Summer Course Mathematics

Facts Friday

1. Extreme points:

   - If a differentiable function goes from decreasing to increasing at the point $x = c$, the first derivative is zero ($f'(c) = 0$) and the extreme is a minimum.
   - If a differentiable function goes from increasing to decreasing at the point $x = c$, the first derivative is zero ($f'(c) = 0$) and the extreme is a maximum.
   - But: If at c the first derivative is zero ($f'(c) = 0$) and the function goes from increasing to increasing or the function goes from decreasing to decreasing, there is not an extreme.
   - A maximum point c is a global maximum point of a function f with domain D if

     $$f(x) \leq f(c) \text{ for all } x \in D.$$

   - A minimum point c is a global minimum point of a function f with domain D if

     $$f(x) \geq f(c) \text{ for all } x \in D.$$

   - If an extreme point is not a global extreme point, it is a local extreme point.

2. A procedure to find the extreme points of a differentiable function f defined on an open interval I:

   - Solve $f'(x) = 0$. The solutions are possible locations for extreme points.
   - Determine (using a sign diagram) the sign variation of f'.
   - Conclude where the function is increasing and where it is decreasing.
   - Indicate which stationary point is a maximum, a minimum, or neither.