Part IV

Evaluation of the Decomposition Methods

10

Decomposition Techniques, Revisited

10.1 Introduction

The last part of this book consists of two chapters. The first chapter, on *Decomposition Techniques, Revisited*, includes four sections. These sections correspond to selected decompositions presented in Part II, which are compared in light of the direct vs. compositional decomposition.

In the first section, we compare Kitagawa's initial formulation with our main equation (6.4). Kitagawa's formulation was further developed by Cho and Retherford, Kim and Strobino, Das Gupta and structural decomposition. In an additional section, we compare the extensions of Kitagawa's formula with the formulations of Chapter 8. Next we include a section on applications of decomposition methods in mortality, fertility and population growth rates, where we compare existing methods with direct vs. compositional decomposition. Finally, alternative decomposition methods, regression decomposition, the purging and delta methods are compared with direct vs. compositional decomposition.

Several of the applications are examined again in these sections. This allows us to distinguish differences or similarities between the methods. We combine some parts of tables from Part II with parts of tables from direct vs. compositional decomposition of the same application.

The second chapter of this part comprises the final conclusions of the book.

10.2 Kitagawa's Decomposition Method, Revisited

The first method introduced in the book is the technique proposed by Kitagawa (1955). Her proposal includes two components presented in equations (3.1) and (3.2). As shown earlier, the first component corresponds to the change in the variable of interest while the second component is the contribution of the change in the structure of the population. In Kitagawa's words, these terms are respectively the rate component and the composition component of change.

Equation (6.4) of direct vs. compositional decomposition also presents two components. Here, the components express a change in the demographic average due to a direct change in the characteristic of interest and a compositional change.

In cases where the variable under study is a demographic rate, as the crude death rate, the direct change of direct vs. compositional decomposition is equal to a rate effect, as in Kitagawa's proposal. In both methods, the compositional term accounts for the total change due to changes in the population composition. Kitagawa's structural change is simply the difference of the normalized weights. In direct vs. compositional decomposition this component is a covariance between the variable of interest and the intensity of the population structure. On one hand, the first method used a more simple term, on the other hand, the term in the second method describes the dynamic relationship between the variables better.

A second option for direct vs. compositional decomposition is introduced in equation (6.9). This equation uses normalized weights and two components

$$\dot{\bar{v}} = \bar{\dot{v}} + \overline{v\dot{c}}.$$

As in Kitagawa's formulation, the compositional component accounts for the effect of change in the normalized weights.

A linear change, as explained in Chapter 9, in both equations (6.4) and (6.9) leads precisely to Kitagawa's expression. As mentioned in Chapter 9, direct vs. compositional decomposition can be estimated assuming many other types of change. Kitagawa's proposal is therefore a particular case of the more general decomposition (6.4).

Table 10.1 presents the results of Tables 3.1 and 6.1 on the Mexican crude death rate (CDR) for both decompositions. Both models yield the same results. Nevertheless, direct

Table 10.1: Crude death rate, $d(t)$, per thousand, and the annual change over time in 1985-1995 for Mexico. The change is decomposed by the direct vs. compositional decomposition and the method suggested by Kitagawa.

Direct vs. compositional		*Kitagawa*	
$d(1990)$	5.100		
$d(1985)$	5.532	$d(1985)$	5.532
$d(1995)$	4.755	$d(1995)$	4.755
$\dot{d}(1990)$	-0.078	$\Delta d(1985)$	-0.078
$\bar{\dot{\mu}}$	-0.116	$\Delta\ m_a$	-0.116
$C(\mu, r)$	0.039	$\Delta\ \frac{N_a}{N.}$	0.038
$\dot{d} = \bar{\dot{\mu}} + C(\mu, r)$	-0.077	$\Delta d(1985) = \Delta\ m_a + \Delta\ \frac{N_a}{N.}$	-0.078

Source: Tables 3.1 and 6.1.

vs. compositional decomposition shows that the structure of the population contributes to the change in the CDR through a covariance between age-specific death rates and growth rates.

As shown in Figure 7.3, the mortality increases with age after age 10. The positive covariance in Table 10.1 of 0.038 implies a shift in the population structure towards older ages.

More is learned from direct vs. compositional decomposition by looking at the extensions of the change in the Mexican CDR in Chapter 7 and in Tables 6.4 and 6.3. All these alternative sections used direct vs. compositional decomposition to study age decomposition, relative change and the sex difference of the Mexican CDR, respectively.

10.3 Further Decomposition Research, Revisited

Following Kitagawa's proposal, in Chapter 3 we introduced four general decompositions for the case of numerous compositional components. Analogously, Chapter 8 extended direct vs. compositional decomposition (6.4) to the case of multidimensional decomposition.

Table 10.2 displays the crude death rate, $d_E(t)$, and the decomposition of the change over time for 1992-1996 for selected European countries, which is derived from Tables 3.2, 3.3, 3.4, 3.5 and 8.3. As in Table 10.1, there is hardly any difference in the results of the different

Table 10.2: Crude death rate, $\bar{d}_E(t)$, per thousand, and decomposition of the annual change over time in 1992-1996 for selected European countries. The decompositions are direct vs. compositional decomposition and the methods suggested by Cho and Retherford, Kim and Strobino, Das Gupta and Oosterhaven and Van der Linden.

Direct vs. compositional			*CR*	*KS*	*DG*	*OV*
$\bar{d}_E(1994)$	11.187					
$\bar{d}_E(1992)$	10.916	$\bar{d}_E(1992)$	10.917	10.917	10.917	10.917
$\bar{d}_E(1996)$	11.595	$\bar{d}_E(1996)$	11.596	11.596	11.596	11.596
$\dot{\bar{d}}_E(t)$	0.170	$\Delta\bar{d}_E(1992)$	0.170	0.170	0.170	0.170
$\bar{\dot{m}}$	0.036	Δm_{ac}	0.047	0.047	0.047	0.047
$C(m,r)$	0.133					
$C(m,r_a)$	0.134	$\Delta\frac{N_{a\cdot}}{N_{\cdot\cdot}}$	0.131	0.131	0.129	0.130
$C(m,r_c)$	-0.001	$\Delta\frac{N_{ac}}{N_{a\cdot}}$	-0.008	-0.008	-0.006	-0.007
$\dot{\bar{d}}_E = \bar{\dot{m}} + C(m,r_a) + C(m,r_c)$		$\Delta\bar{d}_E(t) = \Delta\frac{N_{a\cdot}}{N_{\cdot\cdot}} + \Delta\frac{N_{ac}}{N_{a\cdot}} + \Delta m_{ac}$				
	0.169		0.170	0.170	0.170	0.170

Source: Tables 3.2, 3.3, 3.4, 3.5 and 8.3. The headings in the table correspond to CH for Cho and Retherford, KS for Kim and Strobino, DG for Das Gupta, and OV for Oosterhaven and Van der Linden.

decompositions in Table 10.2. Particularly, the first four methods by Cho and Retherford, Kim and Strobino, Das Gupta and Oosterhaven and Van der Linden, respectively, obtain the same direct change and compositional component. The difference in death rates is $\Delta m_{ac} = 0.047$ and the addition of the two compositional components is $\Delta\frac{N_{a\cdot}}{N_{\cdot\cdot}} + \Delta\frac{N_{ac}}{N_{a\cdot}} = 0.123$. The only method that differs is direct vs. compositional decomposition where we get a level-1 effect of

0.036 and a level-2 effect of 0.133. Nevertheless, for direct vs. compositional decomposition the estimated results of Table 8.3 varied slightly from the observed changes.

There is no doubt that the biggest advantage of direct vs. compositional decomposition compared to the other methods is its simplicity. Equation (8.21) is, as (6.4), simple and elegantly expressed as

$$\dot{\bar{v}} = \bar{\dot{v}} + C(v, \acute{w}_1) + C(v, \acute{w}_2) + ... + C(v, \acute{w}_n).$$

The first term on the right-hand side captures the change in the characteristic of interest, this is the direct change. The other components are the covariances between the underlying variable of interest and each of the intensities of the weighting function. These are the structural or compositional components of change due to each of the compositional factors. It seems inconvenient to assume that the weighting function can be expressed as a product of weighting functions $w = w_1 w_2 ... w_n$. But this is not the case as is immediately proved.

If instead of equation (6.4) we extend its alternative (6.9), $\dot{\bar{v}} = \bar{\dot{v}} + \overline{v\acute{c}}$, then the assumption of the weighting function as a product of weights is avoided. Equation (6.9) uses the normalized weights as weights. This is shown in its extension in equation (10.3). In the case of two compositional components x and z, the normalized weights are expressed as $c(x, z, t)$, that is the proportion of the total values of the weights that belong to the category x and z at time t, $c(x, z, t) = \frac{w(x,z,t)}{\int_0^\infty \int_0^\infty w(x,z,t) dx dz} = \frac{w(x,z,t)}{w(t)}$. The second component in (6.9) is the average of the variable of interest multiplied by the intensity of the normalized weights, $\overline{v\acute{c}}$. The normalized weights can be further separated into a product of normalized weights as suggested by Kim and Strobino in (3.12) as

$$c(x, z, t) = \frac{w(x, z, t)}{w(t)} = \frac{w(x, z, t)}{w(x, t)} \frac{w(x, t)}{w(t)}, \tag{10.1}$$

where $\frac{w(x,z,t)}{w(x,t)}$ and $\frac{w(x,t)}{w(t)}$ correspond to the weights due to the components z and x, respectively. Let these two terms be $c_z = \frac{w(x,z,t)}{w(x,t)}$ and $c_x = \frac{w(x,z,t)}{w(z,t)}$. Likewise, Das Gupta's equation (3.15) can be used to separate the weights into multiplicative terms.

The property of the relative derivative of a product (8.19) and the average property of separating additions

$$\overline{u_1 + u_2} = \overline{u_1} + \overline{u_2}, \tag{10.2}$$

lead us to achieve the extension of the decomposition when numerous compositional components are included. From equation (6.9) and by using (10.1) and (10.2) we obtain

$$\begin{aligned} \dot{\bar{v}} &= \bar{\dot{v}} + \overline{v\acute{c}} \\ &= \bar{\dot{v}} + \overline{v\left(\acute{c}_z + \acute{c}_x\right)} \\ &= \bar{\dot{v}} + \overline{v\acute{c}_z} + \overline{v\acute{c}_x}, \end{aligned} \tag{10.3}$$

where $\overline{v\acute{c}_z}$ and $\overline{v\acute{c}_x}$ are the compositional components accounting for the change in z and x, respectively. By using the methods proposed by Kim and Strobino, Das Gupta or fellow innovators, such as the log-linear model used in Chapter 8, separating the weighting function is a minor step in the exercise. As shown in Chapter 9, derivatives and relative derivatives with respect to time can be estimated in several ways. These approximations correspond to different types of changes and not only to linear assumptions as assumed in the other methods. Therefore, we conclude that direct vs. compositional decomposition is a general option that allows for diverse types of changes over time.

10.4 Applications of Decomposition Methods, Revisited

In Chapter 4 we presented three sections on decomposition applications in mortality, fertility and population growth. The same three sections are included here as subsections. Each subsection contrasts the decompositions used in these areas of demography with direct vs. compositional decomposition.

10.4.1 Mortality Measures, Revisited

In Chapter 4 we studied mortality measures, such as life expectancy. Here we concentrate on the decompositions of the change over time of this measure.

We start by looking at Keyfitz's equation (4.5) for the relative change in life expectancy. The $\acute{e}^o(0,t)$ is equal to the product of improvements in mortality and the entropy of the survival function,

$$\acute{e}^o(0,t) = \rho(t)\mathcal{H}(t).$$

In the work of Keyfitz (1985), the change over time in life expectancy corresponds to equal proportional changes in mortality at all ages. Keyfitz assumes constant mortality improvement at all ages, $\rho(a,t) = \rho(t)$ for all a. An obvious generalization would then be to allow the improvements to change over age. Vaupel and Canudas Romo (2003) proved that the new proposed decomposition of life expectancy (6.35) is this generalization. We mention here some of the crucial points of that demonstration.

The average number of life-years lost as a result of death, $e^\dagger$, introduced in (6.33) can be written as the product of life expectancy at birth and the entropy of the survival function:

$$e^\dagger(t) = e^o(0,t)\mathcal{H}(t). \tag{10.4}$$

By substituting this result in equation (6.35) and dividing by life expectancy at birth, the general formula for the relative change in life expectancy is obtained

$$\acute{e}^o(0,t) = \frac{\dot{e}^o(0,t)}{e^o(0,t)} = \bar{\rho}(t)\mathcal{H}(t) + \frac{C_f(\rho, e^o)}{e^o(0,t)}. \tag{10.5}$$

No constraints were made on the improvement in mortality in equation (6.35). Consequently, (10.5) is the generalization of Keyfitz's (4.5) since the improvement in mortality here is not necessarily equal for all ages.

The next comparison is between the Vaupel-Canudas decomposition (6.35) and the proposals by Pollard (1982, 1988), Arriaga (1984), Pressat (1985) and Andreev (1982; Andreev et al. 2002). Their work focused on the discrete difference in life expectancy between two periods of time. As already mentioned in Chapter 4, these proposals for the difference in life expectancies were developed independently. However, they all have a similar decomposition for the difference in life expectancies. We present them here as Arriaga's decomposition.

Table 10.3 combines the results of applying Arriaga's method in Table 4.1 and Vaupel and Canudas Romo's (2003) result for Sweden, from 1995 to 2000. A similar application of Vaupel and Canudas Romo's method is found in Table 6.7 for Sweden from 1990 to 1999. When we

Table 10.3: Life expectancy at birth, $e^o(0,t)$, and decomposition suggested by Vaupel and Canudas and Arriaga for the annual change over time from 1995 to 2000 for Sweden.

Vaupel–Canudas		*Arriaga*	
$e^o(0,1998)$	79.262		
$e^o(0,1995)$	78.784	$e^o(0,1995)$	78.784
$e^o(0,2000)$	79.740	$e^o(0,2000)$	79.740
$\dot{e}^o(0,1998)$	0.191	$\Delta e^o(0,1995)$	0.191
$\bar{\rho}$ (%)	1.586		
$e^\dagger$	10.042		
$\bar{\rho}e^\dagger$	0.159	Δ_D	0.007
$C_f(\rho,e^o)$	0.032	Δ_I	0.184
$\dot{e}^o(0,1995) = \bar{\rho}e^\dagger + C_f(\rho,e^o)$	0.191	$\Delta e^o(0,1995) = \Delta_D + \Delta_I$	0.191

Source: Tables 4.1 and Vaupel and Canudas Romo (2003).

compare the two methods, we see that both methods divide the change into two components. Arriaga's decomposition consists of a direct and an indirect component (interaction included here). The second component accounts for all the change; in Table 10.3 this is 0.184 years of the annual change. The direct component is generally not at all significant. Here it was only 0.007 years of the annual change. A component in the Vaupel-Canudas method accounts for the general effect of the reduction in the death rates. The second component, the covariance captures the effect of heterogeneity on mortality improvements over age. The direct component was $\bar{\rho}e^\dagger = 0.159$ while the distribution of the improvements in mortality was $C_f(\rho, e^o) = 0.032$. The two components of the decompositions can be interpreted similarly, but there are big differences in the results of the components. It can, therefore, be concluded that the Vaupel-Canudas decomposition distributes Arriaga's interaction effect into both its components.

Vaupel and Canudas Romo (2003) show that their and Arriaga's formula allocate the same contribution by ages in the total change in life expectancy. But, even the contribution by age of Arriaga's direct and indirect components are far from equal to the results obtained by the direct and compositional components of the Vaupel-Canudas method.

Other tables, which complement each other, are the decomposition of the average age at death in Table 8.4 and the decomposition by age and cause of death proposed by Pollard in Table 4.2.

We conclude here that the new decomposition method complements the previous studies of changes in life expectancy. As in some of the previous decompositions, the Vaupel-Canudas method also allows for an age decomposition, a cause of death decomposition and a decomposition of the difference in life expectancies.

10.4.2 Fertility Measures, Revisited

The second section in Chapter 4 focuses on the study of fertility measures. We will concentrate here on comparing the decompositions of the crude birth rate, as proposed by Zeng et al. (1991) and direct vs. compositional decomposition.

Tables 4.3, 4.4 and 6.5 are combined in Table 10.4. The first two columns display the result of the relative change in the crude birth rate using direct vs. compositional decomposition. The remaining columns show the decomposition suggested by Zeng et al. (1991) for married, unmarried and for all women. All the results have previously been discussed in their respective

Table 10.4: Crude birth rate, $CBR(t)$, in percentage, for the total population and by marital status (married, unmarried and total). The decompositions are direct vs. compositional decomposition for the relative annual change over time in CBR and the method suggested by Zeng for the difference over time, in percentage, from 1992 to 1997, for the Netherlands.

Direct vs. compositional		*Zeng*			
		Marital status	*Married*	*Unmarried*	*Total*
$CBR(1995)$	1.258				
$CBR(1992)$	1.295	$CBR_s(1992)$	1.134	0.161	1.295
$CBR(1997)$	1.232	$CBR_s(1997)$	0.997	0.236	1.233
$C\acute{B}R(1995)$	-0.996	$\Delta CBR_s(1992)$	-0.027	0.015	-0.012
$\tilde{\acute{b}}$	-0.285	Δb_{sa}	0.015	0.012	0.027
$\tilde{\acute{c}}_f$	0.060	$\Delta \pi_{fa}$	-0.005	-0.003	-0.008
r_B	-0.212	$\Delta \pi_{sa}$	-0.037	0.007	-0.030
r	0.554				
$r_B - r$	-0.766	$\Delta CBR_s(1992) = \Delta b_{sa} + \Delta \pi_{sa} + \Delta \pi_{fa}$			
$C\acute{B}R = \tilde{\acute{b}} + \tilde{\acute{c}}_f + [r_B - r]$	-0.991		-0.027	0.016	-0.011

Source: Tables 4.3, 4.4 and 6.5. The subindex s corresponds for the marital status: married or unmarried.

chapters. Here we concentrate on some comparisons between these methods. The terms denoting the average of the relative change in birth rates, $\tilde{\acute{b}}$, and the differences in birth rates, Δb_{sa}, have opposite signs. Both depict the change over time of birth rates. If we only look at one of these results, it would lead us to conclude that birth rates are declining or increasing depending on whether we observe the first or second decomposition. The intensity of the birth rates, $\tilde{\acute{b}}$, uses as weights the number of births to women. The weights for the difference in births, Δb_{sa}, are the proportion of women (married or unmarried) to the total population. These different weights account for the discrepancy in the results.

The simplicity of the decomposition proposed by Zeng is generalized in the book by applying it to unmarried women. Further generalization is shown in equations (6.16) and (6.17) where the direct vs. compositional decomposition method is used and the residual term in Zeng's formulation is eliminated. Table 8.6 presents the multidimensional decomposition of the general fertility rate when the compositions due to age, marital status and country are included. We want to stress here that the use of these formulas allows for other types of change in the variables involved in the decomposition. Direct vs. compositional decomposition also permits a study of the relative changes. This is an alternative change that can help us detect variations in rates that are not visible in other decompositions due to their weights.

10.4.3 Growth Measures, Revisited

The growth rate of the population is highly relevant for demographic research. The close relation between growth rates and direct vs. compositional decomposition is already exhibited in Chapter 7. This relation is a simple substitution of any of the systems of the age-specific growth rates, from Chapter 4, in the covariance of direct vs. compositional decomposition. By substituting Preston and Coale's (1982) system in equation (4.30), $r = \nu - \mu$, in the decomposition of the crude death rate

$$\dot{\bar{\mu}} = \bar{\dot{\mu}} + C(\mu, r),$$

we obtain

$$\dot{\bar{\mu}} = \bar{\dot{\mu}} + C(\mu, \nu) - C(\mu, \mu) = \bar{\dot{\mu}} + C(\mu, \nu) - \sigma^2(\mu). \tag{10.6}$$

By substituting Arthur and Vaupel's (1984) system in equation (4.33), $r = r_0 - \varphi$, in the decomposition of the crude death rate we obtain

$$\dot{\bar{\mu}} = \bar{\dot{\mu}} + C(\mu, r_0) - C(\mu, \varphi). \tag{10.7}$$

equations (10.6) and (10.7) explain not only why the structure of the population has changed, but also how this change affects the change in the crude death rate.

Another useful formula is the average growth rate and its change over time. Table 10.5 includes the results of the decompositions proposed by Keyfitz and direct vs. compositional decomposition for the change in the average growth rate of the world, found in Tables 4.9 and 7.2. As a consequence of Keyfitz's (1985) assumption on fixed age-specific growth rates, the observed growth rates for 1980 and 1983 are different from those observed during the period shown in the first two columns. The direct vs. compositional decomposition is another generalization of Keyfitz's results where not only the variance but also a direct change appear. In Table 10.5 the new term is the most relevant of the two components and essential to understand the observed variation in the world's growth rate.

10.5 Alternative Decomposition Methods, Revisited

Chapter 5, on *Alternative Methods of Decomposition*, presented an alternative separation procedure for the case of parametric models. Little connection has been made between these decompositions and direct vs. compositional decomposition. The first two alternative methods were developed for cases that consider samples rather than population totals (from censuses

Table 10.5: Population growth rate of the world, $\bar{r}(t)$, and direct vs. compositional decomposition and the method suggested by Keyfitz for the annual change over time around January 1, 1982.

Direct vs. compositional		*Keyfitz*	
$\bar{r}(1980)$	1.732 %	$\bar{r}(1980)$	1.697 %
$\bar{r}(1983)$	1.711 %	$\bar{r}(1983)$	1.722 %
$\dot{\bar{r}}(1982)$	-0.716 *	$\dot{\bar{r}}(1982)$	0.832 *
$\bar{\dot{r}}$	-1.545 *		
$\sigma^2(r)$	0.829 *	$\sigma^2(r)$	0.832 *
$\dot{\bar{r}} = \bar{\dot{r}} + \sigma^2(r)$	-0.716 *	$\dot{\bar{r}} = \sigma^2(r)$	0.832 *

Source: Tables 4.9 and 7.2.

or vital statistics). Direct vs. compositional decomposition is used for measures of population totals.

The regression decomposition is based on the difference of means, but, if the same population is studied over time, derivatives could be applied. As in equation (5.3), by carrying out the following

$$\begin{aligned}\bar{y}_2 - \bar{y}_1 &= (\alpha_2 - \alpha_1) + \sum_{k=1}^{K} \left(\frac{\bar{x}_{1k} + \bar{x}_{2k}}{2}\right)(\beta_{2k} - \beta_{1k}) \\ &+ \sum_{k=1}^{K} \left(\frac{\beta_{1k} + \beta_{2k}}{2}\right)(\bar{x}_{2k} - \bar{x}_{1k}),\end{aligned}$$

and using derivatives, we can write the change over time in groups' means as the sum of changes in intercepts, coefficients and independent variables $\bar{x}_{ik}$,

$$\dot{\bar{y}} = \dot{\alpha} + \sum_{k=1}^{K} \dot{\beta}_k \bar{x}_k + \sum_{k=1}^{K} \beta_k \dot{\bar{x}}_k. \qquad (10.8)$$

These components can also be interpreted as the effects of change on the influence of group membership on the dependent variable, $\dot{\alpha} + \sum_{k=1}^{K} \dot{\beta}_k \bar{x}_k$, and as the effect on the dependent variable of changes in the characteristics of the groups over time, $\sum_{k=1}^{K} \beta_k \dot{\bar{x}}_k$. Nevertheless, as with regression decomposition, next to the calculations, a test of statistical hypotheses should also be considered.

The log-linear model formulation used by Clogg (1978) is also suggested as an option for separating the compositional component for direct vs. compositional decomposition in Chapter 8. Again, here it is necessary to include indicators of the precision of the adjusted frequencies.

Both regression decomposition and the purging method are important statistical models that account for sampling variability. Direct vs. compositional decomposition is not immediately applicable in sampling cases. The advantage of the direct vs. compositional decomposition method compared to other decomposition methods mentioned is that there is no need to develop procedures for hypothesis testing or confidence intervals.

The delta method is a general decomposition method, which sometimes consists of terms that are difficult to interpret. Similar to direct vs. compositional decomposition, it uses derivatives. In the direct vs. compositional decomposition method the derivative is always with respect to time. For the delta method, the derivative is with respect to all parameters of the model. If the parameters of the delta model also depend on time, then the delta method could be used as a general formulation of change over time.

10.6 Conclusions

This chapter evaluates the various decomposition methods. The first point to note is that one should talk of complementary methods rather than of competing methods.

Even in the first method with its further extensions we find similarities with direct vs. compositional decomposition. The simplicity of Kitagawa's method is only comparable to the continuous formulation of direct vs. compositional decomposition in equation (6.4). Moreover, it is proven that the earlier method is a particular case of direct vs. compositional decomposition.

We also compared the extensions of Kitagawa's work, by Cho and Retherford, Kim and Strobino, Das Gupta and structural decomposition, in direct vs. compositional decomposition. An alternative formulation of direct vs. compositional decomposition helped us to demonstrate the similarity among all these techniques. The flexibility of direct vs. compositional decomposition is especially useful for adapting techniques of separation used by the other methods. Direct vs. compositional decomposition can also assist in implementing other hypotheses concerning types of changes over time of the variables involved in the decomposition: e.g. linear, exponential, logistic, or others.

Studies of changes in life expectancy, crude birth rate and population growth rate can potentially benefit from the contributions of direct vs. compositional decomposition. However, it will always be in the researcher's interest to find the proper decomposition for the measure under study. Here again, complementary application of the methods is the final advice.

The decomposition methods are applied to population totals, while regression decomposition and the purging method are used in cases of samples of the population. We have suggested some of the possible relations in both directions here by applying direct vs. compositional decomposition method in regression decomposition and by using log-linear models for separating cross-tabulated data into compositional components of direct vs. compositional decomposition. Therefore, more exchange between these methodologies, as those suggested here, can facilitate the study of changes in demographic measures.