Answers Sample Entrance Exam Mathematics

Duration: 3 hours

For entrance to the studies Economics or Business: Solve the problems 1-5.
For entrance to the study Econometrics and Operations Research: Solve the problems 1-6.

If something is not clear or when you have the idea that a problem contains a mistake, ask!
It is quite possible that in your book a subject has a different name than used in this exam.

It is not allowed to use a sheet with formulas, a graphical calculator, a symbolic calculator or a calculator with an alpha-numeric keyboard. A simple calculator is allowed and is recommended!

The weight of the respective problems are 25, 25, 18, 9, and 18 points. The total number of points is 95. The grade equals the points earned multiplied with 9/95 plus 1, rounded.
For student Econometrics: Problem 6 (20 points) is added. The total number of points is 115. The grade equals the points earned multiplied with $\frac{9}{115}$ plus 1, rounded. You should obtain at least 11 points for problem 6 to pass the exam.

A good or wrong answer is only a small part of the solution. The quality and completeness of your detailed solutions determine the points you will get. You should end an exercise with a conclusion or an answer.

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1. Basics, I.

   a. (2) $x = 2$.

   b. (3)
   (i) $2 < -4$ is not true, so $x = 0$ is not a solution.
   (ii) $-x + 2 < 2x - 4 \Rightarrow 6 < 3x \Rightarrow x > 2$.

   c. (4)
   (i) $x^2 + 3x + 2 = 0 \Rightarrow (x + 2)(x + 1) = 0 \Rightarrow x = -2$ or $x = -1$,
   (ii) $4x^2 - 16 = 0 \Rightarrow (2x + 4)(2x - 4) = 0 \Rightarrow x = -2$ or $x = 2$.

   d. (4)
   (i) $x^3 - 25x = x(x^2 - 25) = x(x + 5)(x - 5)$,
   (ii) $x^2 + 5x + 6 = (x + 2)(x + 3)$.

   e. (6)
   (i) $16x^2 = 64 \Rightarrow x^2 = 4 \Rightarrow x = 2$ or $x = -2$,
   (ii) $10 = \dfrac{20}{2x+4}, x \neq -2 \Rightarrow 10(2x + 4) = 20, x \neq -2 \Rightarrow 20x = -20, x \neq -2 \Rightarrow x = -1$,
   (iii) $27^{2x-2} = 81^x \Rightarrow 3^{3(2x-2)} = 3^{4x} \Rightarrow 6x - 6 = 4x \Rightarrow 2x = 6 \Rightarrow x = 3$,
   (iv) $\ln(x) + \ln(2x) = \ln(8), x > 0 \Rightarrow 2x^2 = 8, x > 0 \Rightarrow x = 2$.

   f. (6)
   (i) $x = 0.63$,
   (ii) $\dfrac{3}{4x+7} = \dfrac{6}{2x-5} \Rightarrow 2x - 5 = 8x + 14, x \neq -1\frac{3}{4}, x \neq 2\frac{1}{2} \Rightarrow x = -3.17$,
   (iii) $7^{3x+1} = 98 \Rightarrow 3x + 1 = \frac{\ln 98}{\ln 7} \Rightarrow x = 0.45$,
   (iv) $\log_3 x = 5.5 \Rightarrow x = 3^{5.5} = 420.89$.

2. Basics, II.

   a. (5) $x = 2, y = 1$.

   b. (5) $\dfrac{2}{x+3} + \dfrac{7}{x+2} = -1, x \neq -3, x \neq -2$
   $\Rightarrow 2(x+2) + 7(x+3) + (x+2)(x+3) = 0, x \neq -3, x \neq -2$
   $\Rightarrow x^2 + 14x + 31 = 0, x \neq -3, x \neq -2$
   $\Rightarrow x = -11.24$ or $x = -2.76$.

   c. (5) $(x-2)\sqrt{x-1} = 0 \Rightarrow x - 2 = 0$ or $x - 1 = 0, x \geq 1 \Rightarrow x = 2$ or $x = 1$.

   d. (5) $\ln(\frac{1}{3}x^{-2}) = \ln(\frac{1}{3}) + \ln(x^{-2}) = \ln 1 - \ln 3 - 2\ln x = -\ln 3 - 2\ln x,\ x > 0$.

   e. (5) $y = ax + b, a = \frac{19-9}{14-9} = 2 \Rightarrow 9 = 2 \cdot 9 + b \Rightarrow b = -9 \Rightarrow y = 2x - 9$.

3. Differentiation and shifting graphs.

   a. (5) $f'(x) = \frac{1}{2\sqrt{x}} + 4x^3, x \geq 0$.
   $f'(1) = 4\frac{1}{2} > 0 \Rightarrow f$ is increasing at $x = 1$.

   b. (5) $g'(x) = (2x+2)(x^2+x) + (x^2+2x+1)(3x^2+1)$.

   c. (5) $h'(x) = 3(x^4+4x^2+1)^2 \cdot (4x^3+8x)$.
   $h'(0) = 3(1)^2 \cdot 0 = 0 \Rightarrow h$ is neither increasing or decreasing, but stationary.

   d. (3) Shift 3 units to the left, stretch the graph in the positive Y-direction with a factor 3, shift 2 units upwards.

4. Growth processes.

   a. (3) $15000(1.024)^{10} = 11832.91$,

   b. (3) $15000(1.0255)^5 = 17012.56$,

   c. (3) $25500(0.88)^5 = 13457.16$.

5. Extremes.

   a. (3) f is negative for $x < -3$, f is positive for $-3 < x < 0$, and f is negative for $0 < x < 3$, thus f has a maximum somewhere in the interval from -3 to 0.
   f is positive for $-3 < x < 0$, f is negative for $0 < x < 3$, and f is positive for $x > 3$, thus f has a minimum somewhere in the interval from 0 to 3.

   b. (2) $x(x-3)(x+3) = x(x^2-9) = x^3 - 9x$.

   c. (4) $f'(x) = 3x^2 - 9$.
   $f'(x) = 3x^2 - 9 = 0 \Rightarrow x^2 = 3 \Rightarrow x = \pm\sqrt{3} \Rightarrow x = \pm 1.7$.

   d. (4) Make a sign diagram of $f'(x) = 3(x^2-3) = 3(x-\sqrt{3})(x+\sqrt{3})$. It follows that f is increasing for $x \leq -\sqrt{3}$ and for $x \geq \sqrt{3}$ and that f is decreasing for $-\sqrt{3} \leq x \leq \sqrt{3}$.

   e. (2) At $x = -\sqrt{3}$, f goes from increasing to decreasing. Thus, f has a maximum at $x = -\sqrt{3}$.
   At $x = \sqrt{3}$, f goes from decreasing to increasing. Thus, f has a minimum at $x = \sqrt{3}$.

   f. (3) Use the points $(-\sqrt{3}, 10.4)$ and $(\sqrt{3}, -10.4)$.

6. **For aspirant students Econometrics and Operations Research only!**

a. (3)
(i) $\sin(212^o) \approx -0.53$,
(ii) $\sin(3.2) \approx -0.06$,
(iii) $\cos(\frac{1}{3}\pi) = 0.5$.

b. (4) The range of sin is $[-1, 1]$, so the range of $5 - 3\sin(t - 2)$ is $[2, 8]$. The equation does not have a solution because 0 is not contained in this range.
$5 - 6\sin(t - 2) = 0 \Rightarrow \frac{5}{6} = \sin(t - 2) \Rightarrow$ possibility: $t - 2 \approx 0.9851 \Rightarrow t \approx 2.9852$.

c. (3) We can only take the square root of nonnegative numbers $\Rightarrow x + 2 \geq 0 \Rightarrow x \geq -2$. The outcome of a square root is always nonnegative $\Rightarrow 1 - x \leq 0 \Rightarrow x \geq 1$. By combining these two conditions, it follows that the domain is $x \geq 1$.
$-2\sqrt{x+2} = 1 - x \Rightarrow 4(x + 2) = (1 - x)^2 \Rightarrow x^2 - 6x - 7 = 0 \Rightarrow (x - 7)(x + 1) = 0 \Rightarrow$ $x = 7$ or $x = -1$. Only $x = 7$ falls within the domain $x \geq 1$.
Conclusion: The unique solution of $-2\sqrt{x+2} = 1 - x$ is $x = 7$.

d. (4) $f(x) = x\mathrm{e}^x \Rightarrow f'(x) = (1 + x)\mathrm{e}^x \Rightarrow f''(x) = (2 + x)\mathrm{e}^x$.
Stationary point: $x = -1$. $f''(-1) = \mathrm{e}^{-1} > 0 \Rightarrow$ The stationary point is a minimum.

e. (2)
(i) $\lim\limits_{x\to 1} \dfrac{x-1}{x^2-1} \lim\limits_{x\to 1} \dfrac{x-1}{(x+1)(x-1)} = \lim\limits_{x\to 1} \dfrac{1}{x+1} = \frac{1}{2}$,
(ii) $\lim\limits_{x\to -\infty} \dfrac{x|x|-2}{x^2+2} = \lim\limits_{x\to -\infty} \dfrac{-x^2-2}{x^2+2} = \lim\limits_{x\to -\infty} \dfrac{-x^2}{x^2} = -1$.

f. (4)
(i) $\displaystyle\int (6x^2 + 5)\ \mathrm{d}x = 2x^3 + 5x + C$,
(ii) $\displaystyle\int_0^2 (6x^2 + \sqrt{x})\ \mathrm{d}x = \Big|_0^2 (2x^3 + \tfrac{2}{3}x^{1\frac{1}{2}}) = 2 \cdot 2^3 + \tfrac{2}{3} \cdot 2^{1\frac{1}{2}} - 0 \approx 17.89$.