A geometric approach to multi-modal and multi-agent systems
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INTRODUCTION

Dynamical systems with nonsmooth components arise in various settings. For example, nonsmoothness may occur due to dry friction, impacts or backlash in mechanical systems, or due to diode elements in electrical circuits. The governing differential equations of nonsmooth dynamical systems have right-hand sides which are not differentiable or even discontinuous. Consequently, the classical control theory for smooth systems cannot be applied; a different and more careful approach is needed. This approach includes extending differential equations to differential inclusions and adopting different solution concepts.

In this thesis, we focus our attention on two types of nonsmooth dynamical systems. First, we study linear multi-modal systems. These systems consist of a collection of linear systems, each of which is active on a polyhedral region of the state space. Although these systems exhibit linear behavior locally, they are nonsmooth, since their vector fields lack differentiability. Second, we study multi-agent systems with a nonlinear communication protocol. The nonlinear functions in these systems do not have the property that they are locally linear, but they do satisfy a property that we call sign-preservation.

In the rest of this chapter, we have a closer look at both types of nonsmooth dynamical systems and we introduce the problems that are studied in this thesis for these systems.

1.1 LINEAR MULTI-MODAL SYSTEMS

Linear multi-modal systems form a class of hybrid systems; they are a combination of continuous-time linear systems, the modes, together with the discrete dynamics of switching between these modes. In this thesis, we study two general classes of linear multi-modal systems.

First, we consider continuous piecewise affine systems. The state space of a piecewise affine system is divided into solid polyhedral regions, on each of which a different linear or affine system is active. We call the regions with their corresponding linear systems the modes. Switching between different modes
of a piecewise affine system is state-dependent: if the state lies in a certain region, then the corresponding linear or affine system is active at that moment. The resulting vector field is not differentiable, but we do assume continuity of the right-hand side of the governing differential equations. Piecewise affine systems can be used to approximate nonlinear systems, but they can also appear naturally, for example in systems that deal with friction.

Second, we generalize piecewise linear systems to linear multi-modal systems for which the polyhedral regions may overlap and do not have to cover the full state space. As a consequence, we have to replace the differential equations by differential inclusions. Examples of such linear multi-modal systems are switched linear systems, conewise linear systems, and linear complementarity systems.

In this thesis, we study two geometric control theory problems for linear multi-modal systems, namely the disturbance decoupling problem and the fault detection and isolation problem.

1.2 THE DISTURBANCE DECOUPLING PROBLEM

Annihilating or reducing the effects of disturbances is of major importance in many real-life control problems. Designing feedback laws that decouple the disturbances from a certain to-be-controlled output constitutes the well-known disturbance decoupling problem. An input/state/output system is called disturbance decoupled if for each fixed initial condition and zero input, the output corresponding to one disturbance is exactly the same as the output corresponding to another disturbance. The disturbance decoupling problem amounts to finding a feedback law that renders the system disturbance decoupled by eliminating the effect of disturbances on the output. The investigation of this problem for linear and (smooth) nonlinear systems has been the starting point for the development of geometric control theory [Basile and Marro, 1969a,b; Wonham and Morse, 1970]. For both linear and (smooth) nonlinear systems, geometric control theory has been proven to be very efficient in solving various control problems, including the disturbance decoupling problem (see e.g. [Wonham, 1985; Nijmeijer and van der Schaft, 1990; Basile and Marro, 1992; Isidori, 1995; Trentelman et al., 2001]). In this thesis, we use tools from geometric control theory to study the disturbance decoupling
problem for both piecewise affine systems and the more general linear multi-modal systems.

So far, in the context of hybrid dynamical systems, the results on the disturbance decoupling problem are limited to jumping hybrid systems [Conte et al., 2015] and switched linear systems [Conte et al., 2014; Otsuka, 2010, 2011, 2015; Yurtseven et al., 2012; Zattoni and Marro, 2013; Zattoni et al., 2016]. The major difference between piecewise affine systems and switched linear systems is the nature of the switching behavior. For piecewise affine systems the switching behavior is state-dependent whereas it is state-independent for switched linear systems.

For the case of state-independent switching, the solution of the disturbance decoupling problem can be obtained by following mainly the footsteps of the (non-switching) linear case. An interesting consequence of the state-independent switching is that the set of reachable states under the influence of disturbances is a subspace. This allows one to generalize the so-called controlled invariant subspaces of linear systems to switched linear systems. Such a generalization leads to elegant necessary and sufficient conditions [Otsuka, 2010; Yurtseven et al., 2012] for a switched linear system to be disturbance decoupled. In the same papers, disturbance decoupling problems by different feedback schemes have also been solved based on these necessary and sufficient conditions.

However, a similar approach breaks down in the case of state-dependent switching as the set of reachable states under the influence of disturbances is not anymore a subspace, not even a convex set in general. As such, neither the results nor the approach adopted for the state-independent case can be applied to the state-dependent switching case.

In this thesis, we develop a new approach that takes into account the state-dependent switching behavior of piecewise affine systems. In Chapter 2, based on the conference paper [Everts and Camlibel, 2014b], we start by studying a simple class of piecewise affine systems, namely bimodal linear systems. Our approach provides easily verifiable geometric necessary and sufficient conditions for these systems to be disturbance decoupled. Based on these conditions, we study the disturbance decoupling problem for both state feedback controllers and dynamic feedback controllers. For both feedback schemes, we consider mode-independent and mode-dependent controllers, and provide necessary and sufficient conditions for the solvability of the disturbance decoupling problem. These conditions amount to checking certain subspace inclusions very much anal-
ogous to linear systems and linear state-independent switching systems.

In Chapter 3, we study the disturbance decoupling problem for general continuous piecewise affine systems, based on the conference paper [Everts and Camlibel, 2014a]. We provide a set of necessary conditions and a set of sufficient conditions under which such a system is disturbance decoupled. Although these conditions do not coincide in general, we point out some special cases in which they do coincide. Furthermore, we present conditions for the existence of mode-independent static feedback controllers that render the closed-loop system disturbance decoupled. All the conditions we present are geometric in nature and easily verifiable.

Next, we consider a particular linear complementarity system and study when such a system is disturbance decoupled in Chapter 4, based on the book chapter [Everts and Camlibel, 2015]. Linear complementarity systems are nonsmooth dynamical systems that are obtained by taking a standard linear input/output system and imposing certain complementarity relations on a number of input/output pairs at each time instant. A wealth of examples, from various areas of engineering as well as operations research, of linear complementarity systems can be found in [Camlibel et al., 2004; Schumacher, 2004; van der Schaft and Schumacher, 2000; Heemels and Brogliato, 2003]. For the work on the analysis of linear complementarity systems, we refer to [Camlibel et al., 2003; Heemels et al., 2002; Camlibel et al., 2002; van der Schaft and Schumacher, 1996; Camlibel, 2007; van der Schaft and Schumacher, 1998; Heemels et al., 2000]. Particular linear complementarity systems can be written as linear multi-modal systems, namely those of index zero [Camlibel, 2001, Chapter 2]. Different from the piecewise affine systems treated before, the polyhedral regions on which the modes are active, can now be non-solid, and together they do not cover the full state space. It turns out that the resulting linear subsystems share a certain geometric structure, which we exploit to find necessary and sufficient conditions for disturbance decoupledness that are crisp and easily checkable.

Finally, in Chapter 5, we study the disturbance decoupling problem for general linear multi-modal systems. We recover almost all results from Chapters 2, 3 and 4, as well as known results for a particular class of switched linear systems. Moreover, this general approach gives us the possibility to treat the disturbance decoupling problem for another class of linear complementarity systems, namely linear passive-like complemen-
1.3 THE FAULT DETECTION AND ISOLATION PROBLEM

The second geometric control problem that we consider in this thesis is the fault detection and isolation problem, which we first study for bimodal piecewise linear systems. Given a system that is prone to faults, the fault detection and isolation problem amounts to finding an observer that detects when a fault occurs. Moreover, if a fault occurs then the observer should identify what kind of fault it is. Examples of faults are the complete failure of an actuator, a biased actuator, or changes in the system dynamics.

Fault detection and isolation (FDI) is an active area of research in control theory, due to the essential requirement of high reliability for many applications of control systems. Various types of FDI techniques have been proposed for linear systems and for some classes of nonlinear ones. There is a large number of contributions in this area, and consequently we direct the interested reader to the comprehensive survey papers [Frank, 1990; Hwang et al., 2010; Isermann, 2006; Isermann and Bailé, 1997]. On the other hand, research on FDI for hybrid and switched systems, and in particular for piecewise linear systems, has been less intensive and fruitful (see [Balluchi et al., 2002; Cocquempot et al., 2004; Narasimhan et al., 2000; Wang et al., 2009]).

In Chapter 6 of this thesis, based on the paper [Everts et al., 2016], we use the classical geometric control theory framework (see [Basile and Marro, 1992; Wonham, 1985]) to investigate the problem of fault detection and isolation for bimodal linear systems. Our approach is inspired by the ideas pioneered in [Massoumnia, 1986a], where several formulations of the fault detection and isolation problem were stated and solved in geometric terms for linear systems. We give a sufficient condition for solving the fault detection and isolation problem for bimodal linear systems.

As a by-product, and before we continue with the second class of nonsmooth dynamical systems, we consider the FDI problem for a class of linear dynamical systems defined over
an undirected graph. Two disjoint sets of agents are identified in the network: the faultable agents, which are prone to failure, and the observer agents, whose output is measurable. Fault detection is performed by an unknown input observer, and stated in the geometric language of [Massoumnia, 1986a], i.e. output separability of fault subspaces. In Chapter 7, which is based on the paper [Rapisarda et al., 2015], we present a characterization of the smallest conditioned invariant subspaces that are generated by the faults. This characterization is exploited in order to give graph-theoretical conditions guaranteeing output separability in terms of distances between faultable agents and observer ones. In addition, we study the case where two faultable vertices share exactly the same neighbors in order to present a condition under which fault detectability fails.

In this thesis, we make extensive use of geometric control theory for both the disturbance decoupling problem and the FDI problem. Although the resulting conditions often are somewhat similar to those of the linear case, the methods to obtain these conditions are fundamentally different, since we have to take state-dependent switching into account.

As a last subject in this thesis, we study a truly nonlinear system, that does not exhibit linear behavior locally.

1.4 NONLINEAR CONSENSUS PROTOCOLS FOR DIGRAPHS

The second class of nonsmooth dynamical systems that we consider in this thesis arises in the context of nonlinear consensus protocols. We consider a network of agents that communicate according to a fixed communication topology, represented by a directed graph containing a directed spanning tree. For the well-known linear consensus protocol these graph-theoretical are known to be a sufficient and necessary condition for reaching state consensus. In Chapter 8 of this thesis, we generalize this result and study the consensus problem for a general nonlinear consensus protocol. A nonlinear consensus protocol may arise due to the nature of the controller [Jafarian and De Persis, 2015; Saber and Murray, 2003], or may describe the physical coupling existing in the network [Bürger et al., 2014; Monshizadeh and De Persis, 2015]. The nonlinear functions in our model are assumed to be sign-preserving and are allowed to have possible discontinuities. Examples of such functions are the saturation function and the sign function. To deal with the possible discontinuities, we will need to employ Filippov solutions...
1.5 Preliminaries and Notation

and replace the differential equations by differential inclusions [Filippov, 1988; Smirnov, 2002; Aubin and Cellina, 1984]. In this framework, we first study the case that these nonlinearities happen at the level of the nodes. Next, we consider the case that we have nonlinearities at the edges. Finally, we combine these results. Chapter 8 is based on the paper [Wei et al., 2016].

The last section of this chapter is devoted to discussing some notation and notions in geometric control theory, which we use throughout this thesis.

1.5 Preliminaries and Notation

First, we fix some notation and definitions that we use throughout this thesis. Let $\mathbb{R}$ denote the set of real numbers. For a vector $v$ we denote its transpose by $v^T$ and its dimension by $n_v$. For two vectors $v$ and $w$, we let $\text{col}(v, w)$ denote the column vector that is obtained by stacking $v$ and $w$. We denote the set $\{1, 2, \ldots, m\}$ by $I_m$. For a subset $\alpha$ of $I_m$, $\alpha^c$ denotes the subset $I_m \setminus \alpha$. A cone is a subset of a vector space that is closed under multiplication by positive scalars.

Let $S$ be a set in $\mathbb{R}^n$. The Minkowski sum of two subsets $S_1$ and $S_2$ of $S$ is given by

$$S_1 + S_2 = \{x + y \mid x \in S_1, y \in S_2\}.$$

The affine hull of $S$ is the smallest affine set containing $S$ in $\mathbb{R}^n$ and is denoted by $\text{aff}(S)$. The relative interior of $S$ is defined as

$$\text{rint}(S) := \{x \in S : \exists \epsilon > 0, N_\epsilon(x) \cap \text{aff}(S) \subseteq S\},$$

where $N_\epsilon(x)$ is an $\epsilon$-neighborhood of $x$. We call the set $S$ solid if the affine hull of $\text{rint}(S)$ is $n$-dimensional. With $S^\perp$ we denote the orthogonal complement of $S$ with respect to the inner product $v^T w$ for $v, w \in \mathbb{R}^n$.

1.5.1 Geometric control theory

Geometric control theory, illustrated in depth in e.g. [Basile and Marro, 1992] and [Trentelman et al., 2001], plays an important role in this thesis. The rest of this section quickly summarizes some definitions and results from geometric control theory that are relevant to this thesis.
Consider the linear, time-invariant system $\Sigma = \Sigma(A, B, C, D)$ given by

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (1.1a)$$

$$y(t) = Cx(t) + Du(t), \quad (1.1b)$$

where $x \in \mathbb{R}^{n_x}$ is the state, $u \in \mathbb{R}^{n_u}$ is the input, $y \in \mathbb{R}^{n_y}$ is the output, and $A$, $B$, $C$ and $D$ are matrices of appropriate dimensions.

A subspace $V \subseteq \mathbb{R}^{n_x}$ is called $A$-invariant if $V$ satisfies $AV \subseteq V$. The controllable subspace is the smallest $A$-invariant subspace containing $\text{im} B$. It is denoted by $\langle A \mid \text{im} B \rangle$ and satisfies

$$\langle A \mid \text{im} B \rangle = \text{im} B + \text{im} AB + \cdots + \text{im} A^{n_x-1}B. \quad (1.2)$$

Note that for any matrix $K \in \mathbb{R}^{n_u \times n_x}$ we have

$$\langle A + BK \mid \text{im} B \rangle = \langle A \mid \text{im} B \rangle. \quad (1.3)$$

A subspace $V \subseteq \mathbb{R}^{n_x}$ is called controlled invariant with respect to $A$ and $B$, or $(A, B)$-invariant in short, if there exists a matrix $F$ such that $V$ is $(A + BF)$-invariant. Such a matrix $F$ is called a friend of $V$. Equivalently, a subspace is controlled invariant if

$$AV \subseteq V + \text{im} B. \quad (1.4)$$

From (1.4) it follows that the sum of two controlled invariant subspaces is again controlled invariant.

A subspace $V \subseteq \mathbb{R}^{n_x}$ is called an output nulling controlled invariant subspace of $\Sigma$ if

$$\begin{bmatrix} A \\ C \end{bmatrix} V \subseteq (V \times \{0\}) + \text{im} \begin{bmatrix} B \\ D \end{bmatrix}.$$

The weakly unobservable subspace of $\Sigma$ is the largest (with respect to the subspace inclusion) output nulling controlled invariant subspace and will be denoted by $V^*(\Sigma)$. In the case that $D = 0$, we sometimes write $V^*(C, A, B)$ to denote $V^*(\Sigma)$, which is then the largest $(A, B)$-invariant subspace contained in $\ker C$.

A subspace $T \subseteq \mathbb{R}^{n_x}$ is called conditioned invariant with respect to $C$ and $A$, or $(C, A)$-invariant in short, if

$$A(T \cap \ker C) \subseteq T.$$

This condition is equivalent to the existence of a matrix $K \in \mathbb{R}^{n_x \times n_y}$ such that $T$ is $(A + KC)$-invariant, i.e.

$$(A + KC)T \subseteq T.$$
We call such a matrix $K$ a friend of $T$. The intersection of two conditioned invariant subspaces is again conditioned invariant.

We call a subspace $T \subseteq \mathbb{R}^n_x$ input containing conditioned invariant if

$$[A \ B]\left((T \times \mathbb{R}^n_u) \cap \ker [C \ D]\right) \subseteq T.$$ 

It is well-known that a subspace $T$ is an input containing conditioned invariant subspace if and only if there exists a matrix $L \in \mathbb{R}^{n_x \times n_y}$ such that

$$(A + LC)T \subseteq T \quad \text{and} \quad \text{im}(B + LD) \subseteq T. \quad (1.5)$$

The strongly reachable subspace of $\Sigma$ is defined to be the smallest (with respect to the subspace inclusion) input containing conditioned invariant subspace and will be denoted by $T^*(\Sigma)$.

Let $K$ and $L$ be $m \times n$ and $n \times p$ matrices, respectively. Denote the system $\Sigma(A + BK + LC + LDK, B + LD, C + DK, D)$ by $\Sigma_{K,L}$. Then we have the following equality:

$$T^*(\Sigma_{K,L}) = T^*(\Sigma). \quad (1.6)$$

It follows from (1.5) with the choice of $L = 0$ that the controllable subspace is an input containing conditioned invariant subspace. Hence, we have

$$T^*(\Sigma) \subseteq \langle A \mid \text{im } B \rangle. \quad (1.7)$$

In the case that $D = 0$, we sometimes write $T^*(B, C, A)$ to denote $T^*(\Sigma)$, which is then the smallest $(C, A)$-invariant subspace containing $\text{im } B$. It can be shown that for a friend $K$ of $T^*(B, C, A)$, we have $T^*(B, C, A) = \langle A + KC \mid \text{im } B \rangle$. The subspace $T^*(B, C, A)$ can be computed by the following subspace algorithm (see e.g. Algorithm 4.1.1 p. 203 of [Basile and Marro, 1992]):

$$T^0 := \text{im } B \quad (1.8a)$$

$$T^k := \text{im } B + A \left(T^{k-1} \cap \ker C\right) \quad (1.8b)$$

for $k \geq 1$. As these subspaces are nested, that is

$$T^k \subseteq T^{k+1},$$

it follows that there exists an integer $\ell$ such that $0 \leq \ell \leq n$ and

$$T^\ell = T^{\ell+1}.$$
It is well-known that
\[ T^*(B, C, A) = T^\ell. \]

A pair of subspaces \((T, V)\) is called a \((C, A, B)\)-pair if \(T\) is \((C, A)\)-invariant, \(V\) is \((A, B)\)-invariant and \(T \subseteq V\). If \((T, V)\) is a \((C, A, B)\)-pair, then there is a linear map \(N : \mathbb{R}^{n_y} \rightarrow \mathbb{R}^{n_u}\) such that \((A + BNC)T \subseteq V\) (see e.g. [Trentelman et al., 2001, Lemma 6.3]).

It is well-known that the transfer matrix \(D + C(sI - A)^{-1}B\) is right-invertible as a rational matrix if and only if
\[ V^*(\Sigma) + T^*(\Sigma) = \mathbb{R}^{n_x} \text{ and } [C \ D] \text{ is of full row rank.} \]

Straightforward linear algebra arguments (see e.g. [Kaba, 2001, Ch. 2, Thm. 4] and [Trentelman et al., 2001, Thm. 8.27]) show that these conditions are equivalent to
\[ \text{im } D + CT^*(\Sigma) = \mathbb{R}^{n_y}. \] (1.9)