SHARED INFORMATION SOURCES
IN EXCHANGES *

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Abstract

In financial and commodity exchanges, traders gain information from shared information sources, such as commonly accessed forecasts and standardized reports, which induce interdependence in forecast errors. In a linear normal model with noisy signals about values, I show that the presence of non-iid. errors can improve trade stability when it counters order shading, and error interdependence can improve price informativeness when it is stronger than value interdependence. From a practical information design perspective, source restrictions can prevent market collapse, and segmentation of trading venues can improve price informativeness. Gains in informativeness depend on the relative trader to trader interdependence of both values and errors.

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1 Introduction

Shared sources of information are prevalent in exchanges. Traders often gain their assessments about how much the good is worth to them from information sources that may reach other traders too. Examples of shared information sources include standardized reports in commodity markets, reports from consulting groups in securities exchanges, and supply predictions based on weather forecasts in electricity exchanges. These examples illustrate that information sources can reach various groups of traders, which, from a modeler’s perspective, motivates signal structures that are richer than the ones with independent errors. Indeed in practice, market operators recognize and report diverse pairwise interdependence in forecast errors: moreover, in response to the structure of error interdependence, they design the reach and format of information sources and facilitate segmentation of trading venues. Not relying mainly on market growth or more information counters economic intuitions which had been developed for independent forecast errors. However, I show here that shared information sources and the related interdependence in forecast errors can justify the information-based designs observed in exchanges, moreover I give sufficient conditions for improving stability and informativeness in exchanges through the design of information sources.

This paper shows that shared information sources alter possibilities of trade stability, informativeness, and learning from price. To show this, I study the effect of error correlations in a linear-normal quadratic exchange model. The results highlight opportunities for information-based market design to prevent market collapse and to enhance informativeness of the price by designing the format or reach of information sources, for example, by introducing reporting requirements or allowing the segmentation of trading venues. Recall that in models of exchanges, independence of errors is a common premise which

1Information sources can be shared among various subsets of traders. Agricultural commodity traders place their orders based on the USDA’s yield forecast, an information source that is shared among all traders. Foreign exchange traders’ most important market information sources are financial wire services, personal connections and financial analysts (Oberlechner and Hocking (2004)), information sources that are shared among many but possibly not all traders.

2As an example of the recognition of spatial error correlations, see electricity futures markets and the European Wind Energy Association, which publishes weather-based forecast errors for wind power generation (Giebel et al. (2007)).

3Regulating information sources is a common practice. For example, the Securities Exchange Commission requires firms to report quarterly results in XBLR, a unified reporting language. As the format is unified, common omissions thus common error biases are prevalent in the reports.

4I examine continuous orders trade with uniform price market clearing in the tradition of Hellwig (1980); Klemperer and Meyer (1989); Kyle (1989); Wilson (1979) and closest to the more recent works Rostek and Weretka (2012); Vives (2011). For related empirical studies, though without common value uncertainty, see, for example, Hortacsu et al. (2016); Hortacsu and Puller (2008); Ito and Reguant (2016).
embodies the implicit assumption that each trader acquires information independently and does not share information with other traders. To understand the consequences of the independence assumption, note that the well-known large-market results, such as perfect information aggregation through price and monotone information aggregation in market size, suggest little room for market design and advise that market growth resolves informational inefficiencies. However, the assumption of independence of errors fundamentally drives these results: when errors are independent, then in the limit they become negligible in large markets and the information that is revealed in the price precisely captures the payoff-relevant value. Whereas when shared information sources are present, then forecast errors shall be correlated and such correlations can be designed via source limitations, format restrictions and trade segmentations.

To explain the results on market design for the stability of exchanges, note that the effect of inference from price can compromise stability, in the sense that it can lead to the collapse of the exchange. Note that in exchanges, diminishing marginal valuations imply that along the curve of price-dependent orders, for higher prices, lower quantities are demanded. Recall also that competition of traders introduces order shading which leads to demand reduction at each price. However, inference from price further affects order shading, importantly, it impacts equilibrium existence. On the one hand, with strong common values the focus on information content in the price can outweigh the weakened effect of diminishing marginal utilities especially when error biases are low. When values are strongly correlated, then orders become upward sloping - at higher prices traders prefer to order more. As a consequence, the downward sloping residual supply precludes tradeoff between quantities and payments and the traders would order more without bounds. This shows that, if values are strongly correlated and the market is not large, then inference from price can undermine existence of equilibrium and can induce trader behavior that leads to instability. On the other hand, with low common values, the focus on information content in price typically does not outweigh the effect of diminishing...
marginal utilities especially not when errors are strongly correlated. In this case, even though traders may learn as much from price through common errors as through common values, inference from price counters order shading from market power, in other words, strengthens the tradeoff between quantities and payments.

For the formal results and the analysis of the inference-related order shading and the instability it can cause, see Propositions 1 and 2 and for the representation of the existence region, see Figures 1 and 2. These results show that when errors are on average strongly correlated then the stability of the exchange is not compromised; a sharp contrast to exchanges with strong common values. While more interdependence in errors supports stability, more variance in errors can compromise stability. Importantly, these results suggest that the design of information sources can prevent downward sloping residual supplies thus stabilize the exchange. As is the case in Example 1 restricting information sources to one information source as opposed to multiple information sources can improve the exchange by stabilizing since it introduces stronger interdependence in errors which reduces the inference-related weight on the price in the price-dependent orders.

To explain the results on market design for informational efficiency, consider comparative statics of price informativeness. Price informativeness is a property of the exchange, which shows by how much more traders learn through information aggregated in the price beyond their previously accessed information. I show here that information design can improve price informativeness and that effectiveness of information-based designs need not diminish in larger exchanges; as demonstrated by the lack of full information aggregation in Proposition 3. Moreover, the lack of full variance reduction through learning from price, as demonstrated in Proposition 4 implies that regulating the format and reach of shared information sources and that restricting trader participation based on error correlations can improve informational efficiency of exchanges. Importantly in the comparative statics exercise, different exchange sizes, different reaches of information sources or different intensities of traders’ informational connections correspond to different growth rates in average value correlation relative to average error correlation. The sufficient conditions that guarantee improvement of price informativeness in Propositions 5 and 6 are useful to develop design recommendations as summarized in Section 3. These conditions require that the difference between average value and error correlations grows faster than the average signal correlation and their application shows that improvements in price informativeness depend on the microstructure of trader to trader interdependencies in errors and values.

To study different reaches of information sources, I examine practically motivated error correlation structures and derive comparative statics and market design recommendations.
The model Common Errors captures information structures of traders who employ the same forecast methodology which induces common forecast bias. The model Group Errors captures distinct groups of traders who employ distinct forecasting technologies. The model Errors with Spatial Decay captures information structures of traders whose interdependence in forecast errors declines with the distance from other traders. The models Errors through Condensing Links, Errors through Strengthening Links and Errors in Expanding Network each capture effects of changes in an underlying network structure or in the strength of interconnectedness across traders.

To relate to the literature that studies the role of forecast errors in exchanges, note that the existing literature typically assumes independent errors therefore the current understanding of the impact of shared information sources and forecast errors is limited. The classic literature in the rational expectation equilibrium tradition attributes forecast errors to "noise traders". If the presence of noise traders decreases, then the informativeness of the price proportionally increases\(^\text{6}\). Another approach considers reduced error variance in the predictions of values, which unambiguously increases informativeness regardless of the specific error correlation structure (see examples in Vives (2008)). In this paper instead, the approach clarifies that the correlation of forecast errors, in particular the relative strength of value correlations to error correlations, affects thresholds both for existence and for improving informativeness, and that comparative statics based on exchange size and strength of interconnectedness depend on the particular microstructure implied by the reach of information sources.

2 Price-dependent Orders and Shared Information Sources

The Exchange and the Traders. Consider the trade of a perfectly divisible good with at least two traders and denote the set of traders by \(I\), the number of traders by \(I\) and a generic trader by \(i\) or \(j\). Denote the index of exchange size by \(\gamma\) where \(\gamma := 1 - 1/(I - 1)\), thus \(\gamma \in [0, 1)\). Traders participate in a uniform price market clearing exchange. That is, each trader submits a price-dependent order \(q_i(p)\)\(^7\) then pays the market clearing unit price \(p^*\) determined by the exchange operator such that \(\sum_{i \in I} q_i(p^*) = 0\) and acquires the

\(^6\)More recently, [Malinova and Smith (2006)] develop the corresponding intuition in a setting where the information structure is based on a Brownian motion process.

\(^7\)Price dependent orders are functions of the endogenously determined price. They can also be seen as continuous approximation to multiple limit orders. In exchanges, limit orders are price and quantity pairs which fill only if the price in the exchange reaches the price in the limit order.
volume based on the submitted order; \( q_i (p^*) \).

Assume that traders’ preferences are such that trader \( i \) derives ex-post utility \( u_i(q_i) = \theta_i q_i - \frac{\mu}{2} q_i^2 - pq_i \) from the exchange, that is traders’ preferences exhibit diminishing linear marginal utility from their assigned trade volume and are quasilinear in payment. Further, assume that the intercept of the marginal utility \( \theta_i \) is uncertain for each trader. Uncertainty of the parameter \( \theta_i \) captures the following. Traders may hold only partial information about, for example, the future resale value or quality of the traded good. I refer to the vector \( \theta = (\theta_1, \theta_2, \ldots, \theta_I) \) as the vector of values.

**Information Structure.** The traders’ information about \( \theta_i \) is incomplete. Assume that each trader observes a noisy signal \( s_i \) about the value, such that

\[
s_i = \theta_i + \varepsilon_i,
\]

and the random vector \((\theta_1, \theta_2, \ldots, \theta_I, \varepsilon_1, \varepsilon_2, \ldots, \varepsilon_I)\) is jointly normally distributed. I refer to the vector \( \varepsilon = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_I) \) as the vector of errors. Assume further that error expectation is 0 for all \( i \) and error variance is \( \sigma^2_{\varepsilon} \) for all \( i \), as well as that value expectation is \( m \) for all \( i \) and value variance is \( \sigma^2_{\theta} \) for all \( i \). Let \( \sigma^2 \) denote the variance ratio such that \( \sigma^2 = \sigma^2_{\varepsilon}/\sigma^2_{\theta} \). The variance ratio captures the relative strength of value variance to error variance in traders’ signals. Assume that values and errors are uncorrelated for all pairs of traders. Use the notation \( \rho^\theta_{ij} \) and \( \rho^\varepsilon_{ij} \) to denote correlation across values and errors, respectively. Use the notation \( [\Sigma^\theta]_{ij} = \sigma^2_{\theta} \rho^\theta_{ij} \) and \( [\Sigma^\varepsilon]_{ij} = \sigma^2_{\varepsilon} \rho^\varepsilon_{ij} \) to denote the covariance matrices, and write the following to summarize the information structure:

\[
\begin{bmatrix}
\theta \\
\varepsilon
\end{bmatrix} \sim \mathcal{N}
\left(
\begin{bmatrix}
mI_l \\
0_I
\end{bmatrix},
\begin{bmatrix}
\Sigma^\theta & 0_{I \times I} \\
0_{I \times I} & \Sigma^\varepsilon
\end{bmatrix}
\right).
\]

Since signal is a linear function of value and error, the vector \((s, \theta_I, \varepsilon_I)\) is jointly normal. Notice that unlike in previous models, errors may be correlated. The following assumption is a generalization of ‘equicommonality’ of Rostek and Weretka (2012).

**Assumption 1 (Equicommonal Correlations).** The information structure is such that for each trader his error’s average correlation with the other traders’ errors and his value’s average correlation with the other traders’ values is independent of the trader’s identity. That is there exist \( r^\varepsilon \) and \( r^\theta \) such that

\[
\frac{\sum_{j \in I} \rho^\varepsilon_{ij}}{I} = r^\varepsilon \quad \text{and} \quad \frac{\sum_{j \in I} \rho^\theta_{ij}}{I} = r^\theta \quad \text{for all } i \in I.
\]

In this paper, I maintain this assumption thus restrict the study to exchanges in
which each trader is symmetrically related to the market, in that, both the value and
the information of each trader identically correlate with the market value and the market
information. This assumption guarantees tractability, and importantly, it allows for a
two-dimensional representation of the equilibrium price-dependent orders, the existence
results and the informativeness measure. The two-dimensional representation is based
on the strength of error interdependence versus the strength of value interdependence.\footnote{Note ahead that comparative results and information-based market designs derive from the average correlations as well as the pairwise interdependencies.}
The notation \( r^\theta \) captures the \textit{average value correlation} and \( r^\varepsilon \) captures the \textit{average error correlation} in the exchange. For later convenience, introduce average commonalities such that
\[
\bar{\rho}^\theta = \frac{\sum_{j \neq i} \rho^\theta_{ij}}{(I - 1)} \quad \text{and} \quad \bar{\rho}^\varepsilon = \frac{\sum_{j \neq i} \rho^\varepsilon_{ij}}{(I - 1)}.
\]
Assumption 1 also ensures that symmetric equilibrium can exist and if it exists. In Subsection 2.1, I give economically motivated examples in which correlations satisfy equicommonality. These examples, motivate the information structure and describe how the two statistics change if economically relevant factors such as the exchange size or market interconnectedness change. Notice that for well-defined covariance matrices, average value correlation and error correlation fall between 0 and 1. The explanation is that correlations are less than 1 and positive semidefiniteness gives
\[
1^T C_\theta 1_I \geq 0 \quad \text{and} \quad 1^T C_\varepsilon 1_I \geq 0,
\]
from which \( 0 \leq r^\theta \leq 1 \) and \( 0 \leq r^\varepsilon \leq 1 \).

\textbf{Solution Concept.} I solve the exchange for equilibrium in symmetric linear price-dependent orders. Let the function \( p^{S_{-i}} \) denote the residual supply based on other traders’ orders. Orders constitute an equilibrium if
\[
q_i (p) = \arg \max_q E (\theta_i | s_i, p (s)) - \frac{\mu}{2} q^2 - p^{S_{-i}} (q) \quad \text{for all } p \text{ and for all } i \in I,
\]
such that market clearing holds, \( \sum_{i \in I} q_i (p^*) = 0 \), and the demand \( q_i (\cdot) \) is linear in \( s_i \) and \( p \) for all \( i \). This equilibrium concept assumes that traders infer from the price, account for their market power and optimize price by price based on inference from their private information and the information in the market clearing price. Proposition 1 establishes a lower bound characterizing existence which derives from the second order conditions representing the requirement that demand slopes be negative.

\textbf{Proposition 1 (Equilibrium Existence).} There exists a lower bounding curve \( r^\varepsilon_{\gamma, \sigma^2} (r^\theta) \) such that a symmetric linear equilibrium with downward sloping demand functions exists if and only if the average error correlation \( r^\varepsilon > r^\varepsilon_{\gamma, \sigma^2} (r^\theta) \). This bound is increasing in average value correlation \( r^\theta \). The bound is more permissive as the exchange size \( \gamma \) increases or as the variance ratio \( \sigma^2 = \sigma^2_\varepsilon / \sigma^2_\theta \) decreases.
The shaded areas in the Figure display the existence regions for different combinations of average value correlation \( r^\theta \) and average error correlation \( r^\varepsilon \). For example, if the number of participating traders \( I \) is 6 instead of 4, then the set of points \((r^\theta, r^\varepsilon)\), for which equilibrium in downward sloping orders exists, expands.

**Proof.** See Proof 2 in the Appendix. \( \square \)

Consider the lower bound \( r_{\gamma, \sigma^2}^\varepsilon (r^\theta) \) as in Proof 2, Equation (15) in the Appendix

\[
r_{\gamma, \sigma^2}^\varepsilon (r^\theta) = \max \left\{ 0, r^\theta \frac{(1 - \gamma) - \frac{2\gamma}{\sigma^2} (1 - r^\theta)}{1 - \gamma r^\theta} \right\}.
\]

Notice that equilibrium exists for all information structures in which \( r^\varepsilon > r^\theta \), regardless of the exchange size \( \gamma \) and variance ratio \( \sigma^2 \). Notice also that exchanges in which values resemble strong common values, that is in which \( r^\theta \) is close to 1, typically fail to admit equilibrium. From a market design perspective, error correlation structures with stronger interdependence can be more beneficial than weak interdependence in that they are more likely to accommodate existence of equilibrium. In other words, the design of shared information sources with regards to the particular pattern of access or the format of information sharing can prevent market instability.

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\(^9\)Proposition 1 contributes to the literature by showing equilibrium existence for both heterogeneously correlated values and errors, and for exchanges with 2 traders provided that \( \rho_{12}^\theta < \rho_{12}^\varepsilon \), a contrast to [Kyle (1985)]. For exchanges with \( r^\varepsilon = 1/I \) and change of notation such that \( \bar{\rho}^\theta = (r^\theta I - 1) / (I - 1) \), the lower bound coincides with the bound in [Rostek and Weretka (2012)].
The shaded areas in the Figure display the existence regions for different combinations of average value correlation $r^\theta$ and average error correlation $r^\epsilon$. For example, if the ratio of error variance to value variance $\sigma^2$ is 2 instead of 6, then the set of points $(r^\theta, r^\epsilon)$, for which equilibrium in downward sloping orders exists, expands.

Figure 1 and Figure 2 illustrate the expansion of the existence region with increasing exchange size $I$ and with decreasing variance ratio $\sigma^2$, respectively, and the inference coefficients in Proposition 2 provide further explanation. Proposition 2 demonstrates the equilibrium price-dependent orders for exchanges with $(\gamma, r^\theta, r^\epsilon, \sigma^2)$ in terms of average correlations. The derivation of the equilibrium price-dependent orders is as follows. Assume that linear symmetric equilibrium exists. Then, the best response of trader $i$ given the linear symmetric orders of traders $j \neq i$ satisfies

$$E (\theta_i|s_i, p) - \mu q_i = p + \lambda q_i$$

for all $p$, (2)

where $\lambda \equiv -((I - 1) \partial q_j (p) / \partial p)^{-1}$ is the slope of the residual supply faced by trader $i$. By summing the conditions in (2) for all $i$, market clearing gives the equilibrium price such that $p^* = \sum_{i \in I} E (\theta_i|s_i, p^*) / I$. The conditional expectation of the value, by the normal information structure, is linear such that $E (\theta_i|s_i, p) = c_m m + c_s s_i + c_p p$, where $c_m$, $c_s$ and $c_p$ are the inference coefficients. By the projection theorem and Assumption 4, $c_s$, $c_p$ and $c_m$ are determined by the average correlations. Then, $\lambda$ attains as the fixed point satisfying the equations system from the first order conditions in (2), from which $\lambda = \mu (1 - \gamma) / (\gamma - c_p)$. 

Figure 2: Existence Region and Variance Ratio - Lower Bounding Curves $r^\epsilon_{0.9, \sigma^2}$
**Proposition 2** (Equilibrium Orders). The equilibrium price-dependent orders, if they exist, are unique and given by

\[
q_i(p) = \frac{\gamma - c_p}{1 - c_p} \frac{c_m}{\mu} m + \frac{\gamma - c_p}{1 - c_p} \frac{c_s}{\mu} s_i - \frac{\gamma - c_p}{\mu} p \quad \text{for all } i \in I,
\]

where the inference coefficients are given by

\[
c_s = \frac{1 - r^\theta}{1 - r^\theta + \sigma^2 (1 - r^\varepsilon)},
\]

\[
c_p = \frac{\sigma^2 (1 - r^\varepsilon)}{1 - r^\theta + \sigma^2 (1 - r^\varepsilon)},
\]

\[
c_m = \frac{\sigma^2 (r^\varepsilon - r^\theta)}{1 - r^\theta + \sigma^2 (1 - r^\varepsilon)}.
\]

*Proof.* See Proof 1 in the Appendix.

The equilibrium orders in Proposition 2 provide further explanation as to how existence is affected by inference from price. First, if error interdependence is stronger than value interdependence, then inference from price counters order shading from competition and strengthens the tradeoff between payments and quantities, thus does not compromise the stability of the exchange. To see this formally, notice that Equation 5 clarifies that if error interdependence is strong, then the inference-related weight \(c_p\) is negative, thus the sign of the order slope, which is given by \(- (\gamma - c_p)\), is also negative. Second, the more traders participate, the more permissive the order slope is to information structures. The effect of exchange size on the existence region is illustrated in Figure 1. The formal explanation is that if more traders participate, then the index of exchange size \(\gamma\) is closer to 1 and allows for higher positive inference coefficient \(c_p\) without resulting in incentives for infinite demand. Third, the lower the variance ratio \(\sigma^2 = \sigma^2_{\varepsilon} / \sigma^2_{\theta}\) is, the more permissive the order slope is to information structures. The effect of variance ratio on the existence region is illustrated in Figure 2. Formally, if the variance ratio is lower, then notice that all signals are more precise and both own signal and the price become more informative; two possibly counteracting effects for order shading. However, from Equations (4) and (5), notice that the weight on price for inference decreases thus the set of information structures which can accommodate downward sloping equilibrium orders expands.

\[10\] In the benchmark case with complete information and initial valuations, \(a_i\), the linear equilibrium demands are such that \(q_i(p) = (\gamma / \mu) (a_i - p)\) that is orders that are shaded compared to the benchmark of the large exchange, \(q_i(p) = (1 / \mu) (a_i - p)\).
Observe, that the conditional expectation $E(\theta_i|s_i, p^*)$ is increasing in the realization of own signal as well as in the value of the unconditional expectation $m$, since the related inference coefficients $c_s$ and $c_m$ are positive for all information structures. However, the conditional expectation need not be increasing in price. It is increasing in price if and only if the average value correlation exceeds the average error correlation, in other words, if $c_p$ is positive. On the other hand, when average error correlation exceeds average value correlation, then higher price induces traders to update for lower expectation behind which the main learning mechanism is learning from price through common errors.

Consider traders who are ignorant about common errors and submit price-dependent orders that neglect the correlation in measurement errors. If values are strongly correlated in the exchange, then traders, mistakenly, place higher weight on the price for their inference about the value, which flattens their price-dependent orders and, as discussed concerning existence of equilibrium, may compromise the stability of the exchange.

A consequence of shared information sources and the induced error interdependence is that learning through the market clearing price is necessary even in exchanges with independent values.

From an empirical perspective, the information structure is partially identified either from the slope of the submitted price-dependent orders or from the slope of the residual supply; routinely estimated measures in exchanges. The slope of the residual supply corresponds to the price impact $\lambda = -((I - 1) \partial q_j(p)/\partial p)^{-1} = \mu(1 - \gamma)/\gamma - c_p$. In the price impact, $c_p$‘s sign is given by the sign of $r^e - r^\theta$, thus price impact identifies if average error correlation is higher than average value correlation or if the reverse is true.

**Corollary 1 (Equilibrium Trade Volumes).** *In equilibrium, the market clearing price and volumes are*

\[ p = \frac{c_m}{1 - c_p} m + \frac{c_s}{1 - c_p} \bar{s}, \]

\[ q_i = \frac{\gamma - c_p c_s}{1 - c_p} \mu (s_i - \bar{s}) \text{ for all } i \in I. \]

Consider the above trade volumes and notice that traders with high signals represent the demand side and traders with low signals represent the seller side. Regarding trade volume, value interdependence and error interdependence play distinct roles in the following way. By looking at the inference coefficients $c_s$ and $c_p$ in Equations (4) and (5), notice that if value interdependence $r^\theta$ increases, then on the buyer side, total trade volume decreases. Formally, the inference coefficient $c_s$ on own signal is strictly decreasing
in $r_\theta$ and the inference coefficient $c_p$ on price is strictly increasing in $r_\theta$, thus volume is decreasing through the term $(\gamma - c_p) / (1 - c_p)$ which is decreasing in $c_p$. This observation can be connected to the empirical finding that high stock market daily return correlation tends to be accompanied by declining volume (Campbell et al. (1993)). However, if error interdependence $r^e$ increases, then the coefficient on own signal $c_s$ in the inference is higher and the weight on price in the inference $c_p$ is lower thereby the volume traded in the exchange increases. Observe also that the effect of higher variance ratio $\sigma^2 = \sigma^2_e / \sigma^2_\theta$ is the opposite compared to the effect of higher error correlation; increasing variance ratio decreases $c_s$ and increases $c_p$, which effects decrease the trade volume in the exchange.

2.1 Shared Information Sources and Error Correlations

Different types of shared information sources induce different error correlation patterns which are relevant from a market design perspective. To suggest information-based market improvements, it is important to model the resulting error correlations such that to incorporate local differences in interdependence thereby to represent systematic mistakes in information, which for example can be related to shared forecasting services, public information sources, reporting regulations, and traders’ influence on other traders’ through social network type relations. From a market design perspective and with a comparative view, it is useful to describe the impact of market parameters on the interdependencies in the exchange. For this reason I present error correlation models along with the corresponding comparative static model of average error correlations in the parameters of comparative interest.

The comparative parameters in the error correlation examples below represent exchange sizes or intensity of connections across traders. The error correlation models are equicommonal so as to accommodate Assumption 1. The models are well-defined as long as they describe appropriate correlation matrices, ones that satisfy positive semidefiniteness and symmetry, and whose elements are from $(-1, 1)$.12

Model 1 (Common Errors). Consider $I$ traders. The error terms in each trader’s signal is determined by a common term plus a trader specific shock $\varepsilon_i = \varepsilon_0 + \eta_i$. Therefore, the

\[ \frac{\partial c_p}{\partial \sigma^2} = \frac{\sigma^2(1+\sigma^2)(1/r^e-1)}{(1-r^e+\sigma^2(1-r^e))^2} < 0 \]

for all exchanges in which equilibrium in downward sloping demands exists.

These models illustrate that local differences qualitatively affect the comparative changes in measures of commonality. To describe the effect of intuitive local changes in the information structure, I focus on the average error correlation function. To compare to identical correlation or to commonality functions as in Rostek and Weretka (2012), see Table A.2 in the Appendix.
pairwise correlations are the same, $\rho_{ij} = \rho^e > 0$ for all $j \neq i$ for all trader pairs. This error correlation model captures situations, for example, in which the bias or mistake in the shared information source impacts all traders.

As a new trader joins the exchange, his correlation with each of the other traders' errors remains equal to $\rho^e$. Then, the average correlation, which is decreasing in $I$ and converges to $\rho^e$ in exchange size, is

$$r^e(I) = \frac{1 + (I - 1)\rho^e}{I}.$$

**Model 2** (Common Errors and Multiple Information Sources). Consider $I$ traders and $k$ information sources. Assume that $k$ divides $I$ and that each trader gains a signal from one information source and that each information source is seen by $I/k$ traders. If traders share the same information source then their errors are correlated $\rho^e \in (-1, 1]$. If two traders gain their signals from different sources of information then their errors are correlated 0. Notice that the previous model "Common Errors" is a special case of this model with 1 information source.

Assume that new traders uniformly join the information sources. Then, the average correlation, which is decreasing in $I$ and decreasing in $k$, is

$$r^e(I) = \frac{1 + (I/k - 1)\rho^e}{I}.$$

**Model 3** (Errors with Spatial Decay). Consider $I$ traders located on a cycle graph. Traders’ errors correlate proportionally to the distance among them. If the shortest distance between trader $i$ and trader $j$ is $d_{ij}$ (measured by the length of the shortest path between the two traders where the distance between two immediate neighbors is 1), then $\rho_{ij}^e = \beta^{d_{ij}}$, where $\beta \in (0, 1)$ is a decay parameter. This error correlation model captures situations, for example, in which the strength of interdependence through information sources diminishes with distance, as is the case in electricity exchanges, see [Giebel et al. (2007)](http://example.com).

As a new trader enters the exchange, the length of the cycle increases by one. Then, the average correlation, which is decreasing in $I$, is

$$r^e(I) = \frac{1 + \beta}{1 - \beta} \cdot \frac{1 - \beta^{\frac{1}{2}}}{I}.$$

**Model 4** (Errors in Two Groups). Consider two groups of traders of equal size $I/2$. Traders’ errors are perfectly correlated within groups, that is $\rho_{ij}^e = 1$ if $i$ and $j$ belong to
the same group. Traders’ errors are correlated \( \alpha \in [-1, 1] \) across groups that is \( \rho_{ij}^{\epsilon} = \alpha \) if \( i \) and \( j \) belong to different groups.

Assume that new traders increase both group sizes and that their errors are perfectly correlated within groups and correlated \( \alpha \) across groups. Then, the average correlation, which is constant in \( I \), is

\[
r^\epsilon(I) = \frac{1 + \alpha}{2}.
\]

Next, I turn to networks based information structures which capture social interaction type relationships among traders. For a comparative view, I describe how the changing intensity of traders’ interconnectedness or the changing topology of traders’ interconnectedness impacts average error correlations. The Condensing Links Model captures situations in which traders’ social network is becoming denser by the formation of new links between traders. The Strengthening Links Model captures situations in which existing correlations between traders’ errors are becoming stronger by switching to information sources that are increasingly similar. The Expanding Network Model captures situations in which the information network of traders is growing and becoming more interconnected at the same time.

**Model 5** (Errors through Condensing Links). Consider a fixed cycle graph of \( I \) traders first without links, with independent errors. At each step \( k \), for each trader, two new links are added to the network that connect the trader to the two closest yet unlinked trading partners on the cycle graph. Along each new link, the error correlation is \( \beta^k \). The parameter \( k \) represents the interconnectedness of the network while \( \beta \) the role of distance for spatial decay. \( I \) is an odd, fixed number and \( k \leq (I - 1)/2 \). Then, the average error correlation, which is increasing in \( k \), is

\[
r^\epsilon(k) = \frac{1 + 2\beta + 2\beta^2 + \cdots + 2\beta^k}{I} = \frac{1 + 2\beta^{1-\beta^k}}{I}.
\]

**Model 6** (Errors through Strengthening Links). Consider a cycle graph of \( I \) traders as in the Spatial Model and introduce the parameter \( \delta \) (\( \delta \in (0, 1) \)). At each step \( k \), the correlations of each trader’s errors are multiplied by \( \delta^{-1} \), thereby \( \delta \) is the parameter of strengthening interconnectedness with respect to information sources across traders. Then, the average correlation, which is decreasing in \( k \), is

\[
r^\epsilon(k) = \frac{1 + \delta^{-k} \beta}{1 - \delta^{-k} \beta} \cdot \frac{1 - (\delta^{-k} \beta)^{\frac{I}{2}}}{I}.
\]
Model 7 (Errors in Expanding Network). Consider a $k$-regular graph of traders. A graph is $k$-regular if each of its nodes is of degree $k$. Let the errors of traders who are linked be correlated $\alpha$ and errors of traders who are not connected to be correlated 0. Assume that as new traders enter the market, the regularity of the market can also increase with the size of the market and that the dependence of regularity on exchange size is described by a non-decreasing function $I(k)$. Then, the average correlation is

$$r^c(k) = \frac{1 + \alpha k}{I(k)}.$$

2.2 Inference from Price

The focus of this paper is to understand how inference from price changes order shading and informational efficiency in exchanges and how shared information sources can possibly be designed so as to ensure better stability and more desirable informational properties of exchanges. In the exchange model, traders infer about the unknown value $\theta_i$ based on the privately observed signal $s_i$ and the price $p$. A market clearing price in exchanges, because of its endogeneity, represents additional information for each trader, as it depends on other traders’ submitted price-dependent orders therefore conveys information about other traders’ signals. The amount information aggregated in the price, which previously has been dispersed across traders, therefore represents the market’s ability to allow for additional learning about the value of the good in trade.

The exchange’s ability to aggregate dispersed information on the market can be evaluated relative to what a trader could learn from all dispersed information on the market. While the exchange’s ability to allow for learning about values to the traders can be evaluated relative to what a trader can learn from his own signal only without participating in the market. The first benchmark based on all available private information is represented by the conditional random variable of values given the profile of all traders’ signals, $s = (s_1, s_2, \ldots, s_I)$. The second benchmark based only on the information, before learning through the exchange, is represented by the conditional random variable of the value given the trader’s signal.

To consider information aggregation of the exchange, the equilibrium price aggregates all trade relevant information for trader $i$ if the two conditional random variables that regard the payoff-relevant uncertainty coincide, that is when the conditional distribution of the values, $\theta$, satisfies that $F\left(\{\theta_i|s_i, p^*\}_{i \in I}\right) = F(\theta|s)$, given the equilibrium price $p^* = p^*(s)$. Proposition 3 establishes that price is perfectly informative if and only if the pairwise correlations of both values and errors in traders’ signals are the same for
all pairs of traders\textsuperscript{13}. This result extends the information aggregation result of Rostek and Weretka (2012) to show an additional limitation to perfect information aggregation, namely that it may no longer hold when the assumption of independent errors is dropped.

Proposition 3 (Information Aggregation). The equilibrium market clearing price, if it exists, aggregates all information dispersed across traders if and only if covariances across traders’ values are identical and covariances across traders’ errors are identical, more formally, if there exist $\rho^\theta$ and $\rho^\varepsilon$ such that for all pairs $(i, j) \in I \times I$ and $j \neq i$, $\rho^\theta_{ij} = \rho^\theta$ and $\rho^\varepsilon_{ij} = \rho^\varepsilon$.

Proof. See Proof 3 in the Appendix.

Thus the familiar argument as proposed in the seminal work Hayek (1945), that price aggregates all payoff relevant information, is a non-generic property in the class of information structures and is lost when the information structure features heterogeneity in correlation of traders’ values or errors. In particular, even if value correlations are pairwise identical, price may not aggregate all payoff relevant information if measurement errors in signals are not independent, which is the typical case given the prevalence of shared information sources.

To consider learning through the price in the exchange, measure price informativeness of an exchange by the variance reduction in the conditional expectation of the value due to inference from price and call the normalized difference the index of price informativeness $\psi^+$ such that

$$\psi^+ \equiv \frac{V\text{ar}(\theta_i|s_i) - V\text{ar}(\theta_i|s_i, p^*)}{V\text{ar}(\theta_i|s_i)}.$$ 

In the linear-normal setting, this measure of informativeness, gives the same qualitative results as the Kullback-Leibler divergence between the two random variables $\theta_i|s_i$ and $\theta_i|s_i, p^*$ since both measures depend only on the second order moments of the conditional normal distributions. Thus the index $\psi^+$ can be seen as an information theoretic measure.

\textsuperscript{13}Proposition 3 relates to the dimensionality argument by Jordan (1982) which demonstrates that information aggregation depends on the relative dimension of the payoff-relevant information and the prices. In the linear normal setting, from the projection theorem, a trader’s sufficient statistic for full information is a linear combination of signals with coefficients that derive from the matrices $C^\theta$ and $C^\varepsilon$. Thus both price and the sufficient statistic are one dimensional. However, in the generic case, the sufficient statistic is a different linear combination for each trader thus the price-system with a single market clearing price can not convey all dispersed information to all traders. For instance, the market clearing price in an equicommonal exchange reveals the average signal $\bar{s}$ (see (8)), thus intuitively, the information learned through this price can match the sufficient statistics for full information for all traders only if signal correlations are identical.
Figure 3: Price Informativeness - Isolines of $\psi^+$

If average value correlation equals average error correlation, $r^\theta = r^\varepsilon$, then $\psi^+ = 0$ and price is uninformative about the value. In both directions away from the 45 degree line, $\psi^+$ increases, yet perfect learning through price, $\psi^+ = 1$, does not attain for any exchange of finitely many traders.

To interpret $\psi^+$, note that an uninformative price does not reduce the variance further than the variance of the baseline conditional expectation given the trader’s signal and $\psi^+ = 0$; whereas a price that together with a private signal, reveals the exact realization of the trader’s value $\theta_i$ would result in full reduction of variance and $\psi^+ = 1$.

**Proposition 4** (Price Informativeness). *There exists no exchange $(\gamma, \sigma^2, r^\theta, r^\varepsilon)$ such that learning through price fully reduces uncertainty about the payoff relevant value $\theta_i$. Given equicommonal information structure as in Assumption 1, price informativeness is given by the average value and error correlations such as

\[
\psi^+ = \frac{\sigma^2 (r^\theta - r^\varepsilon)^2}{(r^\theta + \sigma^2 r^\varepsilon) (1 - r^\theta + \sigma^2 (1 - r^\varepsilon))} < 1.
\] (7)

*Proof. See Proof 4 in the Appendix.*

Consider Price Informativeness geometrically. For each value of $\psi^+ \in [0, 1)$, construct the isoline curve consisting of all combinations of the average correlations $(r^\theta, r^\varepsilon)$ for which price informativeness equals the value in consideration to get the isoline map on Figure 2.2. Recall that for positive semidefinite matrices, the valid range of the average correlations is between 0 and 1, that is $r^\theta \in [0, 1]$ and $r^\varepsilon \in [0, 1]$. The map of informativeness isolines on
Figure 2.2 illustrates learning through price for information structures represented on the 2 dimensional plane, given existence of equilibrium. The $x$ axis represents value correlations ranging from cases of negative value interdependence at $r^\theta < 1/I$ to independent values at $r^\theta = 1/I$ and almost perfect common values at close to $r^\theta = 1$. Similarly, the $y$ axis represents error correlations.

When error interdependence equals value interdependence, then price fails to be informative about own value, for the following reason. Error correlations and value correlations both contribute to the correlation of the payoff-relevant value and the signals, therefore the information in price is difficult to interpret. For example, price may convey that the average signal of traders is high, yet if this is due to high common component in errors or high common component in values remains unclear. From the comovement of values and price, traders can not distinguish the effect of error interdependence from the effect of value interdependence, therefore are unable to use correlations with the price to infer more about their own value.

Exchanges which ensure the highest price informativeness correspond to information structures in which it is easy for traders to distinguish the effect of average error correlation from the effect of the average value correlation in the price. Such exchanges include the ones in which $r^\theta$ is close to 0 while $r^\varepsilon$ is close to 1 and the opposite ones in which $r^\theta$ is close to 1 while $r^\varepsilon$ is close to 0. Due to non-existence in markets with perfect common values, price ensures the most variance reduction when $r^\theta$ is close to 0 and $r^\varepsilon$ is close to 1, that is when the exchange is large and the common component in errors is strong. For given average value correlation, the exchange that ensures the highest informativeness is either the exchange in which the average error correlation is closest to 1 or the exchange in which the average error correlation is closest to 0. In particular, exchanges in which the average value correlation is stronger than 0.5 allow for the best informativeness if the average value correlation is as low as possible which is implied by Equation (7), from which notice that, for all variance ratios and for all exchange sizes, $\psi^+ (r^\theta, 0) > \psi^+ (r^\theta, 1)$ if and only if $r^\theta > 0.5$.

Each isoline curve of $\psi^+$ consists of two segments on the two sides of the 45 degree line. Curves that are located further away from the 0-curve represent exchanges in which price informativeness is higher, in particular, the 1-curve corresponds to the points $(0, 1)$ and $(1, 0)$. The two segments of each $\psi^+$-curve are centrally symmetric to the midpoint $(0.5, 0.5)$ that is $\psi^+$ equals in the exchanges represented by $(r^\theta, r^\varepsilon)$ and the exchanges represented by $(1 - r^\theta, 1 - r^\varepsilon)$. Notice that the average error correlation and the average value correlation affect price informativeness $\psi^+$ individually and not only through the
difference \( r^\theta - r^\varepsilon \), rather they affect an exchange’s informativeness \( \psi^+ \) through both their difference and their weighted sum in which the weight is given by the variance ratio.

Even for fixed value correlations, the information aggregation properties of exchanges are qualitatively influenced by the error correlations induced by shared information sources or commonly used forecasting methodologies. Figure 2.2 and Equation (7) of price informativeness also suggest, that a market designer who aims to improve the informativeness of the exchange can successfully exploit the redesign of information sources to target a more desirable error correlation structure without necessarily being able to change the underlying correlation structure of payoff-relevant values of the traders. For example, the designer can consider producing publicly available databases and public predictions, restricting access to certain information sources, or allowing segmentation of trading venues; and recognize that these interventions are market design tools that shape error correlations induced by shared information sources in the exchange.

Now consider if larger markets aggregate information better through price, given Proposition 3 and the formula for price informativeness in (7). In exchanges in which both error correlations and value correlations are identical, price aggregates all trade relevant information that is dispersed among the traders. The larger the market, the more information is present for aggregation, thus the larger the market, the more informative the price becomes. On the other hand, in exchanges with heterogeneously correlated error, price does not incorporate all trade relevant information, therefore informational monotonicity properties of larger markets depend on the details of the information structure, specifically also on the details of the error correlation structure.

I report two sufficient conditions on the monotonicity of price informativeness. Below the derivatives \( r^\theta \)' and \( r^\varepsilon \)' of the average correlations are understood in an economically relevant factor, which, for example, can represent the size of the exchange \( I \), the index \( \gamma \), market interconnectedness \( k \), or in general, a factor that changes average correlation in the exchange.

**Proposition 5** (Sufficient Condition for Monotonicity of Price Informativeness 1). Assume that the average correlations \( r^\theta \) and \( r^\varepsilon \) are increasing and \( r^\theta > r^\varepsilon \). If the average value correlation grows faster than the average error correlation, \( r^\theta / r^\theta > r^\varepsilon / r^\varepsilon \), then price informativeness \( \psi^+ \) increases.

Assume that the average correlations \( r^\theta \) and \( r^\varepsilon \) are increasing and \( r^\theta < r^\varepsilon \). If the average value correlation grows slower than the average error correlation, \( r^\theta / r^\theta < r^\varepsilon / r^\varepsilon \),

\(^{14}\)With a slight abuse of notation, the derivatives are taken from the natural continuously differentiable approximation of \( r^\theta \) and \( r^\varepsilon \) when the factor of interest may be discrete.
then price informativeness $\psi^+$ increases.

Proof. See Proof 5 in the Appendix. $\square$

Note that this sufficient condition does not depend on $\sigma^2$. Ranking growth rates such that $r^\theta/r^\theta > r^\epsilon/r^\epsilon$ is equivalent to the condition that $(r^\theta - r^\epsilon)/(r^\theta - r^\epsilon) > (r^\theta + \sigma^2 r^\epsilon)/(r^\theta + \sigma^2 r^\epsilon)$, which prescribes that the growth rate of the difference between average correlations be higher than the growth rate of the signal variance.

**Proposition 6** (Sufficient Condition for Monotonicity of Price Informativeness 2). Assume that the average correlations are such that $r^\theta + \sigma^2 r^\epsilon < (1 + \sigma^2)/2$.

If $r^\theta > r^\epsilon$ and average value correlation grows faster than average error correlation, $r^\theta/r^\theta > r^\epsilon/r^\epsilon$, then price informativeness $\psi^+$ increases.

If $r^\theta < r^\epsilon$ and average value correlation grows slower than average error correlation, $r^\theta/r^\theta < r^\epsilon/r^\epsilon$, then price informativeness $\psi^+$ increases.

Proof. See Proof 5 in the Appendix. $\square$

Notice that this latter sufficient condition, in contrast to the one in Proposition 5, covers cases in which one or both of the average value correlations may be decreasing. From a market design viewpoint, both of these sufficient conditions are useful. In Section 3, the market design conclusions in applications to specific models of error interdependencies derive from Propositions 5 and 6.

## 3 Design of Shared Information Sources

This section illustrates how market design can shape stability of exchanges and price informativeness in exchanges by influencing access to information sources or access to trading venues. The analysis thus far demonstrates that, in exchanges with interdependent values or errors, market growth or increasing connectivity of traders’ may have a non-monotone impact on price informativeness. I exploit the understanding of how informational microstructure affects learning through price, Propositions 4 and Proposition 5, to guide information-based market design in the illustrative examples from Section 2.1.

### 3.1 Design for Stability

From an information-based market design perspective, the existence result in Propositions 1 suggests solutions to enhance the stability of an exchange. Exchanges with highly
correlated values fail to exist or collapse, because of the implied flat or upward sloping orders. However, stability can possibly be restored by introducing public forecast services or restricting the number of available information sources, so as to ensure that the remaining forecast errors in traders’ signals about values be strongly correlated. Such an intervention can ensure through the induced change in inference from price that traders’ orders become downward sloping. The following example illustrates the possibility of improving exchange stability by restricting information sources from 3 information sources to 1 information source.

**Example 1 (Restricting the Number of Information Sources).**

Consider \( I = 6 \) traders in an exchange with uniform price market clearing and consider a value correlation structure such that traders’ average value correlation is \( r_{\theta} = 3/4 \); which represents an exchange in which values are strongly correlated. Such average value correlation may emerge, for example, when traders’ values consist of a common value plus an idiosyncratic component such that the pairwise correlations across traders’ values are \( \rho^\theta = 0.8 \). Assume that traders gain information from one of 3 different information sources and that each information source is accessed by 2 of the traders. Assume that information from these 3 information sources results in signals in which the error components are independent across different information sources and correlated \( \rho^\varepsilon \) for traders gaining information from the same source. For simplicity, fix the variance ratio at \( \sigma^2 = 3 \). This information structure is an instance of the Model ‘Common Errors and Multiple Information Sources’. To study market design for exchange stability, recall the corresponding lower bound on average error correlation \( r_{\gamma,\sigma^2} \) given in (1).

In this information structure with 3 different information sources, the stability of the exchange is compromised if the pairwise error correlation \( \rho^\varepsilon \) is not strong enough, more precisely, equilibrium does not exist when the pairwise correlation of errors of traders, who access the same information source, \( \rho^\varepsilon \) is less than 0.5. As a solution to non-existence, the designer may restrict access to only two information sources, such that each information source is accessed by 3 traders. This information structure with 2 different information sources allows for existence for a larger set of error correlation structures; in particular, for information structures in which the error correlation of traders, who obtain information from the same information source, \( \rho^\varepsilon \) is between 0.25 and 1. Restricting to only 1 information source allows for existence for an even larger set of error correlation structures, in particular, for information structures in which the pairwise error correlation \( \rho^\varepsilon \) is between 0.1 and 1.

This example shows that there may be a robust range of error correlations for which
stability of trade is compromised when traders gain information from one of 3 different information sources. However, the designer can solve the problem of non-existence – interpreted as instability or market collapse – and further accommodate exchange equilibrium by restricting traders to only 2 information sources each of which is accessed by half of the traders. However, if error correlations in traders’ information are very weak then the market may still be unstable with 2 information sources, in which case the designer can further accommodate equilibrium existence by restricting traders to only 1 information source which is then shared across all traders.

The existence result in Propositions 1 also implies that the existence region expands with increasing exchange size. Therefore, attracting more traders to the exchange is another tool that a designer might want to consider to prevent market failure. At the same time, exchange size has a dual effect on the order slope. First market expansion affects market power, second it affects inference through the interplay between average value and error interdependence. Attracting more traders can help existence only if the entrance of the new traders does not overwhelmingly increase the average value interdependence in the exchange. This suggests, that expanding or segmenting exchanges can be seen as designing information sources as it inherently changes informational interdependencies across the participating traders in each venue. The following example illustrates a situation in which the information structure is such that increasing trader participation improves market stability.

**Example 2 (Increasing the Participation of Traders).**

Consider \( I = 4 \) traders in two groups whose payoff-relevant values are perfectly correlated within groups and correlated \( \alpha^\theta = 0.8 \) across groups. For simplicity, fix the variance ratio at \( \sigma^2 = 3 \). Following the Model ‘Errors in Two Groups’, assume also that errors in traders’ signals are correlated \( \alpha^\varepsilon = 0.2 \) across groups and perfectly correlated within groups. Given this information structure, the two groups in two separate exchanges with 2-2 traders can not trade due to perfect common values. Formally, the average value correlation is 1 and the average error correlation is 0.5, which does not exceed the corresponding lower bound \( r^\varepsilon (1) = 1 \) as given by (I).

Now, consider the exchange in which all 4 traders participate. In this exchange the average value correlation is 0.9 and the average error correlation is 0.6. However, the lower bound on average error correlation, given by the number of traders 4 and the average value correlation 0.9, is \( r^\varepsilon_{2/3,3} (0.9) = 0.7 \). The average error correlation is less than the lower bound for equilibrium existence, thus equilibrium in downward sloping demands does not
exists and the stability of the exchange is compromised.

However, if the designer can include 2 more traders, one more trader to each group, then the new exchange with 6 traders is described by the same average correlations as the previous one, yet due to the possibility of smaller market power, the bound on the average error correlation decreases to $r_{4/5,3}^{3/4}(0.9) = 7.8/14$, thus the average correlation of the errors in traders’ signals, which remains 0.6, is more than the lower bound of existence. This shows that, if the designer can attract two more traders to the exchange, then the inclusion of the two new traders accommodates existence of equilibrium in downward sloping demand functions, thus restores stability of the exchange. □

3.2 Design for Improving Price Informativeness

Consider the underlying interdependence of traders’ uncertain values as fixed. From the formula of price informativeness in Equation (7), the error correlation structure which is optimal for informativeness depends on the average value correlations. In particular, in exchanges with low average value correlation, for example due to almost independent values or negative pairwise correlations, the optimal error correlation structure is the one with perfectly common errors, that is when average error correlation equals 1. On the other hand, in exchanges with stronger interdependence in values, for example due to strong common values, the optimal error correlation structure is the one with 0 average error correlation. More specifically, perfectly common errors are optimal for price informativeness if and only if the average value correlation is less than $0.5 (\psi^+(r^\theta, 1) > \psi^+(r^\theta, 0)$ if and only if $r^\theta < 0.5$). The evaluation of information-based market designs, as illustrated in the remaining parts of this section, depends on the interdependence of the payoff-relevant values. Encouraging trader participation or allowing the segmentation of the exchange, restricting the number of available information sources or enhancing informational connections across traders are effective market design tools which, when tailored to the underlying interdependence of payoff-relevant values, can ensure better learning through price in the exchange.

3.2.1 Fundamental Values: Identical Pairwise Correlations

Consider exchanges in which a common value component symmetrically impacts traders such that correlations for all pairs of traders are the same, $\rho_{ij} = \rho$ for all $i, j$. Exchanges with pairwise identical value correlations provide a tractable framework to study common value uncertainty, therefore are often studied in the literature. It is known, also a consequence of Proposition 3, that in exchanges with Fundamental Values and independent
errors, price aggregates all available information. Therefore, increasing the size of the exchange is a way to improve informativeness, since then more information is present, although dispersed among the traders, and all of this information is aggregated in the price. This classic conclusion, which indicates that the larger the exchange the better it aggregates information, according to Proposition 3, carries over to some exchanges with interdependent errors, namely to exchanges with Fundamental Values and Common Errors. Therefore, from a design perspective, attracting more participants is a way to improve price informativeness in these types of exchanges. On the other hand, marginal improvement in price informativeness is diminishing in exchange size if \((\rho^\theta + \sigma^2 \rho^\varepsilon) > 0\)\(^{16}\).

In this latter case, the designer might need to balance marginal cost of attracting more traders to the exchange and marginal benefits from improvements in price informativeness for optimal design. On the other hand, on markets with negative uniform correlations in values and error, the marginal price informativeness is increasing in exchange size; thus, no such considerations occur.

In exchanges with Fundamental Values and Group Errors the optimal design depends on the relationship between the key parameters. First, regarding trader participation, price informativeness can increase (when \(\rho^\theta \leq \alpha\)), decrease (when \(\rho^\theta > (1 + \alpha) / 2\)), or exhibit a U-shape, dropping to zero for an intermediate exchange size. Thus, if \(\rho^\theta \leq \alpha\) then the classic argument holds and encouraging trader participation improves price informativeness, whereas this might not be the case for other parameter settings. For example with \(\rho^\theta > (1 + \alpha) / 2\), the optimal design is the one that restricts access to trading venues and segments traders into the smallest possible groups for which trade is feasible. Second, the designer might also consider compulsory switch to one forecasting service or produce and encourage the use of public forecasts thereby influence the error correlation structure to get close to the Common Error model with close to 1 average error correlation. This der-

\(^{15}\)To show monotonicity in the number of traders \(I\), one can directly derive the property form the formula of price informativeness. In exchanges with Fundamental Values and Common Errors,

\[
\psi^+ = \frac{\sigma^2 (\rho^\theta - \rho^\varepsilon)^2}{\left(1 + \sigma^2 + \rho^\theta + \sigma^2 \rho^\varepsilon\right) (1 - \rho^\theta + \sigma^2 (1 - \rho^\varepsilon))},
\]

which is increasing in \(I\), regardless of the parameters \(\rho^\theta\) and \(\rho^\varepsilon\).

\(^{16}\)The second order derivative of price informativeness with respect to market size, in exchanges with Fundamental Values and Common Errors (treating \(\psi^+\) as a polynomial of \(I\)), is

\[
\psi^{+''} = -\left(\rho^\theta + \sigma^2 \rho^\varepsilon\right) \frac{2\sigma^2 (1 + \sigma^2) (\rho^\theta - \rho^\varepsilon)^2}{\left(\rho^\theta + \sigma^2 (1 - \rho^\varepsilon)\right) (1 + \sigma^2 + (I - 1) (\rho^\theta + \sigma^2 \rho^\varepsilon))},
\]

which is negative, thereby \(\psi^+\) is concave in \(I\), if and only if \((\rho^\theta + \sigma^2 \rho^\varepsilon) > 0\).
The directed curves in the Figure display the impact of increasing the number of participants on the average value correlation and on the average error correlation in the exchange. For example, when the exchange size increases and values are close to independent, $\rho^\theta = 0.2$, and errors are correlated according to the spatial decay model, $\beta^\varepsilon = 0.9$, then for smaller exchanges, the average value correlation decreases faster than the average error correlation and for larger exchanges the reverse is true.

Sign obviously improves informativeness in the case of low value interdependence, whereas it could hinder informativeness in markets with high error correlations, for example, in the case with $\rho^\theta = 0.8$ and $\alpha = 0.3$.

In exchanges with Fundamental Values and Spatial Errors, depending on the relative correlation of value and error, price informativeness may increase to reach a maximum value in a small exchange, then, as the exchange grows further, diminish to zero and then increase again. This happens for example in the case when $\rho^\theta = 0.2$ and $\beta^\varepsilon = 0.9$; thus optimal design regarding exchange size shall support the size where the peak occurs. On the other hand, in the example when $\rho^\theta = 0.6$ and $\beta^\varepsilon = 0.8$ there is a small exchange maximum, yet this maximum is local and price informativeness is globally maximal in the large limit exchange; thus optimal design shall support participation in the exchange.

For illustrations on how price informativeness in exchanges with Fundamental Values depend on the error correlation structure, see Figures 4 and 5.
Figure 5: Price Informativeness $\psi^+$ in Exchanges with Fundamental Values

The graphs in the Figure display the measure of learning through price in exchanges with fundamental values and with different interdependencies in errors in traders’ signals. For example, when values are close to independent, $\rho^\theta = 0.2$, and errors are correlated according to the spatial decay model, $\beta^\varepsilon = 0.8$, then price informativeness is optimal for a small exchange, which suggest market designs that allow for the segmentation of trading venues. However, when values are more strongly correlated and errors are correlated according to the spatial decay model, $\beta^\varepsilon = 0.9$, then price informativeness is optimal in the largest possible exchange.
3.2.2 Networked Information Structures

First I turn to information structures, where both value interdependencies and error interdependencies follow a network model. Interestingly, for both the Condensing Links and Strengthening Links, one finds that price informativeness improves with market interconnectedness and link intensity, respectively, regardless of the specific parameters in those models. The explanation to these phenomena is driven by Proposition 5.

When new link formation among traders in an exchange corresponds to a Condensing Links model for both values and errors, then price informativeness is always increasing in the number of links among traders. The fact that this holds regardless of the specific parameter settings is surprising as, in general, the average correlations in value and in error work against each other. I can show that the attribute with the higher average correlation gives rise to a higher growth rate in the average correlation, thus by Proposition 5 price informativeness is monotone increasing in market interconnectedness. Therefore, in Exchanges with Condensing Links, market design that aims for improving learning through price shall enhance trader interconnectedness.

Similarly, in networks where the link intensity across traders evolves according to the Strengthening Links Model and this link intensity is the primary determinant of interdependencies both for values and errors, then price informativeness is again always increasing regardless of the specific parameters in the model. This is again driven by the intuition behind Proposition 5 namely that the signal component (either value or noise) that displays higher average correlation also follows higher growth rate, while both

\[ r'(k) = \frac{-2\ln \beta}{\beta^{-k-1} + \beta^{-k} - 2} \]

Differentiating with respect to \( \beta \), I derive

\[ \frac{\partial r'(k)}{\partial \beta} = \frac{-\beta^{-k-2} - \beta^{-k-1} + 2\beta^{-1} - \ln \beta^{k+1} \beta^{-k-2} + \ln \beta^k \beta^{-k-1}}{2^{-1}(\beta^{-k-1} + \beta^{-k} - 2)^2} > 0, \]

where I used the inequality \( \ln x x < 1 - \frac{1}{x} (x \in (0,1)) \) in the terms \( \ln \beta^{k+1} \beta^{-k-2} < \frac{1}{\beta} - \frac{1}{\beta^{k+1}} \) and \( \ln \beta^k \beta^{-k-1} < \frac{1}{\beta} - \frac{1}{\beta^{k+1}} \). Now, if both values and error follow the Condensing Links model, then \( \beta^\theta > \beta^\epsilon \) is equivalent to \( r^\theta > r^\epsilon \) and to \( r^{\theta'} > r^{\epsilon'} \), which, by using the increasing average correlations proves that, for any choice of \( \beta^\theta \) and \( \beta^\epsilon \), the conditions of Proposition 5 hold. Thus, price informativeness is monotone increasing.
average correlations are increasing,\footnote{The growth rate of the average correlation in the Condensing Links Model is \[ \frac{r'}{r} = \frac{\beta(1/2)\delta(1/2k)\ln \delta}{1 - \beta(1/2)\delta(1/2k)} . \] If $\beta^0 < \beta^e$, then $r^0 < r^e$ and $\frac{r^0'}{r^0} < \frac{r^e'}{r^e}$. Thus, by Proposition 5 the monotonicity of price informativeness in $k$ holds.\footnote{Assume fixed exchange size $I$ and $\alpha > 0$. Then the average correlation $r(k) = \frac{1+\alpha k}{I}$ is increasing in $k$. The growth rate of the average correlation, $\frac{r'}{r} = \frac{\alpha}{1+\alpha k}$, is increasing in $\alpha$, thus the model satisfies the conditions of Proposition 5.\footnote{Assuming a fixed exchange size $I$ and $\alpha < 0$, the average correlation $r(k) = \frac{1+\alpha k}{I}$ is decreasing in $k$, implying $r(k) < r(1) = \frac{1+\alpha}{I} < 1$. Thus, $r^0 + \sigma^2 r^e < \frac{1+\alpha^0}{I} + \sigma^2 \frac{1+\alpha^e}{I} < \frac{1+\alpha^0}{I} + \sigma^2 \frac{1+\alpha^e}{I} < \frac{1+\sigma^2}{2}$. This condition, together with the increasing growth rate, ensures monotone price informativeness by Proposition 6} which, along with increasing average correlations, by Proposition 5 implies the monotonicity of price informativeness, therefore implies that regardless of the specific parameters, a designer can improve price informativeness by enhancing the strength of informational links across traders in the exchange.

Now, such a clear answer for market design does not hold for exchanges with Fundamental Values and Errors through Condensing Links. In this case, as average value correlation is not affected by the strength of the links and exchange size is fixed, the parameters $\rho$ and $\delta$ play important distinguishing roles for price informativeness. For example, fix the parameter $\rho = 0.8$ in the Fundamental Value Model. If the error correlation parameter $\beta$ is small, then price informativeness decreases with increasing interconnectedness. On the other hand, if $\beta$ is large, then price informativeness increases with increasing interconnectedness. If the information structure in an exchange is such that only error correlations follow the Condensing Links model, for example, due to more and more information sharing along the underlying social network of traders or due to targeted advertisement, but interdependence regarding fundamentals are not affected, then the market design that enhances or restricts link formation might both be beneficial, depending on the current strength of value interdependence versus error interdependence.

In networks described by Expanding Network for Values with Expanding Network for Errors, although connectedness of the network, $k$, increases, this does not necessarily increase price informativeness as the exchange grows. Price informativeness may diminish for a small number of individual links for small exchange sizes, achieve a minimum at a non-zero level, and increase with further increases in $I$ and $k$ (this is the case for small exchanges, $I \leq 10$, with $I = 2k$, $\alpha^0 = 0.099$ and $\alpha^e = 0.8$). For fixed exchange size, $I (k) = I$, price informativeness is monotone by Proposition 5 for positive $\alpha$ and by Proposition 6 for negative $\alpha\footnote{Assuming a fixed exchange size $I$ and $\alpha < 0$, the average correlation $r(k) = \frac{1+\alpha k}{I}$ is decreasing in $k$, implying $r(k) < r(1) = \frac{1+\alpha}{I} < 1$. Thus, $r^0 + \sigma^2 r^e < \frac{1+\alpha^0}{I} + \sigma^2 \frac{1+\alpha^e}{I} < \frac{1+\alpha^0}{I} + \sigma^2 \frac{1+\alpha^e}{I} < \frac{1+\sigma^2}{2}$. This condition, together with the increasing growth rate, ensures monotone price informativeness by Proposition 6}. See the illustration on Figure 3.2.2.}
This figure illustrates that the relative size of the exchage $I$ to the number of informational connections $k$, although both nondecreasing, can alter the monotonicity of price informativeness in exchanges.

### 4 Conclusion

The sympathetic view of markets’ ability to aggregate dispersed information through the price system is expressed, for example, in Hayek (1945) as follows. "The mere fact that there is one price for any commodity ... brings about the solution ... which might have been arrived at by one single mind possessing all the information which is in fact dispersed among all the people involved in the process." In this paper I show that exchanges with one price and optimal trader behavior can result in a price which may not aggregate all information held by the traders and thus may not implement the best choice of a benevolent planner who hypothetically would possess all information. This paper indeed highlights the limits of Hayek’s view in the presence of shared information sources which induce error correlations in traders’ information. In particular, the results in this paper clarify that if the implied interdependence in errors and the underlying interdependence in values are not sufficiently distinguishable then information aggregation through price is weak in the, otherwise well-functioning, exchange. However, if interdependence in errors is distinguishable and not ignored, then it can be as valuable for information aggregation as interdependence in values.

As a behavioral implication, this paper also shows that the impact of error correlations on order shading is opposite to the impact of common values. In particular, if common errors are strong, then the stability of the exchange is preserved; a contrast to
the destabilizing effect of strong common values.

The results also contribute to the recent literature on information aggregation in the presence of correlation neglect (for financial market experiments, see Eyster and Weisacker (2012) and Enke and Zimmermann (2015), for voting, see the survey in Ashworth and de Mesquita (2016)). The analysis in this paper is complementary to the current literature on correlation neglect as it focuses on correlations across traders’ errors as opposed to correlations across multiple signals available to the traders. Levy and Razin (2015) shows for information aggregation in voting, that correlated information as aggregate shock to voters’ preferences along with behavioral voting, that is naive voting which neglects correlation in information, may enhance information aggregation in the electorate and be beneficial to the voting system for which the explanation is that when correlations are neglected, then votes are more heavily based on own information as opposed to party preference. A similar intuition is confirmed in the current analysis only when common values are strong. Then the neglect of error correlations increases the weight on own signal in the inference about value and ensures higher informativeness of the exchange and lower volatility of the price. However when value correlations are weak, then the reverse may hold; traders’ ignorance of error correlations lowers price informativeness of the exchange through the lost opportunity to learn from the error correlations conveyed by the price.

The information-based designs in this paper can be seen as the market design applications of the theory of ‘information design’ (see Taneva (2016) and Bergemann and Morris (2016)). Just as mechanism design finds best solutions and benchmarks through unrealistic implementations, information design suggests best solutions through unrealistic control of the underlying informational relations of participants. Market design is more practically constrained than to implement first best solutions of mechanism design or information design, and resorts to second best solutions to improve existing markets and institutions. In reality, instead of the full range of action suggestions of the general theory of information design, only a restricted set of tools are available to the designer to influence or govern the information structure. In this paper, I adopt the view that the design of information sources leaves intact the underlying interdependencies of payoff-relevant values yet may alter error correlations in traders’ noisy signals about values. For example, if the designer restricts electricity traders to the same information source, whereas previously they accessed different information sources, then intuitively, the spatially driven interdependencies across traders’ values remain the same yet common components in forecast errors become stronger. Note that, in many markets, the designer may restrict the available information sources or mandate reporting formats, however she may not have the power to change how information sources relate to the traders’ payoff relevant uncertain
values.

For information-based market design, the paper provides the following insights. First, to increase informativeness of the price in an exchange in which a good with a strong common value component is traded, it is beneficial to design access to information sources so that error correlations be less prominent only so long as it does not diminish error correlations to become negligible. Low error correlations are harmful for stability because of the lost counter effect on order shading which may lead to orders that are too flat or upward sloping. These orders require traders to respond by demands that overwhelm the market and compromise the stability of the exchange, which needs to be avoided when designing information, especially when better informativeness would require to reduce error interdependence. Second, more interdependence in errors contributes to exchanges’ ability to function, while more variance in errors does not, thus reducing error variance can be, in general, a useful tool to prevent instability in exchanges. Third, the particulars of pairwise interdependencies in the measurement errors of traders’ information determine the exchange’s ability to aggregate dispersed information, therefore studying the pairwise trader to trader commonality in error biases is necessary to ensure the success of information-based market designs.
A Appendix

A.1 Proofs

**Proof 1** (Proposition 2: Equilibrium Orders). By market clearing and the first order conditions in (2), the equilibrium price satisfies 
\[ p^* = \frac{1}{1 - c_p} m + \frac{c_s}{1 - c_p} \bar{s}, \]
where \( \bar{s} \) is the average signal, that is, \( \bar{s} = (1/I) \sum_{i \in I} s_i \). By (8), the random vector \((\theta, s, p^*)\) is jointly normally distributed such that
\[
\begin{pmatrix}
\theta_i \\
s_i \\
p^*
\end{pmatrix}
\approx N
\begin{bmatrix}
m \\
m \\
m
\end{bmatrix}
\begin{pmatrix}
\sigma^2_\theta & \sigma^2_\theta & \text{cov}(\theta_i, p^*) \\
\sigma^2_\theta & \sigma^2_\theta & \text{cov}(s_i, p^*) \\
\text{cov}(p^*, \theta_i) & \text{cov}(p^*, s_i) & \text{Var}(p^*)
\end{pmatrix}.
\]

By (8), the covariances with price in the joint distribution are given by
\[
\begin{align*}
\text{Cov}(\theta_i, p^*) &= \frac{c_s}{1 - c_p} r^\theta \sigma^2_\theta, \\
\text{Cov}(s_i, p^*) &= \frac{c_s}{1 - c_p} (r^\theta \sigma^2_\theta + r^\varepsilon \sigma^2_\varepsilon), \\
\text{Var}(p^*) &= \left( \frac{c_s}{1 - c_p} \right)^2 (r^\theta \sigma^2_\theta + r^\varepsilon \sigma^2_\varepsilon).
\end{align*}
\]

The projection theorem gives that\footnote{The Projection Theorem for Normal Random Variables states the following. Let \( \theta \) and \( s \) be random vectors such that \((\theta, s) \sim N(\mu, \Sigma)\) and
\[
\mu \equiv \begin{pmatrix} \mu_\theta \\ \mu_s \end{pmatrix} \quad \text{and} \quad \Sigma \equiv \begin{pmatrix} \Sigma_{\theta,\theta} & \Sigma_{\theta,s} \\ \Sigma_{s,\theta} & \Sigma_{s,s} \end{pmatrix},
\]
and \( \Sigma_{s,s} \) is positive definite. The distribution of \( \theta \) conditional on \( s \) is normal and is given by \((\theta|s) \sim N(\mu_{\theta} + \Sigma_{\theta,s} \Sigma_{s,s}^{-1}(s - \mu_s), \Sigma_{\theta,\theta} - \Sigma_{\theta,s} \Sigma_{s,s}^{-1} \Sigma_{s,\theta})\).}
By equating coefficients, the inference coefficients \( c_s \) and \( c_p \) satisfy

\[
\begin{align*}
    c_s &= \frac{1 - r^\theta}{1 - r^\theta + \sigma^2 (1 - r^\varepsilon)} \quad \text{and} \\
    c_p &= \frac{\sigma^2 (1 - r^\varepsilon)}{1 - r^\theta + \sigma^2 (1 - r^\varepsilon)}. 
\end{align*}
\]

By \( E (\theta_i) = E (s_i) = E (p) = m, c_m + c_s + c_p = 1 \), which gives

\[
    c_m = \frac{\sigma^2 (r^\varepsilon - r^\varepsilon)}{1 - r^\theta + \sigma^2 (1 - r^\varepsilon)}. 
\]

By the first order condition as in Equation (2), the equilibrium price-dependent orders are such that

\[
q_i (p) = \frac{c_m m + c_s s_i - (1 - c_p) p}{\left( \mu - (\partial q_i (p) / \partial p)^{-1} / (I - 1) \right)}, \tag{13}
\]

where the order slope \( \partial q_i (p) / \partial p \) satisfies the fix point equation for the slope such that \( \partial q_i (p) / \partial p = -(1 - c_p) / (\mu - (\partial q_i (p) / \partial p)^{-1} / (I - 1)) \). Thus the equilibrium price-dependent orders are

\[
q_i = \gamma - c_p \frac{c_\theta}{1 - c_p} E (\theta_i) + \gamma - c_p \frac{c_s}{1 - c_p} s_i - \gamma - c_p \frac{p}{\mu}. 
\]

Proof 2 (Proposition 1: Existence of Equilibrium). The order profile with price-dependent orders as in (3) from Proposition 2 constitutes an equilibrium with downward sloping
Proof 3 (Information Aggregation through Price). Recall that the equilibrium price aggregates all information dispersed across traders if and only if the conditional distributions coincide, $F(\theta_i|s_i, p^*(s)) = F(\theta_i|s)$ for all $i$. First, by the projection theorem, the conditional variables given the signal vector are jointly normal with conditional expectation $E(\theta|s) = \text{const} \cdot \mathbf{1} + C_\theta (C_\theta + \sigma^2 C_\varepsilon)^{-1} s$ and conditional variance $\text{Var}(\theta|s) = \left(E - C_\theta (C_\theta + \sigma^2 C_\varepsilon)^{-1}\right) \sigma^2 \tilde{C}_\theta$. Second, by Proposition 2, the market clearing price is a linear combination of the signals, and the conditional random variables $\{\theta_i|s_i, p^*(s)\}_{i \in I}$ are jointly normal with conditional expectation $E(\theta_i|s_i, p^*(s)) = c_{m_m} m + c_s s_i + c_p \bar{s}$.

If the equilibrium price aggregates all information in the exchange, then $E(\theta_i|s) = E(\theta_i|s_i, p^*)$ for all $i$, thus $\left[C_\theta (C_\theta + \sigma^2 C_\varepsilon)^{-1}\right]_{ij} = c_p / I$ for all $i \neq j$ and in the diagonal, $\left[C_\theta (C_\theta + \sigma^2 C_\varepsilon)^{-1}\right]_{ii} = c_s + c_p / I$ for all $i$. Thus the matrix $D := C_\theta (C_\theta + \sigma^2 C_\varepsilon)^{-1}$ has identical elements in its diagonal and identical elements off its diagonal. For further reference, call matrices with this property ‘patterned’ and notice that the inverse of patterned matrices is patterned, and the sum and the product of patterned matrices is also patterned. If the equilibrium price aggregates all information in the exchange, then also the variance-covariance matrices coincide such that $\text{Var}(\theta|s) = \text{Var}(\theta_1|s_1, p^*(s); \theta_2|s_2, p^*(s); \ldots; \theta_I|s_I, p^*(s))$ and by Proposition 2, the latter matrix is patterned, therefore $\text{Var}(\theta|s) = (E - D) \sigma^2 \tilde{C}_\theta$ is patterned, which then, by $D$ being patterned, implies that $C_\theta$ is patterned, which, combined again with $D$ being patterned, implies that $C_\varepsilon$ is also patterned. Thus pairwise identical correlations across values and pairwise identical correlations across errors follow.

orders if and only if $\gamma > c_p > -\infty$ which, by (5) requires that

$$\gamma r^\theta \left(1 - r^\theta + \sigma^2 (1 - r^\varepsilon) \right) > \sigma^2 r^\varepsilon (1 - r^\varepsilon).$$

(14)

Notice that (14) holds when $r^\varepsilon > r^\theta$, regardless of exchange size and variance ratio. By $\gamma r^\theta - 1 < 0$, rearranging (14) gives the lower bounding curve such that

$$r^\varepsilon_{\gamma,\sigma^2}(r^\theta) := \max \left\{0, \frac{r^\theta (1 - \gamma) - \frac{\gamma}{\sigma^2} (1 - r^\theta)}{1 - \gamma r^\theta} \right\}.$$  (15)

The curve $r^\varepsilon_{\gamma,\sigma^2}(r^\theta)$ characterizes existence for exchanges with $(r^\theta, r^\varepsilon, \gamma, \sigma^2)$, that is equilibrium exist if and only if $r^\theta \in (0, 1)$ and $r^\varepsilon_{\gamma,\sigma^2}(r^\theta) < r^\varepsilon < 1$. The existence region shrinks for higher value interdependence, since the bound $r^\varepsilon_{\gamma,\sigma^2}(r^\theta)$ is increasing in $r^\theta$. The existence region expands for lower variance ratio $\sigma^2$. The existence region expands for larger market, since the derivative of the nonzero part of $r^\varepsilon_{\gamma,\sigma^2}(r^\theta)$ with respect to $\gamma$, $-r^\theta (1 + \sigma^{-2}) (1 - r^\theta) (1 - \gamma r^\theta)^{-2}$, is negative. □
If the information structure is such that pairwise identical correlations across values and pairwise identical correlations across errors hold with parameters $\bar{\rho}^\theta$ and $\bar{\rho}^\varepsilon$, then the matrices $C_\theta$ and $C_\varepsilon$ are both patterned and computing their inverses is convenient.\footnote{Consider the $k$ by $k$ patterned matrix $C$ such that $[C]_{ii} = a$ and $[C]_{ij} = b$. Then, 

$$C^{-1} = \frac{1}{(a - b)} \left( E - \frac{b}{a + (k - 1) b} J \right),$$

where $a \neq b$, $a \neq -(k - 1) b$, $E$ is the $k$ by $k$ identity, and $J = 1_k^T 1_k$.}

From the formulas given by the projection theorem one can show that the expected values and the variances of the two conditional distributions coincide. Thus perfect information aggregation follows. □

**Proof 4** (Proposition 4: Price Informativeness). The projection theorem gives conditional variances $\text{Var}(\theta_i|s_i)$ and $\text{Var}(\theta_i|s_i, p^*)$ such that

$$\text{Var}(\theta_i|s_i) = \sigma_\theta^2 - \frac{\sigma_\theta^4}{\sigma_\theta^2 + \sigma_\varepsilon^2} \quad \text{and} \quad \text{Var}(\theta_i|s_i, p^*) =$$

$$\sigma_\theta^2 - \frac{\sigma_\theta^2}{1 - \bar{c}_p r^\theta} \left[ 1 + \sigma_\theta^2 \left( \frac{c_s}{1 - \bar{c}_p} \right)^2 \left( \frac{1 - r^\theta}{1 - r^\varepsilon} \right) \right]^{-1} \left[ \frac{1}{1 - \bar{c}_p r^\theta} \right].$$

Substituting $c_s$ and $c_p$ with the expressions in (5) and (4) gives price informativeness such that

$$\psi^+ = \frac{\text{Var}(\theta_i|s_i) - \text{Var}(\theta_i|s_i, p^*)}{\text{Var}(\theta_i|s_i)} = \frac{\sigma_\theta^2}{1 + \sigma_\theta^2 - \sigma_\varepsilon^2} \frac{r^\theta (1 - r^\theta) + \sigma_\theta^2 r^\varepsilon (1 - r^\varepsilon)}{(r^\theta + \sigma_\theta^2 r^\varepsilon)(1 - r^\theta + \sigma_\theta^2 (1 - r^\varepsilon))},$$

which, when simplified, gives

$$\psi^+ = \frac{\sigma_\varepsilon^2}{1 + \sigma_\theta^2} \left( \frac{r^\theta - r^\varepsilon}{r^\theta + \sigma^2 r^\varepsilon(1 - r^\varepsilon)} \right)^2.$$
terms of average commonalities such that

\[ \psi^+ = \frac{\sigma^2 (\rho - \bar{\rho})^2}{(1 + \sigma^2)^2 (1 - \gamma) + \gamma (\bar{\rho}^2 + \bar{\rho} \sigma^2)(1 + \sigma^2) - (\bar{\rho} + \bar{\rho} \sigma^2)^2}. \]

\[ \square \]

**Proof 5** (Propositions 5 and 6: Sufficient Conditions for Monotone Price Informative-
ness). Monotone price informativeness is implied if the polynomial function \( \psi^+ (r_\theta (x), r_\varepsilon (x)) \) is increasing in factor \( x \). Consider the derivative of \( \psi^+ \), that is, \( \partial \psi^+ (r_\theta (x), r_\varepsilon (x)) / \partial x \), which is positive if and only if

\[ 2 (1 - r_\theta + \sigma^2 (1 - r_\varepsilon)) (r_\theta - r_\varepsilon) (r_\theta' - r_\varepsilon') (r_\theta + \sigma^2 r_\varepsilon) > \]

\[ > (1 - 2r_\theta + \sigma^2 (1 - 2r_\varepsilon)) (r_\theta - r_\varepsilon)^2 (r_\theta' + \sigma^2 r_\varepsilon') . \]

(16)

To show that the conditions in Proposition 5 are sufficient for (16), consider first \( r_\theta > r_\varepsilon \) and notice that it implies the inequality \( 2 (1 - r_\theta + \sigma^2 (1 - r_\varepsilon)) > (1 - 2r_\theta + \sigma^2 - 2\sigma^2 r_\varepsilon) \). The inequality \( (r_\theta' - r_\varepsilon') (r_\theta + \sigma^2 r_\varepsilon) > (r_\theta - r_\varepsilon) (r_\theta' + \sigma^2 r_\varepsilon') \) follows from \( r_\theta' / r_\theta > r_\varepsilon' / r_\varepsilon \). Recall that \( (r_\theta' + \sigma^2 r_\varepsilon') > 0 \) which combined with the previous two inequalities gives (16). In the case when \( r_\theta < r_\varepsilon \), a similar argument holds.

To show that the conditions in Proposition 6 are sufficient for (16), consider first \( r_\theta > r_\varepsilon \) and notice that it implies the inequality \( 2 (1 - r_\theta + \sigma^2 (1 - r_\varepsilon)) > (1 - 2r_\theta + \sigma^2 - 2\sigma^2 r_\varepsilon) \). The inequality \( (r_\theta' - r_\varepsilon') (r_\theta + \sigma^2 r_\varepsilon) > (r_\theta - r_\varepsilon) (r_\theta' + \sigma^2 r_\varepsilon') \) follows from \( r_\theta' / r_\theta > r_\varepsilon' / r_\varepsilon \). Recall that \( (1 - 2r_\theta + \sigma^2 - 2\sigma^2 r_\varepsilon) > 0 \) which combined with the previous two inequalities gives (16). In the case when \( r_\theta < r_\varepsilon \), a similar argument holds. \( \square \)

**A.2 Table**

<table>
<thead>
<tr>
<th>Error Models and Statistics</th>
<th>( r_\varepsilon^2 )</th>
<th>Commonality ( \bar{\rho}_\varepsilon^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Common Errors</td>
<td>( \frac{1+(I-1)\rho}{I} )</td>
<td>( \rho )</td>
</tr>
<tr>
<td>Errors with Spatial Decay</td>
<td>( \frac{1+\beta}{1-\beta} \frac{1-\beta^2 I}{I} )</td>
<td>( (1 - \gamma) 2\beta \frac{1-\beta^2 I}{I} )</td>
</tr>
<tr>
<td>Errors in Two Groups</td>
<td>( \frac{1+\alpha}{2} )</td>
<td>( (2-\gamma)\alpha + \gamma )</td>
</tr>
<tr>
<td>Condensing Links</td>
<td>( \frac{1+2\beta}{1-\beta} \frac{1-\beta^k I}{I} )</td>
<td>( (1 - \gamma) 2\beta \frac{1-\beta^k I}{I} )</td>
</tr>
<tr>
<td>Strengthening Links</td>
<td>( \frac{1+\delta^{-k}\beta}{1-\delta^{-k}\beta} \frac{1-\delta^{-k}I}{I} )</td>
<td>( (1 - \gamma) 2\delta^{-k} \beta \frac{1-\delta^{-k}\beta^2 I}{I} )</td>
</tr>
<tr>
<td>Expanding Network</td>
<td>( \frac{1+\alpha k}{T(k)} )</td>
<td>( (1 - \gamma (k)) \alpha k \frac{2-\gamma (k)}{1-\gamma (k)} )</td>
</tr>
</tbody>
</table>

Table 1: Average Error Correlation and Commonality Function

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References


