

Contents

1	Introduction	1
1.1	Linear programming with random parameters	1
1.2	Examples	2
2	Random objective functions	9
2.1	Some classical approaches to optimization under uncertainty	9
2.1.1	Maximization of expected utility	9
2.1.2	The mean-variance model	14
3	Recourse models	19
3.1	Models with penalty costs in deterministic LP	19
3.2	Recourse models in stochastic linear programming	22
3.2.1	Representations of recourse models	22
3.2.2	Modeling aspects	26
3.2.3	Examples of recourse models	27
3.2.4	Special recourse structures	41
3.2.5	Generalizations	43
3.3	Properties of recourse models	43
3.3.1	Fixed recourse	44
3.3.2	Simple recourse	46
3.3.3	Expected shortage and surplus functions	48
3.3.4	Bounds on the optimal value of the recourse model	54
3.3.5	Approximation of distributions	58
3.4	Algorithms for recourse models	61
3.4.1	Simple recourse	61
3.4.2	Fixed recourse	65
4	Stochastic mixed-integer programming	75
4.1	Mixed-integer recourse models	75
4.1.1	Examples	78
4.1.2	Special mixed-integer recourse structures	85

4.2	Properties of mixed-integer recourse models	87
4.2.1	Fixed mixed-integer recourse	87
4.2.2	Simple integer recourse	90
4.2.3	Integer expected shortage and surplus functions	92
4.3	Algorithms for mixed-integer recourse models	110
4.3.1	Simple integer recourse	110
4.3.2	Fixed mixed-integer recourse	118
5	Chance constraints	133
5.1	Modeling with chance constraints	133
5.2	Examples	136
5.2.1	Water Management in the Bodrog River Area	136
5.3	Mathematical properties of chance constraints	138
5.4	Discrete distributions	147
5.5	Integrated chance constraints	150
5.5.1	Single integrated chance constraint	151
5.5.2	Joint integrated chance constraints	162
5.5.3	Integrated chance constraints and Conditional Surplus-at-Risk	163
5.5.4	Integrated chance constraints and recourse models	166
5.5.5	Conclusions	167
5.6	Algorithms for models with a joint chance constraint	168
5.6.1	Evaluation of the distribution function of a multivariate normal distribution	169
5.7	Algorithms for models with integrated chance constraints	173
6	Multi-stage recourse models	177
6.1	Examples	177
6.1.1	ICC in an ALM model for pension funds	177
A	Maximum calculus	189
B	Probability	191
B.1	Expected functions	191
B.2	Limit sets	194
C	Elementary Convex Analysis	197
D	Deterministic LP	207
E	Algorithms for convex non-linear optimization	211
E.1	Frank & Wolfe's algorithm	212
E.2	Veinott's supporting hyperplane method	214

F Exercises	217
A1 Multiperiod inventory control	217
A2 Flood control reservoir system	219
R1 Probabilistic interpretations of integrals of distribution functions . . .	219
R2 Complete and sufficiently expensive recourse	221
R3 Simple recourse, only RHS random	223
R4 Workforce level planning: an example of multiple simple recourse? . .	224
C1 Chance constraints and expected shortage constraints	226
C2 The expected shortage function for positive random variables	226
C3 Convexity for a simple chance constraint	227
C4 Relation individual and joint chance constraints	228
C5 Joint Chance Constraints, only RHS random, discrete distribution . . .	229
C6 Individual Chance Constraints with One Random Variable	229
U5 Stochastic Dominance and Expected Utility	231
Bibliography	233
Index	239