

Sample Entrance Exam Mathematics

Duration: 3 hours

Note: This test exam does not contain all subjects that have to be studied. Furthermore, the actual exam might contain subjects that are not covered by this test exam.

For entrance to studies Economics or Business: Solve the problems 1-5.

For entrance to the study Econometrics and Operations Research: Solve the problems 1-6.

If something is not clear or when you have the idea that a problem contains a mistake, ask! It is quite possible that in your book a subject has a different name than used in this exam.

It is not allowed to use a sheet with formulas, a graphical calculator, a symbolic calculator or a calculator with an alpha-numeric keyboard. A simple calculator is allowed and is recommended!

The weight of the respective problems are 25, 25, 18, 9, and 18 points. The total number of points is 95. The grade equals the points earned multiplied with $9/95$ plus 1, rounded.

For student Econometrics: Problem 6 (20 points) is added. The total number of points is 115. The grade equals the points earned multiplied with $\frac{9}{115}$ plus 1, rounded. You should obtain at least 11 points for problem 6 to pass the exam.

A good or wrong answer is only a small part of the solution. The quality and completeness of your detailed solutions determine the points you will get. You should end an exercise with a conclusion or an answer.

1. Basics, I.

- a. (2) Solve $7(x - 3) - 2 = -3(x + 1)$.
- b. (3) Given is the inequality $-1(x - 2) < 2x - 4$.
 - (i) Is $x = 0$ a solution?
 - (ii) Solve the inequality.
- c. (4) Solve
 - (i) $x^2 + 3x + 2 = 0$,
 - (ii) $4x^2 - 16 = 0$.
- d. (4) Factorize
 - (i) $x^3 - 25x$,
 - (ii) $x^2 + 5x + 6$.
- e. (6) Solve without using your calculator:
 - (i) $16x^2 = 64$,
 - (ii) $10 = \frac{20}{2x + 4}$,
 - (iii) $27^{2x-2} = 81^x$,
 - (iv) $\ln(x) + \ln(2x) = \ln(8)$, $x > 0$.
- f. (6) You should use your calculator now. Solve and approximate in two decimals:
 - (i) $x^{-5} = 10$,
 - (ii) $\frac{3}{4x + 7} = \frac{6}{2x - 5}$,
 - (iii) $7^{3x+1} = 98$,
 - (iv) $\log_3 x = 5.5$.

2. Basics, II.

- a. (5) Solve the system of equations $\begin{cases} 2x + 3y = 7 \\ 3x - 2y = 4 \end{cases}$ and check your answer.
- b. (5) Solve $\frac{2}{x+3} + \frac{7}{x+2} = -1$.
- c. (5) Determine the domain and solve $(x-2)\sqrt{x-1} = 0$.
- d. (5) Show that $\ln(\frac{1}{3}x^{-2}) = -3 - 2\ln x$ for $x > 0$.
- e. (5) The graph of a linear function passes through the points (9, 9) and (14, 19). Give the equation of this linear function.

3. Differentiation and shifting graphs.

- a. (5) Determine the derivative of $f(x) = 5 + \sqrt{x} + x^4$, $x \geq 0$.
Is f increasing or decreasing at $x = 1$?
- b. (5) Use the product rule to differentiate $y = g(x) = (x^2 + 2x + 1)(x^3 + x)$. Do not simplify or expand.
- c. (5) Use the chain rule to differentiate $y = h(x) = (x^4 + 4x^2 + 1)^3$.
Is h increasing or decreasing at $x = 0$?
- d. (3) Use the rules for shifting a graph to explain in words how you get the graph of $y = 2 + 3(x+3)^2$ from the graph of $y = x^2$. You do not have to sketch the graphs.

4. Growth processes.

Your deposit in a bank at this moment is 15,000 euro.

- a. (3) The interest rate is 2.4% per year. How much should you have deposited in a bank 10 years ago to have this deposit now.
- b. (3) The interest rate changes to 2.55% per year. Calculate the value of your deposit 5 years from now.
- c. (3) You buy a device for 25,500 euro. It depreciates 12% per year. What is its value 5 years later?

5. Extremes.

Given is the function $y = f(x) = x(x-3)(x+3)$, $x \in \mathbb{R}$.

- a. (3) Make a sign diagram of the function f and use it to explain in which interval(s) you expect at least one maximum or one minimum.
- b. (2) Show that $x(x-3)(x+3) = x^3 - 9x$.
- c. (4) Determine the first derivative of f .
Determine where $f'(x) = 0$ (use for calculator for it, 1 decimal is sufficient).
- d. (4) Where is f increasing and where decreasing?
- e. (2) Use d. to classify the extremes (maximum or minimum).
- f. (3) Sketch the graph of f .

6. For aspirant students Econometrics and Operations Research only!

- a. (3) Determine using your calculator:
- (i) $\sin(212^\circ)$,
 - (ii) $\sin(3.2)$,
 - (iii) $\cos(\frac{1}{3}\pi)$.
- b. (4) Explain why the equation $5 - 3 \sin(t - 2) = 0$ does not have a solution. Determine at least one solution of the equation $5 - 6 \sin(t - 2) = 0$.
- c. (3) Determine the domain of and solve $-2\sqrt{x+2} = 1 - x$.
- d. (4) Given is the function $y = f(x) = xe^x$. Determine where the function is convex/concave. Use this to classify the stationary point ($f'(x) = 0$).
- e. (2) Determine:
- (i) $\lim_{x \rightarrow 1} \frac{x-1}{x^2-1}$,
 - (ii) $\lim_{x \rightarrow -\infty} \frac{x|x|-2}{x^2+2}$.
- f. (4) Determine
- (i) $\int (6x^2 + 5) dx$,
 - (ii) $\int_0^2 (6x^2 + \sqrt{x}) dx$.
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