7

Age, Categorical and Cause of Death Decomposition

7.1 Introduction

This chapter shows that direct vs. compositional decomposition can be used to study the contributions of particular ages (levels of the components or categories) to the total change over time of demographic measures. Particularly in the study of life expectancy a cause of death decomposition is shown.

Chapter 4 presented studies analyzing the change in life expectancy over time or between populations. Andreev (1982) (cited by Andreev et al. (2002)), Arriaga (1984), Pressat (1985) and Pollard (1988) decomposed the change in life expectancy into different components. An important part in the analysis of changes in life expectancy is to estimate the contribution of mortality changes for a specific age group to the total difference in life expectancy.

The focus on the contribution in the total change of changes in the different age groups is not exclusive to studies of life expectancy. A similar application can be made with regard to the total fertility rate (TFR). In Chapters 4 and 5 we introduced studies of this kind. The aim is to evaluate the components of change in the TFR, over time or across populations. As in the study of change in life expectancy the analysis focuses on the contributions of the levels to the total change. In the case of the TFR, the area of concern is the contribution of particular parity-specific fertilities to the total difference.

Similar to these studies on TFR and life expectancy we focus here on the crude death rate and the population growth rate of the world. The following are some of the examples of questions which can be asked about the contribution of ages (levels of the components or categories) to both the crude death rate and the population growth rate of the world: How are death rates of different age groups contributing to the change in the Mexican crude death rate? How are different regions of the world contributing to the growth rate of the world's population?

The following section demonstrates a group age decomposition of our main equation (6.4), which is taken to a single age decomposition in the third section. A section on categorical decomposition is then included, where the age decomposition is employed to study categories other than age. The last section in this chapter is a cause of death decomposition of change in life expectancy.

7.2 Age Decomposition

By applying equation (6.4) we obtain two main terms that explain the change of a population average. The change is divided into a term denoting the direct change in the characteristic of interest and a term describing the change in the structure or composition of the population. These two terms can be further decomposed by age.

The age decomposition is accomplished by using the additive property of integrals. In the following text we describe how this property of the integrals is applied to the ages of a given population. The integral from age zero to the last age attained by a person in the population $(0, \omega)$ can be separated into the addition of integrals that account for the different age groups, $(0, x_1)$, (x_1, x_2), (x_2, x_3) and so on until the last age group (x_n, ω). The corresponding equation is

$$\int_0^\omega v(a,t)da = \int_0^{x_1} v(a,t)da + \int_{x_1}^{x_2} v(a,t)da + ... + \int_{x_n}^{\omega} v(a,t)da. \tag{7.1}$$

Both terms in equation (6.4), the average change, $\bar{\dot{v}}$, and the covariance component, $C(v, \acute{w})$, are defined over integrals. From the property (7.1) it follows that these components can be separated by age groups. The numerator in the direct component of equation (6.4) is decomposed by age as follows:

$$\begin{aligned}
\bar{\dot{v}} &= \frac{\int_0^\omega \dot{v}(a,t)w(a,t)da}{\int_0^\omega w(a,t)da} \\
&= \frac{\int_0^{x_1} \dot{v}(a,t)w(a,t)da}{\int_0^\omega w(a,t)da} + ... + \frac{\int_{x_n}^\omega \dot{v}(a,t)w(a,t)da}{\int_0^\omega w(a,t)da} \\
&= [\bar{\dot{v}}]_0^{x_1} + ... + [\bar{\dot{v}}]_{x_n}^{\omega}\,.
\end{aligned} \tag{7.2}$$

It should be noted that the denominator has not changed, while the numerator is partitioned over the different age groups. A similar approach is used to separate the compositional component by age,

$$\begin{aligned}
C(v, \acute{w}) &= \frac{\int_0^\omega [v(a,t) - \bar{v}(t)] \left[\acute{w}(a,t) - \bar{\acute{w}}(t)\right] w(a,t)da}{\int_0^\omega w(a,t)da} \\
&= [C(v,\acute{w})]_0^{x_1} + ... + [C(v,\acute{w})]_{x_n}^{\omega}\,,
\end{aligned} \tag{7.3}$$

where the covariance in the age group x_a to x_b is defined as

$$[C(v,\acute{w})]_{x_a}^{x_b} = \frac{\int_{x_a}^{x_b} [v(a,t) - \bar{v}(t)] \left[\acute{w}(a,t) - \bar{\acute{w}}(t)\right] w(a,t)da}{\int_0^{\omega} w(a,t)da}. \tag{7.4}$$

The covariance in the age group x_a to x_b is a component of the covariance over all ages and not a covariance over the smaller age group, the reason being that the averages $\bar{v}(t)$ and $\bar{\acute{w}}(x,t)$ refer to all ages.

The final decomposition formula (6.4) is expressed by using (7.2) and (7.3) which yields:

$$\dot{\bar{v}} = [\bar{\dot{v}} + C(v,\acute{w})]_0^{x_1} + ... + [\bar{\dot{v}} + C(v,\acute{w})]_{x_n}^{\omega}. \tag{7.5}$$

This decomposition allows us to estimate the contribution of each age group to the total change over time of any demographic variable. Another attribute of the age decomposition is the estimation in each age group of the level-1 and level-2 effects.

Table 7.1 shows the age decomposition of the change in the Mexican crude death rate (CDR) from 1965 to 1975, 1975 to 1985 and 1985 to 1995, denoted in the table by the mid-years 1970, 1980 and 1990, respectively. The selected age groups are from 0 to 9 years, 10 to 59 years, and 60 years and above. In the last row, "All ages", the additions of the level-1 and level-2 effects, columns with the legends $[\bar{\dot{\mu}}]_{x_a}^{x_b}$ and $[C(\mu,r)]_{x_a}^{x_b}$ respectively, are found for the three age groups.

Table 7.1: Age decomposition of the annual change over time of the crude death rate, $d(t)$, per thousand, in 1965-1975, 1975-1985 and 1985-1995 for Mexico.

Ages	$[\bar{\dot{\mu}}]_{x_a}^{x_b}$	$[C(\mu,r)]_{x_a}^{x_b}$	$[\bar{\dot{\mu}}]_{x_a}^{x_b} + [C(\mu,r)]_{x_a}^{x_b}$
1970			
$0-9$	-0.151	-0.003	-0.154
$10-59$	-0.034	-0.008	-0.042
60+	0.024	-0.033	-0.009
All ages	-0.161	-0.044	-0.205
1980			
$0-9$	-0.106	-0.013	-0.119
$10-59$	-0.050	-0.017	-0.067
60+	-0.001	-0.002	-0.003
All ages	-0.157	-0.032	-0.189
1990			
$0-9$	-0.046	0.008	-0.038
$10-59$	-0.047	0.002	-0.045
60+	-0.023	0.029	0.006
All ages	-0.116	0.039	-0.077

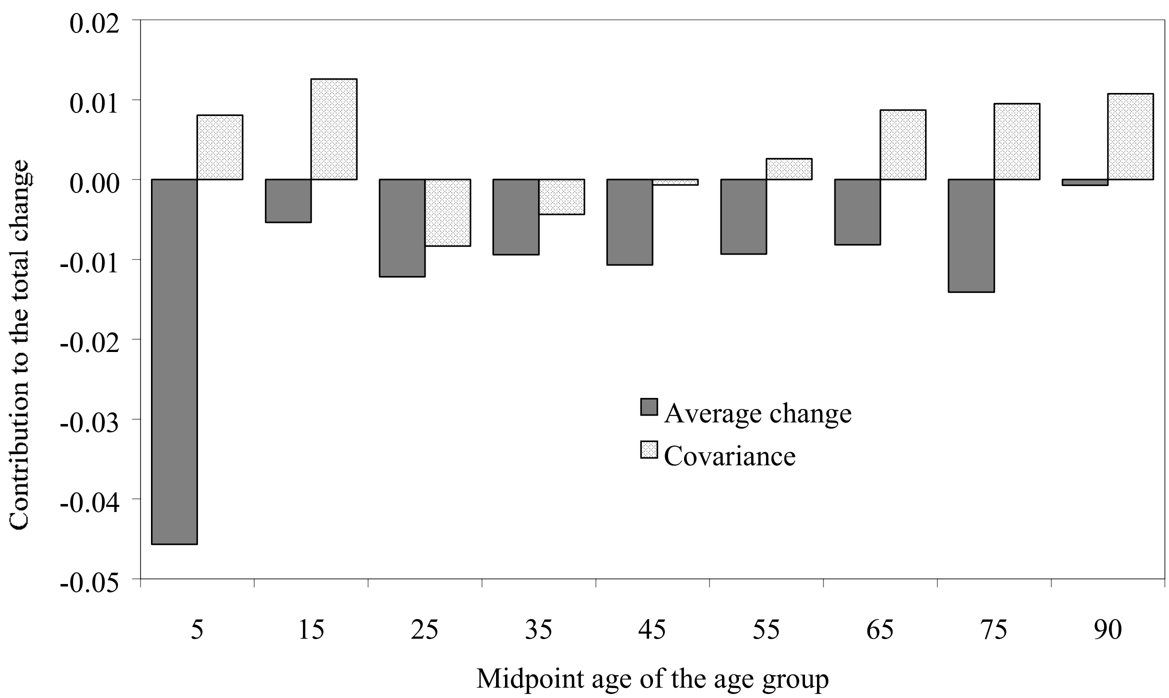
Source: Author's calculations described in Chapter 9, based on the United Nations Data Base (2001).

The results of Table 7.1 provide a more detailed picture of the change over time in the Mexican CDR. As can be seen in the negative values of the average change $[\bar{\dot{\mu}}]_{x_a}^{x_b}$, there are improvements in mortality at all ages and periods, except for the age group 60 years and above in 1970. The covariance component, $[C(\mu, r)]_{x_a}^{x_b}$, which accounts for the effect of the change in the structure of the population, opposes the effect of the decreased mortality for all age groups in the year 1990.

In the last column it is shown how in the first two periods the age group 0 to 9 is the major contributor to the decrease in the Mexican CDR, while persons aged 10 to 59 account for the greatest share of the decrease in 1990. In the last period the opposing effect of the compositional component generates an increase in mortality for people aged 60 years and above.

Figure 7.1 shows a bar chart of the components contributing to the change in the Mexican crude death rate from 1985 to 1995. We have analyzed the effects by dividing the population into ten age groups. Again, we find that the major contribution to the decrease in the Mexican

Figure 7.1: Age decomposition of the annual change over time in the Mexican crude death rate from 1985 to 1995.



CDR in 1990 is due to changes in the ages 0 to 9. In contrast, persons aged 80 years and above, in Figure 7.1 indicated by the bar columns at age 90, show the smallest reduction in mortality due to average changes. In this figure the unbalanced contribution by age of the compositional component becomes evident. Among persons less than 20 years or above age 60 we find positive covariance, while negative values are found for the age groups 20 to 59.

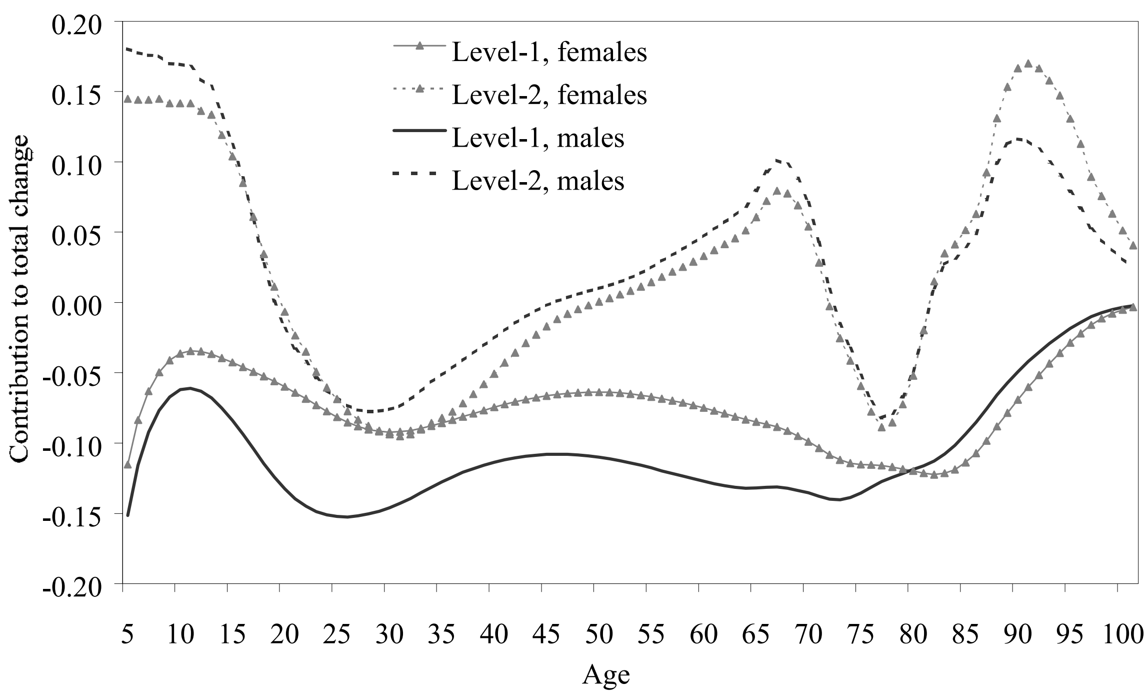
In the next section a single age decomposition is shown. Here, it is shown that the fluctuations over age of the compositional component are due to changes in the age-specific growth rates. These changes are the product of past and present demographic processes.

7.3 Single Age Decomposition

Let the age intervals of the terms in equation (7.5) be as small as one year of age. The direct and compositional components in equation (7.5) can also be calculated for each single age. In this section we examine the reasons for the numerous fluctuations of the compositional component throughout a single-age decomposition.

Figure 7.2 shows a line chart of the values obtained for the direct and the compositional components of the change in the male and female Mexican crude death rates from 1985 to 1995. The values of these components were so large at ages 0, 1 and 2, that the figure is restricted to 5 to 100. The level-1 effects are marked by continuous lines while the level-2 effects are marked by broken lines. Both female components are marked by triangles to distinguish them from their male counterparts. The data used in this section was supplied by Virgilio Partida of the National Population Board of Mexico, Conapo (2002).

Figure 7.2: The direct effect and the compositional effect, in percentages, of the annual change over time in the Mexican male and female crude death rates from 1985 to 1995.



In Figure 7.2 we see that the direct component only has values below zero indicating a decrease in the age-specific death rates at all ages during the analyzed period. The compositional component shows the fluctuations already noted in Figure 7.1. To understand the compositional component it is necessary to look at the elements of this component in equation (7.4).

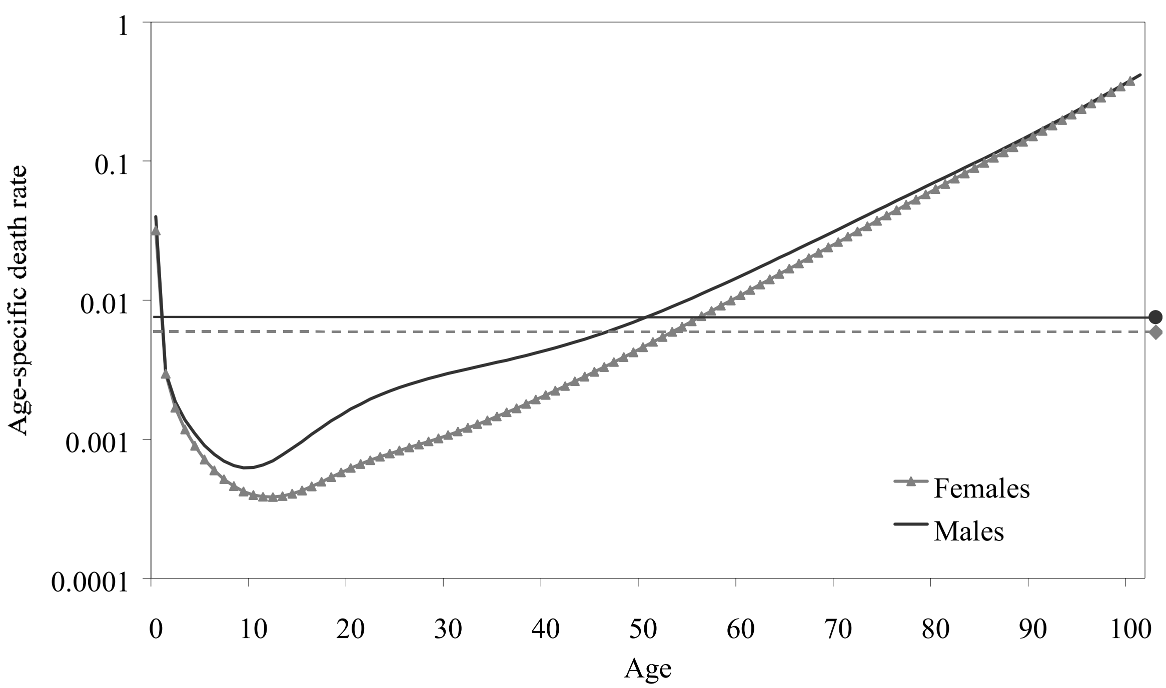
The compositional component of the change over time in the CDR in the single age decomposition is

$$[C(\mu, r)]_{a-0.5}^{a+0.5} \approx [\mu(a,t) - \bar{\mu}(t)]\,[r(a,t) - \bar{r}(t)]\,c(a,t), \tag{7.6}$$

where the term $c(a,t)$ corresponds to the proportion of the population at age a and time t with respect to the total population, $c(a,t) = \frac{N(a,t)}{\int_0^\infty N(a,t)da}$. In other words, the compositional component of a certain age takes the deviation of the age-specific death rate from the crude death rate, $[\mu(a,t) - \bar{\mu}(t)]$, and of the age-specific growth rate from the population total growth rate, $[r(a,t) - \bar{r}(t)]$, as well as the proportion of the total population at that age into account. Fluctuations in the compositional component are due to one of these three factors.

Figure 7.3 shows the age-specific death rates and the Mexican crude death rate for males and females in 1990. The male and female CDRs are indicated by straight lines in the figure, male CDR of 6.233 and female of 4.755 per thousand. After a quite high level of age-specific

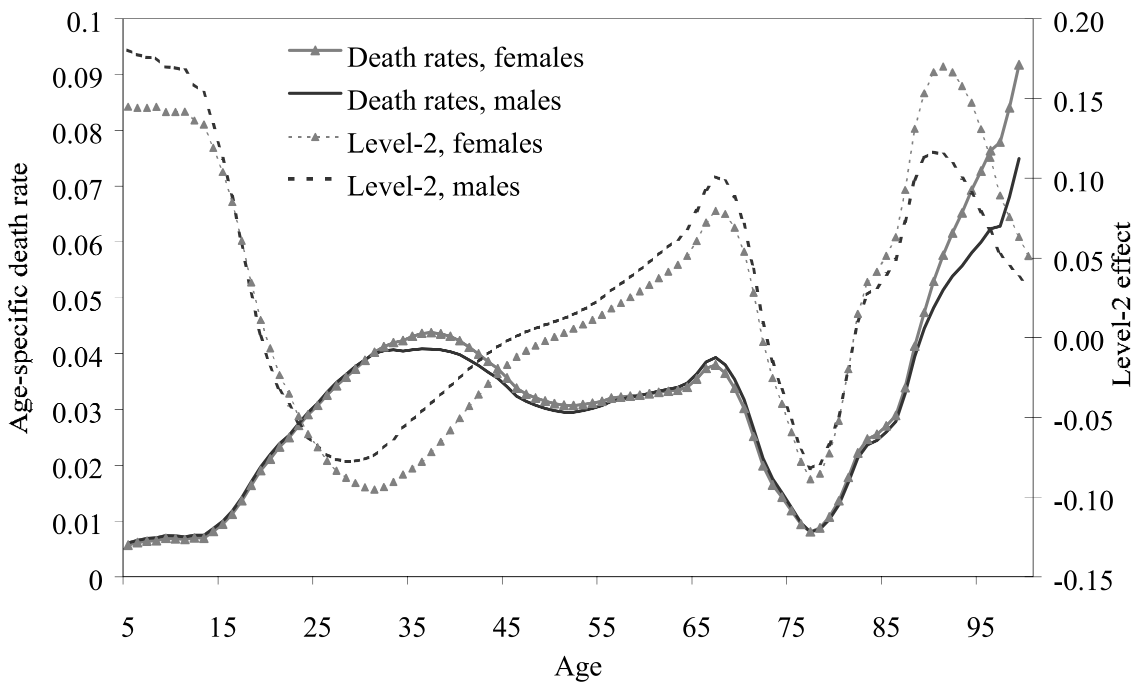
Figure 7.3: The age-specific death rates and the Mexican crude death rate for males and females in 1990.



death rates due to infant mortality, the rates reach their lowest level around age 10. The age-specific death rates for both males and females increase thereafter monotonically. Male age-specific death rates are above the female rates at all ages, with the gap between the sexes most pronounced from age 10 to 60. Around age 50 male and female age-specific death rates have the same values as the crude death rates. This means the difference between the death rates $[\mu(a,t) - \bar{\mu}(t)]$ in equation (7.6) can explain the sign (positive or negative) and magnitude of the compositional component but not the fluctuations we find in Figure 7.2. We reach a similar conclusion for the Mexican population composition, $c(a,t)$, which decreases with age and, therefore, only contributes to the magnitude of the level-2 effect. Further, the product of the difference of death rates and the population composition yields a nearly straight line. We conclude that in order to understand the fluctuations of the compositional component it is necessary to examine the differences between the age-specific growth rates and the population total growth rate, $[r(a,t) - \bar{r}(t)]$.

Figure 7.4 shows the male and female age-specific growth rates together with the level-2 effects depicted in Figure 7.2. At ages below age 50 the compositional components undergo

Figure 7.4: The Mexican male and female age-specific growth rates together with the level-2 effect of the annual change over time in the crude death rate.



exactly the same fluctuations as the age-specific growth rates but in the opposite direction and of higher magnitude. After age 50 the patterns of the fluctuations for both curves follow the same direction, again with higher levels for the compositional component.

In general, the change over time of the population composition does not only depend on the change experienced during the analyzed period, but also on changes of the past. All the information needed to estimate this demographic dynamic is found in a series of age-specific growth rates. Any of the growth rates' decompositions shown in Chapter 4 can be used to examine the components of this change. In other words, much could be learned from substituting the components of the growth rate in the covariance term in equation (6.4). For example, if we recall the decomposition in equation (4.33) for the age-specific growth rates

$$r(a,t) = r(0, t-a) - \varphi(a,t),$$

where $r(0, t-a)$ is the growth rate of the number of births $t-a$ years earlier, and $\varphi(a,t)$ is the cumulation of changes in the cohort age-specific mortality rates up to age a at time t. By substituting equation (4.33) in the decomposition of the crude death rate we obtain

$$\begin{aligned}\dot{\bar{\mu}} &= \bar{\dot{\mu}} + C(\mu, r)\\ &= \bar{\dot{\mu}} + C(\mu, r_0) - C(\mu, \varphi),\end{aligned}$$

where r_0 here denotes $r(0, t-a)$. The CDR can then be decomposed further by age and single age to obtain the contributions of the level-2 effects. These effects are (partly) based on the birth growth rates and the cohort changes in mortality for every age.

When applying the main equation (6.4) to decompose the change of the crude death rate it can be expressed as equation (6.7). The direct component captures mainly the change in the crude death rate due to changes in the age-specific death rates. The compositional component accounts for the change in the crude death rate due to changes in the population structure. By applying the single age decomposition we can see that the compositional component fluctuations are explained by the past and present demographic history within the age-specific growth rates.

7.4 Categorical Decomposition

As shown in the previous sections, equation (6.4) can be further decomposed by age due to the additive property of integrals. It is possible to measure the contribution of a specific age group to the total change over time, as well as the level-1 and level-2 effects of each age group.

Equation (2.1), for a demographic average, has two versions: one for the case of x being continuous and another for the discrete or categorical x. In both cases t was continuous. Age a is a particular case of x being continuous. From this variable we developed the age decomposition. In the same way, it is possible to decompose any category that x stands for. This section presents a categorical decomposition of the change over time in demographic variables.

Let i be the discrete case of x varying from $i = 0, ..., x_n$. Analogous to the property (7.1) of integrals any series of the categories i of the variable $v_i(t)$ can be separated into groups of categories

$$\sum_{i=0}^{x_n} v_i(t) = \sum_{i=0}^{x_1} v_i(t) + \sum_{i=x_2}^{x_3} v_i(t) + ... + \sum_{i=x_{n-1}}^{x_n} v_i(t). \tag{7.7}$$

From equation (6.4), in the case of categorical x, it follows that the direct and compositional components can be further decomposed as

$$\begin{aligned}
\dot{\bar{v}} &= \bar{\dot{v}} + C(v,\acute{w}) \\
&= \left[\frac{\sum_{i=0}^{x_1} \dot{v}_i(t) w_i(t)}{\sum_{i=0}^{x_1} w_i(t)}\right]\left[\frac{\sum_{i=0}^{x_1} w_i(t)}{\sum_{i=0}^{x_n} w_i(t)}\right] + ... \\
&\quad + \left[\frac{\sum_{i=x_{n-1}}^{x_n} \dot{v}_i(t) w_i(t)}{\sum_{i=x_{n-1}}^{x_n} w_i(t)}\right]\left[\frac{\sum_{i=x_{n-1}}^{x_n} w_i(t)}{\sum_{i=0}^{x_n} w_i(t)}\right] \\
&\quad + \left[C(v,\acute{w})\right]_0^{x_1} + ... + \left[C(v,\acute{w})\right]_{x_{n-1}}^{x_n}, \qquad (7.8)
\end{aligned}$$

where the covariance in the categories x_a to x_b is defined as

$$\left[C(v,\acute{w})\right]_{x_a}^{x_b} = \frac{\sum_{i=x_a}^{x_b}\left[v_i(t) - \bar{v}(t)\right]\left[\acute{w}_i(t) - \bar{\acute{w}}(t)\right] w_i(t)}{\sum_{i=0}^{x_n} w_i(t)}. \qquad (7.9)$$

The general categorical decomposition is then

$$\dot{\bar{v}} = \left[\bar{\dot{v}}\pi + C(v,\acute{w})\right]_0^{x_1} + ... + \left[\bar{\dot{v}}\pi + C(v,\acute{w})\right]_{x_{n-1}}^{x_n}, \qquad (7.10)$$

where $[\bar{\dot{v}}]_{x_a}^{x_b}$ is now the average change in the categories x_a to x_b and $[\pi]_{x_a}^{x_b}$ the proportion of the population in these categories $[\pi]_{x_a}^{x_b} = \frac{\sum_{i=x_a}^{x_b} w_i(t)}{\sum_{i=0}^{x_n} w_i(t)}$. Note that the term $[\pi]_{x_a}^{x_b}$ is necessary to obtain a direct component for the category x_a to x_b.

Vaupel (1992) demonstrated a decomposition of the change in the world's population growth rate, $\bar{r}(t)$. By applying equation (6.4) the decomposition of the change over time of $\bar{r}(t)$ is equal to the average change in growth rates of all the countries plus the variance in the growth rates

$$\dot{\bar{r}} = \bar{\dot{r}} + \sigma^2(r). \qquad (7.11)$$

As shown in Table 7.2, the growth of the world population started to decline around 1980. The pace of this decline was slowed by the variance in growth rates among the world's countries. The average change in country growth rates, $\bar{\dot{r}}$, was negative but the variance term changed this in the late 1970s, yielding an increase in the growth rate of 0.328 per 10,000.

To obtain the contribution of n regions of the world we apply the categorical decomposition demonstrated in equation (7.10) to the decomposition of the population growth rate (7.11). We then get

$$\begin{aligned}
\dot{\bar{r}} &= \left[\bar{\dot{r}}\pi + \sigma^2(r)\right]_1 + ... + \left[\bar{\dot{r}}\pi + \sigma^2(r)\right]_n \\
&= \sum_{i=1}^{n}\left[\bar{\dot{r}}\pi + \sigma^2(r)\right]_i, \qquad (7.12)
\end{aligned}$$

where in region i, the average change in the growth rate is denoted as $[\bar{\dot{r}}]_i$, the proportion of the population in the region is $[\pi]_i$, and $[\sigma^2(r)]_i$ is the variance in growth rates. Table 7.3 shows the regional decomposition of the world's population growth rate from 1975-1980 to 1978-1983, around 1979.

Table 7.2: Population growth rate of the world, $\bar{r}(t)$, and decomposition of the annual change over time around January 1, 1979 and January 1, 1982.

t	1979	1982
$\bar{r}(t-1.5)$	1.722 %	1.732 %
$\bar{r}(t+1.5)$	1.732 %	1.711 %
$\dot{\bar{r}}(t)$	0.328 *	-0.716 *
$\bar{\dot{r}}$	-0.459 *	-1.545 *
$\sigma^2(r)$	0.787 *	0.829 *
$\dot{\bar{r}} = \bar{\dot{r}} + \sigma^2(r)$	0.328 *	-0.716 *

Source: Author's calculations described in Chapter 9, based on the U.S. Census Bureau (2001). Note: * denotes per 10,000. Growth rates were calculated over 5-year intervals (1975-1980, 1978-1983, 1981-1986) in order to estimate growth rates for 1977.5, 1980.5 and 1983.5. Growth rates were estimated based on data for all the countries of the world for which data were available.

The row labeled "World" comprises the addition of the level-1 and level-2 effects over all regions. As seen in the last column, $[\dot{\bar{r}}]_i$, the American and European regions countered the increase in the population growth seen in the African and the Asian regions. For the American and European regions, a similar picture is obtained by looking at the average change in the population growth, $[\bar{\dot{r}}\pi]_i$, while Africa has on average an increasing population growth. As a result of these differences between the continents the direct effect on the world growth is -0.459. The Asian and European regions have variances of the order of 0.280, while very little variation is found in the American region. The total change is an increase in the growth rate of the world.

7.5 Cause of Death Decomposition

This section presents a generalization of the decomposition of the change over time in life expectancy in equation (6.35). Let $\mu_i(a,t)$ be the force of mortality from cause of death i at age a and time t. The chance of surviving cause i, i.e., not dying from cause i, is then $\ell_i(a,t)$. For competing, independent causes of death we have $\ell(a,t) = \ell_1(a,t)...\ell_n(a,t)$. Hence,

$$e^o(0,t) = \int_0^\omega \ell(a,t)da = \int_0^\omega \ell_1(a,t)...\ell_n(a,t)da, \tag{7.13}$$

and the change over time is

$$\begin{aligned} \dot{e}^o(0,t) &= \int_0^\omega \dot{\ell}_1(a,t)...\ell_n(a,t)da + ... + \int_0^\omega \ell_1(a,t)...\dot{\ell}_n(a,t)da \\ &= \int_0^\omega \acute{\ell}_1(a,t)\ell_1(a,t)...\ell_n(a,t)da + ... + \int_0^\omega \acute{\ell}_n(a,t)\ell_1(a,t)...\ell_n(a,t)da \\ &= \int_0^\omega \acute{\ell}_1(a,t)\ell(a,t)da + ... + \int_0^\omega \acute{\ell}_n(a,t)\ell(a,t)da. \end{aligned} \tag{7.14}$$

Table 7.3: Regional decomposition of the annual change over time in the world's population growth rate around 1979, per 10,000.

	Average change in growth rate	*Weight*	*Average weighted change in growth rate*	*Variance in growth rate*	*Change in average growth rate*
i	$[\bar{\dot{r}}]_i$	$[\pi]_i$	$[\bar{\dot{r}}\pi]_i$	$[\sigma^2(r)]_i$	$[\dot{\bar{r}}]_i$
Africa	2.328	0.105	0.244	0.157	0.401
America	-2.741	0.138	-0.378	0.070	-0.307
Asia	-0.062	0.577	-0.036	0.276	0.240
Europe	-1.609	0.180	-0.290	0.284	-0.006
World			-0.459	0.787	0.328

Source: Author's calculations described in Chapter 9, based on the U.S. Census Bureau (2001). Growth rates were estimated over intervals of 5 years (1975-1980, 1978-1983, 1981-1986) for all the countries of the world that had available data. The Asian region includes the Middle East and Oceania, and the American region includes North and South America.

Each of the terms in (7.14) can be reexpressed, by using the fact that $\ell_i(a,t) = e^{-\int_0^a \mu_i(x,t)dx}$ and the remaining life expectancy in equation (6.32), we obtain

$$\int_0^\omega \ell(a,t)\acute{\ell}_i(a,t)da = -\int_0^\omega \ell(a,t)\int_0^a \acute{\mu}_i(x,t)dxda$$
$$= -\int_0^\omega \acute{\mu}_i(a,t)\int_a^\omega \ell(x,t)dxda = -\int_0^\omega \acute{\mu}_i(a,t)\ell(a,t)e^o(a,t)da. \tag{7.15}$$

Thus

$$\dot{e}^o(0,t) = -\sum_{i=1}^n \int_0^\omega \acute{\mu}_i(a,t)\ell(a,t)e^o(a,t)da. \tag{7.16}$$

This equation is the continuous version of the discrete difference equation (4.7) presented by Pollard (1982, 1988). The change in expectation of life of a population between time t and $t+h$ in Pollard's formulation is:

$$e^o(0,t+h) - e^o(0,t) = \sum_{i=1}^n \int_0^\omega [\mu_i(a,t) - \mu_i(a,t+h)]\,\frac{\ell(a,t+h)e^o(a,t) + \ell(a,t)e^o(a,t+h)}{2}da. \tag{7.17}$$

Let $\rho_i(a,t)$ denote the pace of reduction of mortality from cause i, $\rho_i(a,t) = -\acute{\mu}_i(a,t)$. The proportion of deaths from cause i at age a and time t is $f_i(a,t) = \mu_i(a,t)\ell(a,t)$. It then follows from (7.16) that

$$\dot{e}^o(0,t) = \sum_{i=1}^n \int_0^\omega \rho_i(a,t)e^o(a,t)f_i(a,t)da. \tag{7.18}$$

Note how concise (and elegant) equation (7.18) is compared with (7.17).

Applying the decomposition of the average of a product (2.14) in equation (7.18), we obtain

$$\overline{\rho_i e^o} = \bar{\rho}_i(t) e_i^{\dagger}(t) + C_{f_i}(\rho_i, e^o), \tag{7.19}$$

which yields

$$\dot{e}^o(0,t) = \sum_{i=1}^{n} \left[\bar{\rho}_i(t) e_i^{\dagger}(t) + C_{f_i}(\rho_i, e^o) \right] F_i(t), \tag{7.20}$$

where

$$F_i(t) = \int_0^{\omega} f_i(a,t) da. \tag{7.21}$$

The average pace of reduction of mortality from cause i, $\bar{\rho}_i(t)$, is

$$\bar{\rho}_i(t) = \frac{\int_0^{\omega} \rho_i(a,t) f_i(a,t) da}{\int_0^{\omega} f_i(a,t) da}, \tag{7.22}$$

$e_i^{\dagger}(t)$ is the average number of life-years lost as a result of cause of death i,

$$e_i^{\dagger}(t) = \frac{\int_0^{\omega} e^o(a,t) f_i(a,t) da}{\int_0^{\omega} f_i(a,t) da}, \tag{7.23}$$

and the covariance is between the rate of improvement in mortality from cause of death i and the remaining life expectancy at various ages,

$$C_{f_i}(\rho_i, e^o) = \frac{\int_0^{\omega} \left[\rho_i(a,t) - \bar{\rho}_i(t)\right] \left[e^o(a,t) - e_i^{\dagger}(t) \right] f_i(a,t) da}{\int_0^{\omega} f_i(a,t) da}. \tag{7.24}$$

The lifetable distribution of deaths in Japan due to different causes of death in 1980 and 1990 is shown in Table 7.4. This is a distribution of causes of death for a lifetable population in which the proportion of people at each age is determined by lifetable probabilities of survival.

Table 7.5 and Figure 7.5 present the results of applying the decomposition formula in (7.20) to the Japanese data.

Over the decade from 1980 to 1990, Japanese life expectancy rose from 75.91 to 78.80 years, with an estimated annual increase of $\dot{e}^o(0, 1985) = 0.288$. Three fifths of this increase in life expectancy at birth can be attributed to a reduction in mortality due to cerebrovascular disease and heart disease. This is indicated in Table 7.5, where $\dot{e}_i^o(0)$ for heart disease is 0.044 and $\dot{e}_i^o(0)$ for cerebrovascular disease is 0.129. The sum, 0.173, accounts for 60% of the total change, $\dot{e}_i^o(0)$, of 0.288.

On average, death rates from malignant neoplasms and infectious diseases increased, yielding negative values of $\bar{\rho}_i$ and negative level-1 changes. As opposed to this, the level-2 changes for these causes of death had positive values, because improvements were made at younger ages with high remaining life expectancy. As a result of the balance between level-1 and level-2

Table 7.4: Causes of death distribution for Japan in 1980 and 1990.

Cause of death	1980 %	1990 %
Heart disease	21.4	23.7
Malignant neoplasm	18.5	21.6
Cerebrovascular disease	24.3	16.1
Infectious diseases	8.6	12.8
Violent deaths	4.6	4.5
Stomach, liver and kidney disorders	4.3	4.3
Senility without psychosis	7.4	5.0
Other causes	10.9	12.0
All causes of death	100.0	100.0

Source: Based on the Berkeley Mortality Database (2001). Heart disease includes hypertensive disease. Other causes of death are those denoted in the Berkeley Mortality Database (2001) as Other Causes, plus congenital malformations and diabetes mellitus. Infectious diseases include pneumonia and bronchitis.

changes, the final column of Table 7.5 shows only positive contributions for all the causes of death.

Similar cause of death decomposition can be applied in other mortality indexes. The crude death rate is the ratio of deaths over population at risk in a particular period. If the number of deaths are divided by cause of death then the CDR can be seen as CDR of different causes of death

$$\bar{\mu} = \bar{\mu}_1 + \bar{\mu}_2 + ... + \bar{\mu}_n, \tag{7.25}$$

where $\bar{\mu}_i$ is the crude death rate of cause of death i,

$$\bar{\mu}_i(t) = \frac{\int_0^\omega \mu_i(a,t)N(a,t)da}{\int_0^\omega N(a,t)da}. \tag{7.26}$$

Equation (6.7) for the decomposition of the CDR then becomes:

$$\dot{\bar{\mu}} = \sum_{i=1}^{n} \left[\bar{\dot{\mu}}_i + C(\mu_i, r)\right], \tag{7.27}$$

where $r(a,t)$ is the age-specific growth rate $r(a,t) = \acute{N}(a,t)$, and the terms $\bar{\dot{\mu}}_i$ and $C(\mu_i, r)$ correspond to the direct and compositional components for cause i.

Another cause of death decomposition is shown in the next chapter for the average age at death. Table 8.4 shows the average age at death in Japan and the decomposition obtained by distribution of deaths and causes of death, between 1980 and 1990.

Table 7.5: Cause of death decomposition for the annual change over time in life expectancy, for Japan from 1980 to 1990.

Cause of death	$\bar{\rho}_i(t)$ %	$e_i^\dagger(t)$	$\bar{\rho}_i(t)e_i^\dagger(t)$	$C_{f_i}(\rho_i, e^o)$	F_i %	$\dot{e}_i^o(0)$
Heart disease	2.058	8.333	0.172	0.022	22.543	0.044
Malignant neoplasm	-0.098	13.276	-0.013	0.088	20.042	0.015
Cerebrovascular disease	6.979	8.594	0.600	0.038	20.226	0.129
Infectious diseases	-0.703	7.942	-0.056	0.104	10.718	0.005
Violent deaths	1.608	23.384	0.376	0.058	4.516	0.020
Stomach, liver and kidney disorders	2.168	11.548	0.250	0.094	4.294	0.015
Senility without psychosis	9.379	4.294	0.403	0.040	6.200	0.027
Other causes	1.432	13.792	0.197	0.137	11.461	0.038
All causes of death	2.675	10.527	0.282	0.007	100.000	0.288

Source: Author's calculations described in Appendix B, based on the Berkeley Mortality Database (2001). The underlying data pertain to five-year age groups. Note that the values of $\bar{\rho}_i(t)$, $e_i^\dagger(t)$, $\bar{\rho}_i(t)e_i^\dagger(t)$ and $C_{f_i}(\rho_i, e^o)$ for all causes of death are complicated functions and not simple sums of the corresponding cause-specific values. The value of $\dot{e}^o(0)$ for all causes of death is slightly different than the sum of the values of $\dot{e}_i^o(0)$ because of approximation errors.

7.6 Conclusion

Age and categorical decompositions are useful extensions of the main decomposition formula (6.4). The contributions of the different ages (or categories) to the total change are complemented with the age and categorical decomposition of the level-1 and level-2 effects.

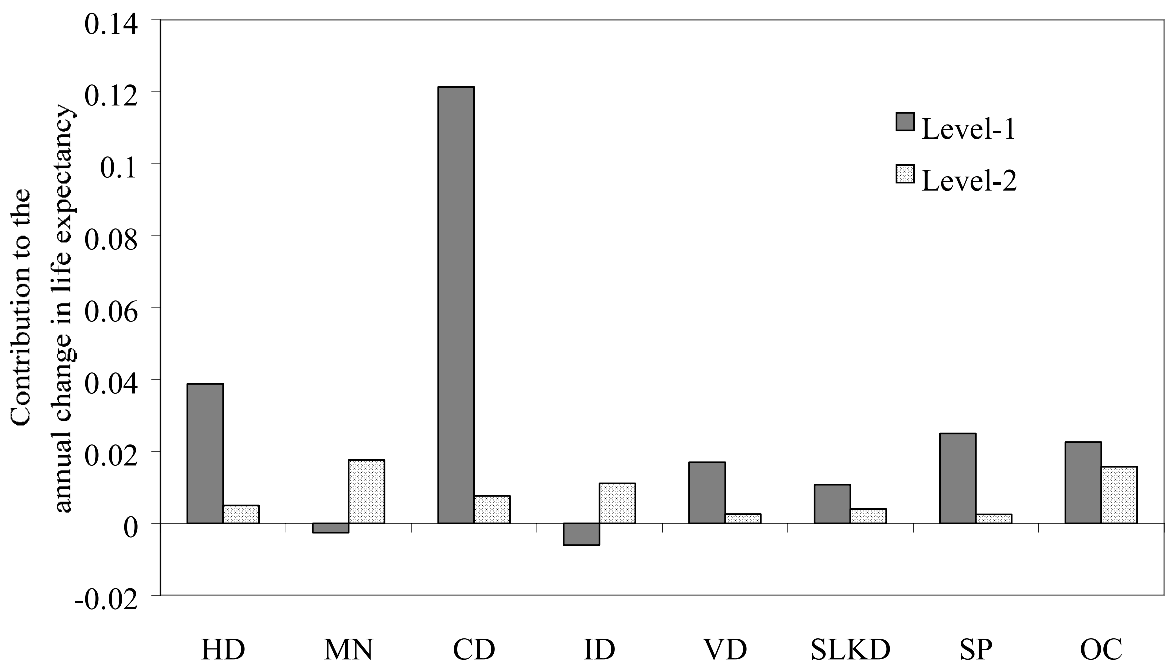
The detailed analysis of the decomposition components also helps the understanding of the changes in the different components. The applications of this chapter illustrate this. When studying the change over time in the CDR, we learned that the past and present demographic history, accounted in the age-specific growth rates, explain the compositional component fluctuations.

In the application of the categorical decomposition, we saw that the total change in the world's population growth rate is based on the changes experienced in the different regions of the world. The regional decomposition allows a detailed analysis of the components responsible for the change in various regions.

As a last decomposition in this chapter a cause of death decomposition is introduced. The change over time in life expectancy is separated into the contributions of the different causes of death. The illustration shows the change over time in the Japanese life expectancy. The direct component comprising cerebrovascular disease and heart disease explained more than half of the total change in life expectancy.

Alternatively it could be possible to combine the cause of death decomposition and the age decomposition to explain the change over time in mortality measures by both categories.

Figure 7.5: Cause of death decomposition of the annual change over time in life expectancy, from 1980 to 1990 for Japan.



Note: The abbreviations correspond to: HD-Heart disease; MN- Malignant neoplasm; CD-Cerebrovascular disease; ID-Infectious diseases; VD-Violent deaths; SLKD-Stomach, liver and kidney disorders; SP-Senility without psychosis; OC-Other causes.