Ordering at Two Length Scales in Comb–Coil Diblock Copolymers Consisting of Only Two Different Monomers

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ABSTRACT: The microphase-separated morphology of a melt of a specific class of comb–coil diblock copolymers, consisting of an AB comb block and a linear homopolymer A block, is analyzed in the weak segregation limit. On increasing the length of the homopolymer A block, the systems go through a characteristic series of structural transitions. Starting from the pure comb copolymer, the first series of structures involve a short length scale followed by structures involving a large length scale. A maximum of two critical points exists. Furthermore, in the two parameter space characterizing the comb–coil diblock copolymer molecules considered, a nontrivial bifurcation point exists beyond which the structure factor can have two maxima (two correlation hole peaks).

I. Introduction

Diblock copolymer melts usually microphase separate with one characteristic length. However, if more than two monomer types are involved, microphase separation frequently occurs at more than one length scale.1–7 Several examples that are of direct interest for the present paper can be found in the experimental work of Ikkala and Ten Brinke and co-workers.4–6 There, comb–coil diblock copolymers are investigated consisting of a poly(4-vinylpyridine)-block-polystyrene (P4VP-b-PS) diblock copolymer with side chains (e.g., pentadecylphenol, PDP) attached by hydrogen bonds to the P4VP block. The resulting comb–coil diblock copolymers show typical two length scale structure-inside-structure morphologies. The PS blocks microphase separate from the P4VP(PDP) blocks, giving rise to the well-known classical morphologies depending on the volume fraction of either block. This structure corresponds to the large length scale ordering, and the order–disorder transition temperature is, if present at all, very high. Inside the P4VP(PDP) domains an additional short length scale lamellar ordering takes place characterized by an easily accessible (ca. 60 °C) order–disorder transition.

Still, two length scale ordering is not restricted to block copolymers involving three or more chemically different monomers. To demonstrate and analyze this, we studied in ref 8 the structure factor of a block copolymer consisting of a linear homopolymer block of A monomers linked to a comb copolymer block with a backbone of the same kind of A monomers and side chains consisting of B monomers. (An experimental example can be found in refs 9 and 10.) For convenience, to reduce the number of free parameters, the discussion was restricted to side chains having a length equal to the backbone of the comb block (see Figure 1). Because of the architecture of the molecule, microphase separation can in principle occur at two different length scales, either “inside” the AB comb block or “between” the linear A block and the AB comb block (see Figure 2). In the latter case the behavior resembles that of a diblock copolymer, where one block

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Figure 1. Model of the comb–coil diblock copolymer molecule studied. Note: for clarity, the homopolymer block is indicated with a thicker line than the backbone of the comb block. In this study both are assumed to be chemically identical. The cartoon corresponds to number of branch points nt = 4, asymmetry parameter t = 0, and one side chain per branch point α = 1. The length of the side chains and length of the backbone between two successive side chains equal d. The length of the homopolymer block equals md, m ∈ R.

Figure 2. Illustration of the two ways in which the system can microphase separate.

is the homopolymer A block and the other block the AB comb block.

In ref 8 the analysis of the structure factor of this system was presented, and the main result was summarized in the form of a so-called classification diagram. In this diagram, the horizontal and vertical axes denote respectively the number of side chains and the length of the homopolymer block; the latter is expressed in units corresponding to the side chain length. Together these two parameters uniquely determine the molecular
structure for the specific class of comb–coil diblock copolymers considered, i.e., length of side chains equal
to length of backbone between consecutive side chains. The position and the value of the absolute minimum of
the inverse structure factor determine the length scale of microphase separation and the temperature at which
the disordered phase becomes unstable against structure formation, i.e., the spinodal. The classification
diagram presented showed which length scale was favored (absolute minimum of the inverse structure factor) and whether the inverse structure factor had one or two minima. One of the most striking features was
the existence of a nontrivial bifurcation point, separating the region in the parameter space where the
structure factor has only one minimum from the region where the structure factor can have two minima (two
(correlation hole peaks)).

In this paper we continue the analysis of the phase behavior of this class of systems by calculating the free
energy up to fourth order in the weak segregation limit and constructing the corresponding phase diagrams.

The paper is organized as follows. In section 2 the system parameters are defined, in section 3 the theory
is outlined, and in section 4 the results are discussed.

II. Parametrization

We consider monodisperse polymer melts of comb–
iole diblock copolymers. The comb–coil diblock copoly-
stipulated that an additional A homopolymer block is linked
to the AB comb copolymer block (see Figure 1).

First the parametrization of the comb block will be
introduced. The phase behavior of pure AB comb
copolymer melts, i.e., without the A homopolymer block,
had already been discussed in several papers.11–14
However, all of these, except for Dobrynin and Erukhin-
movich,14 are restricted to the calculation of the spin-
odals only. Here, we adopt the notation introduced before13 and extend it slightly by incorporating the
possibility of having more than one side chain per branch point.

An AB comb copolymer molecule consists of a back-
bone chain of monomers of type A to which side chains
consisting of monomers of type B are attached. The
monomers (or segments) A and B are assumed to be of
equal size. The number of backbone monomers equals
N2 as the number of monomers per one side chain is
equal to Ns. There are nb branch points which all
have the same functionality z = α + 2. Here, α is the
number of side chains linked to one branch point.
Therefore, the number of side chains equals ns = αnb.
The branch points are assumed to be distributed
regularly along the backbone chain. The number of
backbone segments between consecutive branch points
is assumed to be equal to N2/ns. Obviously, the number
of backbone segments before the first branch point and
after the last branch point must then also add up to
N2/ns. Therefore, the number of backbone segments
before the first side chain and after the last side chain
equals N2/ns and (1 − t)N2/ns, respectively, with t ∈ [0,
1], where t is called the asymmetry parameter.11

The A homopolymer block has N3A segments. Thus,
the total number of A segments in the comb–coil
molecule is N = N2A + N3A, and that of all segments is
N = N2A + N3A + nsNsb. With the above definitions the
volume fractions are fA = N2A/N, fB = N3A/N, and fB =
nbNsb/N. Since the melt is assumed to be incompressible,
fA + fB + fB = 1.

Together these parameters define a 6-dimensional
compositional parameter space: (N2A, N3A, Nsb, ns, α, t).
However, in our discussion of the comb–coil copoly-
melt, we will restrict this to a 3-dimensional subset by assuming Nsb = N3A/ns = d, α = 1, and t = 0,
unless stated otherwise. Hence, the length of the side
chains is equal to the length of the backbone chain
between consecutive side chains, and both will be
denoted by d. For the comb block this implies an equal
amount of A and B segments; fB = 1/2. Furthermore, we
will also express N2A in terms of d by defining N2A = dm,
where m ∈ R+ is a measure for the length of the
homopolymer block in units of the side chain length. In
terms of m and ns the volume fraction of the homopoly-
mer block becomes fB = m/(2n + m). Note that N =
(2n + m)d. The overall volume fraction of A monomers
is given by fA = (m + n)/(m + 2n). Because N cannot be
absorbed in the interaction (Flory–Huggins) parameter
χ, we effectively reduced the number of parameters to
two, namely (ns, m) or (nt, fB).

III. Theory

The Hamiltonian of the copolymer melt is given by
(β = 1/kT)15–18
\[
\beta H = \beta (H_0 + H_i) = \frac{3}{2a^3} \sum_{m=1}^{\infty} \int_0^{N_m} ds \left( \frac{\partial R^{\text{m}}(s)}{\partial s} \right)^2 + \int_{R_s}^{R_b} \frac{1}{2} \sum_{i,j} \epsilon_{ij} \left( \frac{\partial \rho_i}{\partial s} \right) \left( \frac{\partial \rho_j}{\partial s} \right) (1)
\]

The first part, H0, the Edwards Hamiltonian,16 accounts
for the chain connectivity. It is a continuous model of a
Gaussian chain where a configuration of the mth
molecule is represented by a space curve, Rm(t), a is the
effective bond length. We subordinate Rm(t) to the total space
curve in smaller space curves Rm(s). Here, Rm(s) corresponds
to the part of the space curve Rm(t) which is associated
with the ith linear block in which the molecule will be divided. The variable i enumerates the
different number of linear blocks. The variable s is a
continuous variable whose value ranges from 0 to Ni
along the contour of the ith block; Ni denotes the length
of the ith block. We first enumerate the A blocks
(i ∈ {1, 2, ..., ni + 2}) and subsequently the B blocks
(i ∈ {ni + 3, ..., ni + nt + 3}). Note that the B blocks correspond
to the side chains and that the first A block, corresponding
to the homopolymer block, is not present in the case
of pure comb copolymers. Hence, Ni is equal to

\[
N_i = \begin{cases}
N_i^A & i = 1 \\
N_i^A + 3 & i = 2 \\
N_i^A + \left( 3 - 3 \right) & i = 3, \cdots, ni + 1 \\
N_i^A + \left( n_i + 3 \right) & i = ni + 2 \\
N_i^B & n_i + 3 \leq i \leq n_i + ni + 3
\end{cases}
\]

The configuration, i.e., the whole space curve Rm(t)
is obtained by linking the subchains Rm(s) together. This
translates into the following constraint on Rm(s), rep-
resented by the function δc in eq 1.15
The concentration profile \( \rho_\alpha (r) \) is the microscopic monomer concentration of monomer type \( \alpha \) at position \( r \) and is given by

\[
\rho_\alpha (r) = \sum_{m=1}^{\infty} \sigma_m \int_0^{R_m} \delta \left( r - R_m(n) \right) \left( \frac{d^3 r}{\sqrt{2 \pi}} \right) \psi^2 (r) + C
\]

(6)

where \( C \) is an unimportant constant and \( \chi \) is the Flory-Huggins parameter defined by \( \chi = (\epsilon_{AB} - \frac{1}{2} \epsilon_{AA} + \epsilon_{BB})/k_B T \). \( \psi \) represents the concentration profile and equals the deviation of the microscopic concentration of monomer \( A \) from its average value: \( \psi(r) = \rho_A(r) - \rho_A \).

The Landau mean-field free energy corresponds to an expansion in powers of the concentration profile \( \psi(q) \)

\[
F[\psi] = \frac{1}{2} \sum_{n=2}^{\infty} \sum_{n \neq n'} \Gamma_n (q_1, ..., q_{n}) \psi(q_1) ... \psi(q_n)
\]

(7)

The coefficients of the free energy expansion and the vertex functions \( \Gamma_n \) can be related to the chemical single-chain correlation functions, \( g_{\alpha,\beta} = \langle \psi(r) \rangle \), either by the random phase approximation or equivalently by a saddle point approximation of the free energy functional (see Appendix).19-23

For a more detailed discussion of the computational details of the correlations functions, we refer to the Appendix. Through minimizing eq 7 with respect to the concentration profile \( \psi(q) \) the equilibrium free energy is obtained.

The concentration profile \( \psi(r) \) is expanded in a set of wave functions that obey the symmetry of a given (periodic) structure.

\[
\psi(r) = \sum_{m=1}^{\infty} \sum_{q \in H_m} A_m \frac{1}{\sqrt{N_m}} e^{i q r} \psi(q) + \phi(q)
\]

(8)

where \( H_m \) is a set of wave vectors \( Q \) which describes the symmetry of the structure. \( A_m \) and \( \phi(q) \) correspond to the amplitudes and phases of the concentration profile. \( m \) denotes the number of harmonics or shells, \( H_m \) and \( n_m \) is half the number of vectors of the mth harmonic. Vectors belonging to the same harmonic have the same length. The amplitudes belonging to these vectors are all equal. The length of the vectors in the first harmonic is denoted by \( q_0 \). Then the length of the vectors in the higher harmonics are fixed because they are multiples of \( q_0 \) (depending on the symmetry of the phase). By the above ansatz for the concentration profile the minimization of the free energy corresponds to minimizing \( F \) with respect to \( A_m \) and \( q_0 \).

The free energy expansion eq 7 is only valid in the weak segregation limit (WSL), i.e., close to the critical point and \( |q| \ll 1 \). In the WSL the free energy is dominated by the wave vector \( q^* \), where \( q^* \) is equal to the value of \( q \) for which \( \Gamma_2(q) \) attains its minimum. Thus, in the WSL we can set \( q_0 \) equal to \( q^* \).

In this paper we will restrict ourselves to the first harmonic approximation (FHA)19 of the concentration profile. Because of this, only the classical structures, i.e., disordered, lamellar, hexagonal, and spherical (bcc), can be considered. The inclusion of higher harmonics is required to discuss more complex structures such as the gyroid structure.18,24-30 Because of the complicated nature of our structure factor, including the possibility of two maxima, we decided to consider at first only the FHA.

Two important quantities are the spinodal and the critical point. The spinodal denotes the line of instability of the disordered phase, the line in phase space where the disordered phase \( (\psi = 0) \) becomes absolutely unstable against fluctuations. It is given by the absolute minimum of \( \Gamma_2(q) \)

\[
\frac{d \Gamma_2(q) }{dq} \left|_{q^*} \right. = 0 \quad \text{and} \quad \frac{d^2 \Gamma_2(q) }{dq^2} \left|_{q^*} \right. > 0 \quad \text{and} \quad \Gamma_2(q^*) = 0
\]

(9)

Note that the \( \Gamma_2(q) \) is inversely proportional to the structure factor.15

The critical point is given by

\[
\text{spinodal} \wedge \Gamma_3(q', q', q'^*) = 0 \quad \text{or} \quad |q'| = |q| = |q''| = |q^*|
\]

(10)

It is a point on the spinodal curve where also \( \Gamma_3 \) becomes zero. At the critical point the phase transition is continuous, i.e., with zero amplitude. Therefore, close to the critical point, the amplitude will be small, thereby ensuring the validity of the WSL.

IV. Discussion

A. Spinodals. In our previous article8 the spinodals of a specific class of comb–coil diblock copolymers, for which the length of the side chains equals the length of the backbone between two successive side chains, were investigated in some detail. We start the discussion here by recalling the main results, presenting them in a slightly different manner. Figure 3 shows the spinodal surface of \( (\gamma N)_s \) vs \( n_t \) and \( m \) corresponding to the absolute minimum of \( \Gamma_2(q) \). The change of \( (\gamma N)_s \) as a function of \( m \) for a fixed value of \( n_t \) is in line with the change in length scale of the microphase separation, which changes from short to long as the size of the homopolymer block is increased. This fact is most clearly demonstrated by Figure 4, where \( y = a^2 q^3 d^6/\phi \), which is a measure of the inverse length scale squared, has been plotted as a function of \( n_t \) and \( m \).
values for which \( \Gamma_2(q) \) has two minima. Figure 5 is a projection of the above-mentioned boundary lines onto the \((n_t, m)\) plane summarizing this behavior.

Notice that, as already observed before, a pure comb copolymer melts essentially microphase separates on the length scale of its repeat unit. Therefore, the total number of monomers of the comb copolymer \( N \) must be divided by \( n_t \) in order to correctly account for the length of the repeat unit. This fact explains why the spinodal surface of Figure 3, corresponding to the short length scale (microphase separation within the comb copolymer block; Figure 2), increases linearly with \( n_t \). On the other hand, the spinodal surface corresponding to the long length scale (large value of \( m \)) remains essentially "constant" with changing \( n_t \). The number of monomers involved in the short length scale is \( N/n_t \), which for the long length scale is \( N \).

**B. Critical Points.** We consider the specific class of comb–coil diblock copolymers, characterized by side chains of a length equal to the length of the backbone between consecutive side chains. Therefore, an increase in \( f_s \) corresponds to an increase in the length of the homopolymer block. Because of this, critical points will only be found for special values of \( n_t \) and \( m(f_s) \). In Figure 5 these critical points are presented by the dashed and dotted line. The curve has two branches, an upper and a lower branch. The upper branch is associated with the large length scale structure, i.e., the diblock scale separation. The lower branch corresponds to the short length scale, i.e., separation within the comb block. The part of the lower branch located inside region III, indicated by the dotted line, does not correspond to true critical points anymore. In region III the absolute minimum corresponds to the large length scale. The critical points correspond to the relative minimum of \( \Gamma_2 \). For the position of the absolute minimum of \( \Gamma_2 \), \( \Gamma_3 \) does not become zero.

For \( n_t \) values ranging from 3 to 15, critical points are present for two values of \( m \). For \( n_t > 15 \), the critical point corresponding to the short length scale ordering
no longer corresponds to the absolute minimum of $\Gamma_2(q)$. In contrast, for $n_t = 2$ there is no critical point at all. Of course, this is simply due to the specific choice of parameters. We selected $N_B = N_B^0/n_t = d$, $\alpha = 1$, and $t = 0$, and for $n_t = 2$ the critical point is located outside this particular subset of parameters.

In Figure 6, the critical points of the comb–coil diblock copolymer melt are presented once more—this time in the $(n_t, f_A)$ plane. This presentation is better suited to show the upper branch corresponding to the critical points of the large length scale structure formation. It clearly illustrates the presence of two critical points (two values of $m$) for $n_t = 2$. Of course, for $n_t > 15$, the lower branch corresponds again to “pseudo”-critical points.

C. Phase Diagrams. As became clear from the spinodal analysis, the phase behavior of comb–coil diblock copolymers will be much richer. One way of presenting a phase diagram for this class of systems is to fix $n_t$ and vary $m$. In this way we start with a pure comb copolymer $m = 0$ and end with nearly pure homopolymer $m \gg 1$. However, the existence of a bifurcation point in parameter space separating two distinct regions complicates matters. If we start with $n_t > 10$, the second-order vertex function will develop two minima on increasing $m$. First the absolute minimum will correspond to the short length scale ordering, then the two minima will attain the same value for some specific value of $m$, then the absolute minimum will correspond to the large length scale, and, finally, the minimum belonging to the short length scale will gradually disappear. As presented in the Theory section, the analysis of the stability of various ordered structures in block copolymer melts is usually based on a free energy expansion using a single dominant wave vector $q^*$, where $q^*$ coincides with the $q$ value for which the second order vertex function attains its minimum. So, it is quite obvious that the analysis of the microphase-separated morphology in the case where the second order vertex function has two minima of nearly equal value requires a completely different analysis involving the $q$ values of both minima. (See ref 31.) Here we will restrict ourselves to the region in the classification diagram left of the bifurcation point where there is always only one minimum and the theory (WSL) outlined before can be applied without any restriction.

The phase diagrams that are presented correspond to a fixed number of branch points ($n_t$) and varying length of the homopolymer block ($m$), i.e., to slices in the $(n_t, m)$ plane for fixed $n_t$. Figures 8 and 7 correspond to $n_t = 2$ and $n_t = 4$, respectively. The phase diagrams are presented as $\chi N$ vs $m$.

For $n_t = 4$ the phase diagram has two critical points. The critical point on the left signals the change from ordering on the short (comb) length scale to ordering on the long (diblock) length scale, Figure 4. The critical point on the right corresponds to the critical point of the effective diblock copolymer. Throughout the phase diagram the lamellar (respectively hexagonal and spherical) structure is indicated with the same character L (respectively H and B). It should be realized, however, that the same symmetry is accompanied by strongly different periodicities in different regions of the phase diagram.

To illustrate this, we will walk through the phase diagram, Figure 8, as a function of the length $m$ of the homopolymer block, keeping the value of $\chi N$ fixed at
60 and 55. The corresponding changes in length scale can be found from Figure 4.

Let us start with \( \gamma N = 60 \). For \( m \) sufficiently small there is a short length scale lamellar ordering, the layers being alternately rich in side chain B monomers and A monomers. Then a short length scale hexagonal structure is found with cylinders rich in side chain B monomers. Then a kind of in between “intermediate” length scale lamellar structure is formed, followed by a large length scale hexagonal structure with cylinders rich in homopolymer A monomers. Next, the sequence of structures follows the usual pattern from lamellar (layers alternately rich in homopolymer block and comb copolymer block) to hexagonal (cylinders rich in comb copolymer block) to bcc (spheres rich in comb copolymer block) to disordered.

For \( \gamma N = 55 \), we start again with a short length scale lamellar structure (layers alternately rich in side chain B monomers and A monomers) followed with a short length scale hexagonal structure (cylinders rich in side chain B monomers). A sequence of short length scale bcc (spheres rich in B monomers), disordered state, and large length scale bcc (spheres rich in comb copolymer block) is then found. Next, the structures follow the same order as discussed above for \( \gamma N = 60 \).

For \( n_t = 2 \) the phase diagram does not have any critical point. As discussed before, this is simply due to the choice of parameters. Going through the phase diagram at constant value of \( \gamma N \), e.g., 40, there is a much more gradual change in length scale.

V. Concluding Remarks

Phase diagram 7 seems quite characteristic of the class of comb-coil diblock copolymers considered. For sufficiently large values of \( \gamma N \) a sequence of structures is found as a function of the homopolymer block length \( m \). The first part involves the short length scale followed by large length scale ordered structures. This phase diagram was calculated for values of \( n_t \) close to the left of the bifurcation point in the classification diagram (Figure 5). To the left of the bifurcation point there is gradual change from the short to the large length scale as a function of \( m \). To the right of the bifurcation point a similar change from short to long length scale ordering will take place. However, on increasing \( m \), we also traverse through the region where the second-order vertex function has two minima. On crossing the point where the minima are equal, a discontinuous change in length scale occurs. Here, for some values of \( n_t \) and \( m \) the occurrence of stable structures which give rise to two minima should be expected, e.g., HML, HPL, or still more complex. The free energy expansion should involve both wave vectors, and this is the main topic of our current research efforts. Preliminary calculations with two different length scales, thus with two amplitudes, led to relatively large amplitudes, thereby violating the assumption of WSL. Presumably, the higher harmonics of both length scales cannot be ignored in the free energy expansion. This suggests that the WSL may not be applicable there when two competing length scales are present (region III), and we should recourse to other theories, like e.g., SCFT.17,18

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Appendix: Correlation Functions

In this appendix we present in more detail the calculation of the correlation functions. The n-point chemical correlation function \( g_{n_1,...,n_n}(r_1, ..., r_n) \) denotes the probability that at point \( r \) a monomer of type \( a \) is present for \( 1 \leq i \leq n \).

\[
g_{n_1,...,n_n}(r_1,...,r_n) = \frac{1}{N} \rho_{n_1}^{\alpha_1}(r_1) ... \rho_{n_n}^{\alpha_n}(r_n) \tag{A1}
\]

\[
\langle ... \rangle_0 = \frac{1}{N} \int \prod_{m} d^{3} R_{m} e^{-H(M(R_{m})))} \tag{A2}
\]

In eq A1 \( \rho_{n_i}^{\alpha} \) denotes the concentration of a single molecule. Consequently, the probability measure of the ensemble average eq A2 is now also over one molecule only. Defining \( \rho_{n_i}^{\alpha}(r) = \sum_{R_i} \rho_{n_i}^{\alpha}(r, R_i) \), \( \rho_{n_i}^{\alpha}(r) \) is the microscopic single molecule concentration of block \( i \) at position \( r \), eq A1 can be rewritten as

\[
g_{n_1,...,n_n}(r_1,...,r_n) = \frac{1}{N} \sum_{R_i} \rho_{n_1}^{\alpha_1}(r_1) ... \rho_{n_n}^{\alpha_n}(r_n) \tag{A3}
\]

\[
\langle \rho_{n_i}^{\alpha_1}(r_1) ... \rho_{n_i}^{\alpha_n}(r_n) \rangle_0 \quad \text{has a similar meaning as} \quad \langle \rho_{n_i}^{\alpha_1}(r_1) ... \rho_{n_i}^{\alpha_n}(r_n) \rangle_0 \quad \text{it is the probability of finding a monomers of block} \ i \ \text{at position} \ r_i \ \text{for} \ 1 \leq i \leq n.
\]

The correlation functions are related to the vertex functions \( \Gamma_{n_1,...,n_n} \) of the Landau free energy expansion eq 7 by

\[
\Gamma_{2}(q_1,q_2) = V \sum_{a,b} g_{a,b}^{\alpha_1}(q_1) z_{a}(q_1) z_{b}(q_2) - 2 \chi \tag{A4}
\]

\[
\Gamma_{3}(q_1,q_2,q_3) = -V \sum_{a,b,c} g_{a,b,c}^{\alpha_1}(q_1) z_{a}(q_1) z_{b}(q_2) z_{c}(q_3) \tag{A5}
\]

\[
\Gamma_{4}(q_1,q_2,q_3,q_4) = V \sum_{a,b,c,d} g_{a,b,c,d}^{\alpha_1}(q_1) \left[ z_{a}(q_1) z_{b}(q_2) z_{c}(q_3) z_{d}(q_4) + \sum_{m,n} \left( \sum_{a,b,c,d} g_{a,b,c,d}^{\alpha_1}(q_1) \right) g_{a,b,c,d}^{\alpha_1}(q_1) z_{a}(q_1) z_{b}(q_2) z_{c}(q_3) z_{d}(q_4) \right]
\]

\[
\Gamma_{5}(q_1,q_2,q_3,q_4,q_5) = V \sum_{a,b,c,d,e} g_{a,b,c,d,e}^{\alpha_1}(q_1) \left[ z_{a}(q_1) z_{b}(q_2) z_{c}(q_3) z_{d}(q_4) z_{e}(q_5) \right] \tag{A6}
\]

where \( z_{a}(q) = g_{a,a}(q) - g_{a,b}(q) \).

We omit the factor \( 1/N \) in the remaining discussion and write \( G = N \). As the generic example, we take the number of side chains equal to two \( (n_t = 2) \). Schematically the molecule is pictured below

Here \( N_1 = N_2 = N_3 = N_4 \) and \( N_4 = N_5 = N_B \). Now we write \( G_{AB} \) in terms of \( G_{ij} \), the block correlation functions.

\[
G_{AB}(q_1,q_2) = G_{14}(q_1,q_2) + G_{15}(q_1,q_2) + G_{25}(q_1,q_2) + G_{24}(q_1,q_2) + G_{34}(q_1,q_2) + G_{35}(q_1,q_2) \tag{A8}
\]

These functions can, with the above picture in mind, be represented graphically as follows.
for a comb parametrized by \((\alpha = 1, n_t, t)\) the correlation function \(G_{AB}\) is

\[
G_{AB}(q) = \sum_j \left( \begin{array}{c} q \cr j \end{array} \right)
\]

(A15)

where \(j\) runs over \(\{t, t + N_A/n_t, t + 2N_A/n_t, \ldots\}\).

So far this method may seem rather clumsy, since the integral and subsequent summations can be done quite easily. However, when we want to calculate third- and fourth-order correlation functions, this method becomes a useful bookkeeping tool and a concise way of writing down correlation functions. As an example, we present two-thirds-order correlation functions in their diagrammatic representation.

\[
G_{AB}(k, p) =
\]

(A16)

\[
G_{AB}(q, k, p) =
\]

(A17)

Note that at all branch points there is momentum conservation: the momentum flowing in must be equal to the momentum flowing out. Black dots represent summation and open dots integration.

All the diagrams consist of a few elementary building blocks, e.g., lines with one, two, or more open dots and one black dot and lines between two black dots. The integrals corresponding to these building blocks can be performed in a straightforward manner. With these building blocks one can construct all the diagrams. Finally, to obtain the correlation function, one has to perform the summation. This can be done either analytically or by explicit summation. Performing the summation analytically is only feasible when the architecture is regular.

This diagrammatic representation of the correlation functions presented here is similar to the diagrammatic techniques used in particular in ref 30 (also ref 32).

References and Notes


