

**Appendixes to:  
Estimating Real Production and Expenditures Across Nations:  
A Proposal for Improving the Penn World Tables**

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**Appendix A: Proof of Theorem**

Denoting nominal GDP<sub>j</sub> by G<sub>j</sub> and real GDP<sub>j</sub><sup>o</sup> by R<sub>j</sub><sup>o</sup>, j = 1, ..., C, we can substitute

PPP<sub>j</sub><sup>o</sup> = R<sub>j</sub><sup>o</sup> / G<sub>j</sub> in (12)- (14) to obtain the reference prices,

$$\pi_i^o = \frac{\sum_{j=1}^C R_j^o \theta_{ij}^q}{\sum_{j=1}^C q_{ij}}, \quad i = 1, \dots, M, \quad (A1)$$

$$\pi_i^x = \frac{\sum_{j=1}^C R_j^o \theta_{ij}^x}{\sum_{j=1}^C x_{ij}}, \quad \pi_i^m = \frac{\sum_{j=1}^C R_j^o \theta_{ij}^m}{\sum_{j=1}^C m_{ij}}, \quad i = M_0 + 1, \dots, M + N, \quad (A2)$$

where we are making use of the definitions of budget shares in (16). Also using the market shares in (17), real GDP<sub>j</sub><sup>o</sup> can be computed from (11) as,

$$R_k^o = \sum_{j=1}^C \sum_{i=1}^M R_j^o \theta_{ij}^q \mu_{ik}^q + \sum_{j=1}^C \sum_{i=M_0+1}^{M+N} R_j^o (\theta_{ij}^x \mu_{ik}^x - \theta_{ij}^m \mu_{ik}^m) = \sum_{j=1}^C R_j^o (\theta_j^q \mu_k^q + \theta_j^x \mu_k^x - \theta_j^m \mu_k^m). \quad (A3)$$

Define w<sub>jk</sub> as the last term in parentheses in (A3), W = [w<sub>jk</sub>] as the corresponding CxC matrix, and R<sup>o</sup> as the row vector (R<sub>1</sub><sup>o</sup>, ..., R<sub>C</sub><sup>o</sup>). Then (A3) can be re-written as R<sup>o</sup>W = R<sup>o</sup>, so that the R<sup>o</sup> is a row eigenvector of the matrix W.

It is readily checked that each row of W sums to unity. Since W is strictly positive by Assumption 2, from the Frobenius theorem it has a positive eigenvalue that lies in-between the minimum and maximum of its row sums, and the associated row or column eigenvector is strictly positive. Since the row sums are all unity, then the Frobenius eigenvalue also equals one, and R<sup>o</sup> is the strictly positive row eigenvector corresponding to that eigenvalue. Using this in (12)-(18) we obtain strictly positive solutions for PPP<sub>j</sub><sup>o</sup> and the reference prices. QED

## Appendix B: Real GDP in PWT and this paper

The comparisons for 1996 made in this paper can be extended through time, using several possible methods. First, if we just repeated the calculation that we have made for 1996 in other years, then we would obtain a cross-country dataset of real GDP<sup>e</sup> and real GDP<sup>o</sup> in each year. Since the reference prices used to evaluate either concept of real GDP would be changing each year, such series are called “current price” real GDP. For time-series studies, however, it is desirable to have measures of real GDP that keep prices constant over time, in what is called “constant price” real GDP.

A fundamental principle of PWT has been that constant-price real GDP should be obtained by extrapolating from the benchmark year using each country’s *national accounts data on the growth rates of components* of GDP.<sup>1</sup> Specifically, PWT takes the benchmark year measure of a real component (C, I, G, X or M), and extrapolates it over time using the national-accounts real growth rate of that component. The extrapolated components are then summed together to obtain constant-price real GDP each year. Below we show how the growth rate of real GDP in PWT differs both from the growth rates in real GDP<sup>e</sup> and real GDP<sup>o</sup> as proposed in this paper. In practice, however, the existing measure of real GDP growth in the PWT is much closer to the growth of real GDP<sup>o</sup> than to the growth of real GDP<sup>e</sup>. *So even though real GDP for a benchmark year in the PWT should be interpreted as an expenditure-based measure, its growth rate is closer to an output-based measure.* That is the main finding of this Appendix.

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<sup>1</sup> This principle ensures that the growth rates of real GDP computed from PWT will not change very much as the benchmark years is updated, which has been viewed as an essential feature of the database. Note however, that this method leads to a growth rate of constant-price real GDP in PWT that is not identical to the growth rate of real GDP in national accounts, even though the growth rates of the components are the same. The reason for the discrepancy is that the benchmark year components of GDP become “weights” in the calculation: the growth rate of real GDP in PWT is a weighted average of the growth rates of the components, but the weights differ from those in the national accounts, as shown below.

### *Real GDP in PWT*

The starting point for all real GDP measures is the benchmark year calculation, which was 1996 for PWT 6.1 and 2000 for PWT 6.2. In the paper we use 1996 as the benchmark year, so let us continue with that convention for this Appendix. We will contrast the real GDP<sup>o</sup> and real GDP<sup>e</sup>, as proposed in the paper, with a commonly used measure of GDP from PWT, the variable RGDPPL, which is a Laspeyres, per-capita measure of real GDP.<sup>2</sup> Multiplying RGDPPL by each country's population, let us call the resulting series RGDPPL<sup>pwt</sup>. In the benchmark year, RGDPPL<sup>pwt</sup> for the US equals nominal GDP of the United States, by choice of numeraire. Let us adopt the same numeraire for RGDP<sup>e</sup>, equation (10) in the paper, and RGDP<sup>o</sup>, equation (11) in the paper, so they both equal nominal GDP for the United States in 1996,<sup>3</sup> and also equal RGDPPL<sup>pwt</sup> for 1996.

To extrapolate real GDP and its components from the benchmark year, we rely on the *nominal national accounts* data for consumption, investment, government expenditures, exports and imports, which are denoted by  $C_{jt}^{nom}$ ,  $I_{jt}^{nom}$ ,  $G_{jt}^{nom}$ ,  $X_{jt}^{nom}$  and  $M_{jt}^{nom}$ . The nominal data are expressed in national currencies and these are converted to “real” terms by dividing by their respective PPPs in the benchmark year, denoted by  $PPP_{j96}^c$ ,  $PPP_{j96}^i$ , and  $PPP_{j96}^g$ .<sup>4</sup> In the paper, we explain how the *overall PPP for domestic absorption*,  $PPP_{j96}^e$ , is constructed, and  $PPP_{j96}^c$ ,

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<sup>2</sup> PWT also includes two other measures of real GDP, based on chaining (RGDPCH) and adjusting for terms of trade (RGDPTT). RGDPCH is the most commonly used variable in PWT for measured real growth, but because it is easier to compare our new measures with RGDPPL, we shall not discuss RGDPCH or RGDPTT any further.

<sup>3</sup> In contrast, in the paper we use the normalization that “world” real GDP<sup>e</sup> equals “world” real GDP<sup>o</sup> in 1996, which also equal “world” nominal GDP in US\$ at 1996 nominal exchange rates, for the countries in the sample.

<sup>4</sup> Note that in PWT, “real” refers to measuring GDP or its components in common, U.S. dollar reference prices across countries. For convenience, in the remainder of this appendix we will use the term “real” to denote either: (i) GDP and its components that have constant 1996 reference prices; or (ii) GDP and its components taken from the national accounts of each country that have constant prices over time.

$PPP_{j96}^i$ , and  $PPP_{j96}^g$  simply reflect the corresponding PPPs constructed over the individual components of consumption, investment and government expenditures. That is, if the first  $M_1$  final goods are for private consumption, the next  $M_2$  for investment, and the final  $M - M_1 - M_2$  are for the government, then the PPPs are:

$$PPP_{j96}^c \equiv \left( \frac{\sum_{i=1}^{M_1} P_{ij} Q_{ij}}{\sum_{i=1}^{M_1} \pi_i^e Q_{ij}} \right), \quad PPP_{j96}^i \equiv \left( \frac{\sum_{i=M_1+1}^{M_2} P_{ij} Q_{ij}}{\sum_{i=M_1+1}^{M_2} \pi_i^e Q_{ij}} \right), \quad PPP_{j96}^g \equiv \left( \frac{\sum_{i=M_2+1}^M P_{ij} Q_{ij}}{\sum_{i=M_2+1}^M \pi_i^e Q_{ij}} \right), \quad (B1)$$

where the numerators in (B1) are equivalently written as  $C_{j96}^{nom}$ ,  $I_{j96}^{nom}$ , and  $G_{j96}^{nom}$ . Notice that the reference prices used in the denominators in (B1) are obtained from the expenditure-based GK system for the benchmark year 1996. In addition, the overall PPP for domestic absorption is obtained by summing over all  $M$  final goods:

$$PPP_{j96}^e \equiv \left( \frac{\sum_{i=1}^M P_{ij} Q_{ij}}{\sum_{i=1}^M \pi_i^e Q_{ij}} \right) = \left( \frac{C_{j96}^{nom} + I_{j96}^{nom} + G_{j96}^{nom}}{\sum_{i=1}^M \pi_i^e Q_{ij}} \right). \quad (B1')$$

Extending the benchmark estimates over time relies on the *real national accounts data* for the components of GDP, which are denoted  $C_{jt}^{rna}$ ,  $I_{jt}^{rna}$ ,  $G_{jt}^{rna}$ ,  $X_{jt}^{rna}$ ,  $M_{jt}^{rna}$ . These national accounts data are expressed as series in constant national prices, so that the ratios such as  $C_{jt}^{rna} / C_{jt-1}^{rna}$ , etc., give the growth rates of each real component of GDP.

With these data definitions, we can describe the existing method in PWT to obtain constant-price real GDP each year. First, we extrapolate the benchmark data for each component:

$$RC_{jt}^{pwt} \equiv \left( \frac{C_{j96}^{nom}}{PPP_{j96}^c} \right) \left( \frac{C_{jt}^{rna}}{C_{j96}^{rna}} \right), \quad RI_{jt}^{pwt} \equiv \left( \frac{I_{j96}^{nom}}{PPP_{j96}^i} \right) \left( \frac{I_{jt}^{rna}}{I_{j96}^{rna}} \right), \quad RG_{jt}^{pwt} \equiv \left( \frac{G_{j96}^{nom}}{PPP_{j96}^g} \right) \left( \frac{G_{jt}^{rna}}{G_{j96}^{rna}} \right), \quad (B2a)$$

while for exports and imports:

$$RX_{jt}^{pwt} \equiv \left( \frac{X_{j96}^{nom}}{PPP_{j96}^e} \right) \left( \frac{X_{jt}^{rna}}{X_{j96}^{rna}} \right), \text{ and } RM_{jt}^{pwt} \equiv \left( \frac{M_{j96}^{nom}}{PPP_{j96}^e} \right) \left( \frac{M_{jt}^{rna}}{M_{j96}^{rna}} \right), \quad (B2b)$$

In (B2a) we deflate the nominal national accounts data by the respective PPP for each components of domestic absorption, and then multiply by the growth rates of the real national accounts series, to derive constant-price consumption, investment and government expenditures. In (B2b) we apply the *overall PPP for domestic absorption* to the nominal export and imports data, and multiply by the growth rate of the real national accounts series, to obtain real exports and imports. Then the Laspeyres measure of real GDP in PWT in year  $t$  is defined as the sum of its components:

$$RGDPL_{jt}^{pwt} \equiv RC_{jt}^{pwt} + RG_{jt}^{pwt} + RI_{jt}^{pwt} + RX_{jt}^{pwt} - RM_{jt}^{pwt}. \quad (B3)$$

If we compute the percentage growth rates of Laspeyres real GDP, relative to the benchmark year, we obtain:

$$\begin{aligned} \frac{RGDPL_{jt}^{pwt}}{RGDPL_{j96}^{pwt}} - 1 &= KC_{j96}^{pwt} \left( \frac{RC_{jt}^{pwt}}{RC_{j96}^{pwt}} - 1 \right) + KG_{j96}^{pwt} \left( \frac{RG_{jt}^{pwt}}{RG_{j96}^{pwt}} - 1 \right) + KI_{j96}^{pwt} \left( \frac{RI_{jt}^{pwt}}{RI_{j96}^{pwt}} - 1 \right) \\ &+ KX_{j96}^{pwt} \left( \frac{RX_{jt}^{pwt}}{RX_{j96}^{pwt}} - 1 \right) - KM_{j96}^{pwt} \left( \frac{RM_{jt}^{pwt}}{RM_{j96}^{pwt}} - 1 \right). \end{aligned} \quad (B4)$$

where:

$$KC_{j96}^{pwt} \equiv RC_{j96}^{pwt} / RGDPL_{j96}^{pwt} = (C_{j96}^{nom} / PPP_{j96}^c) / RGDPL_{j96}^{pwt} \quad (B5a)$$

is the *share of consumption in real RGDPL*<sup>pwt</sup>, and likewise for shares  $KG_{j96}^{pwt}$  and  $KI_{j96}^{pwt}$

appearing in (B4), while:

$$KX_{j96}^{pwt} \equiv RX_{j96}^{pwt} / RGDPL_{j96}^{pwt} = (X_{j96}^{nom} / PPP_{j96}^e) / RGDPL_{j96}^{pwt}, \quad (B5b)$$

$$KM_{j96}^{pwt} \equiv RM_{j96}^{pwt} / RGDPL_{j96}^{pwt} = (M_{j96}^{nom} / PPP_{j96}^e) / RGDPL_{j96}^{pwt}, \quad (B5c)$$

are the *shares of exports and imports in real RGDP*<sup>pwt</sup>. The growth rate of real GDP in (B4) equals a *weighted average* of the growth weights of the components of GDP, where the weights in (B5) reflect the shares of each component in the benchmark year.

### ***New Definition of Real GDP on the Output-Side***

Now we turn to the new definitions of real GDP proposed in this paper,  $RGDP^e$  and  $RGDP^o$ . Starting with the output-based measure, we take as given the PPPs for consumption, investment and government expenditures in 1996, shown by (B1), and treated these as the “prices” of those three aggregate series for each country. That is, we take  $M = 3$  as the number of non-traded final goods, so that real consumption, investment and government expenditures are unchanged from (B2). Then as described in the paper, we compute the PPPs for exports and imports as in (19):

$$PPP_j^x \equiv \left( \frac{\sum_{i=4}^{3+N} p_{ij}^x X_{ij}}{\sum_{i=4}^{3+N} \pi_i^x X_{ij}} \right), \quad PPP_j^m \equiv \left( \frac{\sum_{i=4}^{3+N} p_{ij}^m m_{ij}}{\sum_{i=4}^{3+N} \pi_i^m m_{ij}} \right). \quad (19)$$

Using these PPP's, we re-compute the real exports and imports as:

$$RX_{jt}^o \equiv \left( \frac{X_{j96}^{nom}}{PPP_{j96}^x} \right) \left( \frac{X_{jt}^{rna}}{X_{j96}^{rna}} \right), \quad \text{and} \quad RM_{jt}^o \equiv \left( \frac{M_{j96}^{nom}}{PPP_{j96}^m} \right) \left( \frac{M_{jt}^{rna}}{M_{j96}^{rna}} \right). \quad (B6)$$

Then real GDP on the output-side can be defined as:

$$RGDP_{jt}^o \equiv RC_{jt}^{pwt} + RG_{jt}^{pwt} + RI_{jt}^{pwt} + RX_{jt}^o - RM_{jt}^o. \quad (B7)$$

Notice that real consumption, investment and government expenditures have not changed in this expression from that used in PWT, as in (B4), because we use the PWT PPP's for those GDP components within our calculations of output-based GDP. What has changed between the PWT calculation in (B4) and that in (B7) is the measure of real export and imports in 1996: in PWT,

the PPP for *domestic absorption* is used to convert nominal exports and imports into real values, whereas in the output-base system we have used the PPP's for exports and imports computed from the unit-values in trade, as in (B6).

If we compute the percentage growth rates of the output-based real GDP, relative to the benchmark year, we obtain:

$$\begin{aligned} \frac{\text{RGDP}_{jt}^o}{\text{RGDP}_{j96}^o} - 1 &= \text{KC}_{j96}^o \left( \frac{\text{RC}_{jt}^{\text{pwt}}}{\text{RC}_{j96}^{\text{pwt}}} - 1 \right) + \text{KG}_{j96}^o \left( \frac{\text{RG}_{jt}^{\text{pwt}}}{\text{RG}_{j96}^{\text{pwt}}} - 1 \right) + \text{KI}_{j96}^o \left( \frac{\text{RI}_{jt}^{\text{pwt}}}{\text{RI}_{j96}^{\text{pwt}}} - 1 \right) \\ &+ \text{KX}_{j96}^o \left( \frac{\text{RX}_{jt}^o}{\text{RX}_{j96}^o} - 1 \right) - \text{KM}_{j96}^o \left( \frac{\text{RM}_{jt}^o}{\text{RM}_{j96}^o} - 1 \right), \end{aligned} \quad (\text{B8})$$

where:

$$\text{KC}_{j96}^o \equiv \text{RC}_{j96}^o / \text{RGDP}_{j96}^o = (\text{C}_{j96}^{\text{nom}} / \text{PPP}_{j96}^c) / \text{RGDP}_{j96}^o, \quad (\text{B9a})$$

is the *share of consumption in real GDP<sup>o</sup>*, and likewise for the shares  $\text{KG}_{j96}^o$  and  $\text{KI}_{j96}^o$

appearing in (B8), while:

$$\text{KX}_{j96}^o \equiv \text{RX}_{j96}^o / \text{RGDP}_{j96}^o = (\text{X}_{j96}^{\text{nom}} / \text{PPP}_{j96}^x) / \text{RGDP}_{j96}^o, \quad (\text{B9b})$$

$$\text{KM}_{j96}^o \equiv \text{RM}_{j96}^o / \text{RGDP}_{j96}^o = (\text{M}_{j96}^{\text{nom}} / \text{PPP}_{j96}^m) / \text{RGDP}_{j96}^o, \quad (\text{B9c})$$

are the *shares of exports and imports in real GDP<sup>o</sup>*.

If we compare the growth rates of real GDP in PWT and the output-based system, or (B4) and (B8), we notice that the only difference is in the *shares used* within these expression. That is, the *growth rates* of real exports and imports in PWT, which are  $\text{RX}_{jt}^{\text{pwt}} / \text{RX}_{j96}^{\text{pwt}}$  and  $\text{RM}_{jt}^{\text{pwt}} / \text{RM}_{j96}^{\text{pwt}}$ , equal that for exports and imports in the output-based system, which are  $\text{RX}_{jt}^o / \text{RX}_{j96}^o$  and  $\text{RM}_{jt}^o / \text{RM}_{j96}^o$ , as can be seen by comparing (B2b) and (B6). Real exports and imports differ *in their levels* in PWT and the output-based system, but that difference in the

benchmark year is maintained in all other years through extrapolating at the same rates (i.e. the national accounts growth rates of real exports and imports) to all other years. Likewise, the growth rates of real consumption, investment and government expenditures in PWT equal that for real consumption, investment and government expenditures in the output-based system. But the weights used to obtain the growth rates in (B5) and (B8) differ. Thus the difference between real GDP from PWT and from our output-based measure is given by:

$$\begin{aligned} \left( \frac{\text{RGDP}_{jt}^{\text{pwt}}}{\text{RGDP}_{j96}^{\text{pwt}}} - \frac{\text{RGDP}_{jt}^{\text{o}}}{\text{RGDP}_{j96}^{\text{o}}} \right) &= (\text{KC}_{j96}^{\text{pwt}} - \text{KC}_{j96}^{\text{o}}) \left( \frac{\text{RC}_{jt}^{\text{pwt}}}{\text{RC}_{j96}^{\text{pwt}}} \right) + (\text{KG}_{j96}^{\text{pwt}} - \text{KG}_{j96}^{\text{o}}) \left( \frac{\text{RG}_{jt}^{\text{pwt}}}{\text{RG}_{j96}^{\text{pwt}}} \right) \\ &+ (\text{KI}_{j96}^{\text{pwt}} - \text{KI}_{j96}^{\text{o}}) \left( \frac{\text{RI}_{jt}^{\text{pwt}}}{\text{RI}_{j96}^{\text{pwt}}} \right) + (\text{KX}_{j96}^{\text{pwt}} - \text{KX}_{j96}^{\text{o}}) \left( \frac{\text{RX}_{jt}^{\text{pwt}}}{\text{RX}_{j96}^{\text{pwt}}} \right) - (\text{KM}_{j96}^{\text{pwt}} - \text{KM}_{j96}^{\text{o}}) \left( \frac{\text{RM}_{jt}^{\text{pwt}}}{\text{RM}_{j96}^{\text{pwt}}} \right). \quad (\text{B10}) \end{aligned}$$

Expression (B10) shows that the difference in growth rates between real GDP in PWT and the output-based system is due entirely to differences in the shares used in expressions (B4) and (B8). If these shares are close, then so are the growth rates.

### ***New Definition of Real GDP on the Expenditure-Side***

Now turn to our proposed measure of real GDP from the expenditure side (real GDP<sup>e</sup>). As in the PWT measure of real GDP, exports and imports are deflated by the domestic absorption PPP given in (B2), but unlike real GDP from PWT or the output-based measure, extrapolation of the benchmark exports and imports is not done by their respective real growth rates from the national accounts. Instead growth rates of exports and imports are derived by deflating with national prices of *domestic absorption* rather than prices of exports and imports. Denoting the national price index for domestic absorption by  $P_{jt}^a$ , and the national price indexes for exports and imports by  $P_{jt}^x$  and  $P_{jt}^m$ , then real exports and imports from the expenditure-side in year t are given by:



$$RX_{jt}^e \equiv \left( \frac{X_{j96}^{nom}}{PPP_{j96}^x} \right) \left( \frac{X_{jt}^{nom} / P_{jt}^a}{X_{j96}^{nom} / P_{j96}^a} \right) = RX_{jt}^{pwt} \left( \frac{P_{jt}^x / P_{jt}^a}{P_{j96}^x / P_{j96}^a} \right), \quad (B11a)$$

and

$$RM_{jt}^e \equiv \left( \frac{M_{j96}^{nom}}{PPP_{j96}^m} \right) \left( \frac{M_{jt}^{nom} / P_{jt}^a}{M_{j96}^{nom} / P_{j96}^a} \right) = RM_{jt}^{pwt} \left( \frac{P_{jt}^m / P_{jt}^a}{P_{j96}^m / P_{j96}^a} \right) \quad (B11b)$$

where the second equalities in (B11a) and (B11b) are obtained by noting that  $X_{jt}^{rna} = X_{jt}^{nom} / P_{jt}^x$  and  $M_{jt}^{rna} = M_{jt}^{nom} / P_{jt}^m$ , while using (B2b).

Then real GDP on the expenditure-side is defined as:

$$RGDP_{jt}^e \equiv RC_{jt}^{pwt} + RG_{jt}^{pwt} + RI_{jt}^{pwt} + RX_{jt}^e - RM_{jt}^e. \quad (B12)$$

Notice that real consumption, investment and government expenditures have not changed in this expression from that used in RGDP from PWT or output-based GDP. What has changed is the measure of real export and imports: in both RGDP and GDP<sup>O</sup>, growth rates of exports and imports are derived by deflating exports and imports by their respective national accounts deflator. But in this case, the deflator for *domestic absorption* is used to convert nominal exports and imports into real values.

If we compute the percentage growth rates of the expenditure-based real GDP, relative to the benchmark year, we obtain an expression similar to the growth of the Laspeyres real GDP in PWT in (B4), except that the growth rates of exports and imports differ:

$$\begin{aligned} \frac{RGDP_{jt}^e}{RGDP_{j96}^e} - 1 &= KC_{j96}^{pwt} \left( \frac{RC_{jt}^{pwt}}{RC_{j96}^{pwt}} - 1 \right) + KG_{j96}^{pwt} \left( \frac{RG_{jt}^{pwt}}{RG_{j96}^{pwt}} - 1 \right) + KI_{j96}^{pwt} \left( \frac{RI_{jt}^{pwt}}{RI_{j96}^{pwt}} - 1 \right) \\ &+ KX_{j96}^{pwt} \left( \frac{RX_{jt}^e}{RX_{j96}^e} - 1 \right) - KM_{j96}^{pwt} \left( \frac{RM_{jt}^e}{RM_{j96}^e} - 1 \right), \end{aligned} \quad (B13)$$

As discussed in the main text, the level of real GDP in 1996, as measured by RGDP<sup>L</sup> in PWT, is equal to the level in the expenditure-approach. Hence the weights appearing in (B13) are not different from the weights in (B4). If we compare the growth rates of real GDP in PWT and the expenditure-based system, which is (B5) and (B13), we notice that there are only differences in the growth rates of real exports and imports. Thus the difference in growth rates between real GDP from PWT and from the expenditure side is given by:

$$\frac{\text{RGDP}_{jt}^{\text{PWT}}}{\text{RGDP}_{j96}^{\text{PWT}}} - \frac{\text{RGDP}_{jt}^{\text{e}}}{\text{RGDP}_{j96}^{\text{e}}} = \text{KX}_{j96}^{\text{PWT}} \left( \frac{\text{RX}_{jt}^{\text{PWT}}}{\text{RX}_{j96}^{\text{PWT}}} \right) \left( 1 - \frac{\text{P}_{jt}^{\text{x}} / \text{P}_{jt}^{\text{a}}}{\text{P}_{j96}^{\text{x}} / \text{P}_{j96}^{\text{a}}} \right) - \text{KM}_{j96}^{\text{PWT}} \left( \frac{\text{RM}_{jt}^{\text{PWT}}}{\text{RM}_{j96}^{\text{PWT}}} \right) \left( 1 - \frac{\text{P}_{jt}^{\text{m}} / \text{P}_{jt}^{\text{a}}}{\text{P}_{j96}^{\text{m}} / \text{P}_{j96}^{\text{a}}} \right). \quad (\text{B14})$$

(B14) shows that the difference between the growth rates of the expenditure-based real GDP and the Laspeyres real GDP in PWT will depend on the relative movements of domestic, export and import prices. If all grow at the same rate, real GDP will be the same in the two approaches. But when the growth rates of these price indexes differ, then the growth rates of the respective real GDP measures will also diverge.

In the Excel-file of the data Appendix, we report the levels and logarithmic growth rates of real GDP<sup>e</sup>, real GDP<sup>o</sup> and real GDP<sup>PWT</sup> computed as described above. In practice, we find that the existing measure of constant-price real GDP growth in the PWT is much closer to the growth of real GDP<sup>o</sup> than to the growth of real GDP<sup>e</sup>. The correlation of growth rates of the Laspeyres real GDP from PWT with growth in real GDP<sup>e</sup> is 0.647, while it is 0.867 with GDP<sup>o</sup>. So even though real GDP for a benchmark year in the PWT should be interpreted as an expenditure-based measure, its *growth rate* is closer to an output-based measure.

### Appendix C: A New Measure of Real Openness

PWT provides also measures of openness to international trade. The one which is most frequently used is called nominal, or “current-price,” openness (OPENC), which is the ratio of nominal exports plus imports to nominal GDP in each year:

$$\text{OPENC}_{jt} = (\text{X}_{jt}^{\text{nom}} + \text{M}_{jt}^{\text{nom}}) / \text{GDP}_{jt}^{\text{nom}}. \quad (\text{C1})$$

PWT also provides a measure of “constant price” openness (OPENK) based on the ratio of real exports and imports over real GDP. In the benchmark year, these two are equal as real exports and imports in PWT, as defined by (B2b), use the *PPP of domestic absorption* to convert the nominal to real values, just as real GDP is obtained using the PPP of domestic absorption to convert the nominal to real values. So  $\text{OPENC} = \text{OPENK}$  in the benchmark year. However, it can be argued that for many applications of the openness indicator, it is preferable to deflate exports and imports with specific export and import PPPs to obtain “real openness”:

$$\text{OPENR}_{j96} \equiv \frac{(\text{X}_{j96} / \text{PPP}_{j96}^{\text{X}}) + (\text{M}_{j96} / \text{PPP}_{j96}^{\text{M}})}{\text{Real GDP}_{j96}^{\text{O}}}. \quad (\text{C2})$$

Real openness in (C2) differs from constant-price and nominal openness in PWT, even in the benchmark year, because the PPPs for exports and imports are used to deflate rather than the PPP for domestic absorption. In the data Appendix, we report series of real openness for 1996, and also extend this series over time so that it can be easily merged with other data in PWT 6.2.

### Appendix D:. Extending the Results of Rigobon and Rodrik (2005)

We conclude this Appendix by showing how our proposed real openness measure in (C2), or the terms of trade, can influence the results in studies of trade and income. We choose just one study to re-estimate, that of Rigobon and Rodrik (2005). To avoid sensitivity to the choice of instruments, these authors instead estimate the relationship between trade, income and

other variables using “identification through heteroskedasticity” (Rigobon, 2003). This technique requires having sub-samples whose second moments differ. Rigobon and Rodrik split their sample along two lines: by former colonial status, following work on the role of institutions (Acemoglu, Johnson and Robinson, 2001); and by geography, following the suggestion of Diamond (1997) that it is easier for seed varieties and agricultural technologies to migrate on an east-west rather than a north-south axis. The geography split therefore separates those countries on continents that are aligned along an east-west axis (Eurasian countries) from those on continents aligned along a north-south axis (Africa and the Americas).

We have replicated the results of Rigobon and Rodrik (2005), who find that nominal openness has a negative and significant impact on real income under both splits of the sample. In Table A1 we show the results are changed by using real openness in 1996 rather than nominal openness. For the colony split, real openness become positive but is insignificant in its impact on real income. For the geography split, real openness has a positive and significant impact on real income in the lower-portion of Table 3. Furthermore, its positive impact on the rule of law increases by more than four times as compared to Rigobon and Rodrik, and the rule of law has a positive and highly significant impact on real income, so that real openness plays both a positive direct and indirect role. When we use the terms of trade in 1996 rather than real openness, as shown in Table A2, then we find that the terms of trade has a positive and significant direct impact on real income under either split of the sample. The terms of trade also has a positive impact on the rule of law in the geography split, and therefore a positive direct and indirect impact on income in that case.<sup>5</sup>

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<sup>5</sup> Surprisingly, the rule of law has a negative and significant impact on income under the colonial status split of the sample in Table A2 (whereas it is insignificant in that split using either real openness or nominal openness).

**Table A1: Results from Using Real Openness**

	Income	Democracy	Rule of law	Real Openness	Distance To Equator	Area	Population
<b><i>Using colonial status to split sample of countries</i></b>							
Income		0.36 (2.86)	0.20 (1.55)	0.03 (0.34)	0.25 (2.62)	-0.17 (2.52)	0.15 (3.05)
Democracy	-0.16 (0.48)		-0.08 (0.38)	0.23 (1.33)	0.58 (2.88)	-0.33 (2.30)	0.23 (1.91)
Rule of Law	-0.17 (0.96)	0.36 (3.87)		0.94 (6.55)	0.31 (2.29)	0.14 (2.09)	0.34 (4.34)
Openness	1.05 (4.41)	0.05 (0.28)	-1.08 (3.25)		0.07 (0.58)	-0.10 (1.13)	-0.31 (3.98)
<b><i>Using geography to split sample of countries</i></b>							
Income		-0.08 (1.08)	0.79 (10.7)	0.18 (3.11)	0.03 (0.65)	-0.01 (0.09)	0.18 (3.24)
Democracy	0.82 (4.90)		0.30 (2.32)	-0.46 (2.90)	-0.10 (0.85)	-0.001 (0.02)	-0.27 (2.93)
Rule of Law	-0.43 (3.21)	-0.32 (2.45)		0.73 (6.04)	0.95 (8.44)	-0.06 (1.16)	0.31 (3.88)
Openness	-0.13 (1.27)	-0.02 (0.28)	0.15 (1.65)		0.01 (0.93)	-0.17 (2.16)	-0.36 (4.88)

**Table A2: Results from Using Terms of Trade**

	Income	Democracy	Rule of law	Terms Of Trade	Distance To Equator	Area	Population
<b><i>Using colonial status to split sample of countries</i></b>							
Income		1.03 (3.36)	-0.94 (2.29)	0.33 (4.14)	0.74 (6.43)	-0.32 (4.96)	0.29 (5.31)
Democracy	0.35 (1.01)		-1.11 (3.32)	-0.10 (0.98)	0.94 (5.55)	-0.62 (5.36)	0.18 (2.19)
Rule of Law	1.08 (4.34)	0.13 (1.21)		0.12 (1.16)	-0.22 (1.03)	0.33 (3.62)	-0.23 (3.68)
Openness	0.70 (3.61)	0.16 (1.30)	-0.24 (1.20)		-0.31 (2.24)	-0.04 (0.46)	0.03 (0.30)
<b><i>Using geography to split sample of countries</i></b>							
Income		-0.08 (1.02)	0.54 (6.00)	0.26 (6.10)	0.09 (1.70)	0.04 (0.67)	0.02 (0.37)
Democracy	0.64 (3.27)		0.14 (0.91)	-0.04 (0.26)	0.17 (1.36)	0.02 (0.30)	-0.02 (0.27)
Rule of Law	-0.004 (0.03)	-0.14 (1.06)		0.21 (2.00)	0.64 (5.95)	-0.13 (2.54)	-0.04 (0.81)
Openness	-0.08 (0.51)	-0.24 (1.89)	0.42 (2.00)		0.03 (0.19)	-0.15 (1.98)	0.04 (0.47)

### Notes to Tables A1 and A2:

Table A1 modifies the regressions in Rigobon and Rodrik (2005) by using real openness in 1996 rather than nominal openness, while Table A2 modifies the regressions by using the terms of trade in 1996 rather than nominal openness. Both those 1996 variables are computed as reported in the main text and the data Appendix.

Identification through heteroskedasticity is used as the estimation method, which requires splitting the sample along lines where the second moments will differ in the sub-samples. Two splits are used here: by former colonial status, and by geography (as described in the text). Dependent variables are shown down the first column and independent variables along the top row, so each row is a regression. T-statistics are shown in parentheses.

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