households.

Due to the absence of financial intermediaries, households hold the complete stock of corporate securities and government bonds in equilibrium. Therefore, the market clearing conditions for corporate securities, government bonds, and the asset in zero net supply are given by:

$$
\begin{aligned}
k_{t} & =s_{t}^{k, h} \\
b_{t} & =s_{t}^{b, h} \\
d_{t} & =0
\end{aligned}
$$

## G. 2 Calibration

My model features a quarterly frequency. Below I discuss the calibration of parameters from both the model with and without financial sector.

## G.2.1 Model without financial frictions and central bank lending operations

Now that I have specified the modifications, I discuss the calibrated parameter values for the model version without financial frictions and central bank lending operations. These parameters can be divided into two groups. The first group consists of parameters whose value is either directly chosen or taken from the literature, while the second group consists of parameters that are manually adjusted to match specific first order moments in the data. Several of these targets are taken from Bocola (2016), who employs a mix of calibration and estimation to match an RBC-model enriched with financial frictions to the Italian economy. A list of calibration targets can be found in Table 1 while Table 2 contains the resulting parameter values.

I start with the parameter values that are either directly chosen or taken from the literature. The first of these is the capital income share $\alpha$, for which I take the parameter value used by Bocola (2016). Next I pick the feedback parameter from government debt to lump sum taxes $\kappa_{b}$, as this parameter cannot be identified in a Bayesian estimation because of the presence of

Ricardian equivalence in a model without financial frictions. I manually set this value to 0.05 , which is larger than the net real return on government bonds, in line with Bohn (1998). I also handpick the steady state elasticity of substitution between different retail goods producers $\bar{\epsilon}_{p}$, and between different labor types $\bar{\epsilon}_{w}$. Both are set at 11 , which implies a steady state markup of $10 \%$. Parameter values for the Taylor-rule feedback coefficients for inflation $\kappa_{\pi}$ and output $\kappa_{y}$ as well as the interest rate smoothing parameter $\rho_{r}$ and the standard deviation of the interest rate shock $\sigma_{r}$ are set to values commonly employed in the literature. I abstain from estimating these parameters as Italian monetary policy is conducted by the ECB, which conducts its monetary policy based on macrodevelopments in the Eurozone as a whole rather than on macrodevelopments in Italy. As such, an estimation of the Taylor rule might produce biased estimates.

Next I discuss the parameters that are manually adjusted to match specific first order moments. I set the subjective discount factor such that the steady state gross interest rate on deposits is the same as in Bocola (2016). I also follow Bocola (2016) in setting the steady state investment-GDP ratio and government spending-GDP ratio equal to 0.213 and 0.198 respectively. I set steady state government debt equal to $100 \%$ of annual GDP, which is a lower bound for the Italian debt-GDP ratio over the 1999Q1-2007Q4 period. The steady state gross inflation rate is set to 1.005 , implying a steady state net inflation rate of $2 \%$ per year, which is in line with the ECB's inflation target. I set the coupon payment $x_{c}$ on long-term government debt equal to $1 \%$ per quarter (which amounts to an annual coupon rate of $4 \%$ per year, in line with pre-crisis average interest rates on long-term government debt), while the average maturity of government debt is set to 6 quarters by adjusting $\rho$ (Bocola, 2016), ${ }^{30}$

I also target the steady state credit spread (difference between the return on corporate securities and deposits) by taking the mean of the empirical time series for the credit spread over the period 2000Q1-2007Q4. This time series is obtained by taking the difference between the interest rate on loans and the Money Market 3-month Euribor interest rate, see Appendix F I find an average annual credit spread of 108 basis points, which amounts to an average quarterly spread of 27 basis points. The time series for the credit spread starts in 2000Q1, as no data are available for the interest rate on loans before 2000Q1. I also calculate the spread between the yield on long-term government bonds and the Money Market 3-month Euribor, and find that the average spread is slightly above the credit spread. Using this bond yield-deposit spread would result in a diversion rate for government bonds $\lambda_{b}$ that is (slightly) larger than the steady state diversion rate on corporate securities $\bar{\lambda}_{k}$ within the model version that includes financial frictions. As this sharply contrasts with most of the literature featuring financial intermediaries with a portfolio choice between corporate securities and government bonds (Gertler and Karadi, 2013, Kirchner and van Wijnbergen, 2016), I reduce the steady state bond yield-deposit spread to ensure that it is equal to the steady state credit spread, which I both set at 27 basis points per quarter. A second reason why it is reasonable to reduce the bond yield-deposit spread is the fact that the

[^22]| Target | Definition | Value | Data |  |
| :--- | :--- | :--- | :--- | :--- |
| Households |  |  |  |  |
| $1 / \beta$ | Interest rate on deposits | 1.003 | Bocola | $(2016$ |
| $\bar{h}$ | Labor supply | 0.318 | Bocola | $(2016$ |
| Financial targets |  |  |  |  |
| $E\left[\bar{r}_{k}-\bar{r}_{d}\right]$ | Credit spread (quarterly) | 0.0027 | See text |  |
| $E\left[\bar{r}_{b}-\bar{r}_{d}\right]$ | Bond yield-deposit spread (quarterly) | 0.0027 | See text |  |
| Aggregate targets |  |  |  |  |
| $\alpha$ | Capital share | 0.300 | Bocola | $(2016$ |
| $\bar{i} / \bar{y}$ | Investment-output ratio | 0.213 | $\overline{\text { Bocola }}$ | $(2016$ |
| $\bar{g} / \bar{y}$ | Gov't spending-output ratio | 0.198 | Bocola | 2016 |
| $\bar{q}_{b} \bar{b} / \bar{y}$ | Gov't liabilities-output ratio (quarterly) | 4 | Lower bound 1999Q1-2007Q4 |  |
| $G o v e r n m e n t ~ p o l i c y ~$ |  |  |  |  |
| $1 / \rho$ | Maturity bonds (in quarters) | 6 | Bocola | $(2016$ |
| $x_{c}$ | Periodic cash-flow payment bonds | 0.01 | Annual net coupon rate of 4\% |  |
| $\bar{x}$ | Inflation target central bank | 1.005 | $2 \%$ annual net inflation |  |

Table 1: Calibration targets for the model version without financial intermediaries and central bank lending operations.
empirical time series for bond yields applies to government bonds with a maturity of 10 years, see Appendix F, whereas the maturity in my model is only 6 quarters. Typically, government bonds with a shorter maturity feature lower bond yields.

Finally, I set the coefficient in front of the (dis)utility function from labor $\Psi$ such that the steady state labor supply is equal to 0.318 following Bocola (2016). In addition, I adjust the depreciation rate $\delta$ such that the above-mentioned steady state investment-GDP ratio and credit spread are matched. Observe, though, that the value of $\Psi$ and $\delta$ are affected by the parameter values for the probability of changing prices $\left(\psi_{p}\right)$ and wages $\left(\psi_{w}\right)$, as well as the degree of indexation for retail goods prices $\left(\gamma_{p}\right)$ and wages $\left(\gamma_{w}\right)$, which are estimated in a Bayesian estimation. To be able to hit the above mentioned calibration targets during this estimation procedure, I manually adjust $\Psi$ and $\delta$ in a separate file that calculates the steady state for every possible combination of $\psi_{p}, \psi_{w}, \gamma_{p}$, and $\gamma_{w}$, see Pfeifer (2018).

## G.2.2 Model with financial frictions and central bank lending operations

Next I discuss the calibration of the model version that includes financial frictions and central bank lending operations. First, this model version employs all the parameter values from the model version without financial frictions, see Table 2. To calibrate parameters that only feature in the model version with financial frictions and central bank lending operations, I target the first order moments in Table 3.

Credit institutions in the Eurozone play a crucial role in the provision of credit to the real economy, as they intermediate approximately $80 \%$ of debt financing to non-financial corporations (European Central Bank, 2015). I set the stock of government bonds held by financial

| Parameter | Value | Definition |
| :--- | :--- | :--- |
| Households |  |  |
| $\beta$ | 0.997 | Discount rate |
| $\Psi$ | 30.5484 | Relative utility weight of labor |
| Goods producers |  |  |
| $\alpha$ | 0.300 | Effective capital share |
| $\bar{\epsilon}_{p}$ | 11 | Elasticity of substitution (producers) |
| $\bar{\epsilon}_{w}$ | 11 | Elasticity of substitution (workers) |
| Capital good firms |  |  |
| $\delta$ | 0.0203 | Depreciation rate |
| Government policy |  |  |
| $\kappa_{b}$ | 0.050 | Tax feedback parameter from government debt |
| $\tau_{k}$ | 0.0027 | Tax on gross return on corporate securities |
| $\tau_{b}$ | 0.0027 | Tax on gross return on government bonds |
| $\rho$ | 0.167 | Maturity parameter bonds |
| $\kappa_{\pi}$ | 1.500 | Inflation feedback on nominal interest rate |
| $\kappa_{y}$ | 0.125 | Output feedback on nominal interest rate |
| $\rho_{r}$ | 0.800 | Interest rate smoothing parameter |
| $\sigma_{r}$ | 0.0025 | Standard deviation interest rate shock |

Table 2: Calibrated parameters for the model version without financial sector and central bank lending operations. The parameter values for $\Psi$ and $\delta$ depend on the value of parameters estimated in the Bayesian estimation. The reported values for these two parameters are the values that arise when I take the mean of the parameters estimated in the Bayesian estimation.

| Target | Definition | Value | Data |
| :--- | :--- | :--- | :--- |
| $\bar{s}_{k} / k$ | Fraction of corp. securities held by intermediaries | 0.8 | See text |
| $\bar{s}_{b} / \bar{b}$ | Fraction of gov't bonds held by intermediaries | 0.10 | See text |
| $\bar{J}_{\bar{\phi}} /(1-\sigma)$ | Average life-time bankers | 20 | Gertler and Kiyotaki $(2015)$ |
| $\lambda_{b} / \lambda_{k}$ | Leverage ratio | Relative diversion rate bonds over securities | 5 |
| Bocola | 1 | See text |  |
| $\Gamma_{c b}$ | Interest spread $\bar{r}_{n, d}-\bar{r}_{n, c b}$ | 0 | See main text |
| $\theta^{k}$ | Collateral parameter corp. securities | 0.4 | See main text |
| $\theta^{b}$ | Collateral parameter gov't bonds | 0.95 | See main text |
| $\bar{\delta}_{c b}$ | Fraction of pre-dividend CB net worth | 0.10 | See text |

Table 3: Calibration targets for the model version including financial intermediaries and central bank lending operations. The abbreviation "CB" refers to central bank.

| Parameter | Value | Definition |
| :--- | :--- | :--- |
| Financial intermediaries |  |  |
| $\sigma$ | 0.95 | Probability of continuing as intermediary |
| $\chi_{b}$ | 0.0382 | Transfer share to new intermediaries |
| $\lambda_{b}$ | 0.3502 | Diversion rate gov't bonds |
| $\bar{\lambda}_{k}$ | 0.3502 | Steady state diversion rate corp. securities |
| Central bank |  |  |
| $\theta^{k}$ | 0.4 | Collateral parameter corp. securities |
| $\theta^{b}$ | 0.95 | Collateral parameter gov't bonds |

Table 4: Calibrated parameters for the model version including financial intermediaries and central bank lending operations.
intermediaries equal to $10 \%$ of the total stock of government debt. I follow Gertler and Kiyotaki (2015) by setting the average life-time of bankers to be equal to 20 quarters, which results in a probability of being allowed to continue operating of $\sigma=0.95$. I explained in Section G.2.1 why I set the steady state credit spread equal to the steady state bond yield-deposit spread, which results in the relative diversion ratios of government bonds and corporate securities to be equal to $1{ }^{31}$ Afterwards, I adjust the level of $\lambda_{k}$ and $\lambda_{b}$ such that the steady state leverage ratio is equal to 5 , see Bocola (2016). To ensure that the law of motion for intermediaries' aggregate net worth holds, I adjust the fraction of previous period net worth $\chi_{b}$ that families provide to new financial intermediaries as starting net worth. I set the steady state fraction $\bar{\delta}_{c b}$ of pre-dividend net worth $\bar{n}_{c b}^{*}$ equal to $10 \%$, while I already motivate my choices for $\theta_{t}^{k}$ and $\theta_{t}^{b}$ in the main text. Table 4 reports the resulting values for parameters relating to the financial sector and central bank lending operations.

The remaining parameters are determined in the moment matching exercise.

## G.2.3 Model version including sovereign default risk

The model version with sovereign default risk contains four additional parameters, namely $\alpha_{b}, \beta_{b}$, $\kappa_{\tau, y}$, and $\bar{b}_{\text {max }}$. The last parameter can be interpreted as the maximum level of debt enshrined in the Maastricht Treaty, which is $60 \%$ of annual GDP. Therefore I set the market value of maximum debt $\bar{q}_{b} \bar{b}_{\max }$ equal to $60 \%$ of annual steady state GDP.

The first two parameters are pinned down by targeting a steady state default probability $\bar{p}_{d e f}$ of 200 annual basis points, or 50 quarterly basis points. In addition, I target the first derivative of the default probability function and ensure that it is equal to 0.2 in steady state. These two targets and the steady state values of $\bar{b}, \bar{b}_{\text {max }}$, and $\bar{y}$ allow me to pin down $\alpha_{b}=32.2543$ and $\beta_{b}=16.5907$. The spread between the ex ante default steady state return on bonds $\bar{r}_{b}$ and

[^23]| Parameter | Value | Definition |
| :--- | :--- | :--- |
| Sovereign default risk |  |  |
| $\alpha_{b}$ | 32.2543 | Parameter default probability function 260 |
| $\beta_{b}$ | 16.5907 | Parameter default probability function 260 |
| $\kappa_{\tau, y}$ | 5 | Feedback from output on level of lump sum taxes |
| $\bar{q}_{b} \bar{b}_{\max } / \bar{y}$ | 2.4 | "Maximum level" of debt, Maastricht Treaty |

Table 5: Calibrated parameters for the model version including sovereign default risk.

| Parameter | Value | Definition |
| :--- | :--- | :--- |
| Small open economy |  |  |
| $v_{c}$ | 0.5 | Import share domestic consumption bundle |
| $v_{i}$ | 0.5 | Import share domestic investment bundle |
| $\eta_{c}$ | 7.512 | Consumption elast. of subst. dom.\& for. goods Burriel et al. 2010 |
| $\eta_{i}$ | 7.851 | Investment elast. of substit. dom.\& for. goods Burriel et al. 2010) |
| $v_{c}^{*}$ | 0.025 | Import share foreign consumption bundle |
| $v_{i}$ | 0.025 | Import share foreign investment bundle |
| $\gamma_{c}^{*}$ | 1 | For. consumption elast. of subst. between goods from different countries |
| $\gamma_{i}^{*}$ | 1 | For. investment elast. of subst. between goods from different countries |

Table 6: Calibrated parameters for the small open economy model version.
the ex post default return on government bonds $\bar{r}_{b}^{*}$, which should be equal to the return on a government bond that is not subject to default risk like the German Bund, is in that case equal to 50 quarterly basis points, or 200 annual basis points.

Finally, the parameter is $\kappa_{\tau, y}$ in the level of lump sum taxes 264 is set to 5 , implying a substantial drop in taxes when the financial crisis hits. As a result, the government has to issue more debt, which subsequently increases sovereign default risk.

An overview of the additional parameters can be found in Table 5

## G.2.4 Small open economy member of a currency union

I take the same value for parameters that also feature in the closed economy. Therefore I only need to discuss the parameters that are new with respect to the closed economy model. Specifically, I take the elasticity of substitution between domestic and foreign goods for consumption $\eta_{c}$ and investment $\eta_{i}$ from Burriel et al. (2010). These are set at 7.512 and 7.851 , respectively. As Italy is part of the European Union's single market, and therefore trades a lot with the rest of the Eurozone, I set the steady state share of foreign goods in the consumption and investment bundles equal to $v_{c}=v_{i}=0.5$. I set the foreign elasticity of substitution between goods from different countries for consumption and investment equal to $\gamma_{c}^{*}=\gamma_{i}^{*}=1$. Finally, I adjust $v_{c}^{*}$ and $v_{i}^{*}$ such that steady state foreign consumption and investment are 20 times domestic consumption and investment, respectively.

An overview of the additional parameters can be found in Table 6

## G. 3 Bayesian estimation of remaining parameters

## G.3.1 Main estimation

I report the priors and posteriors for the parameters that are estimated using Bayesian techniques in Tables 7 and 8. I follow Gerali et al. (2010) for the priors of the $\operatorname{AR}(1)$ coefficients and standard deviations of the exogenous processes. However, Gerali et al. (2010) do not estimate deep parameters that affect the steady state like I do. Therefore, I look to Darracq-Pariès and Kühl (2017) for the priors of the habit formation parameter and the inverse Frisch elasticity, while choosing priors that are less informative for the remaining deep parameters, see Table 7 .

|  | Parameter | Distrib. | Mean | Stdv. | Mean | $10 \%$ | Mode | $90 \%$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $v$ | Habit formation | Normal | 0.7 | 0.1 | 0.4678 | 0.3524 | 0.4503 | 0.5858 |
| $\varphi$ | Inverse Frisch elast. | Gamma | 2 | 0.75 | 1.9880 | 0.7843 | 1.6450 | 3.1438 |
| $\psi_{p}$ | price-stickiness | Beta | 0.5 | 0.1 | 0.6776 | 0.6046 | 0.6681 | 0.7514 |
| $\psi_{w}$ | wage-stickiness | Beta | 0.5 | 0.1 | 0.4599 | 0.3077 | 0.4543 | 0.6109 |
| $\gamma_{P}$ | price-indexation | Beta | 0.5 | 0.2 | 0.1504 | 0.0129 | 0.0809 | 0.2848 |
| $\gamma_{W}$ | wage-indexation | Beta | 0.5 | 0.2 | 0.4836 | 0.1581 | 0.4701 | 0.8110 |
| $\gamma$ | Invest. adj. cost | Gamma | 2.5 | 1 | 2.6849 | 1.3011 | 1.9037 | 4.0339 |

Table 7: Priors (columns 3-5) and posteriors (columns 6-9) of the parameters that are estimated with Bayesian techniques. The results are based on 2 chains,each with 800,000 draws based on the Metropolis-Hastings algorithm.

|  | Parameter | Distrib. | Mean | Stdv. | Mean | $10 \%$ | Mode | $90 \%$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\operatorname{AR}(1)$ |  |  |  |  |  |  |  |  |
| $\rho_{z}$ | Productivity | Beta | 0.8 | 0.1 | 0.3948 | 0.2501 | 0.3756 | 0.5387 |
| $\rho_{\xi}$ | Capital quality | Beta | 0.8 | 0.1 | 0.6829 | 0.5403 | 0.7249 | 0.8277 |
| $\rho_{g}$ | Gov't spending | Beta | 0.8 | 0.1 | 0.6983 | 0.4996 | 0.6978 | 0.9073 |
| $\rho_{c}$ | $c$-preferences | Beta | 0.8 | 0.1 | 0.7300 | 0.5538 | 0.7417 | 0.9117 |
| $\rho_{i}$ | Invest. adj. cost | Beta | 0.8 | 0.1 | 0.6996 | 0.5260 | 0.7516 | 0.8829 |
| $\rho_{p}$ | price-stickiness | Beta | 0.8 | 0.1 | 0.7098 | 0.5605 | 0.7271 | 0.8614 |
| $\rho_{w}$ | wage-stickiness | Beta | 0.8 | 0.1 | 0.8002 | 0.6485 | 0.8461 | 0.9568 |
|  |  |  |  |  |  |  |  |  |
| Stdv. |  |  |  |  |  |  |  |  |
| $\sigma_{z}$ | Productivity | Invg. | 0.01 | 0.05 | 0.0175 | 0.0140 | 0.0169 | 0.0210 |
| $\sigma_{\xi}$ | Capital quality | Invg. | 0.01 | 0.05 | 0.0058 | 0.0024 | 0.0039 | 0.0094 |
| $\sigma_{g}$ | Gov’t spending | Invg. | 0.01 | 0.05 | 0.0200 | 0.0158 | 0.0192 | 0.0239 |
| $\sigma_{c}$ | $c$-preferences | Invg. | 0.01 | 0.05 | 0.0097 | 0.0036 | 0.0103 | 0.0148 |
| $\sigma_{i}$ | Invest. adj. cost | Invg. | 0.01 | 0.05 | 0.0203 | 0.0088 | 0.0130 | 0.0320 |
| $\sigma_{p}$ | $p$-stickiness | Invg. | 0.01 | 0.05 | 0.3008 | 0.2122 | 0.2686 | 0.3869 |
| $\sigma_{w}$ | $w$-stickiness | Invg. | 0.01 | 0.05 | 0.0096 | 0.0023 | 0.0047 | 0.0175 |

Table 8: Priors (columns 3-5) and posteriors (columns 6-9) of the parameters that are estimated with Bayesian techniques. The results are based on 2 chains,each with 800,000 draws based on the Metropolis-Hastings algorithm.

Tables 7 and 8 also report the summary statistics of the posterior distributions. I apply
the convergence statistics proposed by Brooks and Gelman (1998) to check that convergence is reached after having estimated the model.

We see that most of the posterior means differ from the prior mean, except the parameters related to the standard deviation of the preference shock and the labor market: the posterior means of the inverse Frisch elasticity $\varphi$, the probability of changing wages $\psi_{w}$, the degree of wage-indexation $\gamma_{W}$, and the $\operatorname{AR}(1)$ coefficient $\rho_{w}$ and standard deviation $\sigma_{w}$ for the exogenous process for the elasticity of substitution between different labor types hardly differ from the prior mean. This is probably caused by the fact that I only employ a time series for the number of hours worked, and do not have a time series for the wage rate. However, as the focus of my paper is on the interactions between an undercapitalized financial sector and central bank lending operations, I think it is reasonable to argue that my qualitative results are not driven by the particular values for these parameter estimates.

## G.3.2 Robustness Bayesian estimation

The introduction of households' transaction costs in the model version with financial frictions contrasts with the model version without financial frictions, where I instead employ a tax on the gross return on corproate securities and government bonds. As such, the dynamics of the model version with financial frictions in which intermediaries' incentive compatibility constraints are not binding and the collateral effect is eliminated by setting $r_{t}^{n, r}=r_{t}^{n, c b}$ will not be the same as the model version without financial frictions and central bank lending operations. To check whether the error I thus introduce into the parameter estimates is large, I redo the Bayesian estimation for a model version without financial frictions where the tax on the gross returns has been replaced by the quadratic adjustment costs, and take the point estimates for $\kappa_{s_{b, h}}$ and $\kappa_{s_{k, h}}$ that I find in the moment-matching exercise. The subsequent results can be found in Tables 9 and 10, which show that the parameter estimates are close to those in Tables 7 and 8 .

|  | Parameter | Distrib. | Mean | Stdv. | Mean | $10 \%$ | Mode | $90 \%$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $v$ | Habit formation | Normal | 0.7 | 0.1 | 0.5065 | 0.3878 | 0.5089 | 0.6217 |
| $\varphi$ | Inverse Frisch elast. | Gamma | 2 | 0.75 | 2.1085 | 0.7714 | 2.1170 | 3.3585 |
| $\psi_{p}$ | price-stickiness | Beta | 0.5 | 0.1 | 0.7050 | 0.6354 | 0.7049 | 0.7723 |
| $\psi_{w}$ | wage-stickiness | Beta | 0.5 | 0.1 | 0.4712 | 0.3135 | 0.5062 | 0.6318 |
| $\gamma_{P}$ | price-indexation | Beta | 0.5 | 0.2 | 0.1201 | 0.0115 | 0.0607 | 0.2297 |
| $\gamma_{W}$ | wage-indexation | Beta | 0.5 | 0.2 | 0.4380 | 0.1160 | 0.2974 | 0.7592 |
| $\gamma$ | Invest. adj. cost | Gamma | 2.5 | 1 | 2.5782 | 1.1980 | 2.0905 | 3.8959 |

Table 9: Priors (columns 3-5) and posteriors (columns 6-9) of the parameters that are estimated with Bayesian techniques. The results are based on 2 chains,each with 800,000 draws based on the Metropolis-Hastings algorithm.

|  | Parameter | Distrib. | Mean | Stdv. | Mean | $10 \%$ | Mode | $90 \%$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\operatorname{AR}(1)$ |  |  |  |  |  |  |  |  |
| $\rho_{z}$ | Productivity | Beta | 0.8 | 0.1 | 0.4220 | 0.2694 | 0.3945 | 0.5762 |
| $\rho_{\xi}$ | Capital quality | Beta | 0.8 | 0.1 | 0.7037 | 0.5336 | 0.8200 | 0.8805 |
| $\rho_{g}$ | Gov't spending | Beta | 0.8 | 0.1 | 0.6951 | 0.5027 | 0.6823 | 0.9117 |
| $\rho_{c}$ | $c$-preferences | Beta | 0.8 | 0.1 | 0.7598 | 0.6004 | 0.8292 | 0.9264 |
| $\rho_{i}$ | Invest. adj. cost | Beta | 0.8 | 0.1 | 0.7513 | 0.6017 | 0.8150 | 0.9018 |
| $\rho_{p}$ | price-stickiness | Beta | 0.8 | 0.1 | 0.7817 | 0.6526 | 0.8048 | 0.9153 |
| $\rho_{w}$ | wage-stickiness | Beta | 0.8 | 0.1 | 0.7988 | 0.6459 | 0.8462 | 0.9569 |
|  |  |  |  |  |  |  |  |  |
| Stdv. |  |  |  |  |  |  |  |  |
| $\sigma_{z}$ | Productivity | Invg. | 0.01 | 0.05 | 0.0178 | 0.0142 | 0.0168 | 0.0213 |
| $\sigma_{\xi}$ | Capital quality | Invg. | 0.01 | 0.05 | 0.0037 | 0.0022 | 0.0034 | 0.0051 |
| $\sigma_{g}$ | Gov't spending | Invg. | 0.01 | 0.05 | 0.0201 | 0.0160 | 0.0193 | 0.0241 |
| $\sigma_{c}$ | $c$-preferences | Invg. | 0.01 | 0.05 | 0.0093 | 0.0032 | 0.0048 | 0.0144 |
| $\sigma_{i}$ | Invest. adj. cost | Inv. | 0.01 | 0.05 | 0.0252 | 0.0134 | 0.0202 | 0.0370 |
| $\sigma_{p}$ | $p$-stickiness | Invg. | 0.01 | 0.05 | 0.3401 | 0.2349 | 0.3006 | 0.4428 |
| $\sigma_{w}$ | $w$-stickiness | Invg. | 0.01 | 0.05 | 0.0126 | 0.0022 | 0.0047 | 0.0248 |

Table 10: Priors (columns 3-5) and posteriors (columns 6-9) of the parameters that are estimated with Bayesian techniques. The results are based on 2 chains,each with 800,000 draws based on the Metropolis-Hastings algorithm.

## G. 4 Moment matching

Finally, I estimate the remaining parameters using a moment-matching exercise. Specifically, these parameters are the coefficient for households' transaction costs for purchasing corporate securities $\kappa_{s_{k, h}}$ and government bonds $\kappa_{s_{b, h}}$, the reference level of corporate securities and government bonds for households' transaction costs $\hat{s}_{k, h}$ and $\hat{s}_{b, h}$ respectively, and the $\operatorname{AR}(1)$ coefficient $\rho_{\lambda_{k}}$ and standard deviation $\sigma_{\lambda_{k}}$ for the exogenous process that governs the diversion rate for corporate securities $\lambda_{t}^{k}$ :

$$
\begin{equation*}
\log \left(\frac{\lambda_{t}^{k}}{\bar{\lambda}_{k}}\right)=\rho_{\lambda_{k}} \log \left(\frac{\lambda_{t-1}^{k}}{\bar{\lambda}_{k}}\right)+\varepsilon_{\lambda_{k}, t} \tag{317}
\end{equation*}
$$

I allow $\lambda_{t}^{k}$ to vary over time because it is likely that financial constraints were binding during the estimation period (2008Q1-2011Q4) in which two severe financial crises hit the Italian banking system. Within my model such crises are captured by binding incentive compatibility constraints (21), in which case shocks to $\lambda_{t}^{k}$ affect the equilibrium allocation. This contrasts with the 1999Q12007Q4 period, in which such constraints were non-binding and changes in $\lambda_{t}^{k}$ consequently do not affect the equilibrium. This is the reason why $\rho_{\lambda_{k}}$ and $\sigma_{\lambda_{k}}$ cannot be estimated in the Bayesian estimation procedure.

Note, however, that the reference levels $\hat{s}_{k, h}$ and $\hat{s}_{b, h}$ are implictly pinned down by the coefficients for households' transaction costs and households' steady state stocks of corporate
securities and government bonds through the following relation:

$$
\hat{s}_{a, h}=\bar{s}_{a, h}-\left[\beta\left(1+\bar{r}_{a}\right)-1\right] \frac{\bar{q}_{a}}{\kappa_{s_{a, h}}},
$$

which is a rewritten version of households' steady state first order conditions for corporate securities ( $a=k$, equation (315) and government bonds ( $a=b$, equation 316). Therefore, the reference level is effectively a function of the coefficient for households' adjustment costs, which I can write as $\hat{s}_{a, h}=f_{a}\left(\kappa_{s_{a, h}}\right)$, as all the other parameters and steady state values have already been pinned down in earlier stages. As a result, I effectively estimate four parameters in this moment-matching exercise: $\theta=\left\{\kappa_{s_{k, h}}, \kappa_{s_{b, h}}, \rho_{\lambda_{k}}, \sigma_{\lambda_{k}}\right\}$.

I choose to minimize the distance between the empirical standard errors for real GDP, consumption, investment, and the credit spread over the period 2008Q1-2011Q4 and their counterparts from simulations of the full model with financial intermediaries and central bank lending operations. I set $r_{t}^{n, r}=r_{t}^{n, c b}$, and weigh each distance with the inverse of the squared empirical standard error of the relevant variable. Mathematically, I can describe this problem as:

$$
\begin{equation*}
\min _{\theta}\left(M^{D}-M(\theta)\right)^{\prime} W^{-1}\left(M^{D}-M(\theta)\right) \tag{318}
\end{equation*}
$$

where $M^{D}$ is a vector with the empirical standard errors for real GDP, consumption, investment, and the credit spread. $M(\theta)$ denotes the model counterpart of $M^{D}$, and $W$ is a diagonal matrix with the empirical squared standard errors for real GDP, consumption, investment, and the credit spread on the diagonal. For each gridpoint $\theta$, I construct $M(\theta)$ by performing 1,000 simulations. Each simulation starts from the non-stochastic steady of the model, and lasts for 10,016 periods. I subsequently discard the first 10,000 observations as a burn-in, so that I am left with 16 observations, which is equivalent to the number of observations of the empirical data. Next, I take the natural logarithm of real GDP, consumption, and investment, and subsequently filter the resulting time series as well as the time series for the credit spread using a one-sided HPfilter with smoothing parameter 1,600. Finally, I calculate the standard errors for each filtered time series.

The initial grid consists of $\kappa_{s_{k, h}}=\{0.01,0.1,1\}, \kappa_{s_{b, h}}=\{0.0001,0.001,0.01\}, \rho_{\lambda_{k}}=\{0.4,0.5,0.6\}$, and $\sigma_{\lambda_{k}}=\{0.10,0.15\}$. I limit the grid size because of the curse of dimensionality. The lower bounds for $\kappa_{s_{k, h}}$ and $\kappa_{s_{b, h}}$ are established to prevent intermediaries' bond holdings from dropping below $-100 \%$ in the financial crisis simulations. After performing an initial moment-matching exercise on this grid I find that $\kappa_{s_{k, h}}$ and $\kappa_{s_{b, h}}$ are at their lower bounds of 0.01 and 0.0001 , respectively. I construct a second grid in which $\kappa_{s_{k, h}}$ starts at 0.01 and increases to 0.1 with steps of 0.01 , while $\kappa_{s_{b, h}}$ starts at 0.0001 and increases to 0.001 with steps of 0.0001 . The grid points for $\rho_{\lambda_{k}}$ and $\sigma_{\lambda_{k}}$ are the same as in the initial moment-matching exercise.

The second moment-matching exercise finds that the distance between the empirical and simulated moments is minimized for $\kappa_{s_{k, h}}=0.01, \kappa_{s_{b, h}}=0.0001, \rho_{\lambda_{k}}=0.6$ and $\sigma_{\lambda_{k}}=0.10$. The resulting moments and their standard errors can be found in Table 11

|  | Standard deviation |  | Autocorrelations |  |
| :--- | :--- | :--- | :--- | :--- |
|  | Data | Model | Data | Model |
| GDP | 0.0117 | 0.0098 | 0.7747 | 0.6080 |
|  |  | $[-0.5197]$ |  | $[-0.8879]$ |
| Consumption | 0.0107 | 0.0086 | 0.6323 | 0.6430 |
|  |  | $[-0.6468]$ |  | $[0.0639]$ |
| Investment | 0.0221 | 0.0288 | 0.8221 | 0.6335 |
|  |  | $[0.5639]$ |  | $[-1.0183]$ |
| Credit spread | 0.0012 | 0.0036 | 0.6517 | 0.3129 |
|  |  | $[2.6343]$ |  | $[-1.4110]$ |
| Inflation | 0.0034 | 0.0063 | -0.1206 | 0.3426 |
|  |  | $[1.7955]$ |  | $[2.0268]$ |
| Government spending | 0.0130 | 0.0143 | 0.4452 | 0.3589 |
|  |  | $[0.3469]$ |  | $[-0.3689]$ |
| Labor | 0.0098 | 0.0208 | -0.2272 | 0.3208 |
|  |  | $[2.0275]$ |  | $[2.2608]$ |
| Nominal rate | 0.0019 | 0.0028 | 0.8789 | 0.5509 |
|  |  | $[1.0155]$ |  | $[-1.7473]$ |

Table 11: List of standard deviations of data and model (columns 2 and 3), and first order autocorrelations of data and model (column 4 and 5). $t$-statistics are reported between square brackets. Obtained for $\kappa_{s_{k, h}}=0.01, \kappa_{s_{b, h}}=0.0001, \rho_{\lambda_{k}}=0.6$, and $\sigma_{\lambda_{k}}=0.10$.

We see that the resulting model moments match the data quite well. Not only is the difference between the empirical and simulated standard errors statistically not significant except for the credit spread, I also find that this is the case for inflation, government spending, and the nominal interes rate, variables that I did not target. In addition, I also find that the difference between the empirical and simulated first order autocorrelations is not statistically significant for all variables except inflation and labor. As I did not target the first order autocorrelations of any of these variables, I conclude that my model captures the dynamics of the Italian economy over the estimation period reasonably well.

## H Additional figures

In this section I show that my results are robust under alternative setups and different parameter values. First, I show that the results from the main text carry over to a model including sovereign default risk (Figures 8-12). A description of the model including sovereign default risk can be found in Appendix E. 1

Second, I check that my results carry over to a small open economy member of a currency union (Figures 13-17). The accompanying description of the small open economy can be found
in Appendix E. 2
Third, I check that the model version of the main text with $\bar{\lambda}_{b} / \bar{\lambda}_{k}=0.5$ rather than $\bar{\lambda}_{b} / \bar{\lambda}_{k}=1$ in the main text (Figures 18-22.

Finally, Figure 23 shows the absolute level of domestic government bond holdings of aggregate monetary financial institutions in Italy, Spain and Portugal normalized at 100 in December 2011.

DomesticBondholdingsNorm

Central bank funding shock 35 bps.: base case vs. $\theta_{k}=\theta_{b}=0.425$ (sovereign default risk)


Figure 8: Impulse response functions for a central bank funding shock of 35 basis points ("Interest rate difference") for the base case (blue, solid) versus a model version in which $\theta_{t}^{k}=\theta_{t}^{b}=0.425$. The panel "Corporate securities (b)" denotes the value of intermediaries' corporate securities holdings $q_{t}^{k} s_{t}^{k}$. Similarly, the panel "Government bonds (b)" denotes the value of intermediaries' government bond holdings $q_{t}^{b} s_{t}^{b}$. Central bank funding is expressed in terms of annual steady state output.


Figure 9: Impulse response functions for a central bank funding shock of 35 basis points ("Interest rate difference") for the base case $\kappa_{s_{b, h}}=0.0001$ (blue, solid) versus the case where $\kappa_{s_{b, h}}=$ 0.00005 (red, slotted). The panel "Corporate securities (b)" denotes the value of intermediaries' corporate securities holdings $q_{t}^{k} s_{t}^{k}$. Similarly, the panel "Government bonds (b)" denotes the value of intermediaries' government bond holdings $q_{t}^{b} s_{t}^{b}$. Central bank funding is expressed in terms of annual steady state output.

Central bank funding shock $35 \mathrm{bps} .: \theta_{t}^{k}=0.40$ vs. $\theta_{t}^{k}=0.50$ (sovereign default risk)


Figure 10: Impulse response functions for a central bank funding shock of 35 basis points ("Interest rate difference") for the base case $\theta_{t}^{k}=0.40$ (blue, solid) versus the case where $\theta_{t}^{k}=0.50$ (red, slotted). The panel "Corporate securities (b)" denotes the value of intermediaries' corporate securities holdings $q_{t}^{k} s_{t}^{k}$. Similarly, the panel "Government bonds (b)" denotes the value of intermediaries' government bond holdings $q_{t}^{b} s_{t}^{b}$. Central bank funding is expressed in terms of annual steady state output.

Financial crisis: no policy vs limited LTRO vs limited LTRO $\theta_{k}=0.5$ (sovereign default risk)


Figure 11: Impulse response functions for a financial crisis, initiated by a capital quality shock of $3.5 \%$. The figure compares a scenario with no additional policy (blue, solid) with a scenario in which the central bank lowers the nominal interest rate on central bank funding with respect to the nominal interest rate on deposits with 35 quarterly basis points in line with the capital quality shock (red, slotted), and with the same intervention with $\theta_{k}=0.5$ (black, dashed). The panel "Corporate securities (b)" denotes the value of intermediaries' corporate securities holdings $q_{t}^{k} s_{t}^{k}$. Similarly, the panel "Government bonds (b)" denotes the value of intermediaries' government bond holdings $q_{t}^{b} s_{t}^{b}$. Central bank funding is expressed in terms of annual steady state output.

Financial crisis: no policy vs LTRO vs LTRO with $\theta_{k}=0.5$ (sovereign default risk)


Figure 12: Impulse response functions for a financial crisis, initiated by a capital quality shock of $3.5 \%$. The figure compares a scenario with no additional policy (blue, solid) with a scenario in which the central bank lowers the nominal interest rate on central bank funding with respect to the nominal interest rate on deposits with 35 quarterly basis points for 12 quarters, capturing the three-year LTROs (red, slotted), and the same central bank intervention with $\theta_{k}=0.5$ (black, dashed). The panel "Corporate securities (b)" denotes the value of intermediaries' corporate securities holdings $q_{t}^{k} s_{t}^{k}$. Similarly, the panel "Government bonds (b)" denotes the value of intermediaries' government bond holdings $q_{t}^{b} s_{t}^{b}$. Central bank funding is expressed in terms of annual steady state output.

Central bank funding shock 35 bps.: base case vs. $\theta_{k}=\theta_{b}=0.425$ (small open economy)


Figure 13: Impulse response functions for a central bank funding shock of 35 basis points ("Interest rate difference") for the base case (blue, solid) versus a model version in which $\theta_{t}^{k}=\theta_{t}^{b}=0.425$. The panel "Corporate securities (b)" denotes the value of intermediaries' corporate securities holdings $q_{t}^{k} s_{t}^{k}$. Similarly, the panel "Government bonds (b)" denotes the value of intermediaries' government bond holdings $q_{t}^{b} s_{t}^{b}$. Central bank funding is expressed in terms of annual steady state output.

Central bank funding shock $35 \mathrm{bps} .: \kappa_{s_{b, h}}=0.0001 \mathrm{vs} . \kappa_{s_{b, h}}=0.00005$ (small open economy)


Figure 14: Impulse response functions for a central bank funding shock of 35 basis points ("Interest rate difference") for the base case $\kappa_{s_{b, h}}=0.0001$ (blue, solid) versus the case where $\kappa_{s_{b, h}}=0.00005$ (red, slotted). The panel "Corporate securities (b)" denotes the value of intermediaries' corporate securities holdings $q_{t}^{k} s_{t}^{k}$. Similarly, the panel "Government bonds (b)" denotes the value of intermediaries' government bond holdings $q_{t}^{b} s_{t}^{b}$. Central bank funding is expressed in terms of annual steady state output.

Central bank funding shock 35 bps.: $\theta_{t}^{k}=0.40$ vs. $\theta_{t}^{k}=0.50$ (small open economy)


Figure 15: Impulse response functions for a central bank funding shock of 35 basis points ("Interest rate difference") for the base case $\theta_{t}^{k}=0.40$ (blue, solid) versus the case where $\theta_{t}^{k}=0.50$ (red, slotted). The panel "Corporate securities (b)" denotes the value of intermediaries' corporate securities holdings $q_{t}^{k} s_{t}^{k}$. Similarly, the panel "Government bonds (b)" denotes the value of intermediaries' government bond holdings $q_{t}^{b} s_{t}^{b}$. Central bank funding is expressed in terms of annual steady state output.

Financial crisis: no policy vs limited LTRO vs limited LTRO $\theta_{k}=0.5$ (small open economy)


Figure 16: Impulse response functions for a financial crisis, initiated by a capital quality shock of $3.5 \%$. The figure compares a scenario with no additional policy (blue, solid) with a scenario in which the central bank lowers the nominal interest rate on central bank funding with respect to the nominal interest rate on deposits with 35 quarterly basis points in line with the capital quality shock (red, slotted), and with the same intervention with $\theta_{k}=0.5$ (black, dashed). The panel "Corporate securities (b)" denotes the value of intermediaries' corporate securities holdings $q_{t}^{k} s_{t}^{k}$. Similarly, the panel "Government bonds (b)" denotes the value of intermediaries' government bond holdings $q_{t}^{b} s_{t}^{b}$. Central bank funding is expressed in terms of annual steady state output.

Financial crisis: no policy vs LTRO vs LTRO with $\theta_{k}=0.5$ (small open economy)


Figure 17: Impulse response functions for a financial crisis, initiated by a capital quality shock of $3.5 \%$. The figure compares a scenario with no additional policy (blue, solid) with a scenario in which the central bank lowers the nominal interest rate on central bank funding with respect to the nominal interest rate on deposits with 35 quarterly basis points for 12 quarters, capturing the three-year LTROs (red, slotted), and the same central bank intervention with $\theta_{k}=0.5$ (black, dashed). The panel "Corporate securities (b)" denotes the value of intermediaries' corporate securities holdings $q_{t}^{k} s_{t}^{k}$. Similarly, the panel "Government bonds (b)" denotes the value of intermediaries' government bond holdings $q_{t}^{b} s_{t}^{b}$. Central bank funding is expressed in terms of annual steady state output.

Central bank funding shock 35 bps.: base case vs. $\theta_{k}=\theta_{b}=0.425\left(\bar{\lambda}_{b} / \bar{\lambda}_{k}=0.5\right)$


Figure 18: Impulse response functions for a central bank funding shock of 35 basis points ("Interest rate difference") for the base case (blue, solid) versus a model version in which $\theta_{t}^{k}=\theta_{t}^{b}=0.425$. The panel "Corporate securities (b)" denotes the value of intermediaries' corporate securities holdings $q_{t}^{k} s_{t}^{k}$. Similarly, the panel "Government bonds (b)" denotes the value of intermediaries' government bond holdings $q_{t}^{b} s_{t}^{b}$. Central bank funding is expressed in terms of annual steady state output.

Central bank funding shock 35 bps.: $\kappa_{s_{b, h}}=0.0001$ vs. $\kappa_{s_{b, h}}=0.00005\left(\bar{\lambda}_{b} / \bar{\lambda}_{k}=0.5\right)$


Figure 19: Impulse response functions for a central bank funding shock of 35 basis points ("Interest rate difference") for the base case $\kappa_{s_{b, h}}=0.0001$ (blue, solid) versus the case where $\kappa_{s_{b, h}}=0.00005$ (red, slotted). The panel "Corporate securities (b)" denotes the value of intermediaries' corporate securities holdings $q_{t}^{k} s_{t}^{k}$. Similarly, the panel "Government bonds (b)" denotes the value of intermediaries' government bond holdings $q_{t}^{b} s_{t}^{b}$. Central bank funding is expressed in terms of annual steady state output.

Central bank funding shock $35 \mathrm{bps} .: \theta_{t}^{k}=0.40$ vs. $\theta_{t}^{k}=0.50\left(\bar{\lambda}_{b} / \bar{\lambda}_{k}=0.5\right)$


Figure 20: Impulse response functions for a central bank funding shock of 35 basis points ("Interest rate difference") for the base case $\theta_{t}^{k}=0.40$ (blue, solid) versus the case where $\theta_{t}^{k}=0.50$ (red, slotted). The panel "Corporate securities (b)" denotes the value of intermediaries' corporate securities holdings $q_{t}^{k} s_{t}^{k}$. Similarly, the panel "Government bonds (b)" denotes the value of intermediaries' government bond holdings $q_{t}^{b} s_{t}^{b}$. Central bank funding is expressed in terms of annual steady state output.

Financial crisis: no policy vs limited LTRO vs limited LTRO $\theta_{k}=0.5\left(\bar{\lambda}_{b} / \bar{\lambda}_{k}=0.5\right)$


Figure 21: Impulse response functions for a financial crisis, initiated by a capital quality shock of $3.5 \%$. The figure compares a scenario with no additional policy (blue, solid) with a scenario in which the central bank lowers the nominal interest rate on central bank funding with respect to the nominal interest rate on deposits with 35 quarterly basis points in line with the capital quality shock (red, slotted), and with the same intervention with $\theta_{k}=0.5$ (black, dashed). The panel "Corporate securities (b)" denotes the value of intermediaries' corporate securities holdings $q_{t}^{k} s_{t}^{k}$. Similarly, the panel "Government bonds (b)" denotes the value of intermediaries' government bond holdings $q_{t}^{b} s_{t}^{b}$. Central bank funding is expressed in terms of annual steady state output.

Financial crisis: no policy vs LTRO vs LTRO with $\theta_{k}=0.5\left(\bar{\lambda}_{b} / \bar{\lambda}_{k}=0.5\right)$


Figure 22: Impulse response functions for a financial crisis, initiated by a capital quality shock of $3.5 \%$. The figure compares a scenario with no additional policy (blue, solid) with a scenario in which the central bank lowers the nominal interest rate on central bank funding with respect to the nominal interest rate on deposits with 35 quarterly basis points for 12 quarters, capturing the three-year LTROs (red, slotted), and the same central bank intervention with $\theta_{k}=0.5$ (black, dashed). The panel "Corporate securities (b)" denotes the value of intermediaries' corporate securities holdings $q_{t}^{k} s_{t}^{k}$. Similarly, the panel "Government bonds (b)" denotes the value of intermediaries' government bond holdings $q_{t}^{b} s_{t}^{b}$. Central bank funding is expressed in terms of annual steady state output.

## Domestic bond holdings



Figure 23: Domestic government bond holdings of Monetary Financial Institutions (MFIs) excluding the European System of Central Banks in Italy (IT), Spain (ES), and Portugal (PT) from January 2011 to January 2013. The bond holdings are normalized at 100 in December 2011. Source: ECB.

# List of research reports 

15001-EEF: Bao, T., X. Tian, X. Yu, Dictator Game with Indivisibility of Money
15002-GEM: Chen, Q., E. Dietzenbacher, and B. Los, The Effects of Ageing and Urbanization on China's Future Population and Labor Force

15003-EEF: Allers, M., B. van Ommeren, and B. Geertsema, Does intermunicipal cooperation create inefficiency? A comparison of interest rates paid by intermunicipal organizations, amalgamated municipalities and not recently amalgamated municipalities

15004-EEF: Dijkstra, P.T., M.A. Haan, and M. Mulder, Design of Yardstick Competition and Consumer Prices: Experimental Evidence

15005-EEF: Dijkstra, P.T., Price Leadership and Unequal Market Sharing: Collusion in Experimental Markets

15006-EEF: Anufriev, M., T. Bao, A. Sutin, and J. Tuinstra, Fee Structure, Return Chasing and Mutual Fund Choice: An Experiment

15007-EEF: Lamers, M., Depositor Discipline and Bank Failures in Local Markets During the Financial Crisis

15008-EEF: Oosterhaven, J., On de Doubtful Usability of the Inoperability IO Model
15009-GEM: Zhang, L. and D. Bezemer, A Global House of Debt Effect? Mortgages and Post-Crisis Recessions in Fifty Economies

15010-I\&O: Hooghiemstra, R., N. Hermes, L. Oxelheim, and T. Randøy, The Impact of Board Internationalization on Earnings Management

15011-EEF: Haan, M.A., and W.H. Siekman, Winning Back the Unfaithful while Exploiting the Loyal: Retention Offers and Heterogeneous Switching Costs

15012-EEF: Haan, M.A., J.L. Moraga-González, and V. Petrikaite, Price and Match-Value Advertising with Directed Consumer Search

15013-EEF: Wiese, R., and S. Eriksen, Do Healthcare Financing Privatisations Curb Total Healthcare Expenditures? Evidence from OECD Countries

15014-EEF: Siekman, W.H., Directed Consumer Search
15015-GEM: Hoorn, A.A.J. van, Organizational Culture in the Financial Sector: Evidence from a Cross-Industry Analysis of Employee Personal Values and Career Success

15016-EEF: Te Bao, and C. Hommes, When Speculators Meet Constructors: Positive and Negative Feedback in Experimental Housing Markets

15017-EEF: Te Bao, and Xiaohua Yu, Memory and Discounting: Theory and Evidence
15018-EEF: Suari-Andreu, E., The Effect of House Price Changes on Household Saving Behaviour: A Theoretical and Empirical Study of the Dutch Case

15019-EEF: Bijlsma, M., J. Boone, and G. Zwart, Community Rating in Health Insurance: Trade-off between Coverage and Selection

15020-EEF: Mulder, M., and B. Scholtens, A Plant-level Analysis of the Spill-over Effects of the German Energiewende

15021-GEM: Samarina, A., L. Zhang, and D. Bezemer, Mortgages and Credit Cycle Divergence in Eurozone Economies

16001-GEM: Hoorn, A. van, How Are Migrant Employees Manages? An Integrated Analysis

16002-EEF: Soetevent, A.R., Te Bao, A.L. Schippers, A Commercial Gift for Charity
16003-GEM: Bouwmeerster, M.C., and J. Oosterhaven, Economic Impacts of Natural Gas Flow Disruptions

16004-MARK: Holtrop, N., J.E. Wieringa, M.J. Gijsenberg, and P. Stern, Competitive Reactions to Personal Selling: The Difference between Strategic and Tactical Actions

16005-EEF: Plantinga, A. and B. Scholtens, The Financial Impact of Divestment from Fossil Fuels

16006-GEM: Hoorn, A. van, Trust and Signals in Workplace Organization: Evidence from Job Autonomy Differentials between Immigrant Groups

16007-EEF: Willems, B. and G. Zwart, Regulatory Holidays and Optimal Network Expansion

16008-GEF: Hoorn, A. van, Reliability and Validity of the Happiness Approach to Measuring Preferences

16009-EEF: Hinloopen, J., and A.R. Soetevent, (Non-)Insurance Markets, Loss Size Manipulation and Competition: Experimental Evidence

16010-EEF: Bekker, P.A., A Generalized Dynamic Arbitrage Free Yield Model
16011-EEF: Mierau, J.A., and M. Mink, A Descriptive Model of Banking and Aggregate Demand

16012-EEF: Mulder, M. and B. Willems, Competition in Retail Electricity Markets: An Assessment of Ten Year Dutch Experience

16013-GEM: Rozite, K., D.J. Bezemer, and J.P.A.M. Jacobs, Towards a Financial Cycle for the US, 1873-2014

16014-EEF: Neuteleers, S., M. Mulder, and F. Hindriks, Assessing Fairness of Dynamic Grid Tariffs

16015-EEF: Soetevent, A.R., and T. Bružikas, Risk and Loss Aversion, Price Uncertainty and the Implications for Consumer Search

16016-HRM\&OB: Meer, P.H. van der, and R. Wielers, Happiness, Unemployment and Self-esteem

16017-EEF: Mulder, M., and M. Pangan, Influence of Environmental Policy and Market Forces on Coal-fired Power Plants: Evidence on the Dutch Market over 2006-2014

16018-EEF: Zeng,Y., and M. Mulder, Exploring Interaction Effects of Climate Policies: A Model Analysis of the Power Market

16019-EEF: Ma, Yiqun, Demand Response Potential of Electricity End-users Facing Real Time Pricing

16020-GEM: Bezemer, D., and A. Samarina, Debt Shift, Financial Development and Income Inequality in Europe

16021-EEF: Elkhuizen, L, N. Hermes, and J. Jacobs, Financial Development, Financial Liberalization and Social Capital

16022-GEM: Gerritse, M., Does Trade Cause Institutional Change? Evidence from Countries South of the Suez Canal

16023-EEF: Rook, M., and M. Mulder, Implicit Premiums in Renewable-Energy Support Schemes

17001-EEF: Trinks, A., B. Scholtens, M. Mulder, and L. Dam, Divesting Fossil Fuels: The Implications for Investment Portfolios

17002-EEF: Angelini, V., and J.O. Mierau, Late-life Health Effects of Teenage Motherhood
17003-EEF: Jong-A-Pin, R., M. Laméris, and H. Garretsen, Political Preferences of (Un)happy Voters: Evidence Based on New Ideological Measures

17004-EEF: Jiang, X., N. Hermes, and A. Meesters, Financial Liberalization, the Institutional Environment and Bank Efficiency

17005-EEF: Kwaak, C. van der, Financial Fragility and Unconventional Central Bank Lending Operations

17006-EEF: Postelnicu, L. and N. Hermes, The Economic Value of Social Capital
17007-EEF: Ommeren, B.J.F. van, M.A. Allers, and M.H. Vellekoop, Choosing the Optimal Moment to Arrange a Loan

17008-EEF: Bekker, P.A., and K.E. Bouwman, A Unified Approach to Dynamic MeanVariance Analysis in Discrete and Continuous Time

17009-EEF: Bekker, P.A., Interpretable Parsimonious Arbitrage-free Modeling of the Yield Curve

17010-GEM: Schasfoort, J., A. Godin, D. Bezemer, A. Caiani, and S. Kinsella, Monetary Policy Transmission in a Macroeconomic Agent-Based Model

17011-I\&O: Bogt, H. ter, Accountability, Transparency and Control of Outsourced Public Sector Activities

17012-GEM: Bezemer, D., A. Samarina, and L. Zhang, The Shift in Bank Credit Allocation: New Data and New Findings

17013-EEF: Boer, W.I.J. de, R.H. Koning, and J.O. Mierau, Ex-ante and Ex-post Willingness-to-pay for Hosting a Major Cycling Event

17014-OPERA: Laan, N. van der, W. Romeijnders, and M.H. van der Vlerk, Higher-order Total Variation Bounds for Expectations of Periodic Functions and Simple Integer Recourse Approximations

17015-GEM: Oosterhaven, J., Key Sector Analysis: A Note on the Other Side of the Coin
17016-EEF: Romensen, G.J., A.R. Soetevent: Tailored Feedback and Worker Green Behavior: Field Evidence from Bus Drivers

17017-EEF: Trinks, A., G. Ibikunle, M. Mulder, and B. Scholtens, Greenhouse Gas
Emissions Intensity and the Cost of Capital
17018-GEM: Qian, X. and A. Steiner, The Reinforcement Effect of International Reserves for Financial Stability

17019-GEM/EEF: Klasing, M.J. and P. Milionis, The International Epidemiological Transition and the Education Gender Gap

2018001-EEF: Keller, J.T., G.H. Kuper, and M. Mulder, Mergers of Gas Markets Areas and Competition amongst Transmission System Operators: Evidence on Booking Behaviour in the German Markets

2018002-EEF: Soetevent, A.R. and S. Adikyan, The Impact of Short-Term Goals on LongTerm Objectives: Evidence from Running Data

2018003-MARK: Gijsenberg, M.J. and P.C. Verhoef, Moving Forward: The Role of Marketing in Fostering Public Transport Usage

2018004-MARK: Gijsenberg, M.J. and V.R. Nijs, Advertising Timing: In-Phase or Out-ofPhase with Competitors?

2018005-EEF: Hulshof, D., C. Jepma, and M. Mulder, Performance of Markets for European Renewable Energy Certificates

2018006-EEF: Fosgaard, T.R., and A.R. Soetevent, Promises Undone: How Committed Pledges Impact Donations to Charity

2018007-EEF: Durán, N. and J.P. Elhorst, A Spatio-temporal-similarity and Common Factor Approach of Individual Housing Prices: The Impact of Many Small Earthquakes in the North of Netherlands

2018008-EEF: Hermes, N., and M. Hudon, Determinants of the Performance of Microfinance Institutions: A Systematic Review

2018009-EEF: Katz, M., and C. van der Kwaak, The Macroeconomic Effectiveness of Bank Bail-ins

2018010-OPERA: Prak, D., R.H. Teunter, M.Z. Babai, A.A. Syntetos, and J.E. Boylan, Forecasting and Inventory Control with Compound Poisson Demand Using Periodic Demand Data

2018011-EEF: Brock, B. de, Converting a Non-trivial Use Case into an SSD: An Exercise
2018012-EEF: Harvey, L.A., J.O. Mierau, and J. Rockey, Inequality in an Equal Society
2018013-OPERA: Romeijnders, W., and N. van der Laan, Inexact cutting planes for twostage mixed-integer stochastic programs

2018014-EEF: Green, C.P., and S. Homroy, Bringing Connections Onboard: The Value of Political Influence

2018015-OPERA: Laan, N. van der, and W. Romeijnders, Generalized aplhaapproximations for two-stage mixed-integer recourse models

2018016-GEM: Rozite, K., Financial and Real Integration between Mexico and the United States

2019001-EEF: Lugalla, I.M., J. Jacobs, and W. Westerman, Drivers of Women Entrepreneurs in Tourism in Tanzania: Capital, Goal Setting and Business Growth

2019002-EEF: Brock, E.O. de, On Incremental and Agile Development of (Information) Systems

2019003-OPERA: Laan, N. van der, R.H. Teunter, W. Romeijnders, and O.A. Kilic, The Data-driven Newsvendor Problem: Achieving On-target Service Levels.

2019004-EEF: Dijk, H., and J. Mierau, Mental Health over the Life Course: Evidence for a U-Shape?

2019005-EEF: Freriks, R.D., and J.O. Mierau, Heterogeneous Effects of School Resources on Child Mental Health Development: Evidence from the Netherlands.

2019006-OPERA: Broek, M.A.J. uit het, R.H. Teunter, B. de Jonge, J. Veldman, Joint Condition-based Maintenance and Condition-based Production Optimization.

2019007-OPERA: Broek, M.A.J. uit het, R.H. Teunter, B. de Jonge, J. Veldman, Joint Condition-based Maintenance and Load-sharing Optimization for Multi-unit Systems with Economic Dependency

2019008-EEF: Keller, J.T. G.H. Kuper, and M. Mulder, Competition under Regulation: Do Regulated Gas Transmission System Operators in Merged Markets Compete on Network Tariffs?

2019009-EEF: Hulshof, D. and M. Mulder, Renewable Energy Use as Environmental CSR Behavior and the Impact on Firm Profit

2019010-EEF: Boot, T., Confidence Regions for Averaging Estimators

2020001-OPERA: Foreest, N.D. van, and J. Wijngaard. On Proportionally Fair Solutions for the Divorced-Parents Problem

2020002-EEF: Niccodemi, G., R. Alessie, V. Angelini, J. Mierau, and T. Wansbeek. Refining Clustered Standard Errors with Few Clusters

2020003-I\&O: Bogt, H. ter, Performance and other Accounting Information in the Public Sector: A Prominent Role in the Politicians' Control Tasks?

2020004-I\&O: Fisch, C., M. Wyrwich, T.L. Nguyen, and J.H. Block, Historical Institutional Differences and Entrepreneurship: The Case of Socialist Legacy in Vietnam

2020005-I\&O: Fritsch, M. and M. Wyrwich. Is Innovation (Increasingly) Concentrated in Large Cities? An Internatinal Comparison

2020006-GEM: Oosterhaven, J., Decomposing Economic Growth Decompositions.
2020007-I\&O: Fritsch, M., M. Obschonka, F. Wahl, and M. Wyrwich. The Deep Imprint of Roman Sandals: Evidence of Long-lasting Effects of Roman Rule on Personality, Economic Performance, and Well-Being in Germany

2020008-EEF: Heijnen, P., On the Computation of Equilibrium in Discontinuous Economic Games

2020009-EEF: Romensen, G.J. and A.R. Soetevent, Improving Worker Productivity Through Tailored Performance Feedback: Field Experimental Evidence from Bus Drivers

2020010-EEF: Rao, Z., M. Groneck, and R. Alessie, Should I Stay or Should I Go? Intergenerational Transfers and Residential Choice. Evidence from China

2020011-EEF: Kwaak, C. van der, Unintended Consequences of Central Bank Lending in Financial Crises


[^0]:    *The first draft of this paper (2015) circulated under the title "Financial Fragility and Unconventional Central Bank Lending Operations". I acknowledge the generous support of the Dutch Organization for Sciences, through the NWO Research Talent Grant No. 406-13-063. I am grateful to Sweder van Wijnbergen, Wouter den Haan, Nicola Gennaioli, Jose Victor Rios-Rull, Ricardo Reis, Franklin Allen, Ethan Ilzetzki, Lukas Schmid, Thomas Eisenbach, Toni Ahnert, Christian Stoltenberg, Bjoern Bruggeman, Petr Sedlacek, Omar Rachedi, Anatoli Segura, Nuno Palma, Agnese Leonello, Enrico Mallucci, Alex Clymo, Patrick Tuijp, Oana Furtuna, Damiaan Chen, Lucyna Gornicka and Egle Jakucionyte, as well as seminar participants at the University of Manchester, the University of Kent, the National Bank of Hungary, the University of Groningen, Bank of Lithuania, the University of Amsterdam, and the Tinbergen Institute for helpful comments and suggestions.
    ${ }^{\dagger}$ Affiliation: Rijksuniversiteit Groningen. Address: Nettelbosje 2, 9747 AE Groningen. Email address: c.g.f.van.der.kwaak@rug.nl. Telephone: +31(0)50 3633760.

[^1]:    ${ }^{1}$ Italian macrodevelopments affect the policy rate one for one in a closed economy, while they do not affect the policy rate at all in a small open economy that is a member of a currency union. In reality, Italy comprises approximately $15 \%$ of Eurozone GDP, implying that Italian macrodevelopments will affect the policy rate of the ECB. However, their influence will be much smaller than that in a closed-economy model.

[^2]:    ${ }^{2}$ The ECB refers to its lending operations as 'refinancing operations'. In this section I will follow the ECB's terminology.
    ${ }^{3}$ MFIs include "credit institutions and non-credit institutions (mainly money market funds) whose business is to receive deposits from entities other than MFIs and to grant credit and/or invest in securities" (European Central Bank 2011b).

[^3]:    ${ }^{4}$ This information can be found at https://www.ecb.europa.eu/mopo/implement/omo/html/index.en.html

[^4]:    ${ }^{5}$ The fixed-rate full allotment policy consisted of the ECB providing as much funding as demanded by MFIs as long as sufficient collateral was pledged. This policy was introduced in October 2008 in response to the Great Financial Crisis, and was supposed to be temporary. However, it is still in place today.

[^5]:    ${ }^{6}$ In reality, commercial banks can also pledge other assets as collateral, such as corporate bonds, covered bonds, and certain types of corporate loans. The central bank, however, typically provides less liquidity for one euro of those assets than for one euro of government bonds. To simplify the analysis and be able to obtain closed-form analytical expressions, I omit the possibility to pledge corporate loans as collateral in this section, as the key objective is to disentangle the different mechanisms that influence credit provision to the real economy. However, intermediaries will be able to pledge corporate loans in the infinite-horizon DSGE model in subsequent sections, in which it turns out that the qualitative results from this section carry over as long as one euro of corporate loans provides less liquidity than one euro of government bonds.
    ${ }^{7}$ See https://www.ecb.europa.eu/mopo/decisions/html/index.en.html.

[^6]:    ${ }^{8}$ Note that $\lambda_{a}$ does not refer to legal risk weights as in the Basel III regulations. Instead, this is a requirement imposed by one group of private agents (depositors) on another group of private agents (financial intermediaries), rather than a requirement imposed by the government.
    ${ }^{9}$ As central bank reserves are electronic accounts administered by the central bank, I assume that it is impossible to divert these reserves.

[^7]:    ${ }^{10}$ Another reason for assuming $r_{0}^{d} \geq r_{0}^{c b}$ which does not feature in my model but is relevant for the real world is the fact that credit risk on unsecured funding is larger than on secured funding, and therefore carries a higher interest rate, everything else equal.

[^8]:    ${ }^{11}$ I do so to smooth debt issue by the fiscal authority over time.

[^9]:    ${ }^{12}$ Financial intermediaries are not subject to limited liability in the Gertler and Karadi 2011) framework, and will therefore always repay their creditors. In addition, I calibrate the model in such a way that intermediaries never have negative net worth in equilibrium.
    ${ }^{13}$ Central banks typically operate with positive net worth, and pay part of their profits as dividends to the fiscal authority. To ensure that both features are incorporated, I set $0<\delta_{t}^{c b}<1$. Note that this implies that the fiscal authority (partially) recapitalizes the central bank when pre-dividend net worth $n_{t}^{c b *}$ turns negative.

[^10]:    ${ }^{14}$ Note that $\lambda_{t}^{k}$ and $\lambda_{t}^{b}$ do not represent the legal capital requirements from Basel III (according to which $\lambda_{t}^{b}$ should be equal to zero), but capture an agency problem between two groups of private agents (namely depositors and financial intermediaries). Such an interpretation is consistent with $\lambda_{t}^{b}>0$, which will be the case in my simulations.

[^11]:    ${ }^{15}$ Before October 2008 the interest rate on reserves was (approximately) equal to the interest rate at which MFIs borrowed from the ECB; ECB liquidity would be auctioned to Eurozone MFIs, allowing the interest rate on the ECB loans to be slightly higher than the interest rate on required reserves (MRO-rate). In October 2008 the ECB switched to a fixed rate full allotment procedure, under which Eurozone MFIs can borrow at the MRO-rate (European Central Bank 2011b). The last case exactly corresponds with $r_{t}^{n, r}=r_{t}^{n, c b}$ in my model.

[^12]:    ${ }^{16}$ Bocola 2016 estimates a model version with financial constraints that are occassionaly binding by solving the model using global solution methods and estimating it by using particle filters. Unlike Bocola (2016), my model has too many state variables to employ such a strategy.
    ${ }^{17}$ For example, in 2010 haircuts varied from $1.5 \%$ for investment grade corporate bonds with a maturity less than one year to $64.5 \%$ for non-investment grade credit claims with a maturity of more than 10 years, see also https://www.ecb.europa.eu/press/pr/date/2010/html/sp090728_1annex.en.pdf.
    ${ }^{18}$ I can write equation 20 as $\frac{\lambda_{b}}{\lambda_{k}}=\frac{\bar{r}^{b}-\bar{r}^{d}}{\bar{r}^{k}-\bar{r}^{d}}$ in the non-stochastic steady state, since the second term on the right hand side is equal to zero because $\bar{\Gamma}^{c b}=\bar{r}^{n}-\bar{r}^{n, c b}=0$.
    ${ }^{19}$ Although $\kappa_{s_{k, h}}$ did not feature in the analysis of Section 3 one can argue along similar lines as for $\kappa_{s_{b, h}}$ that it will be a key parameter for the strength of the collateral effect.

[^13]:    ${ }^{20}$ The reason why the haircut parameter $\theta_{t}^{k}$ increases by so little is the fact that the market value of intermediaries' steady state government debt $\bar{q}_{b} \bar{s}_{b}$ is less than $5 \%$ of the market value of intermediaries' steady state corporate securities $\bar{q}_{k} \bar{s}_{k}$.

[^14]:    ${ }^{21}$ The ECB announcement of this collateral relaxation can be found at https://www.ecb.europa.eu/press/pr/date/2011/html/pr111208_1.en.html
    ${ }^{22}$ Note, however, that in reality a relaxation of collateral requirements increases the central bank's credit risk, a feature that is not in my model as intermediaries do not default. In reality, however, credit risk considerations can be a factor preventing the central bank from relaxing collateral requirements.

[^15]:    ${ }^{23}$ I match the net uptake of ECB funding rather than the gross uptake, as a substantial amount of the threeyear LTROs was used by the Italian commercial banking system to repay shorter-term ECB funding from Main Refinancing Operations (MROs) and regular-maturity LTROs.

[^16]:    ${ }^{24}$ Note that the growth rate reported here is not the same as the difference between the red, slotted line on the one hand and the blue, solid line on the other. This last difference is expressed as percentage of the steady state value, whereas my growth rate is calculated as the increase in the respective variable with respect to the level of that variable under the base case scenario without three-year LTROs. This is in line with Carpinelli and Crosignani 2018, who report the growth rates with respect to the counterfactual of no three-year LTROs.

[^17]:    ${ }^{25}$ See https://www.ecb.europa.eu/mopo/implement/omo/tltro/html/index.en.html.

[^18]:    ${ }^{26}$ GIIPS-countries consist of Greece, Ireland, Italy, Portugal, and Spain.

[^19]:    ${ }^{27}$ The average maturity of government debt is given by $\frac{\sum_{j=1}^{\infty} j(1-\rho)^{j-1} x_{c}}{\sum_{j=1}^{\infty}(1-\rho)^{j-1} x_{c}}=1 / \rho$, which is therefore effectively

[^20]:    ${ }^{28}$ The case where $\vartheta>1$ happened in the Netherlands, where financial intermediaries received government

[^21]:    ${ }^{29}$ Note that $\bar{b}_{\max }$ is a parameter determining the probability of default, and does not refer to a maximum level of debt. In both Corsetti et al. (2013) and my setup there is only a stochastic maximum level of taxation, while there is no limit to the amount of debt that the sovereign can issue.

[^22]:    ${ }^{30}$ Average maturity is calculated as $\frac{\sum_{j=1}^{\infty} j(1-\rho)^{j}}{\sum_{j=1}^{\infty}(1-\rho)^{j}}=\frac{1}{\rho}$

[^23]:    ${ }^{31}$ Note that $\lambda_{t}^{k}$ and $\lambda_{t}^{b}$ are not the legal capital requirements from Basel III (according to which $\lambda_{t}^{b}$ should be equal to zero). Rather, $\lambda_{t}^{b}>0$ arises from an agency problem between two groups of private agents (namely depositors and financial intermediaries). In this model, a positive steady state spread between the return on bonds and deposits can only be attained when $\lambda_{t}^{b}>0$, see also Gertler and Karadi 2011) and Gertler and Karadi (2013) for an elaborate discussion on diversion parameters.

