

A REJECTION MECHANISM IN 2D BOUNDED CONFIDENCE PROVIDES MORE CONFORMITY

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This paper explores the dynamics of attitude change in two dimensions resulting from social interaction. We add a rejection mechanism into the 2D bounded confidence (BC) model proposed by Deffuant *et al.* (2001). Individuals are characterized by two-dimensional continuous attitudes, each associated with an uncertainty u , supposed constant in this first study. Individuals interact through random pairs. If their attitudes are closer than u on both dimensions, or further than u on both dimensions, or closer than u on one dimension and not further than $u + \delta u$ on the other dimension, then the rules of the BC model apply. But if their attitudes are closer than u on one dimension and further than $u + \delta u$ on the other dimension, then the individuals are in a dissonant state. They tend to solve this problem by shifting away their close attitudes. The model shows metastable clusters, which maintain themselves through opposite influences of competitor clusters. Our analysis and first experiments support the hypothesis that, for a large range of uncertainty values, the number of clusters grows linearly with the inverse of the uncertainty, whereas this growth is quadratic in the BC model.

Keywords: Opinion dynamics; bounded confidence model; attraction and rejection; consensus; multidimensional.

1. Introduction

Many behaviors, especially in conditions of a high involvement, can be understood as originating from underlying attitudes. For instance, one may vote for an extreme

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nationalist party because of a negative attitude toward immigrants, or consume organic food because of a positive attitude toward environmentally friendly agriculture. Hence attitudes motivate behavior and exert selective effects at various stages of information processing [1]. Consequently, to understand behavioral change it is essential to understand the underlying attitudinal dynamics that give rise to such a change. Attitude is here understood in its psychological meaning as a tendency to evaluate a particular entity with some degree of favor or disfavor. The dynamics of attitudes are closely related to social influence, which includes individual influence on feelings, beliefs and behaviors of others [2]. These dynamics are studied, by experiments in the laboratory on individuals and small groups, and are the subject of a variety of theories and assumptions. The most common assumption is a tendency of attitudes to get closer to already similar ones (attraction). A less usual assumption is a tendency to reject the other's attitude if it is psychologically uncomfortable (rejection).

Whereas an abundance of studies have been published in social psychology on the processes leading to attitude change, relatively little attention has been paid to the interactions between multiple attitudes in social interactions. Yet, the issue of interactions between attitudes in a social interaction context seems to be highly relevant. People often discuss different (unrelated) issues, and shifts on one attitude dimension may have an impact on other dimensions. For example, if a friend, who is having attitudes similar to yours on different issues, is speaking favorably about organic food, to which you have a negative attitude, the resulting dissonance may be resolved by either developing a more positive attitude to organic food as well, or by shifting away from the attitude of your friend on other issues. In contrast, if a person you disagree with on many issues also advocates organic food, your attitudes are not likely to change as no dissonance is experienced.

This paper proposes a simple model, implementing individuals with both these opposite tendencies (attraction and rejection in some conditions), and studies through agent based computer simulations how a population initially uniformly distributed in the attitude space evolves toward different global patterns. Our main result is that we observe fewer clusters than in the case of dynamics only based on attraction for a large range of uncertainty values. Before going through this result in more detail, we briefly present related research in social simulation and social psychology.

To begin, we consider the assumption of homophily. It assumes that people, especially if they are uncertain about their capacity and knowledge to evaluate a particular object, are more likely to adopt opinions and attitudes of similar others. For example, Ref. 3 shows that people like to have opinions similar to those of people they interact with. Similarity between receiver and source has a strong impact on the influence level of word of mouth [4]. Additionally, Ref. 5 suggests that homophily facilitates the flow of information between people because of perceived ease of communication. Secondly, besides a perspective on what drives people's

attitudes toward each other, some experiments and theories focus on the forces that may drive people's attitudes apart. At the individual level, the reactance theory [6], the balance theory [7], the motivation to protect oneself [8], and the social judgment theory [9] indicate that a persuasive effort can induce a rejection reaction: the behavior and/or the attitude changes in the direction opposite to the persuasion effort. In groups, the social identity theory [10], the self-categorization theory [11] and the optimal distinction theory [12] consider a capacity to differentiate from the individuals who are members of the same group by rejecting their opinions. This rejection is usually called the "boomerang effect". The conditions of its occurrence vary from one theory to another. Furthermore, some social psychologists admit that the boomerang effect remains poorly understood [13]. The social judgment theory states that uncertainty plays an important role in both attitude attraction and rejection. The social identity theory stresses that attitude rejection is linked to the salience, at a given time, of the individual social identity. At the individual level, the theories link attitude rejection to loss of control or freedom, or a negative relation with others. From these theories, we retain that attitude rejection occurs when several attitudes are implicitly or explicitly activated. Moreover, it is favored by a "dissonant" situation, such as agreement on some attitudes and disagreement on others. As an example, Ref. 14 reports about students who, informed that their attitudes regarding a particular issue were close to that of the Ku Klux Klan, decided to reinterpret this issue and adopt an attitude further away from that of the Ku Klux Klan.

Another group of interesting results for our purpose comes from the social influence paradigm, which has exhibited two important group behaviors: the average consensus [15, 16] and the polarized consensus [17]. The average consensus occurs when the value of an object given by a group after discussion is close to the average of the values given by individuals before discussion. The polarized consensus takes place when the value given by the group after discussion is significantly more extreme than the average of individual opinions before discussion. Following these studies, Nowak [18], in the social simulation domain, has recommended investigating the tendency of individuals attitudes to become more extreme (polarization) as well as the tendency of individuals to aggregate themselves in groups (clustering).

A large number of computer models are based on homophily. They postulate the existence of an attractive force between agents having close attitudes, which can be formulated using thresholds that determine when agents move toward each other's position [19–23] (see, Ref. 46 for an interesting review of opinion dynamics). This attraction threshold, also called uncertainty, can be fixed or dynamic [24, 25].

Other models, less numerous and more recent, also include a rejection mechanism in addition to assimilation. In formalizing the social judgment theory [26, 27, 35], an individual has two thresholds on an attitude dimension: one for assimilation and one for rejection (the latter is assumed to be higher than the former). In

Ref. 28, based on the theory of self-categorization and the metacontrast principle, an individual tends to minimize the distance to a prototypical opinion which defines his own group and, at the same time, he maximizes the distance to an external group. Moreover, a rejection effect appears in Refs. 29 and 30 as an emerging effect of homophilic individual interactions. This effect is due to the fact that getting closer in the 2D attitude space may in some cases result in a shift away on the global attitude (which is a weighted sum of the attitudes).

Another form of rejection mechanism can be found in the “contrarians” of Galam [38, 39], who tend to adopt an attitude which is opposite to that of the majority (attitudes are supposed binary). The stochastic Sznajd model [40, 41] also includes individuals who oppose the majority. In this model, a social temperature implies with a probability p the application of the appropriate Sznajd rule for the opinion choice of an agent, and the application of the opposite rule with a probability $1 - p$. Both of these models consider 1D binary attitudes and tend to a particular final state, due to the “contrarians” effect, for which 50% of the population adopts one opinion, and the other 50%, the other opinion.

The attitude dynamic model we propose postulates multidimensional attitudes, like in Refs. 27, 29–34, 36, 43, 44. Considering 2D attitudes with equal importance, our main assumption is that, if you strongly disagree with someone on attitude x_1 , and are close on attitude x_2 , you tend to solve the dissonance by shifting away on attitude x_2 . More precisely, when attitudes are both far from or both close to each other, we follow the hypotheses of bounded confidence (BC) models [19, 21, 23–25] (see Ref. 45 for a review): when both are close, the attitudes tend to get closer, and when both are far apart, there is no influence. Two models are usually identified as BC models: the Deffuant–Weisbuch model [19] and the Hegselmann–Krause model [21]. These two models differ regarding their communication regime. Agents of the Hegselmann–Krause model adopt the average opinion of all agents which lie in her area of confidence. Agents of the Deffuant–Weisbuch model meet in random pairwise encounters, after which they comprise or not. The model presented in this paper follows the Deffuant–Weisbuch communication regime. Therefore, our model is similar to a multidimensional BC model, except that we add the rejection mechanism when people are close on one attitude and far apart on the other.

The next part of this paper describes the model in a simplified version of the ODD framework [37], which is a protocol for describing individual and agent-based models in three blocks (overview, design concepts, and details). Following that, we present examples of simulation runs for different parameters, which lead to the hypothesis that the number of clusters grows linearly with the inverse of the uncertainty for a large range of uncertainty values. Other examples show that higher uncertainty values tend to less consensus than the classical 2D BC model. Then, we show results of a systematic exploration of the parameter space which supports these hypotheses. Finally, we will discuss the results and conclude.

2. Overview of the Model

2.1. Purpose of the model

The purpose of the model is to test the collective effects of a particular rejection mechanism in 2D BC models which are based on individual attraction mechanisms. The rejection takes place when individuals are close on one attitude and far on the other.

2.2. State variables and scales

We consider a population of N individuals, each having a 2D attitude or two different attitudes x_1 and x_2 , represented by real numbers between -1 and $+1$, and the related uncertainties u_1 and u_2 . Uncertainty is a term used for convenience, because this variable may represent confidence in one's own attitude position as well as the motivation to comply with other's attitude positions (social susceptibility). It corresponds also to the latitude of acceptance of the social judgment theory and represents the level of ego involvement in the value of the attitude. In the following experiments, all individuals have the same uncertainties U on both attitudes ($u_1 = u_2 = U$).

2.3. Process overview and scheduling

At each time step, we choose a pair of individuals A and B at random, and they may influence each other. More precisely, at each time step, the algorithm is as follows:

N times repeat:

- choose couple of individuals (A,B) at random;
- B may influence A.

The influence depends on the conditions describing the values of attitudes and uncertainties. Suppose that A has attitudes a_1 and a_2 with uncertainties u_1 and u_2 , and B has attitudes b_1 and b_2 with uncertainties v_1 and v_2 . We studied only the case where all individuals have the same uncertainty for all their attitudes. Thus, for the sake of simplicity, we used U instead of u_1 and u_2 or v_1 and v_2 in the following since $u_1 = u_2 = U$. Then, A compares its attitudes with those of B . Three cases arise.

2.3.1. Case 1: B is close to A on both attitudes:

$$|a_1^t - b_1^t| \leq U, \quad |a_2^t - b_2^t|. \quad (1)$$

Then both attitudes of A get closer to those of B :

$$a_1^{t+1} = a_1^t + \mu(b_1^t - a_1^t), \quad a_2^{t+1} = a_2^t + \mu(b_2^t - a_2^t). \quad (2)$$

Here μ is a kinetic parameter of the model, representing the velocity of the attraction or the rejection. In our following study, μ has the same value for all individuals.

2.3.2. *Case 2: B is far from A on both attitudes:*

$$|a_1^t - b_1^t| > U, \quad |a_2^t - b_2^t| > U. \quad (3)$$

Then there is no influence of B on A .

2.3.3. *Case 3: B is far from A on one attitude and close to A on the other:*

$$|a_1^t - b_1^t| \leq U, \quad |a_2^t - b_2^t| > U. \quad (4)$$

We describe only the case where people are close to each other on attitude 1 and far from each other on attitude 2, because the case where people are close on attitude 2 and far on attitude 1 is an analog with a_1 and b_1 interchanged with a_2 and b_2 .

Then two cases arise, depending on whether A and B differ strongly on attitude 2. We introduce the positive parameter δ , ruling the intolerance threshold which globally depends on the uncertainty:

Case 3.1: A and B do not differ strongly on attitude 2:

$$|a_2^t - b_2^t| \leq (1 + \delta)U. \quad (5)$$

Then the disagreement is not strong enough to trigger the rejection. A approaches B on attitude 1 and ignores B on attitude 2:

$$a_1^{t+1} = a_1^t + \mu(b_1^t - a_1^t). \quad (6)$$

Case 3.2: A and B differ strongly on attitude 2:

$$|a_2^t - b_2^t| > (1 + \delta)U. \quad (7)$$

Then A shifts away from B on attitude 1. The movement is proportional to the distance needed to get b_1 out of A 's range of uncertainty around a_1 . The specific form of the equation expresses that people move their own average attitude in order to put the average attitude of the unacceptable other out of their own attitude segment. This means that people try to adopt a new attitude in such a way that they do not judge themselves similar to the unacceptable other.

$$a_1^{t+1} = a_1^t - \mu \operatorname{psign}(b_1^t - a_1^t)(U - |b_1^t - a_1^t|), \quad (8)$$

where $\operatorname{psign}()$ is a particular sign function, which returns -1 if its argument is strictly negative, or $+1$ otherwise. The particularity compared with the standard sign function is that when the argument is 0 psign returns $+1$. Moreover, we confine the attitude within the bounds $(-1, +1)$ of the attitude space:

$$\text{If } |a_1^{t+1}| > 1 \text{ then } a_1^{t+1} := \operatorname{sign}(a_1^{t+1}). \quad (9)$$

The following figures illustrate the different types of interactions (attraction, rejection and indifference).

Figure 1 shows on the left the case where A is not influenced by B : they are far from each other on both dimensions. On the right is the case where A is attracted by B and vice-versa because they are close to each other. This means each one has his attitude in the other's acceptance zone.

Figure 2 (left) shows another case where people are close to each other on only one dimension. People are far from each other on one dimension but not far enough to consider the proximity on the other dimension as unacceptable. Thus, they attract each other on the dimension where they are close. On the contrary, Fig. 2 (right) shows the cases where people are far enough from each other on one

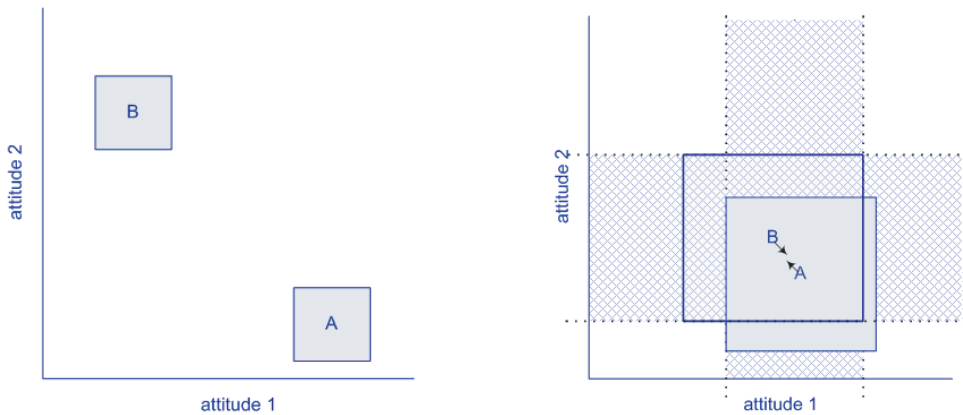


Fig. 1. A and B in a situation of no influence on both dimensions (left) and in a situation of attraction (right).

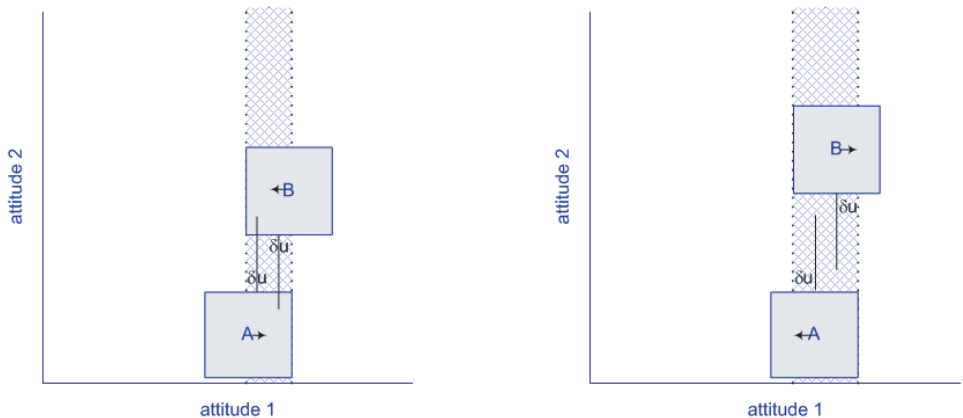


Fig. 2. Left: A and B in a situation of attraction on one dimension (on the attitude 1 dimension here) and indifference on the other dimension. Right: A and B in a situation of rejection on one dimension (on the attitude 1 dimension here) and indifference on the other dimension.

dimension. The proximity on the other dimension is perceived as unacceptable. Thus, they move away from each other on this dimension.

2.4. Initialization

When we do not vary the population size, we consider a population of 1000 individuals with two attitudes. On each dimension, the attitude is randomly initialized following a uniform distribution between -1 and $+1$. Uncertainty U is constant and identical on each dimension.

3. Analysis of Several Examples

In this section, we observe several simulation examples. Their analysis leads in particular to formulating the hypothesis that the number of clusters is a linear function of $1/U$ for weak to average values of uncertainty. Higher uncertainties globally exhibit close final states from those of the 2D BC model.

In Fig. 3, both attitude axes are represented; black spots indicate the attitude position of individual agents. We observe that the population is progressively organized into several clusters. The clusters are not regularly organized on horizontal and vertical lines, as observed with the classical BC model. Rather, they tend to be located on oblique lines, which are not strictly regular. Moreover, the individuals fluctuate in the clusters, with a constant amplitude of fluctuation, leading to a permanent diversity within the cluster. The reason is that individuals are pushed away from the cluster by other clusters, located close on one dimension and far enough on the other. These movements compensate for each other because generally there are several neighboring clusters that reject the cluster in opposite directions. Moreover, individuals are attracted by the cluster itself, especially if it includes many individuals. Therefore one can say that the clusters are metastable, because if there is a strong perturbation (deletion of a neighboring cluster) this may dramatically modify the equilibrium. This is a big difference with the classical BC model, in which, after a while, clusters keep concentrating with time, each independently of the others. A second important difference with the classical BC model is that we get clusters on the border of the attitude domain. With the BC model, the first clusters are always inside the attitude domain, on a distance which is about the uncertainty U . On the contrary, with this model, it appears that there are always some clusters which have a more extreme attitude position than any individual at the initialization. These border clusters are flat, because their neighboring clusters tend to push them away outside the attitude domain. This is a polarization phenomenon in the sense of Nowak: a part of the individuals gets more extreme. If we removed the constraint to remain within the bounds of the attitudes, the global range of attitudes would grow, and we would finally end up with stable clusters, not disturbing each other, in a significantly larger attitude domain.

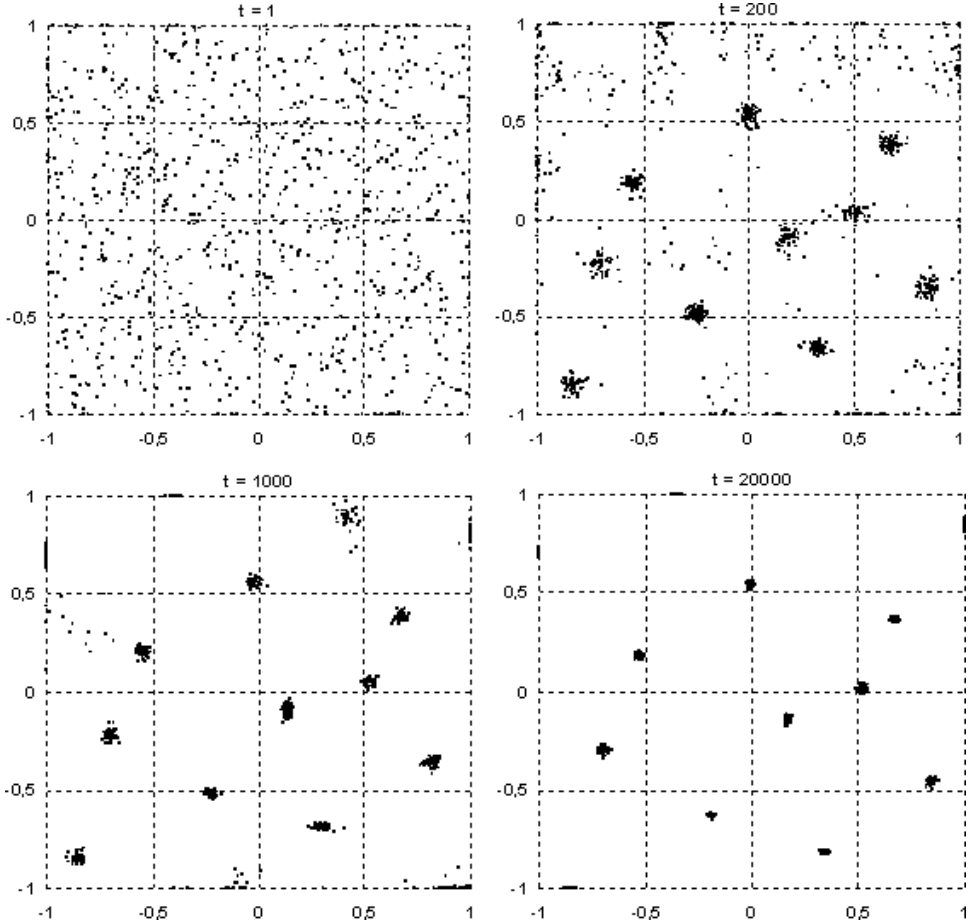


Fig. 3. Initial population uniformly distributed in 2D attitude space. $U = 0.2$, $\mu = 0.3$, $\delta = 0$. We observe the emergence of metastable clusters, with remaining fluctuations of individuals within the clusters. Moreover, some flat clusters are located on the borders of the attitude domain, containing radicalized individuals.

3.1. Evolution with uncertainty $U = 0.2$ and intolerance threshold with $\delta = 0$

Figure 3 shows an example of evolution for uncertainties $U = 0.2$ and intolerance parameter $\delta = 0$, and the kinetic parameter $\mu = 0.3$. The number of time steps t appears at the top of each picture.

3.2. Spatial organization of the clusters and hypothesis of linearity of their number with $1/U$

The spatial organization of the clusters can be further analyzed. In this particular case where $\delta = 0$, we note that there is only one cluster on a horizontal or vertical

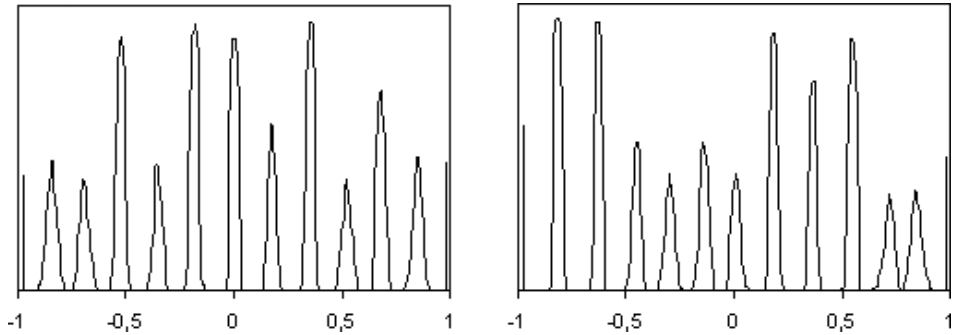


Fig. 4. Kernel density estimator on the horizontal axis (left) and the vertical axis (right), for the final situation of Fig. 3 ($U = 0.2, t = 20,000$). One notes that the 13 final clusters are regularly distributed on each axis.

line. Indeed, two clusters on the same horizontal or vertical line is an unstable situation. If the clusters are far, they tend to push each other from the line. If the clusters are close, they tend to merge. This can be checked by considering the histogram of presence of the individuals on each axis in Fig. 4. We note that 13 clusters appear on the projection of both axes. Moreover, the distance between the clusters is small enough to trigger rejection (11 clusters is the maximum, to provide a distance of at least U between two consecutive clusters), which explains why the individuals fluctuate in the clusters.

In this case, the number of clusters can be analyzed on a single axis: there should be a minimum interval between the clusters on each axis which is about the value of U . As we have seen, because of the metastability, it is possible to get slightly smaller intervals. Nevertheless, one can expect a number of clusters varying linearly with $1/U$.

3.3. Influence of intolerance threshold $\delta > 0$

When the intolerance threshold gets higher, the conditions for rejection are more restricted: the disagreement on one attitude must be higher. Figure 5 shows two examples of final attractors, for $U = 0.2$, $\delta = 1$ (left) and $\delta = 1.5$ (right). The number of clusters appears to increase with δ .

We observe that, for these values of δ , it becomes possible to get two clusters on the same horizontal or vertical line, when they are not too far apart (they remain in the tolerance zone). This explains why there are more clusters. Nevertheless, we can hypothesize that this number should still vary linearly with $1/U$, but with a higher coefficient.

Moreover, for $\delta = 1.5$, we observe flat clusters inside the attitude domain, whereas this did not take place for $\delta = 1$. Such a flat cluster appears when all the neighbor clusters are on the same line in the tolerance zone, or far on both attitudes. The rejection interactions are therefore only in one direction.

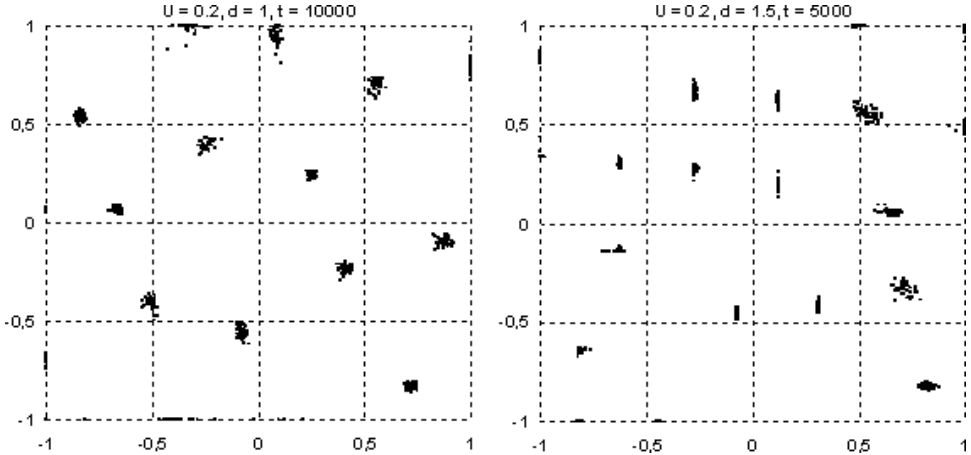


Fig. 5. Example of the final configuration for $U = 0.2$, $\mu = 0.3$, $N = 1000$ and $\delta = 1$ (left), $\delta = 1.5$ (right). It is possible to get two clusters on the same horizontal or vertical line, which is unstable when $\delta = 0$. Moreover, for $\delta = 1.5$, some clusters are flat inside the attitude domain.

3.4. Different values of uncertainty U with intolerance threshold $\delta = 0$

Figure 6 shows several attractor configurations for different values of uncertainty U . This first exploration suggests that the number of clusters decreases while U increases like with the BC model. The observations made on our first simulation extend to these cases: oscillations of individuals remain, with higher oscillations when U increases, and the state is spatially organized to avoid two clusters on the same horizontal or vertical line. In each case, we get flat clusters with the maximum value for one attitude (polarization).

For $U = 0.6$, we observe that the clusters become very concentrated — like in the simple BC model, even if, for the same uncertainty value, the model has only one cluster for a population of 1000 individuals. The reason is that with four clusters, the intervals between the clusters on the same horizontal or vertical line can easily be higher than U , and therefore avoid generating a competition between the attraction in the cluster and the rejection from the neighboring clusters.

4. Systematic Analysis of the Number of Clusters

We are interested in comparing the final number of attitude clusters with that generated by the standard BC model proposed by Ref. 19. First we describe how we compute the final number of clusters. Then, we analyze this final number of clusters regarding two different behavior zones of the BC model (see Refs. 19 and 47 for more details). The first zone is a zone for which the population is organized in several clusters; it is the object of our second point. The second zone is a zone

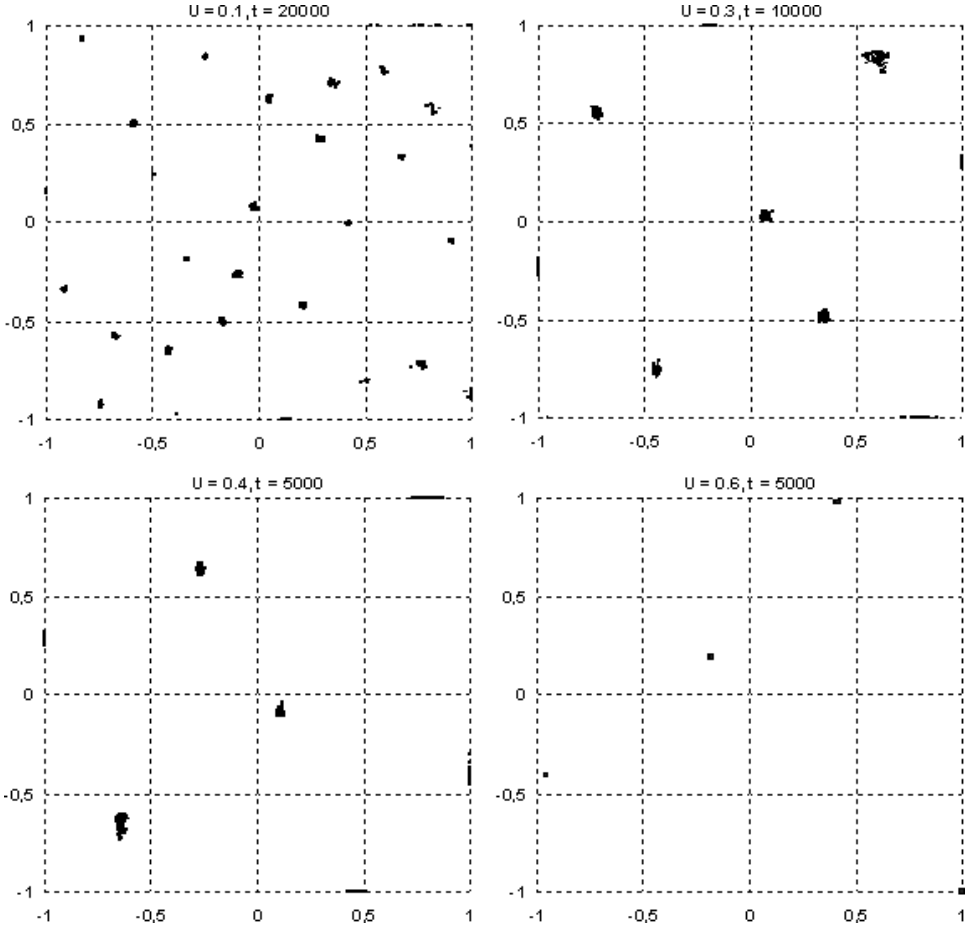


Fig. 6. Examples of attractor configurations for different values of uncertainty U and intolerance parameter $\delta = 0$, $\mu = 0.3$. Population size $N = 1000$.

for which the wide majority of people go in one cluster; it is the object of our third point.

4.1. Computing the number of clusters

From the individual-based simulations, we collect the average, minimum and maximum final number of clusters. To compute the number of clusters, we define a minimum distance ϵ between attitudes, below which we consider that they belong to the same cluster. We compute the clusters as groups of agents such that between any couple of agents of opinions x and x' in the group, there is a list of agents in the group of opinions (x_1, x_2, \dots, x_k) making a chain of couples distanced from each other of at most an Euclidean distance lower than ϵ . The following pseudocode can be used to compute the clusters; `necessaryToLookAt` is a table containing the

identification number of each individual for all the population:

```

for all i of the population
  if necessaryToLookAt[i] > 0
    currentCluster.add(i)
    compt++;
  necessaryToLookAt[i] = 0
  while currentCluster.isNotEmpty()
    for all j of the population
      if necessaryToLookAt[j] > 0
        if distance(pop[currentCluster.get(0)], pop[j] < epsilon)
          necessaryToLookAt[j] = 0
          currentCluster.add(j)
          compt++
    currentCluster.remove(0)
  nbClusters++
if compt = populationSize then i = populationSize

```

In practice, we chose $\epsilon = 0.2U$ and we neglected the clusters of a size lower than or equal to three individuals. The simulations are stopped after 1,000,000 iterations. They can be stopped before that if the number of clusters has not changed after 100,000 iterations. Even though Ref. 47 has demonstrated the interest of minor clusters in wide populations and for a high value of μ [48], we focus only on major clusters in this first study.

4.2. Final number of clusters on the “multiple clusters zone” of the BC model

The BC model, in one dimension, yields a final number of clusters n_c in a population initialized with a uniform law on an attitude space of width $2M$, with all of the same uncertainty U , which can be approximated by

$$n_c \approx \frac{M}{U}. \quad (10)$$

In the 2D case, when both attitude axes are adjusted independently and have the same uncertainty U on both attitude dimensions, this rule is repeated on all lines of the space, and therefore we get

$$n_c \approx \left(\frac{M}{U} \right)^2. \quad (11)$$

This result is confirmed by Fig. 7, which presents on abscissa $1/U^2$ and on the y axis the average number of clusters obtained on 30 replicas.

We focus on the zone where the BC model exhibits a final state including several clusters. For our attitudinal domain (attitudes between -1 and $+1$), it goes from $U = 0$ to $U = 0.54$. Figure 8 shows the number of clusters obtained with rejection

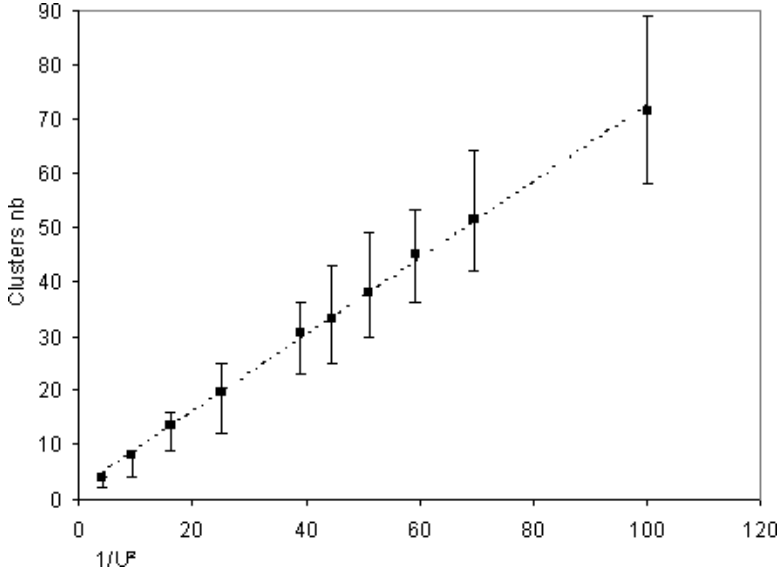


Fig. 7. Average final number of clusters of the 2D BC model as a function of $1/U^2$. Error bars indicate the minimum and maximum obtained on 30 replicas.

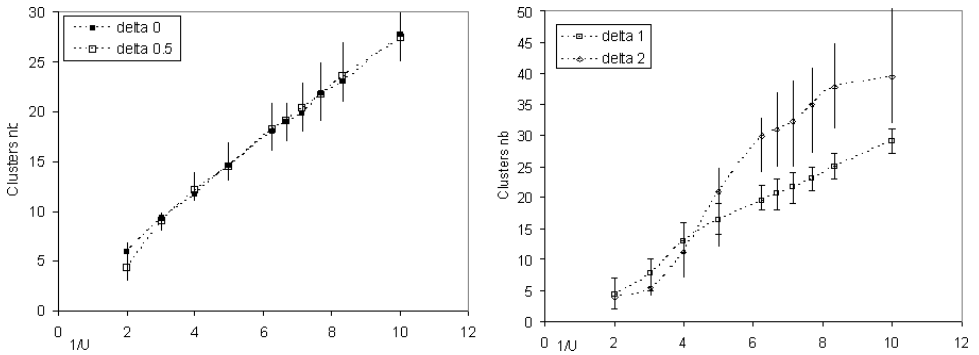


Fig. 8. Mean final number of clusters for the model with rejection as a function of $1/U$, for various values of δ . $N = 1000$ and $\delta = 0.3$. The error bars are the minimum and maximum numbers met in 30 replicas. On the left, for $\delta = 0$ and $\delta = 0.5$, the number of clusters seems linear with $1/U$. On the right, the behavior is not linear for large U .

dynamics, for different values of U and δ . These results confirm the hypothesis of linearity of the number of clusters with $1/U$ for $\delta = 0$ and $\delta = 0.5$ (left) in this zone.

For $\delta = 1, 1.5, 2$ and 3 , there is a nonlinearity for U larger than 1 (only 1 and 2 are presented in the figure). When U is larger than 0.3 , and δ is large, the conditions for rejection are much constrained by the size of the domain: two individuals must be on both sides of the domain. Most of the interactions

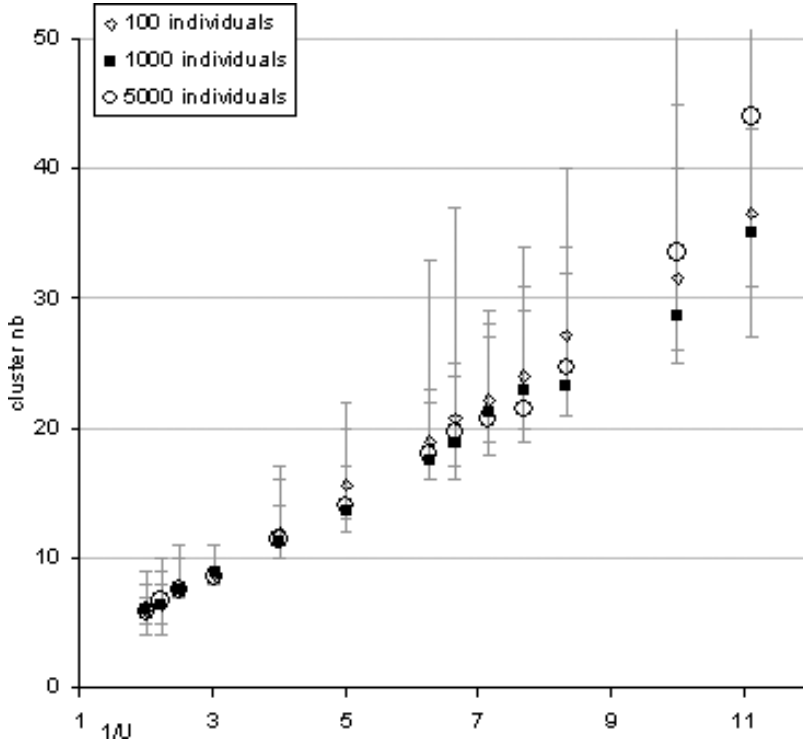


Fig. 9. Mean final number of clusters for the model with rejection as a function of $1/U$, for $\delta = 0$ and various values of population size N (100, 1000, 5000) and U (from $U = 0.09$ to $U = 0.5$). The error bars are the minimum and maximum numbers met in 30 replicas. The number of clusters seems also linear with $1/U$.

correspond therefore to the standard BC, and the curve is thus quadratic. When U decreases ($1/U$ grows), the rejection becomes more common and the curve becomes linear.

Let us now verify if these results are robust when the population size varies (100, 1000 and 5000 individuals). Figure 9 shows the number of clusters obtained with rejection dynamics for $\delta = 0$ and different values of U and population size N . To be able to compare the different population sizes, we count all the clusters (no threshold). From the figure, we note that the population size does not change our conclusion: the final number of clusters tends to be linear with $1/U$.

4.3. Final number of clusters in the “one major cluster” zone of the BC model

We now focus on the zone where the BC model exhibits one major cluster. For our attitudinal domain, it begins for $U > 0.54$. Since for $U > 1$ the rejection mechanism cannot work (all attitudes are at a distance which is within the attraction range), we study only the U value range from 0.54 to 1 (indeed, for $U > 1$, all people go

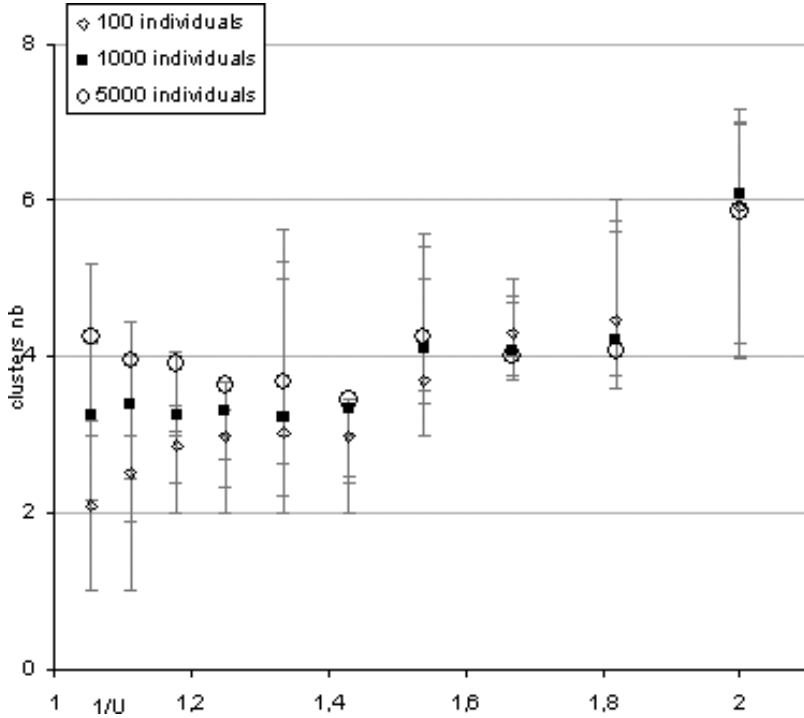


Fig. 10. Mean final number of clusters for the model with rejection as a function of $1/U$, for $\delta = 0$ and various values of N (100, 1000, 5000) and U (from $U = 0.55$ to $U = 0.95$). The error bars are the minimum and maximum numbers met in 30 replicas. The number of clusters is not linear with $1/U$.

in one unique central cluster, exactly as in the BC model). Figure 10 shows the results in this zone for different population sizes. We immediately see that the final number of clusters is not linear with $1/U$.

In the standard BC model proposed by Ref. 19, the final state for this zone is one major cluster containing a large majority of the population with, in some cases, several very minor clusters when the population is very large [46]. Figure 11 shows that our model has also, in the zone of U values, one major cluster containing a majority of the population (from a part of half to the whole population, depending on the parameter value). In our model with a rejection mechanism, we finally obtain between two and six final clusters, as shown in Fig. 10. Are the nonmajor clusters the same as those of the BC model? From Ref. 48, we know that the very small clusters of the standard BC model are very numerous and do not exist for low values of μ . In our attraction–rejection model, minor clusters are not numerous, from one to five on average, and larger than those of the classical BC model. Moreover, they remain when we run simulation with a value of μ equal to 0.01. Finally, our population size is not large enough to really observe “minor clusters” in the sense used by Ref. 47.

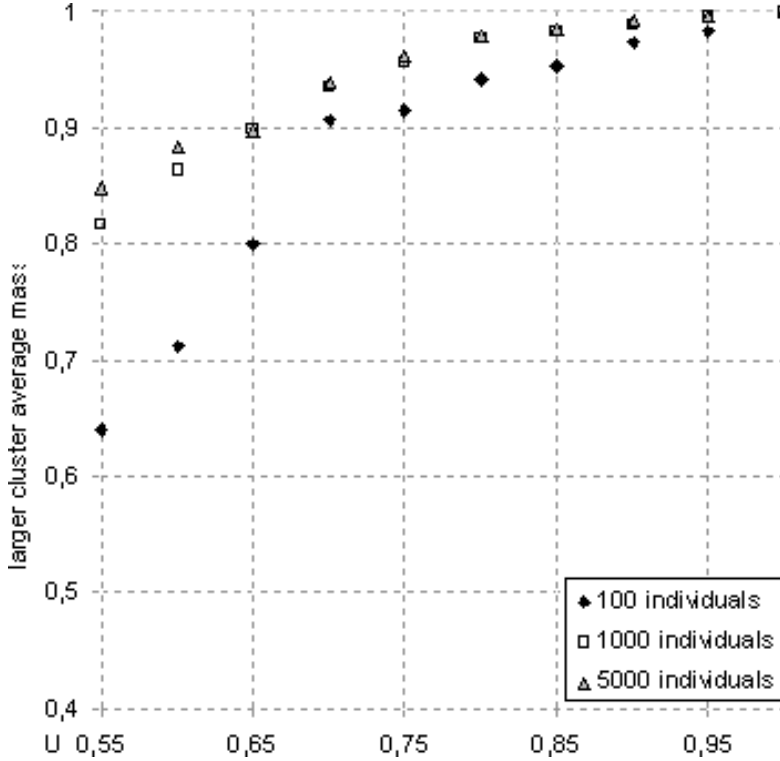


Fig. 11. Average mass of the final larger cluster for $\delta = 0$ and various values of N (100, 1000, 5000) and U (from $U = 0.55$ to $U = 0.95$).

5. Discussion and Conclusion

In the model of 2D attitude dynamics we propose, an agent shifts away from a close attitude on one axis when the interlocutor is far on the other axis. We assume that this is a way to solve the dissonance between the attitude axes. The distance threshold to trigger rejection depends on the intolerance parameter δ and on the uncertainty U , which may define a noncommitment zone, in which the dissonance is tolerated. When the conditions of rejection are not met, i.e. when we exclude the case where two individuals differ strongly on one attitude and are similar on the other attitude, the model behaves exactly like the 2D bounded confidence model.

The first explorations of this model, in the simple case where all uncertainties are the same, have shown several striking results, in comparison with the 2D BC model:

- (i) When the uncertainty is lower than 1, allowing the rejection to occur, the dynamics leads to several metastable clusters, which are generally in competition and tend to reject each other. The stability is due to contradictory rejections from neighboring clusters, which compensate for each other. If one

of its neighboring clusters is removed, the position of a cluster changes significantly, and it may even disappear. Moreover, individuals belonging to a cluster are in constant fluctuation around the cluster center, with amplitudes depending on the cluster size and on the proximity of competing clusters. In this respect, the configuration is very different from the one obtained with the simple BC model where, after a while, clusters keep concentrating with time, each independently of the other.

- (ii) Several clusters are moving toward the limits of the attitude domain. This may be interpreted as a radicalization of a part of the population, which reaches the maximum absolute value of one of the attitudes. This never happens with the 2D BC model.
- (iii) In the case where the intolerance threshold $\delta = 0$, two clusters cannot be maintained on the same horizontal or vertical line. Therefore, the clusters tend to occupy points of the space where they are as far as possible from other clusters on each axis. This analysis suggests a number of clusters growing linearly with $1/U$ for values of U for which the 2D BC model exhibits several clusters called “major” and “central” clusters by Ref. 47. However, for the 2D BC model, for this same range of U values, the cluster number grows quadratically with $1/U$ in the 2D BC model. When δ grows, configurations with more than one cluster on a line may be stabilized, but this number is limited by the size of the tolerance zone. Therefore, the growth of the cluster number should still be linear, but with a factor growing with δ . First systematic experiments support this statement, for different population sizes.
- (iv) For values of U for which the 2D BC model exhibits only one cluster called “central” in Ref. 47, our model does not follow the same law and tends to have less consensus than the 2D BC model. Indeed, depending on the parameter value, it exhibits from two to six clusters on average, with one major cluster containing a majority of people. The other clusters are generally on the limits of the attitude domain. References 47 and 48 show that the 2D BC model has, for a subpart of this zone of U values, numerous very minor clusters when μ is high and when the population size is wide. However, these very minor clusters, even if they are located close to the bound of the attitudinal space, are different from the minor clusters of extremists of our model.

These results suggest several points for discussion:

- (i) The metastability of the clusters is due to the bounds we impose on the attitude values. Indeed, without these bounds, the attitudes grow until the distance between the clusters is higher than the uncertainty in all directions. Then, the clusters do not influence each other, and they keep concentrating as in the BC model. First simulations performed on the same model with an unbounded attitude domain indicate that the final number of clusters is close to that obtained with the bounded domain. However, the unbounded case should be the object of a particular study. In any case, the metastability of the clusters is

- a particular feature of this model, which better fits real group dynamics than the perfect similarity obtained without a bound (or by a standard BC model).
- (ii) Even without bounds, we obtain a global result which shows strong similarities to social identity and self-categorization theories. Our individuals tend to minimize their in-group distance and maximize their out-group distance (to competing groups), while their dynamic has not been modeled for that. We also get some polarized groups (which have more extreme opinions than all the individuals initially). This reminds us of the results of Moscovici and Zavalloni [17]. Therefore, with a model considering only paired interactions, we get group dynamics which seem to make sense from a social psychology perspective.
 - (iii) However, the model stays highly simplified, and a challenge that remains is to check if these interesting properties last when one adds more sophisticated hypotheses. In particular, in our model, all attitudes are considered to have the same weight on the behavior, whereas one expects that only disagreements on attitudes deeply related to social identity can lead to rejection. To take this aspect into account, we should consider attitudes of different types.
 - (iv) We have chosen the communication regime of the Deffuant–Weisbuch model. The communication regime as a parameter of the BC model, as in the formulation proposed by Ref. 42, would be worth investigating. Moreover, it would be a logical extension to relate the chance of interaction to the attitude similarity between the agents, thus reflecting principles of preferential attachment.

In future research, we plan to continue exploring the properties of this model. In particular, introducing extremists like in Ref. 24 could produce unexpected effects.

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