

## Checklist Course Descriptions Mathematics

Please read this document thoroughly!

Please fill in this checklist, merge it with additional (English) documents, and upload this in the University's Progress Portal in PDF. (Note: If you have obtained [one of these qualifications](#), you don't have to provide this checklist and course descriptions).

**Your course descriptions are necessary in order for the Admissions Board to determine whether your background knowledge in the relevant subject(s) is sufficient. The Admissions Board will use the checklist below to easily navigate through your course descriptions to find the relevant aspects of your course(s). Keep in mind that the course descriptions are the only documents the Admissions Board can use to determine your command of the subject, and whether this is sufficient for admission. Therefore, it is very important to provide us with elaborate course descriptions that meet the criteria (see 3. Course descriptions Information).**

Important: You can provide the course descriptions *only once*; providing incomplete or incorrect course descriptions can have consequences for your eligibility for admission. If you have any doubts regarding the required application documents, please contact the Admissions Office via [admissionsoffice@rug.nl](mailto:admissionsoffice@rug.nl). It is not possible to provide additional information about your courses *after* the Admissions Board has decided on your eligibility for admission.

## 1. General information

Bachelor's degree programme: Data Science and Society

Name applicant: [REDACTED]

Student number: [REDACTED]

School(s)/institution(s):	[REDACTED]
Country/countries:	[REDACTED]
Qualification(s)/diploma(s):	[REDACTED]
Diploma(s) obtained:	<input type="checkbox"/> Yes (all diplomas) <input type="checkbox"/> Yes (except for current programme, expected graduation date: <a href="#">Click here to enter a date.</a> ) <input checked="" type="checkbox"/> No, expected graduation date: [REDACTED]

## 2. Course information

Make sure that you include all your relevant courses of your upper secondary *and* higher education (including courses still to be completed this academic year).

*(If more rows are needed, choose 'tab' on your key board when positioned in the last column row)*

Course name: <i>(corresponding with course name on transcript)</i>	Course completed (yes or no): <i>(and indicated as such on transcript)</i>	Contact hours or workload: Specify (per week or per year): <b>Total (contact) hours</b> <b>Theory</b> <b>Practical</b> <b>Expected self-study</b> <b>Other</b>	Literature reference to the course book(s) used:	Website URL to educational institution on which course was taken: <i>(if possible a link to the specific course, preferably in English)</i> <i>(website must be easy accessible)</i>

Advanced Mathematics	No	<p><b>Total (contact) hours:</b> 5h per week</p> <p><b>Expected self-study:</b> 10h per week</p> <p><b>Extra course in preparation for the final exams in the school year 2023-2024:</b> 3h per week</p>	<p><b>Course book:</b> M. Kurczab, E. Kurczab, E. Swida. 2022. "Matematyka 4. Podręcznik do liceów i techników, zakres rozszerzony" ("Mathematics 4. Course book for high schools and technical schools, advanced level"). <i>Oficyna Edukacyjna Krzysztof Pazdro.</i> Warsaw, Poland.</p> <p><b>Exercise book:</b> M. Kurczab, E. Kurczab, E. Swida. 2022. "Matematyka 4. Zbiór zadań do liceów i techników, zakres rozszerzony" ("Mathematics 4. Set of exercises for high schools and technical schools, advanced level"). <i>Oficyna Edukacyjna Krzysztof Pazdro.</i> Warsaw, Poland.</p> <p>(These are books are for the 4th grade, and I also used books from the same series for grades 1-3)</p>	<p><b>Educational institution (high school):</b> <a href="https://www.bohaterki.edu.pl/">https://www.bohaterki.edu.pl/</a></p> <p><b>Course taken</b> (from the website of the <i>Centralna Komisja Edukacyjna</i>, the Central Education Commission): <a href="https://cke.gov.pl/images/EGZAMIN_MATURALNY_OD_2023/podstawa_programowa/matematyka.pdf">https://cke.gov.pl/images/EGZAMIN_MATURALNY_OD_2023/podstawa_programowa/matematyka.pdf</a></p>
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### 3. Course descriptions

(Tip: fill out together with teacher)

Use this checklist and indicate in the table below which topics are discussed and on which page in your actual course descriptions (which you will have to add to this checklist below) they can be found.

The course description **must** include:

- Aims/objectives of the course (learning outcomes);
- Description of the topics taught in your course;

Please note that a list of topics will not be sufficient. The Admissions Board not only needs to know which topics were discussed in your classes, but also to what extent.

- Examples of exercises done as part of this course, especially example exams or tests taken in the last two years;
- Skills on which the course focused.

**Note:** If any of the aforementioned information is missing the admissions request will not be taken into consideration.

**Important:** make sure that you merge additional documents with this checklist. You can add these documents at the end of this checklist (like curriculum or course descriptions, and/or other evidence documents such as the content of study book, summaries of topics, exam exercises and/or projects). Use titles and/or headers to distinguish between your courses and relevant (sub) sections of your courses.

Make sure that the references in the table are correct (i.e. that this checklist document matches the given information in the table). If you use page numbers as reference, first merge all documents then fill out the table so that referred page numbers match.

**Note:** It is possible that not all topics are required for the bachelor programme you are applying for.

Topic	Page and/or (sub)section number(s) (Only <u>specifically</u> refer to the course description pages in this (merged) document). Referring to pages in textbooks or documents that have not been included is not useful.
<b>Skills (example: research skills)</b>	15
<b>Formulas, functions, graphs and (in)equations:</b>	8-9
<b>Sequences:</b>	9
<b>Differential calculus:</b>	12
<b>Integral calculus:</b>	-
<b>Trigonometry:</b>	10
<b>Geometry:</b>	10-12
<b>Counting and probability:</b>	12
<b>Statistics:</b>	12
<b>Other: Real numbers and algebraic expressions (please specify)</b>	7-8

#### Additional information and remarks:

Do you have additional information or comments that could be relevant to the evaluation of your course descriptions?

Max: 600 characters

#### 4. Your Course Descriptions

Add your course descriptions below. These can be documents issued by your school/university or compiled by yourself, as long as they meet the requirements. Make sure that your course descriptions and this checklist are merged into **one** PDF file. Below you can find format suggestions and the requirements for your course descriptions. Documents issued by your school/university may look different from these suggestions based on the number of courses and type of information. Nevertheless, we urge you to make sure all the listed requirements are met.

*Add your course descriptions here (you can remove the sample texts).*

*For example: Name of course (for example: Math 11<sup>th</sup> grade (2017/2018))*

- Aims/objectives of the course (learning outcomes);
- Description of the topics taught in your course;

*Note: a list of topics will not be sufficient; the Admissions Board not only needs to know which topics were discussed in your classes, but also to what extent.*

- Examples of exercises done as part of this course, especially example exams or tests taken in the last two years;

Tip: make sure there are examples for the different topics mentioned if applicable.

- Skills on which the course focused.

*Name of course (for example: Math R102 (2020/2021))*

- Aims/objectives of the course (learning outcomes);
- Description of the topics taught in your course;

*Note: a list of topics will not be sufficient; the Admissions Board not only needs to know which topics were discussed in your classes, but also to what extent.*

- Examples of exercises done as part of this course, especially example exams or tests taken in the last two years;

Tip: make sure there are examples for the different topics mentioned if applicable.

- Skills on which the course focused.

*Higher Education - Name of course (for example: Statistics 101 (2021/2022))*

- Aims/objectives of the course (learning outcomes);
- Description of the topics taught in your course;

*Note, a list of topics will not be sufficient; the Admissions Board not only needs to know which topics were discussed in your classes, but also to what extent.*

- Examples of exercises done as part of this course, especially example exams or tests taken in the last two years;

Tip: make sure there are examples for the different topics mentioned if applicable.

- Skills on which the course focused.

**Name of course:** Advanced Mathematics, grades 1-4 (2020/2024)

**Aims / objectives of the course:**

- Calculative ability: calculations using real numbers; use of calculator; application of mathematic principles to transformation of algebraic expressions
- Use and creation of information: interpretation of mathematic and scientific information; creation of appropriate diagrams, tables and graphs; use of mathematical language to explain and justify the student's reasoning and to present data
- Use and interpretation of representations: use and creation of mathematical models
- Reasoning and argumentation: differentiation between examples and mathematical proofs; noticing analogies and similarities and reaching appropriate conclusions; strategic approach

**Description of the topics:**

**1) Real numbers:**

- equations on sets of real numbers
- mathematical proof related to the divisibility of integers and remainders
- extraction of the root with exponentiation, including roots with any given index; including practical applications, eg. to calculate compound interest, earnings from deposit and credit costs
- monotonicity of exponentiation: especially the rule that when  $x < y$  and  $a > 1$ , then  $a^x < a^y$ , and when  $x < y$  and  $0 < a < 1$ , then  $a^x > a^y$
- Numerical ranges, number lines; geometric and algebraic interpretation of absolute value; equations and inequalities with absolute value
- Logarithms with exponentiation: logarithm of product, logarithm of quotient, logarithm of power; change of the logarithm base formula

**2) Algebraic expressions:**

- Binomial theorem formulas:  $(a+b)^2$ ,  $(a-b)^2$ ,  $a^2 - b^2$ ,  $(a+b)^3$ ,  $(a-b)^3$ ,  $a^3 - b^3$ ,  $a^n - b^n$
- Polynomials and multivariate polynomials: factoring out the common factor from a polynomial; the use of the grouping method; dividing polynomial by binomial in the form of  $x - a$
- Factoring out of monomial from algebraic sum
- Rational functions

- Pascal's triangle and Newton's symbol
- Equating polynomials of integer coefficients to zero to find their roots; with roots of integers and rational numbers
- Formula of  $a^3 + b^3$ ,  $(a+b)^n$  i  $(a-b)^n$

### 3) Equations and inequations:

- Simultaneous equations and inequations
- Identities and contradictions
- Linear inequations with one unknown
- Quadratic equations
- Polynomial equations which can be transformed into quadratic equations; polynomial equations in the form of  $W(x) = 0$  for polynomials in factored form, using different methods
- Rational equations in the form of  $V(x) / W(x) = 0$ , where the polynomials  $V(x)$  and  $W(x)$  are in factored form
- Polynomial inequations such as:  $W(x) > 0$ ,  $W(x) \geq 0$ ,  $W(x) < 0$ ,  $W(x) \leq 0$  for polynomials in factored form
- Rational equations and inequations
- Vieta's formula for quadratic equations
- Equations and inequations with absolute value
- Linear equations and inequations with parameters and quadratic equations with parameters

### 4) Systems of equations

- Systems of linear equations with two unknowns; geometric interpretation of consistent, inconsistent and dependent systems
- Using systems of equations to solve word problems
- Using substitution to solve systems of equations where one equation is linear and the other is quadratic
- Systems of quadratic equations

### 5) Functions

- Using functions in the form of word description, table, chart, or formula; the ability to read and interpret value of the function given in the form of tables, charts or formulas
- Calculating value of the function given in the form of algebraic formula
- Being able to read from graph of a function: domain of a function; set of values; zeros of a function; monotonic intervals; ranges in which the function has values bigger or smaller than a given number; the biggest and the smallest value of the function; arguments of a function for which the value of that function is the biggest or the smallest it can be
- Interpreting the slope and y-intercept in a linear function
- Creates a formula for linear function knowing its graph and properties
- Drawing a graph of quadratic function base on a given formula
- Interpreting the coefficients and y-intercept in the standard form of the quadratic function; interpreting the coefficient and vertex in the vertex form; interpreting the coefficient and roots in the factored form
- Creates a formula for quadratic function knowing its graph and properties
- Using properties of linear and quadratic functions to interpret geometrics and physics questions (also presented in a practical context)
- Using the graph of a function  $y = f(x)$ , being able to draw the graph of functions:  $y = f(x + a)$ ,  $y = f(x) + b$ ,  $y = -f(x)$ ,  $y = f(-x)$
- Using the function of  $f(x) = a/x$ , including its graph
- Using exponential and logarithmic functions, including their graphs, to describe and interpret practical questions
- Using the graph of a function of  $y = f(x)$ , being able to draw the graph of a function of  $y = |f(x)|$
- Function compositions
- Proves monotonicity of a function which is based on a formula

## 6) Sequences

- Finding the nth term in a sequence
- Calculating recurrent sequences
- Checking whether the sequence is increasing or decreasing
- Checking whether the sequence is arithmetic or geometric

- Formulas for the  $n$ th term and the sum of  $n$  terms for arithmetic and geometric sequences
- Using geometric and arithmetic sequences to solve questions, including practical ones
- Calculating the limits of a sequence, using: limits of a sequence such as  $1/n$  and the  $n$ th root of  $a$ ; theorems on the limits of the sum, product, quotient and difference of convergent sequences; the squeeze theorem
- Identifying geometric convergent sequences and calculating their sum

### 7) Trigonometry

- Using the definitions of functions sine, cosine and tangent for angles from  $0-180^\circ$ , especially finding the value of trigonometric function for angles  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$
- Using the formulas:  $\sin^2\alpha + \cos^2\alpha = 1$ ;  $\tan\alpha = \sin\alpha / \cos\alpha$
- Using sine and cosine theorems and the formula for area of a triangle  $P = 1/2 \cdot a \cdot b \cdot \sin \gamma$
- Calculating the angles in a triangle and its sides
- Arc measure; changing arc measure of an angle to degree measure and the other way around
- Graphs of sine, cosine and tangent functions
- Using periodisation of trigonometric functions
- Using reduction formulas for trigonometric functions
- Trigonometry formulas: double-angle formula; addition formula for sine, cosine and tangent
- Solving trigonometric equations and inequations

### 8) Planimetrics

- Circles: diameter, radius, secants, tangents; using the Pythagoras theorem
- Identifying acute, right and obtuse triangles given side lengths; using the theorem that the side opposite the greater angle is the longest side
- Regular polygons
- Using the principles of angles and diagonals in rectangles, rhomboids, rhombus and trapezoid
- Using the principles of inscribed and central angles
- Sector area formula and arc length formula
- Thales theorem, converse Thales theorem, circle theorems (eg. alternate segment theorem)

- Triangle similarity theorems
- Using similarity ratio to calculate areas and perimeters of similar figures
- Points of triangles and their properties
- Using trigonometric functions to calculate the length of a segment in plane figures and the area of figures
- Geometric proofs
- Using the properties of a quadrilateral inscribed into a circle or circumscribed about a circle

### 9) Analytical geometry on Cartesian plane

- Properties of two lines on a plane (eg. their common point)
- Equations of lines on a plane; in the standard and slope-intercept form; equation of a line based on given properties
- Calculating the distance between two points in a coordinate system
- Equation of a circle:  $(x-a)^2 + (y-b)^2 = r^2$
- Calculating the distance between a point and a line; finding common points of a line and a circle, as well as of a line and parable which is the graph of a quadratic function; drawing circles and polygons with their middle point at the beginning of a coordinate system
- Equation of a circle in the standard form; finding common points of two circles; calculating the length and coordinates of a vector, adding vectors and multiplying them by a number

### 10) Stereometry

- recognising the mutual position of lines in space, especially perpendicular lines that do not intersect
- Angle between a line and a plane and dihedral angle between half-planes
- Recognising and calculating angles between walls and segments in prisms and pyramids
- Recognising and calculating angles between segments and angles between segments and planes in cylinders and cones
- Recognising what figure is obtained from the cross-section of a cuboid
- Calculating surface area and volume of prisms, pyramids, cylinders, cones and spheres (also using trigonometry and other theorems)
- Relationships between volumes of similar figures

- Theorem of three perpendiculars and theorem about a line perpendicular to a plane
- Cross-section of equilateral cubes and pyramids and calculating their surface area (also using trigonometry)

### 11) Combinatorial analysis

- Counting objects in simple combinatorial situations
- Counting objects using the rules of multiplication and addition in a given combinatorial situation
- Calculating the number of possible combinatorial situations (which meet given criteria), using the rules of multiplication and addition, as well as the formulas for: permutation, combination and variation
- Using the binomial coefficient and its properties when solving combinatorial problems

### 12) Calculus of probability and statistics

- Classical method of calculating probability
- Using the centile grid
- Calculating the arithmetic mean and weighted arithmetic mean; finding the median and mode
- Calculating standard deviation of a data set; interpreting this parameter for empirical data
- Calculating the expected value
- Conditional probability and Bayes' theorem; Total probability theorem
- Bernoulli's equation

### 13) Optimisation and differential calculus

- Solving optimisation problems which can be described using the quadratic function
- Calculating limits of a function
- Using Darboux's theorem to justify the existence of the zero of a function and to find the approximate value of the zero
- Derivative of a function and its geometric and physical interpretation
- Calculating the derivative of a power function with real index; calculating the derivative using theorems on the derivative of the sum, difference, product, quotient and compound function
- Using the derivative to determine the monotonicity of a function
- Solving optimisation problems using the derivative

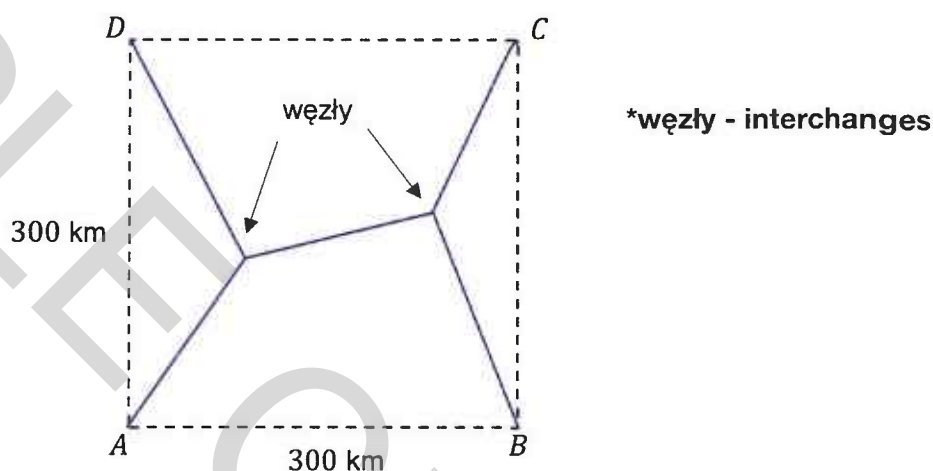
#### Examples of exercises:

- 1) Quadratic function  $f$  is given by the formula  $f(x) = px^2 + (p - 1)x + 1 - 2p$  for every  $x \in \mathbb{R}$ .

Give all values of the parameter  $p$  for which the function  $f$  has exactly two zeros between which the difference equals 1.

2) Calculate the limit  $\lim_{n \rightarrow \infty} \sqrt[n]{6^n + 7^n}$ .

3) Four cities  $A, B, C$  and  $D$  are located in the cusps of a square with a side of length 300 km. A company was given a task to design a net of roads, which would connect each pair of the cities. This net must have two interchanges and the total length of the roads in this net should be as short as possible. (An example of a net of roads with two interchanges, connecting each pair of the cities is given on the drawing below).



Calculate, what must be the length of the shortest possible net of roads and where must the interchanges be located.

4) In an infinite, decreasing geometric sequence  $(a_n)$ , given for  $n \geq 1$ , there is a condition met:

$$\frac{a_5 + a_3}{a_3} = \frac{29}{25}$$

The sum of all expressions of this sequence of even numbers is equal to 6.

Determine the formula for the  $n$ -term of this geometric sequence.

5) Solve the inequality:

$$(2 - \cos x)^2 \leq 4 \sin^2 \frac{x}{2} - 4 \cos^2 \frac{x}{2} + 4,75$$

in the set  $(0, \pi)$ .

6) Sequence  $(a_n)$  is given by the formula:

$$a_n = (n + 5)^2 \cdot \left( \frac{p + 1}{(n + 1)(n + 2)} + \frac{2p + 2}{(n + 2)(n + 3)} \right)$$

for  $n \geq 1$ .

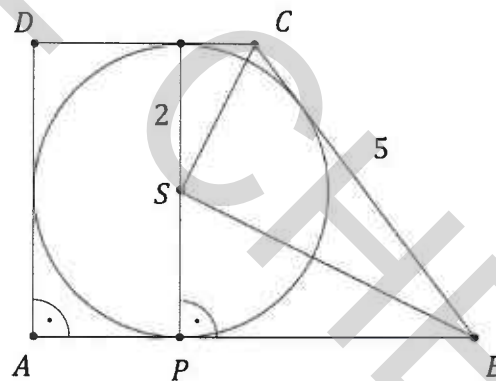
Give all values of the parameter  $p$ , for which the limit of this sequence equals 12.

7) Lines of equations  $2x+y-4m-4=0$  and  $x-3y+5m+5=0$  intersect at point  $P$  with coordinates  $(x_P, y_P)$

Give all values of the parameter  $m$ , for which the coordinates of point  $P$  meet the given conditions:

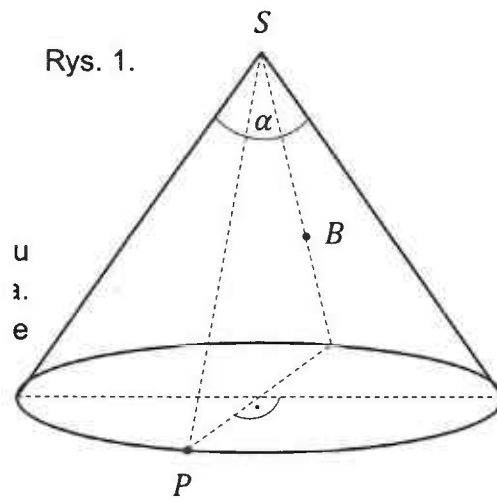
$$x_P > 0, y_P > 0, y_P \geq x_P^2 \quad \text{and} \quad y_P < -\frac{2}{x_P} + 8.$$

8) You are given a right trapezium with right angles by the cusps  $A$  and  $D$ . The  $BC$  side of the trapezium has length of 5. In this trapezium there is a circle inscribed, with its centre at point  $S$  and radius of 2. The point  $P$  is a contact point of the circle and the longer base  $AB$  of the trapezium (see the drawing below).



Show that the triangles  $BPS$  and  $BSC$  are similar triangles, and calculate the scale of their similarity.

9) Tom and Mark want to reach the top  $S$  of a certain mountain. At the beginning they are at point  $P$  located on the slope of the mountain, north of the top at height  $H_0$  metres a.s.l. Tom and Mark want to reach the basecamp  $B$  which is located south of the top on the opposite slope at height  $H_1$  metres a.s.l. Then, from the basecamp they want to reach the top, which is located at height  $H_2$  metres a.s.l. (see the drawing below).



Calculate the length of the shortest possible route to get to basecamp.

Assume that the mountain is a cone with an angle  $\alpha$ .

**Skills:**

- Mathematical reasoning
- Calculative ability
- Communicating information about the properties of mathematical objects
- Constructive thinking
- Analytical thinking
- Unconventional and creative thinking
- Creative problem-solving