ABSTRACT
This paper presents a method for combined interactive filtering and visualization of volumetric data. The user can set the filter parameters of a shape preserving class of morphological filters, called connected filters, efficiently. The filters work by computing some attribute describing the shape or size for each connected component, and then deciding which to keep based on some threshold. We use a method in which the computation of attributes and connected component analysis is separated from the decision stage of the filtering process. After performing the first stage as initialization, we can perform the (much faster) decision stage many times with different threshold values, allowing interactive filtering and visualization of the results. The results indicate that filtering can be performed at about 5 frames per second on a $256^3$ data set using a Pentium 4 at 1.9 GHz.

KEY WORDS
shape filters, volume visualization, interactive filtering

1 Introduction

In this paper, we present a method for combined filtering and visualization of volumetric data in such a way that the user can set the filter parameters efficiently, allowing filtering at interactive rates. We use a shape preserving class of morphological filter called connected filter. Connected filters have received much attention in recent years, in algorithm development [1, 2], and applications [3, 4]. Connected filters are shape preserving, because they never introduce new edges in images. A subclass of these are attribute filters, the first of which were area openings and closings, which remove image detail smaller than a particular area [5]. These in turn were extended to attribute openings which accept or reject image details based on any of a wide range of size parameters [6]. They also put forward the idea of attribute thinnings, which allow image filtering based on shape, rather than size criteria. This idea has been formalized to so called shape filters [7], which have been applied to the problem of vessel enhancement in angiographic volume data sets [4].

In the binary case, attribute filters work by computing some parameter (or attribute) describing the shape or size for each connected component, and then deciding which to keep based on, e.g., some threshold or window on these parameter values. An example is shown in Fig. 1, in which the nuts are separated from the bolts based on the number of holes. In grey scale, attribute filters can be implemented simplistically by thresholding the image at each grey level, applying a binary filter to each, and recombining them. Faster algorithms have been developed [1], but in the case of volumetric data, filtering is still too slow to be interactive in many cases. For example, filtering of a $256^3$ data set for vessel enhancement may take 12 s even on a Pentium 4 at 1.9 GHz with 800 MHz RDRAM. This is a serious drawback when optimal threshold settings are being determined for the filtering process. In this paper, we use a method first proposed by Salembier et al. [2], in which the computation of attributes and connected component analysis (stage 1) is separated from the decision of the filtering process (stage 2). After performing the first stage as initialization, we can then perform the (much faster) decision stage many times with different threshold values, allowing interactive filtering and visualization of the results.

2 Filtering using Max-Trees

An efficient implementation of attribute filters relies on computing both the hierarchy of connected components in the data set, and some attribute for each component to
use as a filter criterion. A Max-Tree representation of the dataset was introduced by Salembier et al. [2] as a more versatile structure to separate the filtering process from the computation of connected components and attributes. The building of this tree structure is called the construction phase, while its use for filtering is called the filtering phase. In this section we will briefly discuss this data representation, and how it can be used to perform filtering.

Let \( M \subseteq \mathbb{R}^n \) be some image domain (\( n = 2 \) for images, \( n = 3 \) for volumes), and \( f : M \to \mathbb{R} \) the grey scale image (volume) under study. Implicitly we assume the existence of some neighborhood graph (i.e. a grid) on \( M \).

A Max-Tree is a tree where the nodes represent sets of flat zones or connected components of \( f \). A set \( F \subseteq M \) is called a flat zone or connected component if for all \( p, q \in F \) there exists a path from \( p \) to \( q \) along which the function value is constant, and the set \( M \) is maximal in size.

The threshold set \( X_h(f) \) of image \( f \) is the set of points that remain after thresholding at level \( h \), i.e.

\[
X_h(f) = \{ x \in M | f(x) \geq h \}.
\]

A peak component at a grey level \( h \) is a connected component of the threshold set \( X_h(f) \). The number of these peak components is finite and can thus be enumerated. We introduce the notation \( P^k_h \) to denote the \( k \)th peak component at level \( h \).

Max-Tree nodes are connected components, and therefore there exists a unique mapping from Max-Tree nodes to peak components. We use the notation \( C^k_h \) to denote the node that consists of the subset of \( P^k_h \) with grey level \( h \).

The root node represents the set of pixels belonging to the background, that is the set of pixels with the lowest intensity in the image. The Max-Tree is a rooted tree: each node has a pointer to its parent, i.e. the nodes corresponding to the components with the highest intensity are the leaves (see Fig. 2). Hence the name Max-Tree: the leaves correspond to the regional maxima. This means that the Max-Tree can be used for filters that process peak components, i.e. starting from the regional maxima. Conversely, a tree in which the leaves correspond to the minima is called a Min-Tree and can be used for filters that process valley components, i.e. starting from the regional minima.

During the construction phase, the Max-Tree is built from the flat zones of the image. After this, the tree is processed during the filtering phase. This filtering removes flat zones based on some property. These properties are defined by an attribute value \( T(P^k_h) \) of a node \( C^k_h \), from an ordered universe (typically \( \mathbb{R} \) or \( \mathbb{Z} \)) on which an order \( \leq \) exists. Given a threshold value \( \lambda \) from this universe, the algorithm decides whether to preserve, or remove a node. Two classes of strategies exist:

- **pruning strategies**, which remove all descendants of \( C^k_h \), if \( C^k_h \) is removed
- **non-pruning strategies**, in which the parent pointers of children of \( C^k_h \) are updated to point at the oldest “surviving” ancestor of \( C^k_h \).

Salembier describes four different rules for the algorithm to filter the tree: the **Min**, the **Max**, the **Viterbi**, and the **Direct** decision. The first three are pruning strategies. In addition, Wilkinson and Urbach [7] introduced another non-pruning strategy, called the **Subtractive** decision. The decisions of these rules are as follows:

- **Min** A node \( C^k_h \) is removed if \( T(P^k_h) < \lambda \) or if one of its ancestors is removed.
- **Max** A node \( C^k_h \) is removed if \( T(P^k_h) < \lambda \) and all of its descendant nodes are removed as well.
- **Viterbi** The removal and preservation of nodes is considered as an optimization problem. For each leaf node the path with the lowest cost to the root node is taken, where a cost is assigned to each transition. In this paper we do not consider this rule. For details see [2].
- **Direct** A node \( C^k_h \) is removed if \( T(P^k_h) < \lambda \); its pixels are lowered in grey level to the highest ancestor which meets the criterion, its descendants are unaffected.

**Subtractive** As above, but the descendants are lowered by the same amount as \( C^k_h \) itself.

Figure 2 shows the peak components of a 1-D discrete signal, their attribute values, and the corresponding...
Max-Tree. The results of applying the Min, Max, Direct and Subtractive methods on this image with \( \lambda = 10 \) are shown in Fig. 3. Which of these rules is the most appropriate depends mainly on the application.

Consider an image with just three nested peak components \( P_3 \subset P_2 \subset P_1 \) at intensity levels 3, 2, and 1, respectively. Furthermore let \( T(P_3) \geq \lambda, T(P_2) < \lambda, \) and \( T(P_1) \geq \lambda. \) No pruning strategy can simultaneously retain \( P_3, P_2, \) and \( P_1 \), while removing \( P_2 \). Using the direct rule, the difference \( f - \phi_\lambda^T(f) \), where \( \phi_\lambda^T(f) \) is the filtered function using criterion \( T \) and threshold \( \lambda \), will consist of a zero background with one or more connected regions at intensity level 1, consisting of those pixels of \( P_1 \) which have intensity level 2, i.e. the members of \( C_1 \) (which need not be connected). In general, a peak component of this image may satisfy the criterion. In the subtractive case, the difference image consists of only those peak components which do not satisfy the criterion.

An example of these properties is shown in Fig. 4. The attribute used is \( I/A^2 \) which is the moment of inertia divided by the square of the area. For a given object, the moment of inertia is minimal for a circle, and increases rapidly as the object becomes more elongated. In the case of the subtractive rule, the filtered image \( \phi_\lambda^S(f) \) contains only elongated structures, and \( f - \phi_\lambda^S(f) \) contains only compact structures.

### 3 Application to visualization

Salembier et al. [2] noted that the building phase of a Max-Tree filter algorithm is by far the most costly. Therefore, given the fact that the Max-Tree algorithm is one of the fastest connected filter algorithms available, we assumed that the decision rule stage alone should work an order of magnitude faster than even the fastest method available [1]. For interactive filtering purposes, this would allow us to build the Max-Tree once, and use it repeatedly for filtering the image with different threshold levels. The speed gain was determined by comparing the time necessary to build the Max-Tree to the decision phase on different volume data sets. The results are shown in Table 1. The results indicate that, given fast rendering hardware, almost 5 frames per second can be reached on the 256\(^3\) data set on a Pentium 4 at 1.9 GHz, with 512 MB RDRAM. The smaller data set with larger number of grey levels gave slower timings, probably due to cache thrashing.

Figure 5 shows an example of an interactively filtered magnetic resonance angiogram volume data set, visualized by maximum intensity projection. The shape filter attribute used was \( I/V^{5/4} \), with \( I \) the moment of inertia and \( V \) the volume. This attribute is a purely shape dependent number, i.e. it is scaling invariant, which has a minimum value for a sphere and increases rapidly with elongation (see [4] for more details). The images show that by increasing the threshold parameter \( \lambda \), more and more structures that are...
not elongated disappear: for low threshold settings, only background noise is affected, and for a high threshold, only thin and very long structures remain.

The user was supplied with a simple GUI containing a slider to set the threshold value $\lambda$. Since the filtering stage takes less than a second on datasets of this size, the user can interactively choose a threshold and view the result. For the particular example shown in Fig. 5, such interactive filtering can be of great assistance to a radiologist when dealing with noisy angiographic volume data.

4 Conclusion

In this paper, we have proposed a method for interactive filtering of volume data sets based on a class of shape preserving filters. We have briefly introduced such filters and how they can be implemented efficiently using Max-Trees. The Max-Tree approach splits the filtering task in two stages. The first stage is a construction of a tree, while the second stage performs actual filtering using this tree. Building the tree takes several seconds for small volumes, and up to 40 seconds for large volumes (i.e. $512^3$). However, after the construction of the tree, we have shown that a volume can be filtered in fractions of a second, allowing interactive filtering and visualization, even on standard commodity hardware like PCs.

References


Figure 5. Magnetic resonance angiogram volume data set (size $256^3$) filtered interactively with an attribute thinning as shape filter. The attribute used was $I/V^{5/3}$, with $I$ the moment of inertia, and $V$ the volume of a peak component; the top left-hand image is the original, in the others the attribute threshold was 0.5, 1.0, 1.5, 2.0, 2.5, 3.0 and 4.0, respectively. This attribute is a shape dependent number that expresses elongation. Visualization was done by maximum intensity projection.