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Brief paper

Asynchronous decentralized event-triggered control☆

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ABSTRACT

In this paper we propose an approach to the implementation of controllers with decentralized strategies triggering controller updates. We consider set-ups with a central node in charge of the computation of the control commands, and a set of not co-located sensors providing measurements to the controller node. The solution we propose does not require measurements from the sensors to be synchronized in time. The sensors in our proposal provide measurements in an aperiodic way triggered by local conditions. Furthermore, in the proposed implementation (most of) the communication between nodes requires only the exchange of one bit of information (per controller update), which could aid in reducing transmission delays and as a secondary effect result in fewer transmissions being triggered.

1. Introduction

Aperiodic control techniques have recently gained much attention due to the opportunities they open to reduce bandwidth and computation requirements in cyber–physical system's implementations (Anta & Tabuada, 2010; Åström & Bernhardsson, 2002; Tabuada, 2007). These savings are especially relevant in the implementation of control loops over wireless channels (Araujo et al., 2011; Rabi & Johansson, 2008). In those set-ups there is not only a limited bandwidth available, but also sensor nodes may have limited energy provided by batteries. It is therefore interesting to explore approaches which may save energy expenditures at the sensors by *e.g.* reducing the number of transmissions from those sensors, or reducing the amount of time the sensor nodes need to keep their radios listening for possible communications from other nodes. While there is an extensive recent literature on event-triggered control aimed at reducing the amount of transmissions necessary to close the control loop while maintaining stability (Cervin & Henningsson, 2008; Heemels, Sandee, & van den Bosch, 2008; Lunze & Lehmann, 2010; Molin & Hirche, 2010; Stöcker, Vey, & Lunze, 2013; Wang & Lemmon, 2011), the problem of reducing listening time has received less attention (Donkers & Heemels, 2012; Mazo & Cao, 2011, 2012; Weimer, Araújo, & Johansson, 2012). Nevertheless, it is a well-known phenomena in the sensor networks community that reducing listening times has a bigger impact on the power burden than reducing transmissions (Ye, Heidemann, & Estrin, 2002). In the present paper we try to bridge this gap by proposing controller implementations focused on reducing listening times. In order to attain this goal, we propose a technique in which the sensors do not need to coordinate with each other, and therefore do not need to listen to each other. Instead, in the proposed implementation the sensors send measurements triggered by local conditions, irrespective of what the other sensors are doing, in contrast with previous work on decentralized triggering (Mazo & Tabuada, 2011). With respect to other work on decentralized or distributed event-triggered control, we do not impose any weak coupling assumptions or very restrictive dynamics, as is often the case in work on multi-agent systems (Dimarogonas, Frazzoli, & Johansson, 2012; Heemels & Donkers, 2013; Li & Lemmon, 2011; Tallapragada & Chopra, 2012; Wang & Lemmon, 2011). Note that also Wang and Lemmon (2011), Dimarogonas et al. (2012) suffer from the drawback of continuous listening. Arguably, the work closest to ours is that presented in Donkers and Heemels (2012), however restricted to the study of linear systems.

The implementation that we propose also enables the stabilization of systems employing communication packets with very small

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payload. In particular, our technique reduces the amount of payload needed to essentially one bit. To appreciate the relevance of reducing the packets payload, besides reducing power consumption (Ye et al., 2002), one must notice that a large portion of delays in communications are due to transmission delay. These transmission delays are dependent on the size of the packages transmitted, and thus reducing the payload will indirectly reduce the communication delays present in the system. Event-triggered implementations of control systems accommodate delays by making more conservative the conditions that trigger communications than those in the delay free case. Employing more conservative conditions results, in general, in more frequent transmissions of measurements. Thus, a reduction on the payload is also expected to result in a reduction on the amount of transmissions from the sensors to the controller.

The ideas in the present paper will remind the reader of dynamic quantizers for control (Liberzon, 2003) and of dead-band control (Otanez, Moyne, & Tilbury, 2002). We have, in a way, combined those ideas with recent approaches to event-triggered control stemming from Tabuada (2007) to provide a formal analysis of implementations benefiting from all those ideas. The current paper is the result of merging previous conference contributions by the authors, providing a unified analysis and removing early mistakes and imprecise statements. As such it should be seen as a more accurate and easier to follow analysis of the proposals by Mazo and Cao (2011, 2012).

2. Preliminaries

We denote the positive real numbers by $\mathbb{R}^+$ and by $\mathbb{R}_0^+ = \mathbb{R}^+ \cup \{0\}$. We use $\mathbb{N}_0$ to denote the natural numbers including zero and $\mathbb{N}^+ = \mathbb{N}_0 \setminus \{0\}$. The usual Euclidean ($l_2$) vector norm is represented by $|\cdot|$. When applied to a matrix $|\cdot|$ denotes the l_2 induced matrix norm. A symmetric matrix $P \in \mathbb{R}^{n\times n}$ is said to be positive definite, denoted by $P > 0$, whenever $x^T Px > 0$ for all $x \neq 0, x \in \mathbb{R}^n$. By $\lambda_m(P), \lambda_M(P)$ we denote the minimum and maximum eigenvalues of P respectively. A function $f : \mathbb{R}^n \to \mathbb{R}^m$ is said to be locally Lipschitz if for every compact set $S \subset \mathbb{R}^n$ there exists a constant $L \in \mathbb{R}_0^+$ such that: $|f(x)-f(y)| \leq L|x-y|,\ \forall x, y \in S$. For a function $f : \mathbb{R}^n \to \mathbb{R}^n$ we denote by $f_i : \mathbb{R}^n \to \mathbb{R}$ the function whose image is the projection of f on its ith coordinate, *i.e.* $f_i(x) = \Pi_i(f(x))$. Consequently, given a Lipschitz continuous function f, we also denote by L_{f_i} the Lipschitz constant of f_i. A function $\gamma : [0, a[\to \mathbb{R}_0^+$, is of class $\mathcal{K}$ if it is continuous, strictly increasing and $\gamma(0) = 0$; if furthermore $a = \infty$ and $\gamma(s) \to \infty$ as $s \to \infty$, then γ is said to be of class $\mathcal{K}_\infty$. A continuous function $\beta : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \to \mathbb{R}_0^+$ is of class $\mathcal{KL}$ if $\beta(\cdot, \tau)$ is of class $\mathcal{K}$ for each fixed $\tau \geq 0$ and for each fixed $s \geq 0$, $\beta(s, \tau)$ is decreasing with respect to τ and $\beta(s, \tau) \to 0$ for $\tau \to \infty$. Given an essentially bounded function $\delta : \mathbb{R}_0^+ \to \mathbb{R}^m$ we denote by $\|\delta\|_\infty$ the $\mathcal{L}_\infty$ norm, *i.e.* $\|\delta\|_\infty = \text{ess sup}_{t\in\mathbb{R}_0^+}\{|\delta(t)|\}$.

The notion of Input-to-State stability (ISS) (Agrachev, Morse, Sontag, Sussmann, & Utkin, 2008) will be central to our discussion:

Definition 1 (*Input-to-State Stability*)**.** A control system $\dot{\xi} = f(\xi, \upsilon)$ is said to be (uniformly globally) input-to-state stable (ISS) with respect to υ if there exist $\beta \in \mathcal{KL}, \gamma \in \mathcal{K}_\infty$ such that for any $t_0 \in \mathbb{R}_0^+$ the following holds:

$$\forall \xi(t_0) \in \mathbb{R}^n, \quad \|\upsilon\|_\infty < \infty,$$
$$|\xi(t)| \leq \beta(|\xi(t_0)|, t - t_0) + \gamma(\|\upsilon\|_\infty), \quad \forall t \geq t_0.$$

Rather than using its definition, in our arguments we rely on the following characterization: a system is ISS if and only if there exists an ISS Lyapunov function (Agrachev et al., 2008).

Definition 2 (*ISS Lyapunov Function*)**.** A continuously differentiable function $V : \mathbb{R}^n \to \mathbb{R}_0^+$ is said to be an ISS Lyapunov function for the closed-loop system $\dot{\xi} = f(\xi, \upsilon)$ if there exist class $\mathcal{K}_\infty$ functions $\underline{\alpha}, \overline{\alpha}, \alpha_v$ and α_e such that for all $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ the following is satisfied:

$$\underline{\alpha}(|x|) \leq V(x) \leq \overline{\alpha}(|x|)$$
$$\nabla V \cdot f(x, u) \leq -\alpha_v \circ V(x) + \alpha_e(|u|). \quad (1)$$

Often we use the shorthand $\dot{V}(x, u)$ to denote the Lie derivative $\nabla V \cdot f(x, u)$, and $\circ$ to denote function composition, *i.e.* $f \circ g(t) = f(g(t))$.

Finally, we employ the following, rather trivial, result in some of our arguments:

Lemma 3. *Given two $\mathcal{K}_\infty$ functions α_1 and α_2, there exists some constant $L < \infty$ such that:*

$$\limsup_{s\to 0} \frac{\alpha_1(s)}{\alpha_2(s)} \leq L$$

if and only if for all $S < \infty$ there exists a positive $\kappa < \infty$ such that:

$$\forall s \in\ ]0, S], \quad \alpha_1(s) \leq \kappa\alpha_2(s).$$

Proof. The necessity side of the equivalence is trivial, thus we concentrate on the sufficiency part. By assumption, we know that the limit superior of the ratio of the functions tends to L as $s \to 0$, and therefore, $\forall\epsilon > 0, \exists\delta > 0$ such that $\alpha_1(s)/\alpha_2(s) < L + \epsilon$ for all $s \in\ ]0, \delta[$. As $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$ we know that in any compact set excluding the origin the function $\alpha_1(s)/\alpha_2(s)$ is continuous and therefore attains a maximum, implying that there exists a positive $M \in \mathbb{R}^+$ such that $\alpha_1(s)/\alpha_2(s) < M$, $\forall s \in [\delta, S]$, $0 < \delta < S$. Putting these two results together we have that $\forall s \in\ ]0, S]$, $S < \infty$, $\alpha_1(s) \leq \kappa\alpha_2(s)$, where $\kappa = \max\{L + \epsilon, M\}$. □

3. Problem definition

The problem we aim at solving is that of controlling systems of the form:

$$\dot{\xi}(t) = f(\xi(t), \upsilon(t)), \quad \forall t \in \mathbb{R}_0^+, \quad (2)$$

where $\xi : \mathbb{R}_0^+ \to \mathbb{R}^n$ and $\upsilon : \mathbb{R}_0^+ \to \mathbb{R}^m$ and the full state is assumed to be measured. In particular, we are interested in finding stabilizing sample-and-hold implementations of a controller $\upsilon(t) = k(\xi(t))$ such that updates can be performed transmitting asynchronous and aperiodic measurements of each entry of the state vector. Furthermore, if possible we would like to do so while reducing the amount of transmissions. This problem can be formalized as follows:

Problem 4. Given system (2) and a controller $k : \mathbb{R}^n \to \mathbb{R}^m$ find sequences of update times $\{t_{r_i}^i\}$, $r_i \in \mathbb{N}_0$ for each sensor $i = 1, \ldots, n$ such that an asynchronous sample-and-hold controller implementation:

$$\upsilon_j(t) = k_j(\hat{\xi}(t)), \quad (3)$$
$$\hat{\xi}_i(t) = \xi_i(t_{r_i}^i),\ t \in [t_{r_i}^i, t_{r_i+1}^i[, \quad \forall i = 1, \ldots, n. \quad (4)$$

renders the closed-loop system:

- uniformly globally practically asymptotically stable (UGPS), i.e. satisfying that for all $\delta > 0$, there exist a controller implementation parameter $\eta(\delta)$[2] and $\beta_\delta \in \mathcal{KL}$ such that for any $t_0 \geq 0$:

  $$\forall \xi(t_0) \in \mathbb{R}^n, \quad |\xi(t)| \leq \beta_\delta(|\xi(t_0)|, t - t_0) + \delta, \quad \forall t \geq t_0;$$

[2] The update times $t_{r_i}^i$ will depend on the selection of η.

- or, uniformly globally asymptotically stable (UGAS), i.e. satisfying that there exists $\beta \in \mathcal{KL}$ such that for any $t_0 \geq 0$:

$$\forall \xi(t_0) \in \mathbb{R}^n, \quad |\xi(t)| \leq \beta(|\xi(t_0)|, t - t_0), \quad \forall t \geq t_0.$$

We address these two problems in what follows under the following two main technical assumptions:

Assumption 5 (*ISS w.r.t. Measurement Errors*)**.** Given system (2) there exists a controller $k : \mathbb{R}^n \to \mathbb{R}^m$ such that the closed-loop system

$$\dot{\xi}(t) = f(\xi(t), k(\xi(t) + \varepsilon(t))), \quad \forall t \in \mathbb{R}_0^+ \tag{5}$$

is ISS with respect to measurement errors ε. Furthermore, we assume the knowledge of an ISS-Lyapunov function V certifying that the system is ISS, i.e. satisfying (1).

Assumption 6 (*Lipschitz Continuity*)**.** The functions f and k, defining the dynamics and controller of the system, are locally Lipschitz.

Note that this last assumption guarantees the (not necessarily global) existence and uniqueness of solutions of the closed-loop system.

4. Uniform global practical stabilization

Representing the effect of the sample-and-hold as a measurement error at each sensor for all $i = 1, \ldots, n$ as:

$$\varepsilon_i(t) = \xi_i(t^i_{r_i}) - \xi_i(t), \quad t \in [t^i_{r_i}, t^i_{r_i+1}[, \; r_i \in \mathbb{N}_0$$

we propose rules defining implicitly the sequences of update times $\{t^i_{r_i}\}$ for each sensor i:

$$t^i_{r_i} := \min\{t > t^i_{r_i-1} \,|\, \varepsilon_i^2(t) = \eta_i\}, \tag{6}$$

where $\eta_i > 0$ are design parameters.

For convenience and compactness of the presentation we introduce the new variable:

$$\eta = \sqrt{\sum_{i=1}^{n} \eta_i}, \tag{7}$$

and consider it as a design parameter that once specified restricts the choices of η_i to be used at each sensor. We remark now that with this definition the update rule (6) implies that $|\varepsilon(t)| \leq \eta$ (with equality attained only when all sensors trigger simultaneously). Furthermore, we assume that the local parameters η_i are defined through an appropriate scaling:

$$\eta_i := \theta_i^2 \eta^2, \quad |\theta| = 1, \tag{8}$$

with θ_i as design constants introduced for analysis purposes only.

Now, we can state the following lemma which will be central in the rest of the discussion:

Lemma 7 (*Inter-Transmission Times Bound*)**.** *If Assumptions 5 and 6 hold, for any $\eta > 0$, a lower bound for the minimum time between transmissions of the sensor i, for $i \in \{1, \ldots, n\}$, for all time $t \geq t_0$, is given by:*

$$\tau_i^* := L_{f_i}^{-1} \theta_i \frac{\eta}{\eta + \underline{\alpha}^{-1}(\max\{V(\xi(t_0)), \alpha_v^{-1} \circ \alpha_e(\eta)\})}, \tag{9}$$

where L_{f_i} denotes the Lipschitz constant of the function $f_i(x, k(x + e))$ for $|x| \leq \underline{\alpha}^{-1}(\max\{V(\xi(t_0)), \alpha_v^{-1} \circ \alpha_e(\eta)\})$ and $|e| \leq \eta$.

Proof. Let us denote in what follows by: $S(y, z) = \{(x, e) \in \mathbb{R}^{n\times n} \mid V(x) \leq y, |e| \leq z\}$ and by $\bar{f}_i(y, z) = \max_{(x,e)\in S(y,z)} |f_i(x, k(x + e))|$. From Assumption 5 we have that: $|e| \leq \eta$, $V(x) \geq \alpha_v^{-1} \circ \alpha_e(\eta) \Rightarrow \dot{V}(x, e) \leq 0$ and thus $\tilde{S} := S(\max\{V(\xi(t_0)), \alpha_v^{-1} \circ \alpha_e(\eta)\}, \eta)$ is forward invariant. Recall that the minimum time between events at a sensor is given by the time it takes for $|\varepsilon_i|$ to evolve from the value $|\varepsilon_i(t^i_{k_i})| = 0$ to $|\varepsilon_i(t^{i-}_{k_i+1})| = \sqrt{\eta_i}$, and thus[3] $\tau_i \geq \sqrt{\eta_i}(\max_{\tilde{S}} \frac{d}{dt}|\varepsilon_i|)^{-1}$. Therefore all that needs to be proved is the existence of an upper bound on the rate of change of $|\varepsilon_i|$. One can trivially bound the evolution of $|\varepsilon_i|$ as: $\frac{d}{dt}|\varepsilon_i| \leq |\dot{\varepsilon}_i| = |f_i(\xi, k(\xi + \varepsilon))|$, and the maximum rate of change of $|\varepsilon_i|$ in $\tilde{S}$ by $\bar{f}_i(\max\{V(\xi(t_0)), \alpha_v^{-1} \circ \alpha_e(\eta)\}, \eta)$. Note that the existence of such a maximum is guaranteed by the continuity of the maps f and k and the compactness of the set $\tilde{S}$. Assumption 6 implies that $f_i(x, k(x+e))$ is also locally Lipschitz, and thus one can further bound $\bar{f}_i(\max\{V(\xi(t_0)), \alpha_v^{-1} \circ \alpha_e(\eta)\}, \eta) \leq L_{f_i}\left(\underline{\alpha}^{-1}(\max\{V(\xi(t_0)), \alpha_v^{-1} \circ \alpha_e(\eta)\}) + \eta\right)$. Finally, recalling that $\eta_i = \theta_i^2 \eta^2$, a lower bound for the inter-transmission times is given by (9) which proves the statement. □

With this result one can state the following theorem:

Theorem 8 (*UGPS*)**.** *If Assumptions 5 and 6 hold, then the closed-loop system (2), (3), (6) is UGPS with respect to the parameter η in (7). Moreover, the time between transmissions of measurements at each sensor i is bounded from below by some $\tau_i^* > 0$.*

Proof. From Lemma 7 we know that there exists some minimum time between the triggering of events at the different sensors. This, together with that the number of sensors is finite, guarantees that there are no Zeno executions of the closed-loop system. Assumption 5 provides the bound: $|\xi(t)| \leq \beta(|\xi(t_0)|, t - t_0) + \gamma(\|\varepsilon\|_\infty)$, and the proposed implementation forces $\|\varepsilon\|_\infty \leq \eta$. Thus, using these two bounds, and ruling out any possible Zeno solution, we know that $|\xi(t)| \leq \beta(|\xi(t_0)|, t - t_0) + \delta$, where $\delta := \gamma(\eta)$, which finalizes the proof. □

5. Uniform global asymptotic stabilization

In general, employing a constant threshold value η establishes a trade-off between the size of the inter-transmission times and the size of the set to which the system converges. In order to achieve asymptotic stability we propose to let the parameter η change over time as:

$$\eta(t) = \eta(t^c_{r_c}), \quad \forall t \in [t^c_{r_c}, t^c_{r_c+1}[, \tag{10}$$

with $\{t^c_{r_c}\}$ being a divergent sequence of times (to be defined later) with $t^c_0 = t_0$; and to use the local update rules:

$$t^i_{r_i} := \min\{t > t^i_{r_i-1} \,|\, \varepsilon_i^2(t) = \eta_i(t)\},$$

$$\eta_i(t) := \theta_i^2 \eta(t)^2, \quad |\theta| = 1. \tag{11}$$

The following lemma establishes some requirements to construct asymptotic stabilizing asynchronous implementations.

Lemma 9. *The closed-loop system (2), (3), (10), (11) is UGAS if Assumptions 5 and 6 are satisfied and the following two conditions hold:*

- *$\{\eta(t^c_{r_c})\}$ is a monotonically decreasing sequence with $\lim_{r_c\to\infty} \eta(t^c_{r_c}) \to 0$;*

[3] The time derivative of $|\varepsilon_i|$ is defined for almost all t (excluding the instants $\{t_{k_i}\}$), which is sufficient to bound the time between events.

- *There exists $\kappa > 0$ such that for all $t^c_{r_c}$* $\frac{\underline{\alpha}^{-1}(\max\{V(\xi(t^c_{r_c})),\alpha_v^{-1}\circ\alpha_e(\eta(t^c_{r_c}))\})}{\eta(t^c_{r_c})} \le \kappa < \infty$.

Proof. In view of Lemma 7 the second condition of this lemma guarantees that there exists a minimum time between events at each sensor when both events fall in an open time interval $]t^c_{r_c}, t^c_{r_c+1}[$, *i.e.* $t^i_{r_i+1} - t^i_{r_i} > \tau^*_i$ for all $i = 1, \dots, n$ and $t^i_{r_i}, t^i_{r_i+1} \in]t^c_{r_c}, t^c_{r_c+1}[$. It could happen however that some sensor update coincides with an update of the thresholds, *i.e.* $t^i_{r_i+1} = t^c_{r_c+1}$, which could lead to two arbitrarily close events of sensor i. Similarly, events from two different sensors could be generated arbitrarily close to each other. Nonetheless, as the sequence $\{t^c_{r_c}\}$ is divergent (by assumption), and there is a finite number of sensors, none of these two effects can lead to Zeno executions.

The second condition of this lemma also implies that at $t^c_{r_c}$ either $V(\xi(t^c_{r_c})) \le \alpha_v^{-1}\circ\alpha_e(\eta(t^c_{r_c}))$ or $V(\xi(t^c_{r_c})) \le \underline{\alpha}(\kappa\eta(t^c_{r_c}))$. From Assumption 5 we have that for all $t \in [t^c_{r_c}, t^c_{r_c+1}[$ the following bound holds:

$$V(\xi(t)) \le \max\{V(\xi(t^c_{r_c})), \alpha_v^{-1}\circ\alpha_e(\eta(t^c_{r_c}))\}$$
$$\le \max\{\underline{\alpha}(\kappa\eta(t^c_{r_c})), \alpha_v^{-1}\circ\alpha_e(\eta(t^c_{r_c}))\}.$$

Thus, using definition (10) results in: $V(\xi(t)) \le \gamma_V(\eta(t)),\ \forall t \ge t_0$, where $\gamma_V \in \mathcal{K}_\infty$ is the function: $\gamma_V(s) = \max\{\alpha_v^{-1}\circ\alpha_e(s), \underline{\alpha}(\kappa s)\}$.

Next, we notice that the first condition of this Lemma implies that $\exists\beta_\eta \in \mathcal{KL}$ such that $\eta(t) \le \beta_\eta(\eta(t^c_0), t - t^c_0)$ for all $t \ge t^c_0$. Putting together these last two bounds, and assuming that the initial threshold is selected as $\eta(t^c_0) = \kappa_0 V(\xi(t^c_0))$, for some constant $\kappa_0 \in]0, \infty[$, one can conclude that:

$$V(\xi(t)) \le \gamma_V(\beta_\eta(\kappa_0 V(t^c_0), t - t^c_0)), \quad \forall t \ge t^c_0.$$

Finally, this last bound guarantees that:

$$|\xi(t)| \le \underline{\alpha}^{-1}(\gamma_V(\beta_\eta(\kappa_0\overline{\alpha}(|\xi(t^c_0)|), t - t^c_0))) \quad (12)$$
$$:= \beta(|\xi(t^c_0)|, t - t^c_0), \quad \forall t \ge t^c_0 \quad (13)$$

with $\beta \in \mathcal{KL}$ which finalizes the proof. □

Remark 10. Note that this lemma, while ruling out Zeno behavior, does not establish a minimum time between transmissions of the same sensor. Such bounds are provided in Proposition 15. Furthermore, neither this lemma nor the results from Section 4 address the occurrence of arbitrarily close transmissions from different sensors. A solution to this last problem is discussed in Section 6.1.

In the remaining part of this section we propose and analyze an update policy for the time-varying threshold $\eta(t^c_{r_c})$ resulting in asymptotic stability employing only asynchronous measurements from all sensors. The proposed update policy for $\eta(t^c_{r_c})$ is given by:

$$\eta(t) = \eta(t^c_{r_c}), \quad t \in [t^c_{r_c}, t^c_{r_c+1}[$$
$$\eta(t^c_{r_c+1}) = \mu\eta(t^c_{r_c}), \quad (14)$$

for some $\mu \in]0.5, 1[$. Given this update policy one can design an event-triggered policy to decide the sequence of times $\{t^c_{r_c}\}$ such that the system is rendered asymptotically stable. Furthermore, as we show later in this section, such a fully event-triggered implementation enables asymptotic implementations only requiring the exchange of one bit of information whenever communication between a sensor and controller, and vice-versa, is necessary. The only exception to this being the transmission of the initial state of the system at t_0. This new strategy uses two independent triggering mechanisms:

- *Sensor to controller:* Sensors send measurements to the controller whenever the local threshold is violated. As explained in the previous section, the update of the control commands is done with the measurements as they arrive in an asynchronous fashion.
- *Controller-to-sensor:* The controller commands the sensors to reduce the threshold used in their triggering condition when the system has "slowed down" enough to guarantee that the inter-sample times remain bounded from below. The controller checks this condition only in a periodic fashion, with period τ^c, and therefore the sensors only need to listen at those time instants.

The mechanism to trigger sensor to controller communication has already been analyzed in Section 3. In what follows we concentrate on describing and analyzing the triggering mechanism for the communication from controller to sensors.

We introduce first the following assumption restricting the type of ISS controllers amenable to the strategy we propose in what follows:

Assumption 11. For some $\epsilon > 1$ the ISS closed-loop system (5) satisfies the following property:

$$\limsup_{s\to 0} \underline{\alpha}^{-1}\circ\overline{\alpha}(\underline{\alpha}^{-1}\circ\epsilon\alpha_v^{-1}\circ\alpha_e(s) + 2s)s^{-1} < \infty. \quad (15)$$

Remark 12. Note that this assumption, as well as Assumptions 5 and 6, are automatically satisfied by linear systems with a stabilizing linear state feedback controller and the usual (ISS) quadratic Lyapunov function.

We remind the reader that $\hat{\xi}$, defined in (4), is the vector formed by asynchronous measurements of the state entries that the controller is using to compute the input to the system. Thus, the controller can compute the following upper bound:

$$|\overline{\xi}|(t) := |\hat{\xi}(t)| + \eta(t^c_{r_c}) \ge |\hat{\xi}(t) - \varepsilon(t)|$$
$$= |\xi(t)|, \quad \forall t \in [t^c_{r_c}, t^c_{r_c+1}[, \quad (16)$$

which also satisfies the bound $|\overline{\xi}|(t) \le |\xi(t)| + 2\eta(t^c_{r_c})$.

The following theorem proposes a condition to trigger the update of sensor thresholds guaranteeing UGAS of the closed-loop system:

Theorem 13 (*UGAS*)**.** *Consider the closed-loop system (2), (3), (10), (11) with the threshold update rule (14) and satisfying Assumptions 5, 6 and 11. Let $\tau_c > 0$ be a design parameter. The sequence of threshold update times $\{t^c_{r_c}\}$ implicitly defined by:*

$$t^c_{r_c+1} := \min\{t = t^c_{r_c} + r\tau^c \mid r \in \mathbb{N}^+, |\overline{\xi}|(t)$$
$$\le \overline{\alpha}^{-1}\circ\underline{\alpha}(\rho\eta(t^c_{r_c}))\}, \quad (17)$$

with any $\rho < \infty$ satisfying:

$$\underline{\alpha}^{-1}\circ\overline{\alpha}(\underline{\alpha}^{-1}\circ\epsilon\alpha_v^{-1}\circ\alpha_e(s) + 2s) \le \rho s \quad (18)$$

for all $s \in]0, \eta(t_0)]$ and some $\epsilon > 1$, renders the closed-loop system UGAS.

Proof. We use Lemma 9 to show the desired result. The first itemized condition of the lemma is satisfied by the employment of the update rule (14) with a constant $\mu \in]0, 1[$ if we can show that the sequence $\{t^c_{r_c}\}$ is divergent. Thus, we must show that this sequence is divergent and that the second itemized condition in the lemma also holds.

First we show that starting from some time $t^c_{r_c}$ there always exists some time $T \ge t^c_{r_c}$ such that for all $t \ge T$ $|\overline{\xi}|(t) \le \overline{\alpha}^{-1}$

$\circ\, \underline{\alpha}(\rho\eta(t^c_{r_c}))$. Showing this guarantees that $\{t^c_{r_c}\}$ is a divergent sequence. From Assumption 5 we know that for every $\bar{\epsilon} > 0$ there exists some $T \geq t^c_{r_c}$ such that $\underline{\alpha}(|\xi(t)|) \leq V(\xi(t)) \leq \alpha_v^{-1} \circ \alpha_e(\eta(t^c_{r_c})) + \bar{\epsilon}$ for every $t \geq T$. Let $\bar{\epsilon} = (\epsilon - 1)\alpha_v^{-1} \circ \alpha_e(\eta(t^c_{r_c}))$, for some $\epsilon > 1$, then:

$$|\xi(t)| \leq \underline{\alpha}^{-1}(\epsilon\alpha_v^{-1} \circ \alpha_e(\eta(t^c_{r_c}))), \quad \forall t > T$$

and thus we have that the proposed norm estimator (16) satisfies the bound:

$$|\bar{\xi}|(t) \leq \underline{\alpha}^{-1}(\epsilon\alpha_v^{-1} \circ \alpha_e(\eta(t^c_{r_c}))) + 2\eta(t^c_{r_c}), \quad \forall t > T.$$

Therefore, if there exists a $\rho > 0$ satisfying (18) for some $\epsilon > 1$ and for all $s \in\,]0, \eta(t_0)]$, a triggering event will eventually happen. Finally, from Assumption 11 and Lemma 3 one can conclude that such a $\rho < \infty$ exists.

The second condition of Lemma 9 is easier to prove. We start remarking that with ρ so that (18) holds, as $\mu < 1, \epsilon > 1$ and $\underline{\alpha}^{-1} \circ \overline{\alpha}(s) \geq s$ for all s, we also have:

$$\frac{\rho}{\mu}s \geq \underline{\alpha}^{-1} \circ \alpha_v^{-1} \circ \alpha_e(s) \tag{19}$$

for all $s \in\,]0, \eta(t_0)]$. Thus, (19) and the triggering condition (17), as $V(\xi(t^c_{r_c})) \leq \overline{\alpha}(|\bar{\xi}|(t^c_{r_c}))$, guarantee that the following holds:

$$\underline{\alpha}^{-1}(\max\{V(\xi(t^c_{r_c})), \alpha_v^{-1} \circ \alpha_e(\eta(t^c_{r_c}))\}) \leq \frac{\rho}{\mu}\eta(t^c_{r_c})$$

at all times $t^c_{r_c} > t^c_0$. Therefore

$$\begin{aligned}\kappa &:= \max\left\{\frac{\rho}{\mu}, \frac{\underline{\alpha}^{-1} \circ V(\xi(t^c_0))}{\eta(t^c_0)}\right\} \\ &\geq \frac{\underline{\alpha}^{-1}(\max\{V(\xi(t^c_{r_c})), \alpha_v^{-1} \circ \alpha_e(\eta(t^c_{r_c}))\})}{\eta(t^c_{r_c})}\end{aligned} \tag{20}$$

for all $t^c_{r_c}$, which concludes the proof. □

Remark 14. Finding controllers to satisfy Assumption 11 might be, in general, an arduous task. However, in practice one is generally only concerned with attaining practical stability and can thus disregard this assumption. In this case, it is enough to guarantee (15) for $s \in [\eta_m, \eta(t^c_0)]$, $\eta_m > 0$, which can always be satisfied given that: for every $\alpha \in \mathcal{K}_\infty$ there always exists $\kappa < \infty$ such that $\alpha(s) \leq \kappa s$ for all $s \in [\eta_m, \eta(t^c_0)]$. Then, one would stop any further updates of the thresholds whenever the threshold reaches η_m. The benefit of this approach is that one would obtain longer inter-transmission times when compared to employing a fixed constant threshold η_m from the beginning (as in Section 3).

The presented implementations of asynchronous event-triggered controllers require only one bit communications: to recover the value of a sensor after a threshold crossing, it is only necessary to know the previous value of the sensor and the sign of the error ε_i when it crossed the threshold:

$$\hat{\xi}_i(t^i_{r_i}) = \hat{\xi}_i(t^i_{r_i-1}) + \mathtt{sign}(\varepsilon_i(t^i_{r_i}))\sqrt{\eta_i(t^c_{r_c})}. \tag{21}$$

Similarly, messages from the controller to the sensors, commanding a reduction of the thresholds, can be indicated with a single bit.

Using this one-bit implementation we can now obtain bounds for the time between updates of a sensor valid globally (not only between threshold updates, but also across such updates):

Proposition 15 (*Inter-Transmission Time Bounds*). *The controller implementation from Theorem 13 with controller updates (21), $\tau^c \geq \max_{i\in[1,n]}\{\frac{\mu L_{f_i}^{-1}\theta_i}{\mu+\rho}\}$ and*

$$\eta(t^c_0) \geq \frac{\mu}{\rho}\underline{\alpha}^{-1} \circ V(\xi(t^c_0)), \tag{22}$$

guarantees that a minimum time between events at each sensor is given, for all $t \geq t^c_0$, by:

$$t^i_{r_i} - t^i_{r_i-1} \geq \tau^*_i \geq (2\mu - 1)\,\frac{L_{f_i}^{-1}\theta_i}{\mu+\rho} > 0, \tag{23}$$

where L_{f_i} is the Lipschitz constant of the function $f_i(x, k(x+e))$ for $|x| \leq \underline{\alpha}^{-1}(\max\{V(\xi(t^c_0)), \alpha_v^{-1} \circ \alpha_e(\eta(t^c_0))\})$ and $|e| \leq \eta(t^c_0)$.

Proof. Theorem 13, by means of Lemma 9, guarantees that:

$$t^i_{r_i} - t^i_{r_i-1} \geq \tau^b_i \geq \frac{L_{f_i}^{-1}\theta_i}{1+\kappa}, \quad \forall\, t^i_{r_i}, t^i_{r_i-1} \in\,]t^c_{r_c}, t^c_{r_c+1}[,$$

with (see proof of Theorem 13) $\kappa := \frac{\rho}{\mu}$ when (22) holds. However, it can happen that some sensors automatically violate their triggering condition when their local threshold is reduced, i.e. some $t^i_{r_i} = t^c_{r_c}$. This can lead to two possible problematic situations: that $t^i_{r_i} - t^i_{r_i-1} < \tau^b_i$; and/or that $t^i_{r_i+1} - t^i_{r_i} < \tau^b_i$. In the first case, one can always bound $t^i_{r_i} - t^i_{r_i-1} \geq \mu\tau^i_b$ following the reasoning in the proof of Lemma 7 with the same bound for the system speed but to reach a threshold $|\varepsilon_i(t^{i-}_{r_i})| = \mu\sqrt{\eta_i(t^c_{r_c-1})}$, as (17) guarantees that no more than one threshold update can occur simultaneously. Note that by employing $\tau^c > \max_i\{\tau^b_i\}$ one also guarantees that threshold updates do not trigger sensor updates closer than τ^b_i. In the second case, the source of the problem is the update of $\hat{\xi}$ following (21). When $|\varepsilon_i(t^{i-}_{r_i})| > \sqrt{\eta_i(t^c_{r_c})}$ the controller is updated with a value:

$$\hat{\xi}_i(t^i_{r_i}) = \hat{\xi}_i(t^i_{r_i-1}) + \mathtt{sign}(\varepsilon_i(t^i_{r_i}))\sqrt{\eta_i(t^c_{r_c})}. \tag{24}$$

Thus, updating the local error accordingly as $\varepsilon_i(t^i_{r_i}) := \hat{\xi}_i(t^i_{r_i}) - \xi_i(t^i_{r_i})$ results in an error satisfying $|\varepsilon_i(t^i_{r_i})| \leq (\frac{1}{\mu} - 1)\sqrt{\eta_i(t^c_{r_c})}$, and not necessarily equal to zero. Reasoning again as in the proof of Lemma 7, but now computing the time it takes $|\varepsilon_i|$ to go from a value of $(\frac{1}{\mu} - 1)\sqrt{\eta_i(t^c_{r_c})}$ to $\sqrt{\eta_i(t^c_{r_c})}$, one can show that $t^i_{r_i+1} - t^i_{r_i} \geq \left(2 - \frac{1}{\mu}\right)\tau^b_i$, whenever $t^i_{r_i} = t^c_{r_c}$. Finally, realizing that $\mu > 2 - \frac{1}{\mu} \geq 0$ for all $\mu \in\,]0.5, 1[$ concludes the proof. □

6. Practical considerations

6.1. Delays

Many effects of a real practical implementation can be abstracted in the form of a delay in the proposed event-triggered implementation. We illustrate this with a specific example: In our implementations controller updates can take place arbitrarily close to each other. This is so because while one sensor cannot trigger updates arbitrarily close to each other, the combination of all sensors can potentially force that to happen. This makes the proposed techniques more suitable for systems with controller(s) and actuators co-located. As in Mazo and Cao (2011), we suggest the use of a periodic subjacent scheme for the update of the controller. The effect of such a scheme is the introduction of an artificial delay in the closed-loop system.

The kind of delays we consider are those between the event-generation at the sensor side and its effect taking place in the control inputs applied to the system. Essentially, what most event-triggered techniques do is control the magnitude of the virtual error introduced by sampling in a digital implementation. If the magnitude of this error signal is successfully kept within certain margins, the controller implementation is stable. This error signal that one must control is defined at the plant side. Therefore, when delays are present, while the sensors send new measurements

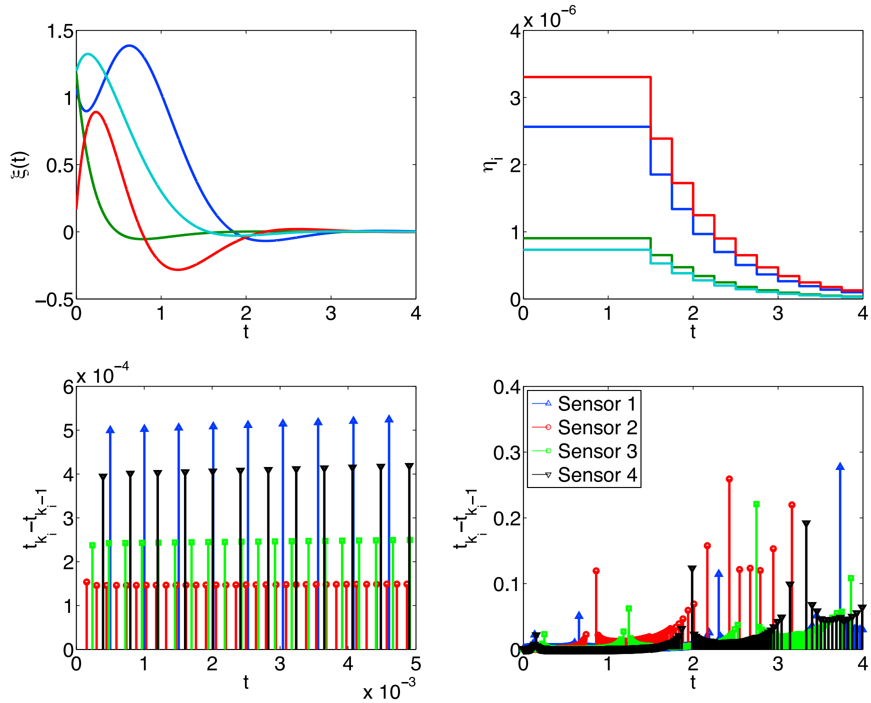

Fig. 1. State trajectory, evolution of the thresholds, and events generated at the sensors.

trying to keep $|\varepsilon_i(t)| = |\xi_i(t^i_{r_i}) - \xi_i(t)| \le \eta_i$, what actually matters is the value of the error at the plant-side $\hat{\varepsilon}_i(t)$, defined as:

$$\hat{\varepsilon}_i(t) = \xi(t^i_{r_i-1}) - \xi(t) \quad t \in [t^i_{r_i}, t^i_{r_i} + \Delta\tau^i_{r_i}[\tag{25}$$

$$\hat{\varepsilon}_i(t) = \varepsilon_i(t) \quad t \in [t^i_{r_i} + \Delta\tau^i_{r_i}, t^i_{r_i+1}[, \tag{26}$$

where $\Delta\tau^i_{r_i}$ denotes the delay between the time $t^i_{r_i}$, at which a measurement is transmitted, and the time $t^i_{r_i} + \Delta\tau^i_{r_i}$, at which the controller is updated with that new measurement. Thus the actual objective to attain UGAS or UGPS is to keep $|\hat{\varepsilon}(t)|$ below the threshold η. From the analysis in the proof of Lemma 7 we know that the maximum speed of the error signal is always kept below $L_{f_i}(\kappa+1)\eta$, with L_{f_i} as in Proposition 15. Thus, given a maximum delay of $\Delta\tau^i_{r_i} \le \Delta\tau < (\kappa+1)^{-1}L_{f_i}^{-1}$ for all i and r_i, reducing the local thresholds as:

$$\theta_i\bar{\eta} = \theta_i\eta\left(1 - L_{f_i}(\kappa+1)\Delta\tau\right)$$

and keeping $|\varepsilon(t)| \le \bar{\eta}$, guarantees that the error at the plant side stays below the desired value $|\hat{\varepsilon}_i(t)| \le \eta$. The more conservative our estimates of κ and L_{f_i} are, the smaller the tolerable delays will be.

6.2. Performance guarantees

Performance guarantees are provided by (13) determined by κ in (20), which in general are very hard to interpret. We provide in the following some intuition on the performance effect of the three design parameters: ρ, μ and τ^c. Reducing ρ, μ or τ^c in general should yield a faster convergence of the system. However, while reducing μ leads to more frequent inter-transmission times from the sensors, reducing ρ may increase the frequency of threshold update requests sent from the controller to the sensors.

7. Example

Consider a nonlinear system of the form:

$$\dot{\xi}(t) = A\xi + B(g(\xi(t)) + \upsilon(t))$$

where g is a nonlinear locally Lipschitz function. Consider the controller affected by measurement errors:

$$\upsilon(t) = -g(\xi(t) + \varepsilon(t)) - K(\xi(t) + \varepsilon(t)),$$

with K such that $A_c = A - BK$ is Hurwitz.

Let $V(x) = x^T Px$, where $PA_c + A_c^T P = -I$, be the candidate ISS-Lyapunov function for the system. It is easy to show that one can set: $\overline{\alpha}(s) = \lambda_M(P)s^2$, $\underline{\alpha}(s) = \lambda_m(P)s^2$, $\alpha_x(s) = a_x s^2$ and $\alpha_\upsilon(s) = \alpha_x \circ \overline{\alpha}^{-1}(s)$ with $\alpha_e(s) = a_e s^2$. Noting that $\epsilon\overline{\alpha}(s) \le \overline{\alpha}(\epsilon s)$, $\forall \epsilon > 1$ and a simple manipulation one can show that Assumption 11 is satisfied and obtain the condition:

$$\rho > \frac{\lambda_M(P)}{\lambda_m(P)}\sqrt{\frac{a_e}{a_x}} + 2\sqrt{\frac{\lambda_M(P)}{\lambda_m(P)}},$$

where $a_x = \frac{1}{2}$ and $a_e = 2(|PBK| + L_g|PB|)^2$ with L_g the Lipschitz constant of g in the compact determined by $|x| \le \underline{\alpha}^{-1}(\max\{V(\xi(t_0)), \alpha_\upsilon^{-1} \circ \alpha_e(\eta(t_0))\}) + \eta(t_0)$. Furthermore, L_{f_i} (as defined in Section 3) can be taken as: $L_{f_i} = \max\{|A_c|, |BK| + |B|L_g\}$. We use in the following simulation the system defined by:

$$A = \begin{bmatrix} 1.5 & 0 & 7 & -5 \\ -0.5 & -4 & 0 & 0.5 \\ 1 & 4 & -6 & 6 \\ 0 & 4 & 1 & -2 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 0 \\ 5 & 0 \\ 1 & -3 \\ 1 & 0 \end{bmatrix},$$

$$g(x) = \begin{bmatrix} x_2^2 \\ \sin(x_3) \end{bmatrix}, \qquad K = \begin{bmatrix} 0.1 & -0.2 & 0 & -0.2 \\ 1.5 & -0.2 & 0 & 0 \end{bmatrix}.$$

Fig. 1 shows the result of a simulation when: $\mu = 0.85$, $\rho = 2140$, $\tau^c = 0.25$ s, $\theta_i = [0.58\ 0.33\ 0.67\ 0.30]^T$ and initialized with $\xi(0) = [1\ 1.18\ 0.16\ 1.19]^T$, $\varepsilon(0) = 0$ and $\eta(0) = 2.9 \cdot 10^{-3}$. Taking initial conditions in $|\xi(t_0)| \le 2$ imposed a $\rho > 2120$. Assuming the maximum delay introduced is $\Delta\tau = 6$ µs, results in a minimum time between transmissions $\tau_i^* \ge 8$ µs. The simulation shows the conservativeness of these bounds, presenting a minimum observed inter transmission time of 145 µs. Furthermore, the average inter transmission time in the simulated time was one order of magnitude larger.

8. Discussion

We have shown how asymptotic stability can be attained with fully decentralized event-triggered conditions. It would also be interesting to combine our approach with decentralized/distributed event-triggered controllers (Wang & Lemmon, 2011). The main assumption adopted was Assumption 5. In fact, this can be replaced by a local version only to be satisfied in the compact of interest for the system operation, which relaxes drastically the requirement (Freeman, 1995). Furthermore, as stated in Remark 14 in practice one can often ignore Assumption 11. Nonetheless, the study of controller designs for non-linear systems to satisfy Assumption 11 is interesting to be followed.

The guarantees that our analysis provide are highly conservative, due to the use of Lipschitz constants in bounding the speed of the system. It would be desirable to study computational methods capable of reducing this conservatism, similarly to those in Donkers and Heemels (2012), Donkers, Heemels, van de Wouw, and Hetel (2011) and Hetel, Kruszewski, Perruquetti, and Richard (2011), and to provide performance guarantees. We also leave as follow-up work the design of numerical methods to select adequate ρ and μ optimizing the controller implementation for given communication bandwidth limitations.

Finally, we suggest studying also the design of protocols for wireless communications exploiting the benefits of the proposed techniques.

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