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Are All Fishers Equal?

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"As against these assertions (of Fisher) I purpose to show: … 2) That the circular test may also be fulfilled by index numbers constructed on a method which does not imply the adoption of a system of uniform weights for all the pairs of times or places for which the comparison is made." (Corrado Gini, 1931, p.5)

Abstract

This paper explores some extensions of the way that Gini formulated the EKS index and weighting of Fisher indexes. It is suggested that when EKS is formulated in terms of least squares minimization, it becomes fairly straightforward, if tedious, to take into account more and less important elements in the matrix of Fisher indexes for groups of countries. This can be done as Gini suggested, using for example, the population of countries. Following up on a feature of the minimum spanning tree method of Robert Hill (1999) and similar concerns of Rao (2001) and Cuthbert (2003), we also take into account different attributes of the cells of a Fisher matrix, namely variances due to data quality or differences in economic structure of countries. The applications of these approaches are illustrated using the 2005 ICP benchmark for both individual countries and the regions used in the ICP. The variance of the error structure in the least squares formulation is handled within a Bayesian framework, and is also illustrated for the weighted CPD or Rao (2002) method, for which it appears more suited. A number of extensions of this research are suggested including taking into account exogenous elements in spatial comparisons such as geographic distance or trade-flows between pairs of countries.

Introduction:

The EKS (Elteto, Koves and Szulc) index is the most common aggregation procedure used in the past 25 years for international comparisons of purchasing power and real product. Diewert (1999) has examined EKS and other indexes used in such comparisons and finds EKS attractive not only because it is superlative but also because it is built up from Fisher bilateral comparisons with well-known properties. Gini (1924) originated the EKS approach and wrote about it both as an index formula and as the solution of an ordinary least squares equation, this latter being one focus of this paper.

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As our title and the quote from Gini suggest, there is no reason to treat every Fisher in a matrix of all pairs of countries as necessarily equal when moving beyond two countries. Gini (1931) suggested using population weights in spatial comparisons, and for example, plutocratic weights have been used (Heston, Aten and Summers, 2001). Another way in which individual Fishers may not be equal is that more error may be attached to some pairs of countries than to others, due for example in differences in economic structure. Hill (1999), Cuthbert (2003) and Rao (2001) have all explored this dimension of EKS, with Hill choosing to develop an alternative approach, the minimum spanning tree. This paper is concerned with the same issues but takes a somewhat different direction. We also illustrate the approach we suggest for EKS applying it to the stochastic formulation that Rao (2002) derived for the Geary-Khamis (G-K) approach and with the fixed effects character of most spatial comparisons including the country product dummy (CPD) approach and hedonic estimates of say rent differentials in different urban areas.

The paper begins with a background section that touches on familiar ground for those involved in purchasing power comparisons except that more emphasis is given to the concerns of Robert Hill (1999) and his minimum spanning tree approach. Part II of the paper describes the underlying approach for estimating and weighted CPD (WCPD) within a least squares framework, and discusses several possible variations based on the assumptions about errors.[2] Results of these exercises using the 2005 ICP are presented at both the Regional and country level. Part III then takes up the concern of Gini, namely that one may want to weight different elements EKS by exogenous factors, such as population. Part IV concludes with some logical extensions of the approach as areas for further evaluation and research.

I Background

This section reviews the last 50 years of international comparisons of purchasing power comparisons with respect to the way that country and world groupings are put

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[2] Cuthbert (2003) examined an approximation to a GLS solution of the variance-covariance matrix structure of Fishers that he judged to support EKS as used for the 32 relatively homogeneous OECD countries in 1996.

together including the recently completed 2005 benchmark. A major new international data set, *Global Purchasing Power Parities and Real Expenditures*, was published in June 2008 by the World Bank. This most recent phase of the International Comparison Program (ICP) covers 146 countries for 2005 and in the version made available to the research community there are 128 basic headings of expenditure on GDP.[3] The ICP world was built up from separate comparisons in 6 regions, Africa, Asia-Pacific, the CIS countries, OECD-EU, South America and Western Asia. The method of joining 5 of the regions was through a group of 18 Ring countries from 5 regions that made separate price comparisons; the CIS countries were linked to OECD-EU through Russia. The Ring country methods were a major improvement on linking in previous ICP rounds, but several problems emerged in carrying out the Global comparison that are being addressed for the 2011 ICP round.[4] This paper is not directly focused on linking at the basic heading level. Rather our proposals focus on aggregating countries and regions into a world comparison in a way that offers more flexibility for countries than in previous ICP rounds in deciding which items they price.[5]

There were several inter-governmental approaches in the 1950s to building multilateral comparisons in the ECLA region (UN,1963; and Braithwaite,1968), the then Organization for European Economic Cooperation led by Milton Gilbert (Gilbert and Kravis, 1954) and by the Council of Mutual Economic Assistance (CMEA). The CMEA and OEEC comparisons were binaries with the Soviet Union and the United States respectively, leading to transitive results by what is termed the star system. ECLA priced the identical specification list of items in each capitol city aggregating by the Walsh method. As a follow up the OEEC (Gilbert, et al., 1958) attempted to make their results less dependent on the US by developing an average price approach.

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[3] The file provided to researchers contains 129 basic headings, of which one, the net foreign balance, we exclude from all our examples in part because of conceptual considerations (See Feenstra, Heston, Timmer and Deng (2009). So in effect our comparisons refer to Domestic Absorption.

[4] Many of the linking procedures were developed by Erwin Diewert (latest draft 2008) and adopted in producing the final Bank estimates (Earlier drafts are on the World Bank website) Many of the linking issues are discussed in the World Bank Report (2008) available online at http://web.worldbank.org/WBSITE/EXTERNAL/DATASTATISTICS/ICPEXT/.

[5] It is anticipated in the 2011 ICP, core item lists will be coordinated across regions, and within some regions like the EU, both a core regional list as well as sub-regional pricing be carried out.

Out of these early efforts the ICP emerged building on CMEA, OEEC, and ECLA experience and also the work on city comparisons of Japan (Tokyo and Shanghai for example) and of Salem Khamis at the Food and Agriculture Organisation and the Indian Statistical Institute. Under Kravis's leadership, the first ICP reported binary results with the United States but mainly sought a multilateral approach that was additive and which would be readily understood. It was analogous to national accounting practice at the time, leading to the adoption of the Geary-Khamis (G-K) approach. EKS, Walsh and other indexes were also reported in Rounds 1 through 3 of the ICP. The EU and the Economic Commission for Europe, which had sponsored a number of binary comparisons between its member countries from the production side, have favored building multilateral comparisons up from binary comparisons using EKS. In their work from the production side beginning with Maddison and continuing with van Ark, Timmer and Inklaar, the group at the University of Groningen have typically built up their multilateral results on the base of binary comparisons[6].

Diewert (1999) and in other writings made clear the limitations of an average price approach like G-K, as applied in the early ICP, namely that the country expenditures are not allowed to adjust to these average prices. The approach of Neary (2) deals with this problem, at the cost of building in a consumption structure. Since the initial phases of the ICP the world has moved to chain indexes for temporal indexes and so non-additivity in a spatial index is now more consistent with national accounting practice. Further the differences between G-K (or Ikle as formulated by Dikhanov, 2) and EKS, whether using plutocratic or democratic weights, is much less in a world of 146 countries than the 10-34 countries of the early phases of the ICP.[7] Basically some of the reasons for and against using an average price approach like G-K have on a practical level become almost moot.

Whatever the method, there is increasing concern in making global comparisons with linking countries with very different economic structures. A frequently used

[6] See work of the Groningen Growth and Development Center (http://www.ggdc.ne).

[7] In the work associated with PWT and the global ICP until 1985 an estimate of the weight of non-included countries has been attributed to included countries on the basis of geographic and economic proximity. These were termed super-country weights and were intended to make results less sensitive to the number of countries serving as the basis for the world, but clearly were not an adequate substitute for participation of more countries in benchmark comparisons.

measure of structure between a pair of regions or countries is the Paasche-Laspeyres spread or PL Spread. The PL Spread can be expressed as the difference in valuing the quantities in two countries at own or other country prices; or weighting the price parities between two countries at own or other country volumes. In 2005 these differences of the PLS for the US range from under 1.10 for many important OECD countries to 1.60 or more for some large countries like China, India or Russia, to over 9.00 for Tajikistan. However, Nigeria has a much smaller range of PL Spreads with respect to other countries, though still in a range of 1.00 to 2.80 for a number of important countries (Deaton and Heston, 2008). Robert Hill (1999) presented one approach to this problem, namely building a minimum spanning tree[8] of binary comparisons so as to include all countries and no countries more than once. The criterion used by Hill was the log of the absolute value of PL Spread, one of many possible measures of similarity.[9]

Operationally the spanning tree approach is probably most suitable for use within regions because they contain many large differences in PL Spreads, and because it is easier for regions to organize binary comparisons between pairs of their countries. The even larger PL Spreads between countries in different regions is due both to differences in economic structures as well as to the greater difficulty in comparing products and services, both in terms of characteristics and qualities. However, within regions like West Asia, the differences in economic structure between Yemen and the gulf oil countries or in Asia between Bhutan and Hong Kong or Singapore are certainly large too. So the problem to which this paper is addressed, namely the linking of diverse countries or regions, remains.

II Stochastic Formulation of EKS and G-K

The GINI Contributions

[8] The idea of a minimal spanning tree (MST) approach was proposed much earlier than Hill, with chain linking descriptions in Fisher (1922), Clark (1940) and Koves (1983) for the ICP round of 1970.

[9] Diewert (2002) and Sergeev (2001 have reviewed a number of similarity measures that one might use. The approach used in early ICP reports (Kravis, Heston and Summers, 1985) was also used in a review of the Hill's spanning tree approach (Heston, Aten and Summers (2001).

The EKS index is attributed to Elteto, Koves and Szulc (Elteo and Koves, 1964 and Szulc (1964), although the authors always acknowledge that the insight came from Gini (1924). We borrow from Gini in that we express EKS in one formulation that he proposed, namely as a least squares estimate (following the notation in Deaton and Friedman (2004)) as presented below in Equation (1):

Equation (1)

$$\ln F_{ij} = \sum_{j=1}^{N} \delta_{ij} \ln P_j^{GEKS} + \varepsilon_{ij}$$

$$\text{Kronecker } \delta \begin{cases} \delta_{ij} = 1 \ \text{ if } i = j, \ 0 \text{ otherwise} \\ \delta_{hs} = 1 \ \text{ if } h = s, \ 0 \text{ otherwise} \end{cases}$$

$$\text{and } \varepsilon_{ij} \sim N(0, \sigma^2 I_n)$$

Also,

$$F_{ij} = \sqrt{LAS_{ij} * PAA_{ij}}$$

$$LAS_{ij} = \sum_{h=1}^{H} w_i^h \frac{p_j^h}{p_i^h}$$

$$PAA_{ij} = \left(\sum_{h=1}^{H} w_j^h \frac{p_i^h}{p_j^h} \right)^{-1}$$

and

$$w_i^h = \frac{\exp_i^h}{\sum_{h=1}^{S} \exp_i^h}$$

Where F is the Fisher index of row country *i* with respect to column country *j* in the matrix of Fishers, and $ln\ P_j^{GEKS}$ is the log of the EKS estimate for each country and ε is the error term. If we divide the Fisher matrix by say the U.S. row, then the $ln\ P_j^{GEKS}$ will be expressed relative to the US.[10]

[10] Because the Fisher matrix is symmetrical with essentially 2 observations per country pair, equation statistics are somewhat messy to interpret. However, the average unexplained variance when you use all 146 countries compared to say the same measure for each of the 6 regions, it is possible to make meaningful inferences. This is a measure that Cuthbert (2003) also used.

The Gini formulation in Equation (1) , which we denote GEKS, can readily take advantage of the size of pair-wise similarities or other variables that may be related to each pair of countries in a Fisher matrix. The Gini formulation is analogous to the type of model used extensively in origin-destination studies such as transport flow modeling (LeSage 2004). These so called 'spatial interaction' models regress pair-wise measures of flows between places against characteristics of the origins and/or destinations, together with a distance variable that represents friction or transport costs. We experiment with this concept by using the Paasche-Laspeyres spread (PL Spread) as a variable, expecting the coefficient to be positive. That is, the larger the spread, the greater the 'distance' in structure and hence the higher the Fisher index.

Another aspect of Equation (1) that has been explored by Rao (2001) and Cuthbert (2003) is the error term. If the errors are correlated, then appropriate weights would still produce best linear unbiased estimates of the coefficients $ln\ P_j^{GEKS}$. Rao looked at weighted least squares estimates using the economic distance between countries as measured by the PL Spreads and related variables. If the diagonal variance terms are heterogeneous, a more robust estimator is needed. Cuthbert explored "idealized" structures for the variance-covariance matrix in order to derive optimal generalized least squares (GLS) estimators. He found that for the 32 relatively homogeneous OECD countries in 1996, excluding headings with negative expenditures, the "idealized" structure was close to the actual structure and thus the EKS was close to the optimal GLS solution. A third option that has been explored by Aten (1997) and Rao ( ) is the use of a spatial 'lag' to capture the correlation in the error term. This means specifying an exogenous relationship between countries, such as geographic distance or trade flows, to remove the error autocorrelation and produce unbiased estimates.

In this paper we explore an approach that does not require the specification of a variance-covariance error matrix, but nevertheless provides robust estimates in a set of heterogeneous observations such as the ICP 2005 countries. It does not take into account autocorrelation, which is something we would like to explore in the next version of the paper.

The G-K Index

The same approach that we are proposing for Gini's EKS formulation applies to the geometric formulation of the Geary-Khamis index, namely Rao's (2002) weighted CPD formulation as expressed below in Equation (2)

Equation (2)

$$\ln P_i^h = \sum_{j=1}^{N} \delta_{ij} \ln P_j^{WCPD} + \sum_{s=1}^{S} \delta_{hs} \beta_s + \varepsilon_i^h$$

$$\text{Kronecker } \delta \begin{cases} \delta_{ij} = 1 \text{ if } i = j, \text{ 0 otherwise} \\ \delta_{hs} = 1 \text{ if } h = s, \text{ 0 otherwise} \end{cases}$$

$$\text{and } \varepsilon_{ij} \sim N(0, \sigma^2 I_n)$$

$$\text{multiply variables by } \sqrt{w_i^h} = \sqrt{\frac{\exp_i^h}{\sum_{h=1}^{S} \exp_i^h}}$$

Note that here the number of observations is equal to the number of countries times the number of headings, while in Equation (1), the number of observations equaled the square of the number of countries. The $ln\ P_j^{WCPD}$ is the estimated price level index for country *j*, which like the GEKS may be expressed relative to the U.S. or any other country. The β_s are the estimated heading coefficients. If we turn to the error structure, it is clear that the variance of the errors is even less likely to be homogeneous than in Equation (1). Unlike the Fisher index, the dependent variable $ln\ P_i^h$ is not an average of weighted price ratios across headings, rather it is the price relative of each heading in each country, weighted by its expenditures share. In addition, there is likely to be correlation among countries and among headings.

First, let us turn to the variance of the diagonal elements of the error terms. A more robust estimator of $ln\ P_j^{WCPD}$ (and of $ln\ P_j^{GEKS}$) is one that takes into account the non-constant variance, or heterogeneity of the diagonal elements, as in Equation (3):

Equation (3)

$$\varepsilon \sim N(0, \sigma^2 V)$$
$$V = diag(v_1, v_2, ..., v_n)$$

The variance terms (v_1 , v_2, ..., v_n) are assumed to be fixed but unknown parameters that need to be estimated. Geweke (1993) and more recently LeSage (1999), propose using an informative prior and Bayesian methods to estimate the v_is: "The prior distribution for the v_i terms takes on an independent χ^2 (r)/r distribution which would allow us to estimate the additional variance parameters with a single parameter r. The prior mean becomes unity and the variance of the prior is 2/r. As r becomes very large, the terms v_i will approach unity, resulting in $V= I_n$, the traditional Gauss-Markov assumption for the errors. The role of $V \neq I_n$ is to protect against outliers and observations containing large variances by placing less weight on these observations. Large r values are associated with a prior belief that outliers and non-constant variances do not exist." (LeSage 1999). In Section II we look at the differences between the ordinary least squares and the Bayesian heteroscedastic formulation for the Gini-EKS and the Weighted CPD indexes applied to the 146 countries and 128 headings of the ICP.

Linking Countries with Diverse Economic Structures

It is instructive to return to Hill's spanning tree and other chaining methods of making multilateral comparisons to motivate discussion of the direction of our approach. The appeal of building up multilateral comparisons from pair-wise relationships or other individual country links is that we feel secure in the building blocks. With countries at a similar economic level, the underlying choice of items to compare, their qualities and types of outlets are likely to be more alike giving us more confidence at the outset in the quality of the exercise. In a chain or spanning tree the differences between countries with different structures are reduced through a series of indirect comparisons. So, what has prevented the international use of these approaches? Certainly inertia is part of the story. Another part is that in terms of applying chains or spanning trees, there is a lot of the devil in the details.

What is the criterion for linking? There are many reasonable ways to describe closeness of countries ex-post, including Hill's absolute value of the log of the Paasche Laspeyres spread, as well as a number of other similarity measures. These are ex-post in the sense that the criterion have generally been applied to previous ICP benchmark comparisons where, outside of what is done in the European Union countries, the quality of any particular binary is not necessarily the best that could be done. Countries have made choices of which items to price or where to price them based upon the regional or world lists, not on the basis of particular pair-wise comparisons. Even in the EU, the elimination of certain items in particular binary comparisons on the basis of lack of representivity is only done ex-post from a multilateral list of items. So there is not agreement on a criterion for chaining, nor will ex-post analysis of previous benchmark comparisons necessarily help choose the best possible pairs of countries that are more likely to be closer in economic structure in the current benchmark. To our knowledge, an ex-ante criterion of similarity of economic structure is more likely to identify one or more clusters of countries rather than particular pairs of countries.

A second problem with chaining procedures is that the final result can be quite sensitive to the particular links chosen. Very small differences in any similarity measure result in the choice of different paths through a chain of countries. The size of errors that can enter into a set of price comparisons, which in the 2005 ICP were subject to a number of validation stages, is not small, even at the regional level. As a consequence there is a good chance that several less satisfactory links are chosen along any particular chain. The conceptual appeal of spatial chaining may be strong, but in our judgment any actual application requires a number of choices that are not that easy to justify. These remarks apply to applications within regions. In large regions the movement is already to move to sub-regional comparisons again requiring a number of decisions if linking countries are to be used.

This led us to examine a less structured approach that does not require an explicit choice of criterion or other decisions about linking of regions and sub-regions. The Bayesian approach to the stochastic formulation of the EKS and the weighted CPD requires no specification of the structure of the indexes, allowing outliers and non-constant variances to be automatically detected during estimation, and adjusting for them.

A Fisher index buries a good part of the difference between countries by taking the geometric mean of the Paasche and Laspeyres indexes so the variance of the errors are likely to be more homogeneous in the GEKS. However, the weighted CPD or Rao index is another matter because the differences between countries at the basic heading level enter into the estimation. Deaton has observed in his work on poverty that compared to EKS or Tornquist, the CPD tends to be the odd index out which he suggests is due to outliers. So the non-constant variance is likely to be more suited to the WCPD than the EKS, a conjecture that we will examine in the next section.

III Results for 2005

Regional Estimates

We begin with the regions because the exposition is easier and because we will have some concluding remarks to make about using the regions as the unit for aggregation in ICP 2005. As a reference to the magnitudes, Table 1.1 presents the summary regional results from ICP 2005. It should be noted that the data set provided to researchers cannot replicate the World ICP report for a number of reasons foremost of which is that the distributed data set has less basic headings than provided by several of the regions.[11] Table 1.2 provides the matrix of regional Fisher Indexes over 128 basic headings, excluding the net foreign balance, while Table 1.3 provides the Paasche-Laspeyres spreads.

Table 1.1: 2005 Regional ICP Results (US$ Reference Currency)

	Africa	Asia	CIS	EU-OECD	South America	West Asia	World
Price Levels OECD=1	2.17	2.44	2.32	1.00	1.91	1.93	1.23
% World GDP at PPP	3.34	21.9	4.1	66.3	5.6	2.5	100.0
% World GDP at Xrate	1.89	11.0	2.2	81.6	3.6	1.6	100.0
pc GDP PPP ($)	2,223	3,592	9,202	26,404	8,415	7,711	8,961
% World Population	13.47	54.6	4.0	20.2	6.0	1.7	100.0

[11] Regions such as the EU-OECD determined their regional results with many more basic headings than provided to researchers. Further, the Ikle, which is very similar to a democratically weighted G-K, was used in Africa for their regional comparison. By agreement these regional results had to be retained in the World Report, but cannot be replicated by researchers; the results for each region appear in their Regional Reports.

Table 1.2: Fisher Price Levels by Region

REGIONS		Africa 1	Asia 2	CIS 3	EU-OECD 4	South America 5	West Asia 6
Africa	1	1	1.240	1.210	0.458	0.848	0.861
Asia	2	0.807	1	0.934	0.387	0.687	0.727
CIS	3	0.827	1.071	1	0.396	0.720	0.739
EU-OECD	4	0.710	2.583	2.527	1	1.845	1.872
South America	5	2.183	1.455	1.389	0.387	1	0.997
West Asia	6	0.781	1.375	1.353	0.396	1.003	1

Table 1.3: Paasche-Laspeyres Spread by Region

REGIONS		Africa 1	Asia 2	CIS 3	EU-OECD 4	South America 5	West Asia 6
Africa	1	1	1.171	1.107	1.407	1.151	1.312
Asia	2	0.854	1	1.330	1.394	1.167	1.199
CIS	3	0.903	0.752	1	1.478	1.232	1.660
EU-OECD	4	0.710	0.717	0.676	1	1.235	1.317
South America	5	0.869	0.857	0.812	0.810	1	1.273
West Asia	6	0.762	0.834	0.602	0.759	0.785	1

First, we can observe that the basic Fisher results for the regions in Table 1.2 (row 4) for the EU-OECD are not that far from the comparable first row for the 2005 ICP in Table 1.1, which is before EKS has been estimated. We can also observe that the Fishers are generated from Paasche and Laspeyres values that vary between a low of 1.107 between Africa and the CIS to high of 1.660 between Western Asia and the CIS. However, the largest PL spreads on average are between the EU-OECD and other regions.

Table 1.4: Regional Price Levels: Gini EKS and Bayesian EKS

Region	Fisher	GEKS	GEKS	GBAYES	Ratio
	EU-OECD =100		World = 100		(3) / (4)
	(1)	(2)	(3)	(4)	(5)
Africa	45.80	46.56	84.16	84.22	0.999
Asia	38.71	37.89	68.49	68.56	0.999
CIS Countries	39.57	39.48	71.36	70.56	1.011
EU-OECD	100	100	180.75	181.66	0.995
South America	54.21	54.49	98.49	98.08	1.004
West Asia	53.41	53.53	96.75	96.93	0.998

Table 1.4 provides EKS estimates for the 6 regions in 2005 and Table A at the end of the paper provides the estimates for each of the 146 countries. Column (1) reproduces the Fisher from Table 1.2. Column (2) provides the basic EKS estimate with the EU-OECD as the reference region, and column (3) gives the GEKS estimate with the world as reference. If one divides column (3) by the EU-OECD value as a percent, it is of course the same as column (2) to illustrate the relationship of GEKS estimates to the Fisher index.

Column (4) is the Bayesian version of the EKS estimate with the strongest assumption about non-constant variances, namely an $r=1$ which allows the v_i s to deviate most from unity. The run was 5000 cases, but it converged with half that number. Column (5) compares GBayes with the GEKS and the largest difference is 1.1% for the CIS region.

Country Estimates of GEKS and WCPD

We have also carried out this exercise using the 146 countries and they are shown in Table A (column (3), (4)), where the world is the base for all series so there is no effect due the particular reference country or region chosen.

The GEKS versus GBayes comparison is graphed in Chart 1 below. The countries are ordered alphabetically by country code within each region, which are also ordered alphabetically as in the previous section. This highlights the regional pattern with respect to the variance assumption. There is little difference in the average adjustment by region, but the individual countries are affected most in Africa, 5 over 2% and 8 over 1%, followed by Asia with 6 over 1%.

The smallest variance adjustment is for the EU-OECD countries. This appears consistent with Cuthbert's work on 32 OECD countries in 1996 where he compares an idealized variance-covariance matrix with the actual. Cuthbert (2003, p.80) concluded that for a homogeneous set of the countries like the OECD that there would be little gained by adopting a robust GLS estimator for the Gini EKS. The weighted CPD is a somewhat different story because the basic heading data entering into the estimation involve more outliers than in a Fisher matrix. When we relax the constant variance assumption and estimate WCPD we find larger adjustments in the coefficients. The only bit of good news is that conjecture was correct. Both sets of estimates are given in Table A (columns (1) and (2)).

Chart 2 compares the two WCPD series, again ordering countries as in Chart 1. What appears clear that the adjustments for WCPD do not move the countries closer to either version of EKS. In fact the WCPD without adjustment estimates are on average considerably closer to GEKS than without the constant variance assumption.

IV Some GEKS Extensions

Merging the PL Spread and GEKS

A fairly simple experiment is to introduce the Paasche-Laspeyres spread as a variable into equation (1) above. It has to be skew-symmetric like the Fisher, as in Table 1.3 above, so that the inverse of the PLS is calculated for the corresponding lower diagonal. As discussed earlier,Rao used the PL Spread as weights in a GEKS estimation, and weighted least squares will provide best linear unbiased estimators if the weights are proportional to the inverse of the variance-covariance of the observations. Here we introduce it as a variable rather than a weight. Inclusion of the PL Spreadsas a variable is similar to a distance variable in the spatial interaction model, as it is expected to capture some of the 'friction' between each pair of observations. Using the PL Spread, which has the same components of prices and expenditures as the Fisher, introduces correlation to the dependent variable, but since the only independent variables are the country dummy coefficients, it also has the effect of capturing some of the residual autocorrelation in the error term. A spatial lag based on the Fisher would probably model this more effectively, but would require specifying the nature of the spatial relationship.

The coefficient on the PL Spread was 0.074 with a standard error of .0062 and the addition of PL Spread does improve the explanatory value of the equation. In every benchmark ICP there is a strong positive association between the per capita income of a country and its price level. So countries with high price levels will generally have economic structures that are different from countries with lower price levels and there will be a larger PL spread between the high and low income countries. For countries that are close in economic structure there is no necessary relationship of their Fisher and their PL Spread. Country pairs that are far apart in structure are likely to have large PL Spreads and smaller Fishers, so their relationship is more likely to be negative. This is true for 30 countries in the 2005 ICP, while for the remaining 115 the correlation was positive. Over all countries, the correlation between the PL Spreads and Fishers countries is positive and significant.

The price levels of countries estimated by the above equation are given in column (5) of Table A. Chart 3 graphs these estimates against the GEKS.[12] The countries are in the same order as in the previous charts and suggest a pattern quite different and more interesting relative to Chart 2. Differences are as large as 9% for Tajikistan and as much as 4% and higher for Burundi, Chad, Sierra Leone, Qatar and the US. These are not unreasonable adjustments to the GEKS estimates for these countries and for a number of others in the 2 and 3% range. While there are more large adjustments in Africa, all regions have countries with changes in the 2 or 3% range including 4 of 10 in both South America and W. Asia. The series in all the charts are normalized so for each series the average of the estimates for all countries is 1.0. These adjustments are both up and down so the 4+ % adjustment in the US implies a significant change in its relationship with most countries, more so for those moving downward. We conclude that this exercise shows promise and merits further investigation.

Moving away from Democratic Weighting of GEKS

Returning to the initial quote from Gini, what happens if we explore his suggestion that not all Fishers be treated equally, for example, using population weights in a GEKS. This could be done in several ways and the method chosen for illustration is simple and parallel to what was done for the PL Spreads. For each country row its population is entered for each country column. This variable, *Pop*, will be introduced on its own and as a variable along with PL Spreads in a GEKS equation. Population is not obviously correlated to the PLS and it would operate to give larger weights to countries

[12] Chart 3 uses GEKS without the Bayes adjustment as the denominator. Both are very close and in fact, there is slightly less variation using the GEKS with Bayes adjustment but the conclusions in the text would hold whichever was used.

like China, India, USA, Indonesia, Brazil, Nigeria with the likely effect of lowering the GEKS estimates for smaller countries

Population has a positive and significant coefficient and by conventional tests improves the equation, reducing the RMSE but less than the PL Spread variable by itself. When both variables are introduced into the GEKS their coefficients are both significant and both remain positive. The equation statistics are not the important part of the story in terms of possible implementation of this form of GEKS. Rather, the question is what sort of country estimates do they produce, namely the α's? This analysis was not completedin time to be included in this draft of the paper.

What Next?

We have looked at some new variations on the EKS and Rao CPD indexes when they are viewed as solutions to a least squares minimization problem and concluded that there are enough interesting results to suggest further directions to explore. Studies such as (Cuthbert) have tried to model the variance-covariance matrix explicitly, using a GLS estimator, while Rao has used a weighted least squares approach in the EKS using the PL spread as weights.

The main innovation introduced at the beginning of the paper is a Bayesian estimation form in an attempt to capture the spatial heterogeneity, or non-constant variance across countries, which we believe is inherent to international comparisons. The advantage of the Bayesian approach is that the variance of the observations does not have to be modeled or specified explicitly. It allows the data to automatically detect outliers and correct for non-constant variances in the errors. We found that in the ICP 2005, allowing for heterogeneity would lead to adjustments of 2 or 3% for a number of African and Asian countries, but much less in relative more homogeneous regions. The heterogeneity adjustment for EU-OECD countries in 2005 would be under 1%, which is consistent with the finding of Cuthbert for 1996.

In addition, our work makes clear that the GEKS approach smoothes out a great deal of the variance in the underlying input data as compared to the WCPD. This occurs because the GEKS index is an average of equally weighted binary comparisons, the Fisher indexes, which are in turn, share weighted price ratios. The WCPD uses the actual price relatives as inputs with expenditure shares as weights. The WCPD share weights have the advantage of reducing spatial autocorrelation in the errors but the disadvantage of increasing heterogeneity in the residuals. While the GEKS residuals are less heterogeneous than the WCPD residuals, the amount of residual autocorrelation may be higher in the GEKS, as evidenced by the inclusion of additional variables which are significant yet do not increase the standard error of the estimated parameters. When the Paasche-Laspeyres spread was considered as an additional variable in estimating a Gini version of EKS, it provided substantial changes for countries across all regions, as did the addition of population as an exogenous weight.

In this paper we do not explicitly treat the autocorrelation of the errors. If there is significant residual autocorrelation, the GEKS and/or WCPD indexes will likely be biased. In the literature the most common and reliable way to remove spatial autocorrelation is to introduce an exogenous variable such as one measuring geographic proximity. Other variables that have been attempted are measures of proximity between groups of countries in formal trading relationships. Environmental variables could also be considered, especially those likely to reflect consumption prices, such as housing and construction. Whether these suggestions have merit are topics for future research.

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Chart 1. Ratio GEKS to GBayes

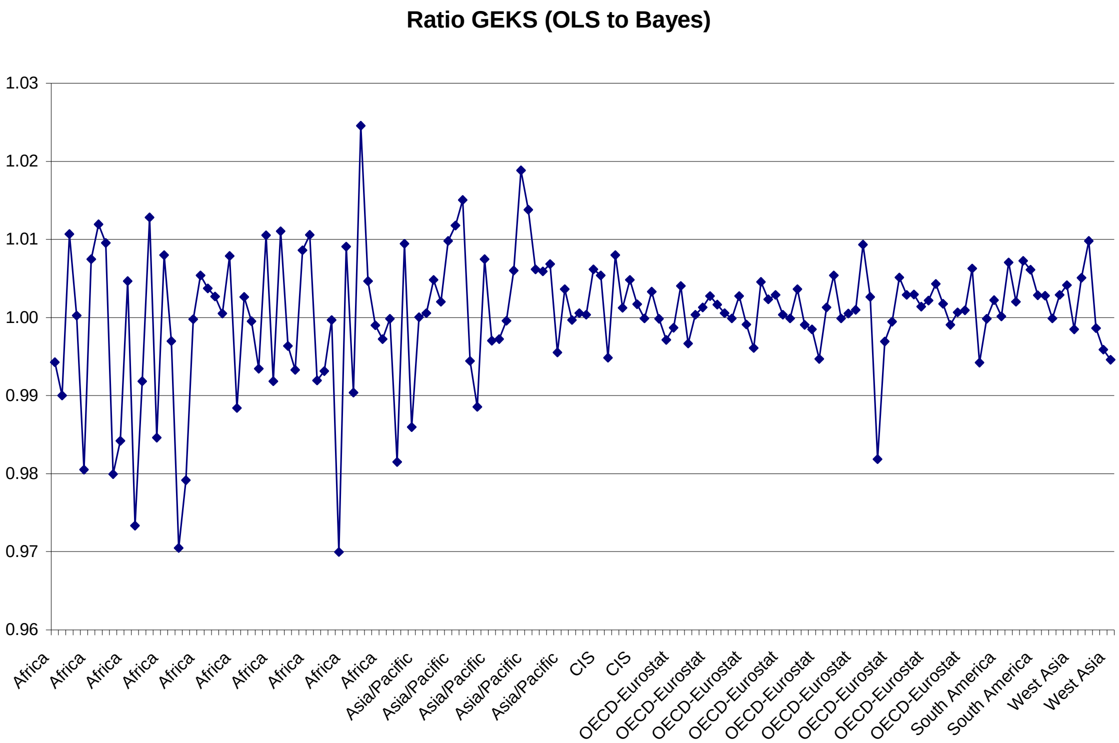

Chart 2. Ratio WCPD (OLS to Bayes)

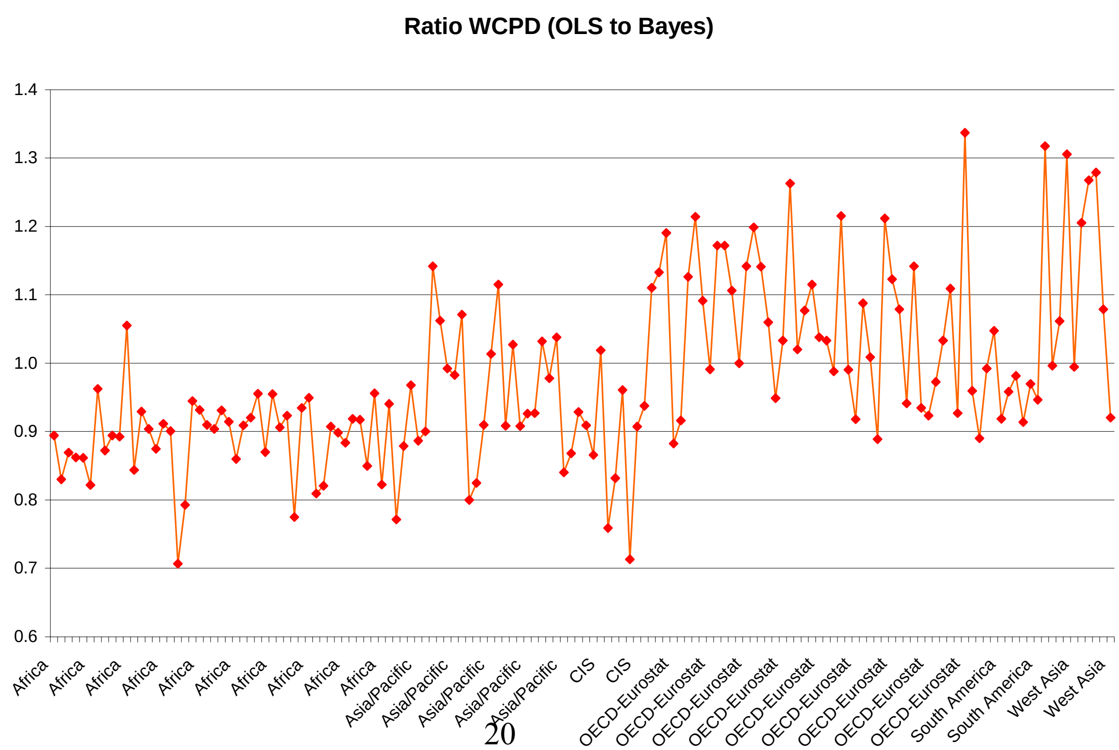

Chart 3. All Indexes relative to GEKS

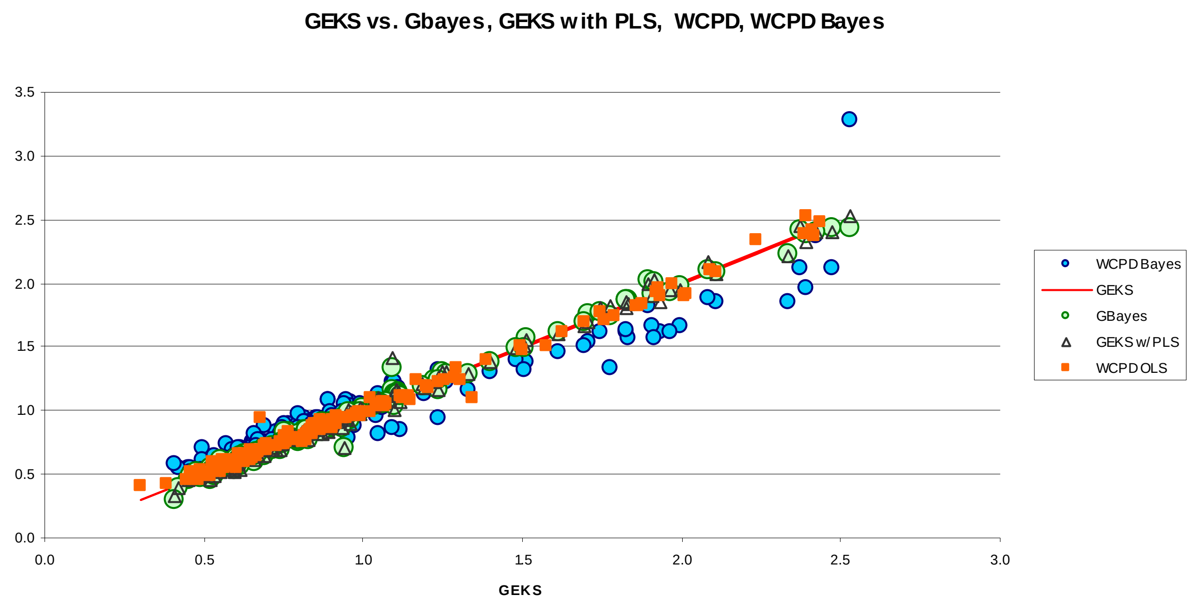

Table A. Country estimates of GEKS and WCPD

			WCPD		GEKS		
			OLS (1)	Bayes (2)	GEKS (3)	GBayes (4)	GEKS w/PLS (5)
1	AGO	Africa	1.089	1.218	1.151	1.158	1.117
8	BDI	Africa	0.576	0.693	0.554	0.560	0.534
10	BEN	Africa	0.725	0.834	0.710	0.703	0.696
11	BFA	Africa	0.653	0.757	0.656	0.656	0.642
21	BWA	Africa	0.768	0.892	0.821	0.837	0.803
22	CAF	Africa	0.894	1.088	0.860	0.854	0.839
27	CIV	Africa	0.996	1.035	1.025	1.013	1.005
28	CMR	Africa	0.817	0.936	0.825	0.817	0.812
30	COM	Africa	0.957	1.071	0.913	0.932	0.895
31	CPG	Africa	1.097	1.229	1.020	1.037	1.002
32	CPV	Africa	1.193	1.130	1.196	1.190	1.179
36	DJI	Africa	0.755	0.895	0.811	0.833	0.794
39	EGY	Africa	0.485	0.522	0.478	0.482	0.471
42	ETH	Africa	0.480	0.531	0.515	0.509	0.507
46	GAB	Africa	0.949	1.084	0.944	0.958	0.931
49	GHA	Africa	0.738	0.810	0.699	0.694	0.694
50	GIN	Africa	0.597	0.663	0.559	0.561	0.554
51	GMB	Africa	0.495	0.700	0.490	0.505	0.482
52	GNB	Africa	0.692	0.872	0.694	0.709	0.686
53	GNQ	Africa	1.239	1.312	1.164	1.165	1.154
69	KEN	Africa	0.689	0.740	0.684	0.681	0.684
76	LBR	Africa	0.851	0.935	0.867	0.864	0.869
78	LSO	Africa	0.857	0.948	0.842	0.839	0.844
83	MAR	Africa	0.958	1.029	0.967	0.966	0.971
85	MDG	Africa	0.600	0.656	0.571	0.566	0.574
89	MLI	Africa	0.746	0.867	0.742	0.751	0.748
93	MOZ	Africa	0.807	0.888	0.818	0.815	0.824
94	MRT	Africa	0.692	0.752	0.679	0.679	0.684
95	MUS	Africa	0.870	0.910	0.873	0.879	0.883
96	MWI	Africa	0.662	0.761	0.606	0.600	0.612
98	NAM	Africa	1.096	1.148	1.124	1.133	1.134
99	NER	Africa	0.754	0.832	0.761	0.753	0.767
100	NGA	Africa	0.827	0.896	0.764	0.766	0.770
115	RWA	Africa	0.568	0.733	0.590	0.594	0.607
117	SDN	Africa	0.786	0.841	0.786	0.779	0.797
118	SEN	Africa	0.839	0.883	0.840	0.831	0.850
120	SLE	Africa	0.657	0.812	0.634	0.639	0.649
122	STP	Africa	0.892	1.087	0.850	0.856	0.871
126	SWZ	Africa	0.900	0.992	0.915	0.916	0.935
128	TCD	Africa	0.944	1.051	0.681	0.702	0.710
129	TGO	Africa	0.794	0.899	0.755	0.748	0.769
132	TUN	Africa	0.766	0.834	0.779	0.786	0.800
135	TZA	Africa	0.648	0.707	0.663	0.647	0.671
136	UGA	Africa	0.589	0.693	0.603	0.600	0.623

			WCPD		GEKS		
			OLS (1)	Bayes (2)	GEKS (3)	GBayes (4)	GEKS w/PLS (5)
143	ZAF	Africa	1.038	1.086	1.064	1.065	1.090
144	ZAR	Africa	0.799	0.972	0.768	0.771	0.791
145	ZMB	Africa	0.903	0.960	0.892	0.892	0.916
146	ZWE	Africa	2.528	3.277	2.394	2.439	2.527
12	BGD	Asia/Pacific	0.518	0.589	0.552	0.547	0.538
19	BRN	Asia/Pacific	0.870	0.898	0.904	0.917	0.886
20	BTN	Asia/Pacific	0.519	0.585	0.554	0.554	0.541
26	CHN	Asia/Pacific	0.626	0.695	0.652	0.652	0.640
44	FJI	Asia/Pacific	1.331	1.165	1.293	1.287	1.284
55	HKG	Asia/Pacific	1.226	1.154	1.238	1.236	1.229
58	IDN	Asia/Pacific	0.616	0.621	0.649	0.642	0.645
59	IND	Asia/Pacific	0.527	0.537	0.534	0.527	0.531
61	IRN	Asia/Pacific	0.522	0.487	0.460	0.453	0.456
71	KHM	Asia/Pacific	0.494	0.617	0.492	0.495	0.492
74	LAO	Asia/Pacific	0.449	0.545	0.448	0.453	0.449
77	LKA	Asia/Pacific	0.552	0.607	0.560	0.556	0.562
82	MAC	Asia/Pacific	1.113	1.098	1.144	1.147	1.148
86	MDV	Asia/Pacific	0.973	0.873	0.980	0.983	0.988
92	MNG	Asia/Pacific	0.564	0.621	0.557	0.557	0.562
97	MYS	Asia/Pacific	0.745	0.726	0.747	0.743	0.753
103	NPL	Asia/Pacific	0.479	0.527	0.521	0.511	0.528
106	PAK	Asia/Pacific	0.498	0.538	0.512	0.505	0.519
108	PHL	Asia/Pacific	0.603	0.651	0.626	0.622	0.633
119	SGP	Asia/Pacific	1.106	1.071	1.144	1.137	1.167
130	THA	Asia/Pacific	0.645	0.660	0.648	0.643	0.658
134	TWN	Asia/Pacific	0.958	0.923	0.969	0.973	0.991
141	VNM	Asia/Pacific	0.459	0.547	0.466	0.464	0.484
4	ARM	CIS	0.617	0.711	0.561	0.561	0.539
7	AZE	CIS	0.599	0.645	0.527	0.527	0.512
16	BLR	CIS	0.550	0.605	0.529	0.529	0.515
48	GEO	CIS	0.611	0.706	0.583	0.580	0.577
68	KAZ	CIS	0.693	0.680	0.645	0.641	0.645
70	KGZ	CIS	0.419	0.552	0.386	0.388	0.386
88	MKD	CIS	0.535	0.643	0.487	0.483	0.492
114	RUS	CIS	0.714	0.744	0.689	0.688	0.699
131	TJK	CIS	0.411	0.576	0.304	0.302	0.332
137	UKR	CIS	0.490	0.541	0.476	0.475	0.491
2	ALB	OECD-Eurostat	0.873	0.931	0.839	0.839	0.822
5	AUS	OECD-Eurostat	1.708	1.538	1.758	1.752	1.701
6	AUT	OECD-Eurostat	1.824	1.610	1.860	1.860	1.804
9	BEL	OECD-Eurostat	1.934	1.624	1.920	1.926	1.859
13	BGR	OECD-Eurostat	0.673	0.763	0.661	0.662	0.649
15	BIH	OECD-Eurostat	0.830	0.906	0.829	0.826	0.815
23	CAN	OECD-Eurostat	1.692	1.502	1.697	1.703	1.657
24	CHE	OECD-Eurostat	2.391	1.969	2.386	2.385	2.322
33	CYP	OECD-Eurostat	1.513	1.386	1.575	1.573	1.554

			WCPD		GEKS		
			OLS (1)	Bayes (2)	GEKS (3)	GBayes (4)	GEKS w/PLS (5)
34	CZE	OECD-Eurostat	0.990	0.999	0.989	0.986	0.979
35	DEU	OECD-Eurostat	1.835	1.565	1.877	1.874	1.837
37	DNK	OECD-Eurostat	2.475	2.111	2.437	2.435	2.395
40	ESP	OECD-Eurostat	1.612	1.457	1.622	1.623	1.604
41	EST	OECD-Eurostat	1.035	1.035	1.040	1.037	1.033
43	FIN	OECD-Eurostat	2.107	1.846	2.092	2.094	2.064
45	FRA	OECD-Eurostat	1.993	1.662	1.968	1.976	1.939
47	GBR	OECD-Eurostat	1.907	1.671	1.931	1.922	1.901
54	GRC	OECD-Eurostat	1.481	1.397	1.497	1.494	1.490
56	HRV	OECD-Eurostat	1.111	1.170	1.131	1.128	1.125
57	HUN	OECD-Eurostat	1.070	1.036	1.069	1.068	1.063
60	IRL	OECD-Eurostat	2.337	1.850	2.232	2.232	2.218
63	ISL	OECD-Eurostat	2.421	2.374	2.412	2.403	2.396
64	ISR	OECD-Eurostat	1.399	1.299	1.384	1.385	1.378
65	ITA	OECD-Eurostat	1.826	1.637	1.862	1.865	1.850
67	JPN	OECD-Eurostat	1.895	1.825	2.010	2.021	1.998
72	KOR	OECD-Eurostat	1.246	1.206	1.305	1.303	1.303
79	LTU	OECD-Eurostat	0.880	0.891	0.879	0.874	0.881
80	LUX	OECD-Eurostat	1.916	1.576	2.015	2.016	2.022
81	LVA	OECD-Eurostat	0.894	0.902	0.895	0.894	0.898
84	MDA	OECD-Eurostat	0.668	0.727	0.674	0.673	0.677
87	MEX	OECD-Eurostat	1.039	0.956	1.062	1.052	1.069
90	MLT	OECD-Eurostat	1.183	1.173	1.203	1.199	1.209
91	MNE	OECD-Eurostat	0.816	0.918	0.835	0.850	0.842
101	NLD	OECD-Eurostat	1.967	1.623	1.929	1.935	1.953
102	NOR	OECD-Eurostat	2.375	2.115	2.413	2.414	2.448
104	NZL	OECD-Eurostat	1.743	1.616	1.788	1.779	1.810
109	POL	OECD-Eurostat	0.937	0.996	0.939	0.936	0.949
110	PRT	OECD-Eurostat	1.509	1.322	1.493	1.488	1.510
113	ROM	OECD-Eurostat	0.840	0.899	0.837	0.836	0.847
121	SRB	OECD-Eurostat	0.717	0.776	0.728	0.727	0.739
123	SVK	OECD-Eurostat	0.921	0.947	0.901	0.898	0.914
124	SVN	OECD-Eurostat	1.259	1.219	1.287	1.285	1.309
125	SWE	OECD-Eurostat	2.085	1.879	2.106	2.108	2.159
133	TUR	OECD-Eurostat	1.047	1.130	1.074	1.074	1.097
139	USA	OECD-Eurostat	1.775	1.328	1.746	1.744	1.822
3	ARG	South America	0.744	0.776	0.751	0.747	0.733
17	BOL	South America	0.455	0.512	0.481	0.484	0.466
18	BRA	South America	0.979	0.987	0.989	0.989	0.971
25	CHL	South America	1.062	1.014	1.061	1.058	1.045
29	COL	South America	0.798	0.869	0.817	0.817	0.803
38	ECU	South America	0.739	0.771	0.747	0.742	0.737
107	PER	South America	0.781	0.795	0.792	0.790	0.802
111	PRY	South America	0.553	0.605	0.561	0.557	0.570
138	URY	South America	0.949	0.979	0.945	0.940	0.966
140	VEN	South America	0.990	1.046	0.988	0.985	1.014

			WCPD			*GEKS*	
			OLS (1)	***Bayes (2)***	***GEKS (3)***	***GBayes (4)***	***GEKS w/PLS (5)***
14	BHR	West Asia	1.119	0.850	1.117	1.114	1.069
62	IRQ	West Asia	0.553	0.554	0.605	0.605	0.602
66	JOR	West Asia	0.822	0.775	0.844	0.841	0.841
73	KWT	West Asia	1.236	0.947	1.253	1.248	1.254
75	LBN	West Asia	0.933	0.938	0.866	0.867	0.867
105	OMN	West Asia	0.952	0.790	0.993	0.988	1.012
112	QAT	West Asia	1.092	0.861	1.344	1.331	1.407
116	SAU	West Asia	1.049	0.821	1.049	1.050	1.074
127	SYR	West Asia	0.606	0.562	0.611	0.614	0.624
142	YEM	West Asia	0.547	0.594	0.568	0.571	0.588
		World	1.00	1.00	1.00	1.00	1.00